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Research Article

Damage Ratio Based on Statistical Damage Constitutive Model for Rock

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The study of damage characteristics of rock mass is of great significance to the analysis of rock mass structure. According to the characteristics of the microscopic unit strength of rock with random distribution, the Weibull distribution is widely used as the statistical functions of the strength of the microunit of rock to establish the damage constitutive model. The concepts of damage ratio D_e and damage index C_c are proposed. Damage ratio is mainly used to describe the law of damage evolution in rock. Damage index can be used to evaluate the damage degree of rock. The influence of confining pressure on distribution parameters and damage ratio is analyzed through uniaxial and triaxial compression tests of sandstone. The results show that damage ratio is an index of structural characteristics of rock damage, which can reflect the evolution characteristics of microcracks in rock under spatial stress. Critical damage ratio refers to the damage ratio corresponding to the peak stress of rock and can be used as a parameter to characterize the strength of rock for corresponding to the peak strain one to one. The critical damage ratio is linearly related to the logarithmic function of confining pressure. Its relationship is as follows: $D_{er} = C_c \ln \sigma_3 + b$. With the increase of σ_3, the increasing trend of D_{er} slows down and gradually tends to a certain value. The larger the damage index is, the more serious the damage of rock is. The smaller the damage index is, the less serious the damage of rock is. Therefore, the damage index can be used to evaluate the damage degree of rock. It will be an important direction of rock damage mechanics research to distinguish the severity of rock damage by using damage index as the limit value.

1. Introduction

Damage research of rock materials has been an important issue in geotechnical engineering. The introduction of damage mechanics theory and damage variables provides a basis for the study of rock damage. At present, the analysis and testing of rock generally assume that it is a homogeneous and isotropic material though there are many subtle and complex defects such as microcracks and cracks inside the rock which have a great influence on the basic mechanical parameters of the rock.

The study of the relationship between rock strength and deformation by using the theory of damage mechanics can accurately reflect the influence of defects, such as microcracks on the mechanical properties of rock, and is considered as one of the most effective methods for studying brittle material, such as rock [1]. In addition, the main difference between rock and rock mass is that rock mass contains discontinuous structural planes such as fissures, joints, bedding, and faults. Rock mass can be regarded as a rock with complex structural planes. And the research on rock damage can promote the development of rock mechanics to a certain extent.

At present, the methods of studying rock damage are mainly as follows. (1) To study the damage of rocks by using the continuum damage mechanics from the macroscopic point of view (e.g., Krajcinovic et al. [2, 3], Marigo et al. [4], Frantziskonis and Desai [5], Kawamoto [6], Cheng and Duseault [7], and Aubertin [8], et al.) In these studies, the damage mechanics was introduced into rock mechanics and the rock damage was assumed to be isotropic.

The damage mechanism was studied based on the structural characteristics of rocks and the corresponding models were established mainly based on the damage research method of metal materials. Therefore, the model established by this method is quite different from the actual situation of rock materials. (2) To deal with the problem of damage using mesodamage theory from the microscopic point of view (e.g., Xia et al. [9], Bai et al. [10], and Horii et al. [11]). In these studies, they believed that it was a good attempt to combine the physical basis of mesomechanics model with macroscopic continuity theory, for the development of microcracks could be described by the theory of mesostructure and was the only phenomenon on the macroscale. (3) The damage theory of rock is studied based on the microscopic phenomenological theory and the damage variable is assumed to be a kind of distribution (e.g., Krajcinovic et al. [12], Cao et al. [13–16], Wang et al. [17], Deng and Gu [18], Zhao et al. [19], Xu et al. [20, 21], Wang et al. [22], Chen et al. [23], and Bian et al. [24]). Statistical damage theory was introduced in these studies. They quantified the degree of damage in rock by microelement strength and established the statistical damage evolution equation and damage softening constitutive model according to the characteristics of random distribution of damage in rock, which made a breakthrough in the study of rock softening constitutive.

A large number of rock damage constitutive models have been obtained based on the methods above. These studies describe the mathematical and mechanical models of rock damage from different angles, which promotes the development of rock mechanics experiments and theories. The constitutive model obtained by the third method above is always called a statistical damage constitutive model. According to the existing published results, the research on damage variable and distribution parameters of statistical damage constitutive model of rock is one of the hotspots.

However, the damage degree of different rocks under specific conditions has not yet been recognized as a quantitative index, and the understanding on damage is different. Though damage variable based on statistical damage constitutive model of rock can be used to describe the degree of damage development, it is process parameter, which will change gradually with the process of rock damage and it is difficult to be quantified.

Based on this, the Weibull distribution commonly used by the predecessors is taken as the statistical functions of the strength of the microunit of rock to establish the damage constitutive model. The damage ratio is defined as a new term based on the damage area to describe the damage degree of rock. The damage ratio is defined in rock analogous to the usual formulation of void ratio in soil mechanics. A curve of relationship between damage ratio and confining pressure (comparable to the compression curve or *e*-*p* curve in soil mechanics) is fitted and the slope of the curve is used to evaluate the damage degree of rock and is defined to be damage index. The damage index can be used as a quantitative index to define the damage degree of rock under test conditions. The research results are of great significance for evaluating the damage degree of rock. To distinguish the damage severity of rock by using damage index as the limit value may be an important direction of rock damage mechanics.

2. Statistical Damage Constitutive Model for Rock

The strength of the rock is also random due to the fact that there are a large number of microcracks, cracks, and other defects; their distribution law, size, and penetration length of them show obvious randomness, and these randomly distributed defects affect the strength of the rock. Therefore, microunit strength can be regarded as a random variable and its probability distribution can be studied.

2.1. Weibull Distribution and Damage Variable. The Weibull distribution is a distribution function proposed by Swedish scholar Waloddi Weibull to describe the distribution of fracture stress in 1939. It is feasible to use Weibull distribution function to calculate the strength of brittle material. Rock is a kind of brittle material; its strength shows a large dispersion; therefore the Weibull distribution function can be used to statistically process the rock strength. Taking the microunit strength parameter $F*$ as the mechanical symbol quantity, the density function expression of the Weibull distribution is

$$\phi(F) = \frac{m}{F_0}\left(\frac{F^*}{F_0}\right)^{m-1} \exp\left[-\left(\frac{F^*}{F_0}\right)^m\right] \quad (1)$$

where m and F_0 are distribution parameters of the Weibull distribution, which reflect the mechanical properties of brittle material, $F*$ is the strength of the microunit, and $\varphi(F)$ is the probability of microunit destruction when the intensity is $F*$, showing the damage rate of microunit in the brittle material like rock during loading.

The definition of damage variable is the core problem to study the damage evolution law of rock and other materials under loading. According to continuum damage mechanics, the damage variable D_* can be defined as the ratio of the damage area S_* to the total area S_m of the material when it is not damaged; that is,

$$D_* = \frac{S_*}{S_m} = \int_0^{F^*} \phi(F)\,dF = 1 - \exp\left[-\left(\frac{F^*}{F_0}\right)^m\right] \quad (2)$$

$$F^* = \alpha I_1{}^* + J_2{}^{*\,1/2} - K \quad (3)$$

where K is the Drucker-Prager failure criterion parameter, which is the parameter related to cohesion C and internal friction angle ψ, $I_1{}^*$ is the first invariant of stress tensor, and $J_2{}^*$ is the second variable of stress deviation; they are as follows:

$$\alpha = \frac{2\sin\varphi}{\sqrt{3}(3-\sin\varphi)} \quad (4)$$

$$K = \frac{6c\cos\varphi}{\sqrt{3}(3-\sin\varphi)} \quad (5)$$

$$I_1{}^* = \sigma_1{}^* + \sigma_2{}^* + \sigma_3{}^* \quad (6)$$

$$J_2^* = \frac{(\sigma_1^* - \sigma_2^*)^2 + (\sigma_2^* - \sigma_3^*)^2 + (\sigma_1^* - \sigma_3^*)^2}{6} \tag{7}$$

where σ_i^* is the effective stress corresponding to the nominal stress σ_i, using the definition of damage:

$$\sigma_i^* = \frac{\sigma_i}{1 - D} \tag{8}$$

2.2. Definition of Damage Ratio. The damage area of the microunit of the material is S_*, and the area with no damaged S_e of the material microunit is the total area minus the damage area; that is,

$$S_e = S_m - S_* \tag{9}$$

Similar to the definition of the void ratio in geotechnical mechanics, the ratio of the damage area S_* to the area with no damaged S_e of the rock microunit is defined as the damage ratio D_e; that is,

$$D_e = \frac{S_*}{S_e} \tag{10}$$

The damage ratio is an indicator of the structural characteristics of rock damage. The damage ratio can reflect the evolution characteristics of microcrack inside the rock under the action of spatial stress and can be used to evaluate the compressive strength and the degree of damage of the rock. Therefore, the study of the damage ratio is also of great significance:

$$D_e = \frac{S_*}{S_e} = \frac{S_*}{S_m - S_*} = \frac{S_m D_*}{S_m - S_m D_*} = \frac{D_*}{1 - D_*} \tag{11}$$

$$D_e = \frac{1}{\exp\left[-(F^*/F_0)^m\right]} - 1 \tag{12}$$

In the process of rock test, internal damage accumulates gradually with the increase of load. When the load stops to increase (that means the stress peak is reached), the internal damage of the rock accumulates to a certain value, and the corresponding damage ratio is defined as the critical damage ratio. Its expression is

$$D_{er} = \frac{1}{\exp\left[-(F_c^*/F_0)^m\right]} - 1 \tag{13}$$

where D_{er} is the critical damage ratio and $Fc*$ is the $F*$ value corresponding to the peak stress of the rock.

2.3. Establishment of Statistical Damage Constitutive Model. In the triaxial compression experiment, it is assumed that the rock microunit conforms to the generalized Hooke's law of an isotropic elastomer:

$$\begin{aligned} \sigma_1 &= E\varepsilon_1 + \mu(\sigma_2 + \sigma_3) \\ \sigma_2 &= E\varepsilon_2 + \mu(\sigma_1 + \sigma_3) \\ \sigma_3 &= E\varepsilon_3 + \mu(\sigma_1 + \sigma_2) \end{aligned} \tag{14}$$

where σ_{ii} and ε_{ii} are the principal strain and stress of the microunit, E is the elastic modulus, and μ *is* Poisson's ratio.

The damage ratio is introduced to analyse the effect of confining pressure on rock, for the random distribution of the microcracks inside the rock changes under the influence of confining pressure.

In continuum damage mechanics, the three-dimensional isotropic linear elastic damage constitutive model can be expressed as [25]

$$\{\sigma\} = [C]\{\varepsilon\}(1 - D_*) \tag{15}$$

where $\{\sigma\}$ is the nominal stress vector, $[C]$ is the material elastic matrix of the material, $\{\varepsilon\}$ is the strain vector, and D_* is the damage variable.

We can get the damage variable D_* according to formula (11):

$$D_* = \frac{D_e}{1 + D_e} \tag{16}$$

when substituting formula (16) into formula (15), we can get

$$\{\sigma\} = [C]\{\varepsilon\}(1 + D_e)^{-1} \tag{17}$$

The statistical damage softening constitutive model of rock is established based on the obedience of microunit strength to Weibull distribution formula (1) in [26]. After introducing damage ratio, the statistical damage constitutive model of rock is as follows:

$$\begin{aligned} \sigma_1 &= E\varepsilon_1(1 + D_e)^{-1} + \mu(\sigma_2 + \sigma_3) \\ \sigma_2 &= E\varepsilon_2(1 + D_e)^{-1} + \mu(\sigma_1 + \sigma_3) \\ \sigma_3 &= E\varepsilon_3(1 + D_e)^{-1} + \mu(\sigma_1 + \sigma_2) \end{aligned} \tag{18}$$

when substituting formula (18) into formula (12), we can get

$$\begin{aligned} \sigma_1 &= E\varepsilon_1 \exp\left[-\left(\frac{F^*}{F_0}\right)^m\right] + \mu(\sigma_2 + \sigma_3) \\ \sigma_2 &= E\varepsilon_2 \exp\left[-\left(\frac{F^*}{F_0}\right)^m\right] + \mu(\sigma_1 + \sigma_3) \\ \sigma_3 &= E\varepsilon_3 \exp\left[-\left(\frac{F^*}{F_0}\right)^m\right] + \mu(\sigma_1 + \sigma_2) \end{aligned} \tag{19}$$

The formula above is a statistical damage constitutive model of rock based on Weibull distribution. The key of the model is to get the two distribution parameters m and F_0 in the formula. The common method is to determine the values by curve fitting according to the rock triaxial test curves of rock as the fitting method is complicated and random. According to the method in [26], the distribution parameters are determined according to the stress strain peaks.

2.4. Determination of Distribution Parameters. According to Drucker-Prager failure criterion,

$$F = \alpha I_1 + J_2^{1/2} - K \tag{20}$$

where F is the macroscopic strength of the material, that is nominal strength, α and K are the Drucker-Prager parameters of failure criterion, I_1 is the first invariant of the stress tensor, and J_2 is the second variable of the stress offset, and the expression of them is as follows:

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 \quad (21)$$

$$J_2 = \frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{6} \quad (22)$$

where σ_i is the nominal stress of the material, that is, the actually measured stress.

According to the damage definition, formula (8) can be converted as follows:

$$\sigma_i^{*} = \frac{\sigma_i}{1-D} = \sigma_i (1 + D_e) \quad (23)$$

therefore,

$$F^{*} = F(1 + D_e) = \frac{(\alpha I_1 + J_2^{1/2} - K) E\varepsilon_1}{\sigma_1 - \mu(\sigma_2 + \sigma_3)} \quad (24)$$

In the conventional triaxial test, the confining pressure is as $\sigma_2 = \sigma_3$, and the strain is as $\varepsilon_2 = \varepsilon_3$; then formula (19) can be simplified as

$$\begin{gathered} \sigma_1 = E\varepsilon_1 \exp\left[-\left(\frac{F^{*}}{F_0}\right)^m\right] + 2\mu\sigma_3 \\ \sigma_3 = \sigma_2 = \frac{E\varepsilon_2 \exp\left[-(F^{*}/F_0)^m\right]}{1-\mu} + \frac{\mu\sigma_1}{1-\mu} \end{gathered} \quad (25)$$

when the rock reaches the peak failure under the triaxial stress, there are $\sigma_1 = \sigma_c$, $\varepsilon_1 = \varepsilon_c$, where σ_c is the peak stress, and ε_c is the strain corresponding to the peak stress, as follows:

$$\frac{\partial\sigma_1}{\partial\varepsilon_1} = 0 \quad (26)$$

$$\sigma_c = E\varepsilon_c \exp\left[-\left(\frac{F^{*}}{F_0}\right)^m\right] + 2\mu\sigma_3 \quad (27)$$

$$F^{*} = \frac{(\alpha I_1 + J_2^{1/2} - K) E\varepsilon_1}{\sigma_1 - 2\mu\sigma_3} \quad (28)$$

The key to solve the distribution parameters is to find the relation of $\partial\sigma_1/\partial\varepsilon_1$. To assume the σ_1 and σ_3 as the function of ε_1 and ε_3, we can get the following:

$$d\sigma_1 = \frac{\partial\sigma_1}{\partial\varepsilon_1} d\varepsilon_1 + \frac{\partial\sigma_1}{\partial\varepsilon_3} d\varepsilon_3 \quad (29)$$

The following expression can be got after differentiating the two equations of formula (25):

$$\begin{gathered} d\sigma_1 = H_1 d\varepsilon_1 + H_2 dF_{\varepsilon_1}{}^{*} + H_3 dm + H_4 dF_0 + 2\mu d\varepsilon_3 \\ d\sigma_3 = L_1 d\varepsilon_3 + L_2 dF_{\varepsilon_3}{}^{*} + L_3 dm + L_4 dF_0 + \frac{\mu}{1-\mu} d\varepsilon_3 \end{gathered} \quad (30)$$

The following expression can be got after differentiating the equation of formula (28):

$$\begin{gathered} dF_{\varepsilon_1}{}^{*} = F_{11} d\varepsilon_1 + F_{12} d\sigma_1 + F_{13} d\sigma_3 \\ dF_{\varepsilon_3}{}^{*} = F_{21} d\varepsilon_3 + F_{22} d\sigma_1 + F_{23} d\sigma_3 \end{gathered} \quad (31)$$

According to the hypothesis of [14], m and F_0 are functions of confining pressure; then

$$\begin{gathered} dm = P d\sigma_3 \\ dF_0 = Q d\sigma_3 \end{gathered} \quad (32)$$

when substituting formulas (31) and (32) into (30), we can get the following:

$$\begin{gathered} (H_2 F_{12} - 1) d\sigma_1 + (H_2 F_{13} + H_3 P + H_4 Q + 2\mu) d\sigma_3 \\ + (H_1 + H_2 F_{11}) d\varepsilon_1 = 0 \\ \left(\frac{\mu}{1-\mu} + L_2 F_{22}\right) d\sigma_1 + (L_2 F_{23} + L_3 P + L_4 Q) d\sigma_3 \\ + (L_1 + L_2 F_{21}) d\varepsilon_3 = 0 \end{gathered} \quad (33)$$

the formula above can be simplified as

$$\begin{gathered} A_1 d\sigma_1 + A_2 d\sigma_3 + A_3 d\varepsilon_1 = 0 \\ B_1 d\sigma_1 + B_2 d\sigma_3 + B_3 d\varepsilon_3 = 0 \end{gathered} \quad (34)$$

when $d\sigma_3$ is eliminated, the formula above can be simplified as

$$d\sigma_1 = \frac{A_3 B_2}{A_2 B_1 - A_1 B_2} d\varepsilon_1 + \frac{A_2 B_3}{A_1 B_2 - A_2 B_1} d\varepsilon_3 \quad (35)$$

we can get the following expression with formula (29):

$$\frac{\partial\sigma_1}{\partial\varepsilon_1} = \frac{A_3 B_2}{A_2 B_1 - A_1 B_2} = 0 \quad (36)$$

From formula (36), we can get that $A_3 = 0$ or $B_2 = 0$. If $B_2 = 0$, then

$$B_1 d\sigma_1 + B_3 d\varepsilon_3 = 0 \quad (37)$$

If there is no $d\sigma_3$ in formula (37) and it is a contradiction for the above formula is to differentiate σ_3. However, it can only be $A_3 = 0$; then

$$A_3 = H_1 + H_2 F_{11} = 0 \quad (38)$$

It can be known from formulas (25) and (30)

$$H_1 = E \exp\left[-\left(\frac{F^{*}}{F_0}\right)^m\right] \quad (39)$$

$$H_2 = -E\varepsilon_1 \exp\left[-\left(\frac{F^{*}}{F_0}\right)^m\right] \cdot \left(\frac{F^{*}}{F_0}\right)^m \left(\frac{m}{F^{*}}\right) \quad (40)$$

$$F_{11} = \frac{\partial F^{*}}{\partial\varepsilon_1} = \frac{(\alpha I_1 + J_2^{1/2} - K) E}{\sigma_1 - 2\mu\sigma_3} \quad (41)$$

Combining formulas (38), (39), (40), and (41), we can get the following:

$$F_0 = F^{*} \cdot m^{1/m} \quad (42)$$

Figure 1: The MTS815 concrete and rock mechanics experiment system.

Figure 2: Failure mode of sandstone under uniaxial compression.

when substituting formula (42) into (27), we can get the following:

$$m = \frac{1}{\ln\left[E\varepsilon_c/\left(\sigma_c - 2\mu\sigma_3\right)\right]} \tag{43}$$

The formula above is the expression of the distribution parameter determined by the conventional triaxial test. When it is determined by uniaxial compression test, for σ_2=σ_3=0, the expression of the parameters determined by the uniaxial compression test is as follows:

$$\begin{aligned} m &= \frac{1}{\ln\left(E\varepsilon_c/\sigma_c\right)} = \frac{1}{\ln\left(\mathrm{E}/\mathrm{E}_S\right)} \\ F_0 &= F^* \cdot m^{1/m} \end{aligned} \tag{44}$$

In formula, E_S is secant modulus of elasticity, i.e., the slope of the straight line connecting the origin and the peak stress point in the stress-strain curve, σ_c is the peak stress in uniaxial compression test, and ε_c is the strain corresponding to the peak stress.

3. Uniaxial and Triaxial Tests for Rock

3.1. Test Equipment. The test was carried out on the MTS815 concrete and rock mechanics experiment system of the College of Water Resources and Hydropower, Sichuan University (Figure 1). This equipment is mainly used for testing the mechanical properties of rock and concrete materials. It can collect both high and low speed data, track the whole process of rock failure, and obtain the stress-strain curve of the whole process of rock. It is one of the most complete rock mechanics testing equipment with the highest technical level in the world. The MTS815 experiment system is an all-digital computer automatic control system that collects load, stress, strain, and displacement values in real time. During the loading process, it can control the loading by means of axial force control, displacement control, and circumferential deformation control and can carry out the whole process test of uniaxial compression and conventional triaxial test, etc. The load-displacement, load-axial deformation, and load-circumferential deformation curves can be drawn in real time.

3.2. Test Scheme. The specimens of fine sandstone were used in the test, and they were processed into Φ50 mm×100 mm standard cylindrical specimens. In order to verify the statistical softening constitutive model of rock and explore the relationship between damage ratio and confining pressure, uniaxial and triaxial compression tests on the specimens were carried out, respectively. In triaxial tests, confining pressures were set as 3 MPa, 6 MPa, 9 MPa, 12 MPa, and 24 MPa. To get the general law of damage ratio and confining pressure, the first four groups of confining pressures were set to increase by 3 MPa. And the last group of confining pressure was set large enough to verify the applicability of the law. A total of 18 specimens were tested for 3 specimens were tested for each confining pressure.

3.3. Test Process. The main process of conventional triaxial compression test: (1) Install the prepared specimen on the test bench. (2) Install the axial and circumferential extensometers to measure the deformation of the specimens. (3) Manually apply the initial axial force of 2 kN to the specimen by means of displacement control to make the specimen in full contact with the indenter. (4) Close the triaxial chamber and the confining pressure to be loaded to the target value at a rate of 3 MPa/min by confining pressure loading system. (5) The axial force is controlled by means of axial force control to be loaded at the rate of 20 kN/min until the stress state of the specimen reaches the limit of proportionality; change the axial force to be loaded at a rate of 0.04 mm/min by the means of circumferential deformation control. (6) Stop the test when the axial force does not decrease significantly with the increase of deformation (Figure 3).

The process of uniaxial compression test: The axial force is controlled by means of force to be loaded at the rate of 5 kN/min. In order to ensure the safety of the test equipment, when the displacement reaches 0.28 mm, change the axial force to be loaded by means of displacement control until the specimen was destroyed (Figure 2).

Table 1: The mechanical parameters subjected to different confining pressures for sandstone.

Confining Pressure (MPa)	σ_1 (MPa)	$\varepsilon_1/10^{-3}$	E (GPa)	E_s (GPa)	μ
0	31.34	5.125	7.556	6.115	0.291
3	52.19	12.783	5.525	4.083	0.292
6	75.38	16.573	6.702	4.548	0.301
9	92.40	17.787	8.052	5.195	0.311
12	102.50	19.539	8.351	5.246	0.318
24	151.61	24.780	10.124	6.118	0.324

Figure 3: Failure mode of sandstone under triaxial compression.

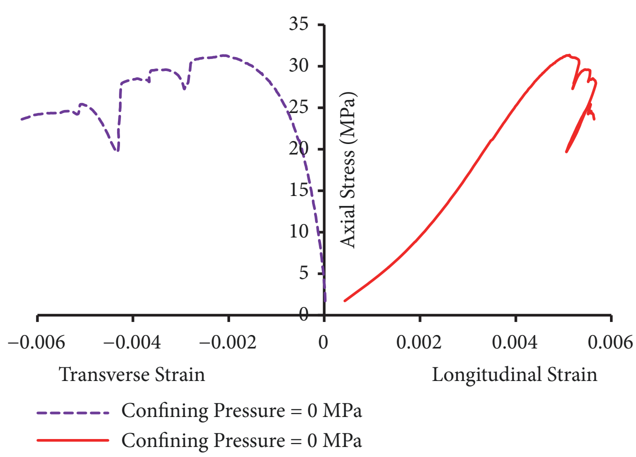

Figure 4: The stress strain curve of rock subjected to uniaxial compression test.

In the whole test process, the data of axial pressure, longitudinal displacement, and transverse displacement in the stress-strain process are collected and recorded automatically by the computer according to the time interval of 3 seconds, the longitudinal displacement of 0.001 mm, and the transverse displacement of 0.005 mm.

3.4. Test Results. The stress strain curve of sandstone under uniaxial stress is shown in Figure 4. It can be seen that the rock quickly destroys after reaching the peak stress and rapidly falls to its residual strength. The brittle failure of rock is obvious under the uniaxial stress. The stress strain curve of sandstone under triaxial stress is shown in Figure 5. It can be seen that the sandstone exhibits different postpeak characteristics under different confining pressures and the brittle failure can be seen under low confining pressure. With the increase of confining pressure, the brittle failure of rock gradually changes into plastic failure. The greater the confining pressures are, the more obvious the plastic deformation is.

In triaxial compression test, the confining pressure is $\sigma_2=\sigma_3$, so it can be obtained by formula (14):

$$\sigma_1 = E\varepsilon_1 + 2\mu\sigma_3 \tag{45}$$

$$\sigma_3 = E\varepsilon_3 + \mu\left(\sigma_1 + \sigma_3\right) \tag{46}$$

we can get the following expression after simultaneous (45) and (46):

$$E = \frac{\sigma_1^2 + \sigma_1\sigma_3 - 2\sigma_3^2}{\left(\sigma_1 + \sigma_3\right)\varepsilon_1 - 2\sigma_3\varepsilon_3} \tag{47}$$

$$\mu = \frac{\sigma_3\varepsilon_1 - \sigma_1\varepsilon_3}{\left(\sigma_1 + \sigma_3\right)\varepsilon_1 - 2\sigma_3\varepsilon_3} \tag{48}$$

The elastic modulus and Poisson's ratio of rock can be obtained by using formulas (47) and (48) combined with the data of the straight line before the peak of the stress-strain relationship curve.

The confining pressure can be considered to be 0 MPa in uniaxial compression test. The mechanical parameters of rock under different confining pressures are shown in Table 1. It shows that as the confining pressure increases, the elastic modulus, the peak stress, and the corresponding strain increase as well as the Poisson's ratio. The confining pressure has a significant influence on the mechanical parameters and deformation characteristics of the rock.

3.5. Solution of Model Parameters and Damage Ratio. The m value can be obtained by substituting the test data in Table 1 into formula (43). $F*$ and F_0 values can be obtained by formulas (24) and (42). In order to obtain $F*$ value, cohesion C and internal friction angle φ of rock should be calculated first according to test data.

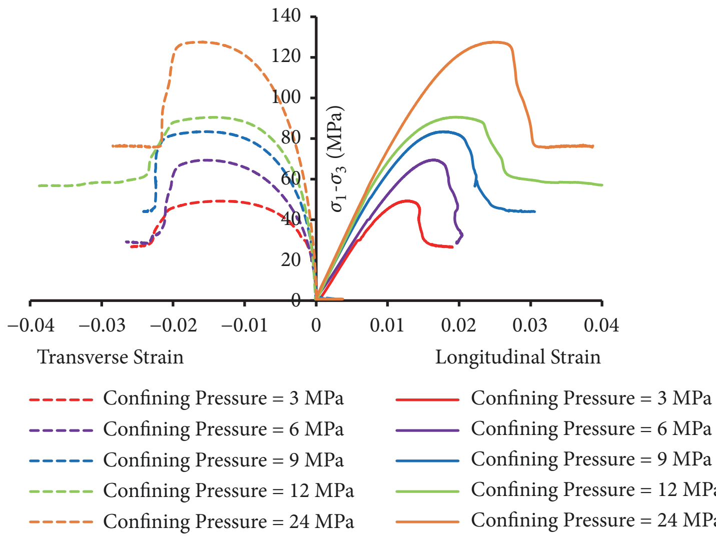

Figure 5: The complete stress strain curve of rock subjected to triaxial compression test.

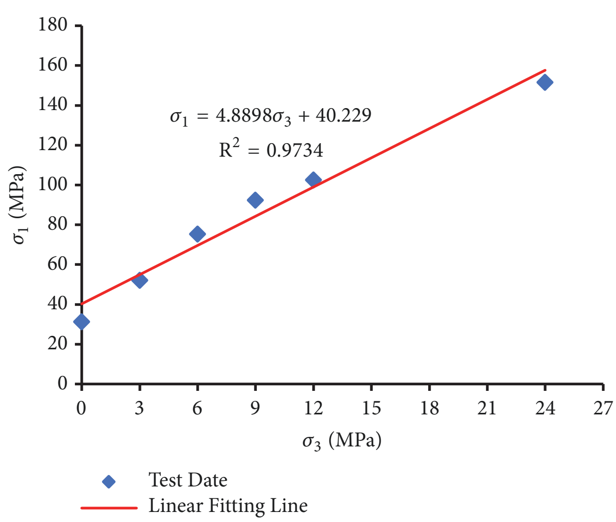

Figure 6: The optimal relation curve of rock.

At present, Mohr-Coulomb criterion is the most widely used strength theory in the study of rock mechanics, and its expression is as follows:

$$\tau_f = c + \sigma \tan \varphi \tag{49}$$

The Mohr-Coulomb criterion can be expressed in another way. The failure criteria of rocks can be expressed in terms of σ_1 and σ_3. The expression is as follows:

$$\sigma_1 = \sigma_3 N_\varphi + 2c\sqrt{N_\varphi} \tag{50}$$

$$N_\varphi = \frac{1}{\tan^2(45^\circ - \varphi/2)} \tag{51}$$

According to the test data, the optimal relation curve is drawn by using least square method with σ_3 as abscissa and σ_1 as ordinate, as shown in Figure 6. It can be seen that the linear relationship between the axial stress and the confining pressure of sandstone is as follows: σ_1=4.8898σ_3+40.229.

Table 2: Correction coefficient and distribution parameters subjected to different confining pressures for sandstone.

σ_3 (MPa)	m	F_0 (MPa)	D_{er}
0	4.726	31.230	0.236
3	2.970	75.256	0.400
6	2.290	129.836	0.548
9	1.997	172.682	0.650
12	1.844	199.821	0.720
24	1.634	320.298	0.844

From the optimal relation curve of rock and formulas (50) and (51), the cohesion C=9.096 MPa and internal friction angle φ=41.33° can be obtained.

The distribution parameters F_0 and m of sandstone under different confining pressures and the critical damage ratio D_{er} of rock under peak stress can be obtained by incorporating C, φ, and test data into formulas (4), (5), (13), (24), (42), and (43), as shown in Table 2.

3.6. Model Verification. Substituting the parameters m and F_0 with values calculated from tests in formula (19), the statistical damage constitutive model under different confining pressures can be obtained. To verify the model, with the experimental strain as the input value of the model, the stress obtained by using the constitutive model should be compared with the experimental results. Figure 7 shows the comparison of stress strain curves between the proposed model-based curve and the test curve under various confining pressures. It can be seen from the figure that the simulated curve can reflect the process of rock failure before peak. The difference of the postpeak curve between test curve and model curve is caused by the change of loading mode, loading speed, etc. during the test and the model needs to be modified as what has been studied in [19].

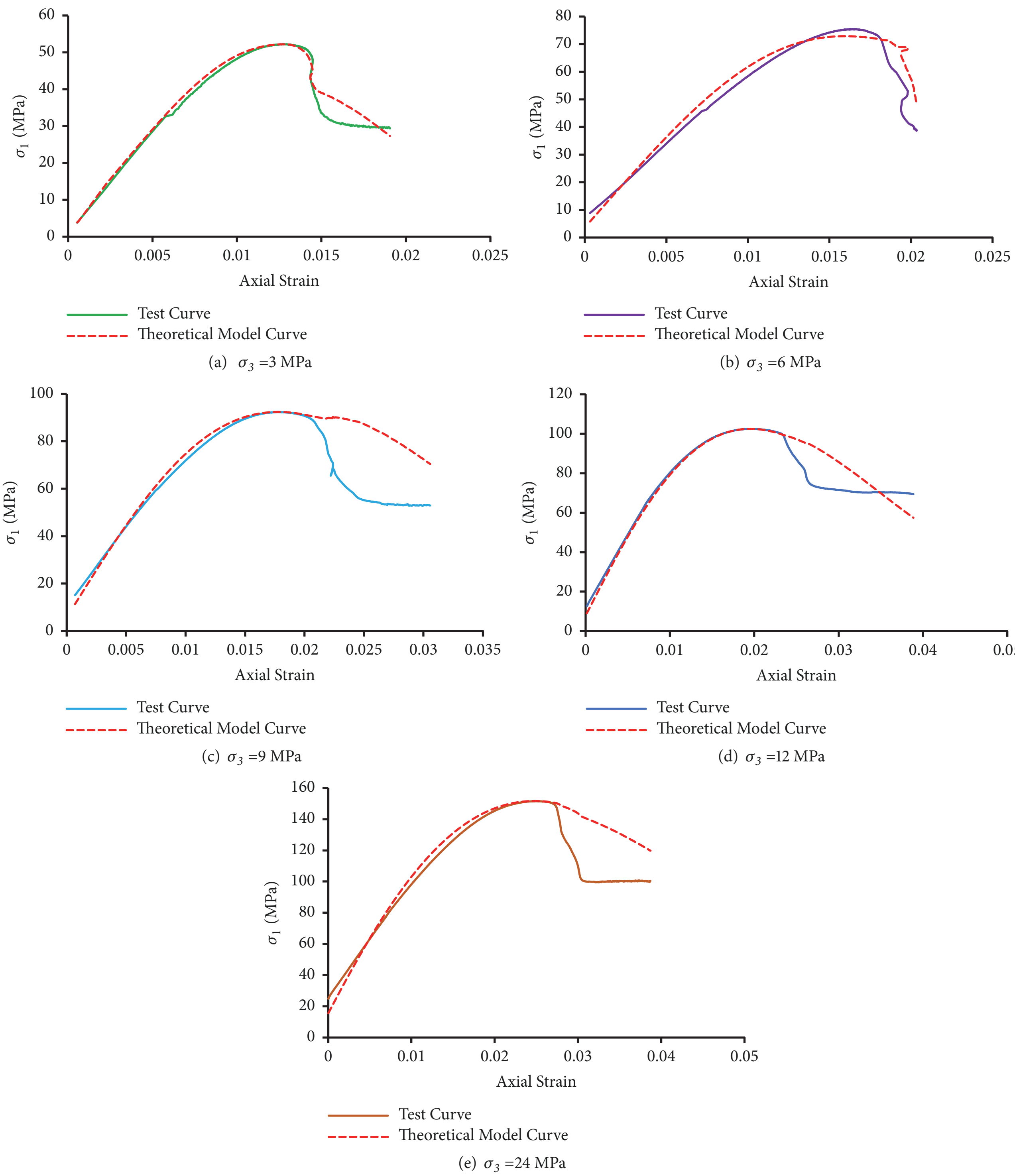

Figure 7: Comparison of stress strain curves between experimental and simulated results under various confining pressures.

4. Discussion

4.1. Influence of Confining Pressure on Distribution Parameters. According to [26], m is a parameter to characterize the brittleness of rock material, and F_0 is a parameter to reflect the macroscopic average strength of rock. From Table 2, it can be found that m decreases with the increase of confining pressure, indicating that brittleness of rock is weakening while plasticity is increasing. Moreover, the m value subject to the triaxial stress is significantly reduced compared with that in uniaxial stress, indicating that the confining pressure has a significant influence on the failure characteristics of the rock,

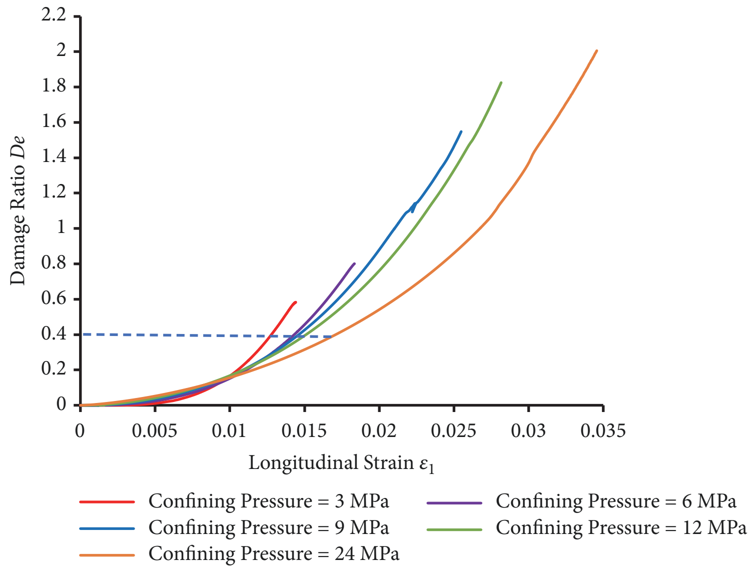

Figure 8: Damage process of sandstone subjected to different confining pressures in triaxial test.

and even a small confining pressure can significantly improve the postpeak plasticity of the rock.

It has been shown in Table 2 that, as the confining pressure increases, F_0 increases, indicating that the macroscopic average strength of rock increases with the increase of confining pressure, for the bearing capacity of the rock is enhanced with the change of the stress of the rock under confining pressure.

4.2. Influence of Confining Pressure on Damage Ratio. Under triaxial stress condition, when the distribution parameters m and F0 are determined, the damage variables can be calculated according to formula (12), and a strain value corresponds to a damage ratio. Therefore, the whole damage ratio curve of sandstone can be obtained by introducing the strain value of the stress-strain curve of sandstone under triaxial compression into (12) (as shown in Figure 7). The curve can reflect the damage development process of the damage of the sandstone under different confining pressures.

From the graph, it can be found that the damage ratio of rock increases with the increase of longitudinal strain, which essentially reflects the process of internal damage expanding and rock failure from loading to gradual failure under the action of axial stress. Furthermore, the increase of longitudinal strain speeds up the increase of the damage ratio. Especially when the rock tends to be damaged, the curve tends to be vertical straight line, the damage ratio develops faster, and the magnitude of increase is more obvious. It can be shown that, when the rock approaches failure, the cumulative area of damage increases and the area of no damage decreases, while the damage ratio increases rapidly as the loading proceeds.

Meanwhile, it can be seen from Figure 8 that, when the damage ratio is 0.4, the tangent value of the curve decreases with the increase of confining pressure, which shows that the increase rate of damage ratio under high confining

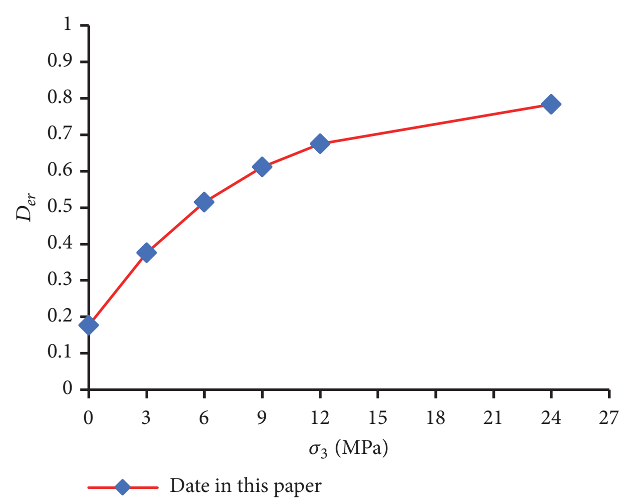

Figure 9: The relationship between critical damage ratio and confining pressure.

pressure is less than that under low confining pressure at the same damage ratio. Therefore, the increase of confining pressure can delay the increase of damage ratio, inhibit the development of damage, increase the strength of rock, slow down the failure process of rock, and make the transition from brittle failure to plastic failure.

4.3. Influence of Confining Pressure on Critical Damage Ratio. Critical damage ratio refers to the damage corresponding to the peak stress of rock for it corresponds to peak strain one to one. To be similar to the strength parameters of rock, the critical damage ratio can be used as a parameter to indicate whether rock is damaged or not. The change curve of critical damage ratio and confining pressure is as shown in Figure 9. It can be found that, with the increase of confining pressure,

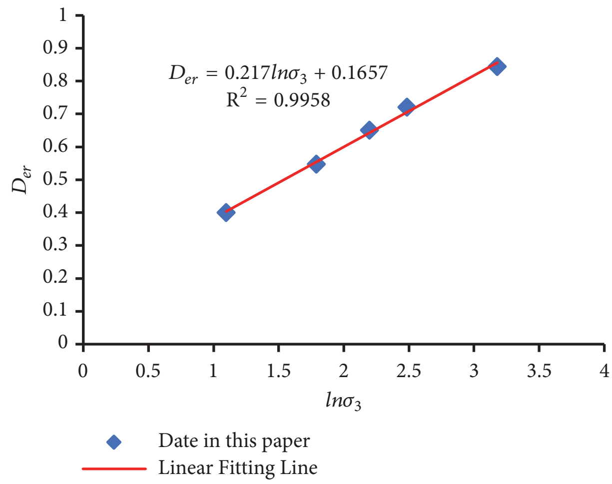

FIGURE 10: D_{er}-$ln\sigma_3$ curve of data in this paper.

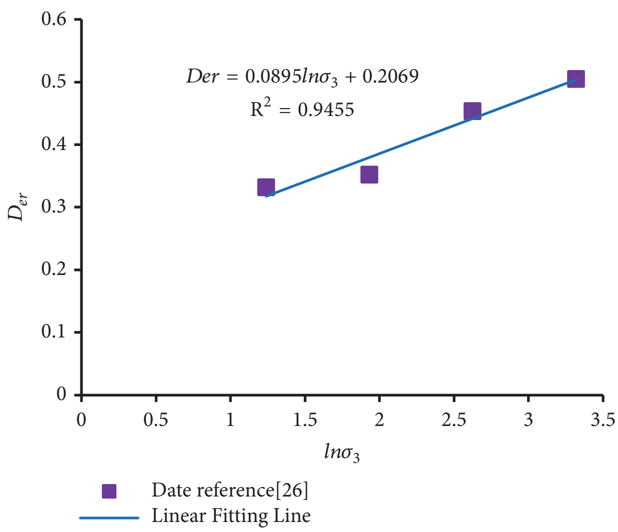

FIGURE 11: D_{er}-$ln\sigma_3$ curve of data in [26].

the critical damage ratio increases gradually in curve and the increase range slows down.

In order to further determine the relationship between critical damage ratio and confining pressure, it is found that the relationship between critical damage ratio and logarithmic function of confining pressure is well linear by data fitting, as shown in Figure 10. In the figure, the Y axis is D_{er} and the X axis is $\ln\sigma_3$. Thus, the expression of critical damage ratio and confining pressure can be obtained as follows:

$$D_{er} = 0.217 \ln \sigma_3 + 0.1657 \tag{52}$$

By introducing the data of [26], the relationship between D_{er} and $\ln\sigma_3$ can be obtained through the methods mentioned above as shown in Figure 11. Therefore, the relationship between critical damage ratio and confining pressure can be obtained as follows:

$$D_{er} = 0.0895 \ln \sigma_3 + 0.2069 \tag{53}$$

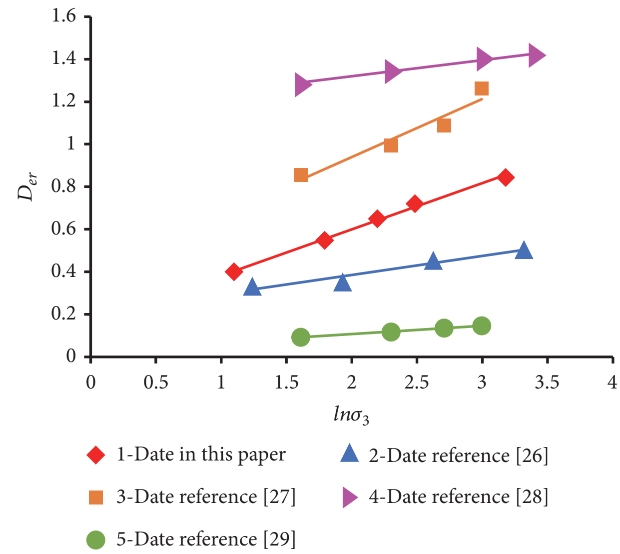

FIGURE 12: D_{er}-$ln\sigma_3$ curves.

TABLE 3: Data fitting results.

Number	a	b	Correlation coefficient (R^2)
1	0.2170	0.1657	0.9959
2 [26]	0.0895	0.2069	0.9455
3 [27]	0.2740	0.3913	0.9305
4 [28]	0.0761	1.1669	0.9829
5 [29]	0.0389	0.0297	0.9965

It can be found that the relationship between critical damage ratio and confining pressure is as follows:

$$D_{er} = a \ln \sigma_3 + b \tag{54}$$

In the formula, a and b are constants, which can be obtained by linear fitting of experimental data.

The following expression can be getting after differentiating the equation of formula (54):

$$\frac{dD_{er}}{d\sigma_3} = \frac{a}{\sigma_3} \tag{55}$$

$$\lim_{\sigma_3 \longrightarrow \infty} \frac{dD_{er}}{d\sigma_3} = \lim_{\sigma_3 \longrightarrow \infty} \frac{a}{\sigma_3} = 0 \tag{56}$$

Formula (55) reflects the tangent of the D_{er}-σ_3 curve. It can be seen from formula (55) that when σ_3 is infinite, (55) equals zero, which means that, with the increase of σ_3, D_{er} tends to a certain value which is related to the category of rock material instead of increasing infinitely.

4.4. Definition and Physical Significance of Damage Index. The D_{er}-$\ln\sigma_3$ curve is obtained from the experimental data in this paper and the data in the literature [26–29], as shown in Figure 12. The results of data fitting are shown in Table 3.

From the D_{er}-lnσ_3 curves shown in Figure 12, we can get the following rule: the curve is close to a straight line. We can define the slope of the curves (the damage index C_c) as

$$C_c = a = \frac{D_{er} - b}{\ln \sigma_3} \tag{57}$$

According to slope of test curve and reference curve, it can be seen that the linear slope of rock in different categories is not the same. The larger the value of a, the greater the influence of confining pressure on damage ratio.

The existence of confining pressure also changes the failure mechanism and the condition of the microcracks in the rock, which leads to the various closing of microcracks or eases the expanding of fractures.

The microcracks can be regarded as internal damage of the rock. Essentially, the effect of confining pressure is to restrain the development of damage ratio in rock; that is the damage area diffusivity of rock under confining pressure is slower than that without confining pressure, and the damage area diffusivity under high confining pressure is lower than that under low confining pressure. Therefore, greater axial force was required for rock failure when confining pressure exists.

It can also be considered that the more microcracks are distributed in the rock, the more serious the rock damage is, and the more obvious the inhibiting effect of confining pressure on the damage development is, for the damage degree can be greatly changed by only a small confining pressure difference. On the contrary, if the rock interior is complete and the damage is slight, the effect of confining pressure on the damage development is not obvious.

Then, the damage index can reflect the effect of confining pressure on the damage ratio. Therefore, the damage index can be used as an index to evaluate the degree of internal damage of rock. The larger the damage index is, the more serious the damage of rock is. The smaller the damage index is, the slighter the damage degree of rock is. Similar to the compression index that can judge the compressibility of soil, the damage index can be used to judge the degree of rock damage.

Because of the limited test data and limited rock types in this paper, it is impossible to give a clear discriminant index, but we can try to predict the following classification:

When $C_c > M$, rock is seriously damaged.

When $M \geqslant C_c \geqslant N$, rock is moderately damaged.

When $C_c < N$, rock is slightly damaged.

Among them, M and N are the limit values of damage index for judging the damage degree of rock, which need to be determined by a large number of tests. A research idea has been provided in this paper, and the follow-up research can be carried out deeper on the basis of this.

5. Conclusions

The study of rock damage characteristics is of great significance to the analysis of rock mass. In this paper, the main conclusions are as follows.

(1) Damage ratio is an index of damage structure characteristics of rock. It not only can reflect the evolution characteristics of microcracks in rock under spatial stress, but also can be used to evaluate the degree of rock damage and the compressive strength of rock. Based on the study of statistical damage constitutive model and damage variable of rock, the definition and solution of damage ratio are obtained.

(2) Critical damage ratio refers to the damage corresponding to the peak stress of rock. For corresponding to the peak strain one to one, the critical damage ratio can be used as a parameter to characterize whether the rock is damaged or not. The critical damage ratio has a good linear relationship with the logarithmic function of confining pressure. Its relationship is as follows: $D_{er} = C_c \ln \sigma_3 + b$. With the increase of σ_3, the increasing trend of D_{er} slows down and gradually tends to a certain value.

(3) The slope of D_{er}-lnσ_3 curve is defined as damage index. The larger the damage index, the more serious the damage of rock. The smaller the damage index, the slighter the damage of rock. The damage index can be used as a quantitative index to define the damage degree of rock under test conditions. The research results are of great significance for evaluating the damage degree of rock. To distinguish the damage severity of rock by using damage index as the limit value may be an important direction of rock damage mechanics.

Data Availability

(1) The data used to support the findings of this study are included within the Supplementary Information file. (Note that the data above is provided to the editor only to use during the review process. And it is not supported for sharing at this stage.) (2) Previously reported data were used to support this study and are available at CNKI. These prior studies are cited at relevant places within the text as [26–29].

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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