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Research Article

Swarming Behaviors in Multiagent Systems with Nonlinear Dynamics and Aperiodically Intermittent Communication

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This paper investigates the stability of a class of swarm model with nonlinear dynamics and aperiodically intermittent communication. Different from previous works, it assumes that the agents obtain information from the neighbors at a series of aperiodically time intervals. Moreover, nonlinear dynamics and time delay are considered. It finds that all agents in a swarm can reach cohesion within a finite time under discontinuous communication, where the upper bounds of cohesion depend on the parameters of the swarm model and communication time. A numerical example is given to demonstrate the validity of the theoretical results.

1. Introduction

In recent years, distributed cooperative control of multiagents has attracted great interest due to its applications in the fields of physics, engineering, social science, etc. Researchers have spent a lot of efforts in understanding how a herd of animals, school of fish, and man-made mobile autonomous agents can produce coordinated collective behaviors due to local interactions among individuals [1–3].

Swarming is an important branch of distributed coordinated control system. Gazi et al. proposed a swarm model which used a special interaction function and studied its aggregation, cohesion, and stability properties [4]. Afterwards, the authors [5] proposed an M-member "individual-based" continuous-time swarm model and investigated collective behaviors of swarm with general nonlinear attraction and repulsion functions. In [6], stability properties of a continuous-time model for swarm aggregation in the n-dimensional space were discussed, and an asymptotic bound for the spatial size of the swarm was computed by using the parameters of the swarm model. For more about the results of the swarm aggregation problem, see [7, 8] and subsequent references therein.

But so far, only a few works have studied swarming behaviors of multiagents with nonlinear dynamical and aperiodically intermittent communication. Swarm systems with nonlinear dynamics are very common in nature, such as fish school and bird flock, whose trajectories are nonlinear, which means the swarm center of an agent is not a fixed constant. Considering this factor, a class of continuous nonlinear swarm models has been proposed and swarm behaviors have been studied [9]. In [10], stability of nonlinear multiagent systems under directed topology has been researched, which proves that if the topology of the underlying swarm is strongly connected, then all agents will converge globally or exponentially to the super ellipsoid in finite time. In [11, 12], mean square exponential synchronization of dynamic networks with random switching and parameter uncertainties has been considered, and the conditions for synchronization under strong connected topology are obtained. Consensus of fractional order nonlinear multiagent systems has been studied in [13, 14]. There is a common feature for achieving stable algorithms of these multiagent systems in [10–14]; that is, each agent can get information from itself and its neighbors without interruption. However, in real life, information about the state of some agents may be unavailable within a certain timetable period; for this reason, intermittent control algorithm was proposed. Each agent can only get information from himself and his neighbors intermittently instead of continuously. At the same time, the intermittent control algorithm has attracted wide attention for it is more effective in saving energy [15–17]. In [14], swarm problem of the first-order nonlinear multiagent systems under periodically intermittent control is discussed, and the proportion of communication time to achieve stability is given. In [18], consensus of the second-order multiagent systems with periodic intermittent communication and nonlinear dynamics is studied. And a new periodic intermittent strategy is proposed to ensure the realization of consensus. In [19], consensus problems for second-order multiagent systems with delayed and intermittent communication were investigated. The distributed consensus tracking problem of multiagent systems with switching directed topologies is studied [20, 21], in which the control inputs to the followers may be temporally missed. Although many intermittent control strategies have been put forward to realize the consensus, swarming and synchronization of multiagents systems, most of the results are based on the periodic intermittent control. And the requirement of periodicity of intermittent control is quite restricted. In fact, the requirement of periodicity is unreasonable and unnecessary, and aperiodically intermittent communication is more general in many real scenarios [22].

To sum up, this paper studies the swarm behaviors of nonlinear multiagents systems with aperiodically intermittent communication, to assume the communication between agents is intermittent and the communication time is arbitrary. By using Lyapunov approach, some sufficient conditions are proposed to guarantee the swarm behaviors of multiagent systems. The contributions of this paper are as follows:

(1) An aperiodically intermittent communication strategy is proposed, which is more general than periodic intermittent communication.

(2) The nonlinear dynamics and time delay are taken into account.

(3) Our conclusion shows that under the strongly connected topology the swarm behaviors can be guaranteed, when the communication ratio reaches a certain condition.

2. Model and Preliminaries

The swarm model considered in [4] is first reviewed. In a swarm of N agents in the n-dimensional Euclidean space, the motion dynamics of the agent ith is described as follows:

$$\dot{x}_i(t) = \sum_{j=1, j\neq i}^{N} g\left(x_i(t) - x_j(t)\right), \quad i = 1, 2, \ldots, N \tag{1}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)]^T \in R^n$ is the position of agent i at time t; $g : R^n \longrightarrow R^n$ represents the interaction force between the corresponding agents in the form of repulsion and attraction given by

$$g(y) = -y\left[g_a(\|y\|) - g_r(\|y\|)\right], \tag{2}$$

where $g_a : R^+ \longrightarrow R^+$ represents the attraction term, whereas $g_r : R^+ \longrightarrow R^+$ represents the repulsion term, and $\|y\| = \sqrt{y^T y}$.

Note that most of the existing protocols are implemented based on a common assumption that all information is transmitted continuously among agents. This assumption might be rare case in practice and noneconomic, since the information exchange among agents does not need to exist all the time. For example, agents may only communicate with their neighbors over some disconnected time intervals due to the unreliability of communication channels, failure of physical devices, etc. On the other hand, in real biological systems, each agent's motion dynamic is determined not only by interagent interactions, but also by each agent's intrinsic dynamics. Therefore, the following generalized swarm model with nonlinear dynamics and intermittent communication is considered:

$$\dot{x}_i(t) = \begin{cases} f\left(t, x_i(t), x_i(t-\tau)\right) + \displaystyle\sum_{j=1, j\neq i}^{N} a_{ij} g\left(x_i(t) - x_j(t)\right), & t \in [t_k, s_k), \\ f\left(t, x_i(t), x_i(t-\tau)\right), & t \in [s_k, t_{k+1}), \end{cases} \tag{3}$$

where $k = 0, 1, 2, \ldots$, $f(t, x_i(t), x_i(t-\tau)) = [f_1(\cdot,\cdot,\cdot), f_2(\cdot,\cdot,\cdot), \ldots, f_n(\cdot,\cdot,\cdot)]^T$ is continuous vector value functions describing the dynamics of agent i and τ is time delay. $A = (a_{ij})_{N\times N}$ is coupled matrix of model (3), if agent i is subjected to the action from agent $j (i \neq j)$, then $a_{ij} > 0$; otherwise, $a_{ij} = 0$. Here, we take $a_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$ and assume that the coupling matrix A is irreducible, which means that the corresponding digraph is strongly connected. $[t_k, t_{k+1})$ denotes the k-th communication period, and $s_k - t_k \geq 0$ is the communication width.

Remark 1. This paper assumes that the communication topology of multiagent system (3) is fixed, symmetrical and irreducible. In fact, the communication topology of the system can be further weakened to be switched topology, and the topology contains the case of a directed spanning tree. Consistency of multiagent systems with switching topology and directed spanning trees is studied in references [20, 21]. However, there are few researches on swarming behavior problem about this kind of multiagent systems.

Remark 2. The purpose of this paper is to minimize the communication time of agents under the condition of guaranteeing the swarming behavior of multiagent systems. At present, there are several communication mechanisms in reducing communication time, such as impulsive communication [23], intermittent communication, and event-triggered communication [24]. As for event-triggered communication mechanism, the system uses sampled data to determine whether the triggering condition is satisfied or not. If the triggering condition is satisfied (the plant state deviates more than a certain threshold from a desired value), the system communicates; otherwise, it will be no need to communicate. Compared with periodic control, event-triggered control has obvious advantages in resolving communication energy, computing, and communication constraints in designing wireless networked control systems [24]. In this paper, we mainly study the swarming behavior of multiagent systems under intermittent communication. Unlike event-triggered communication, intermittent communication is the intermittent exchange of information among agents according to the setting communication strategy (i.e., the interleaving of communication and noncommunication). On the other hand, intermittent communication is related to impulsive communication. When each communication time $T_k = s_k - t_k$ tends to zero, model (3) evolves into interconnected impulsive system [23]. For the interconnected impulsive system, it can be stabilized by choosing the appropriate pulse intensity and pulse time series (the relevant theories can be consulted in literature [23]).

Assume that system (3) satisfies the initial conditions: $x_i^0(t) = [x_{i1}^0(t), x_{i2}^0(t), \ldots, x_{in}^0(t)]^T \in C([t_0 - \tau, t_0], R^n)$. To discuss swarming behaviors, we define the set

$$\overline{x} = \sum_{i=1}^{N} \xi_i x_i(t), \tag{4}$$

as the average position for system (3), where $\xi = (\xi_1, \xi_2, \ldots, \xi_N)^T$ is the left eigenvector of A corresponding to eigenvalue zero and $\sum_{i=1}^{N} \xi_i = 1$.

The dynamical equation of $\overline{x}$ satisfies

$$\dot{\overline{x}} = \begin{cases} \displaystyle\sum_{i=1}^{N} \xi_i f(t, x_i(t), x_i(t-\tau)) + \sum_{i=1}^{N}\sum_{j=1}^{N} \xi_i a_{ij} g\left(x_i(t) - x_j(t)\right), & t \in [t_k, s_k), \\ \displaystyle\sum_{i=1}^{N} \xi_i f(t, x_i(t), x_i(t-\tau)), & t \in [s_k, t_{k+1}). \end{cases} \tag{5}$$

Define the error vectors $e_i(t) = x_i(t) - \overline{x}$, then the error dynamical system can be rewritten as

$$\dot{e}_i(t) = \begin{cases} \displaystyle\tilde{f}(t, e_i(t), e_i(t-\tau)) + \sum_{j=1, j\neq i}^{N} a_{ij} g\left(e_i(t) - e_j(t)\right) + J - \sum_{s=1}^{N}\sum_{j=1}^{N} \xi_s a_{sj} g\left(e_s(t) - e_j(t)\right), & t \in [t_k, s_k), \\ \tilde{f}(t, e_i(t), e_i(t-\tau)) + J, & t \in [s_k, t_{k+1}), \end{cases} \tag{6}$$

where $\tilde{f}(t, e_i(t), e_i(t-\tau)) = f(t, x_i(t), x_i(t-\tau)) - f(t, \overline{x}(t), \overline{x}(t-\tau))$, and $J = f(t, \overline{x}(t), \overline{x}(t-\tau)) - \sum_{s=1}^{N} \xi_s f(t, x_s(t), x_s(t-\tau))$.

For the purpose of the swarming behaviors of multiagent systems (3), we need the following assumptions and lemmas.

Assumption 3. The functions $f(\cdot,\cdot,\cdot)$ satisfy the Lipschitz condition. That is, there exist two constants $l_1, l_2 > 0$ such that

$$\left\| f(t, x, x_\tau) - f(t, y, y_\tau) \right\| \leq l_1 \|x - y\| + l_2 \|x_\tau - y_\tau\|, \quad \forall x, y, x_\tau, y_\tau \in R^n. \tag{7}$$

Assumption 4. For the aperiodically intermittent communication strategy, there exist two positive scalars $0 < \theta < \omega$, such that, for $i = 0, 1, 2, \ldots$

$$\inf_i (s_i - t_i) = \theta > 0, \qquad \sup_i (t_{i+1} - t_i) = \omega < +\infty. \tag{8}$$

Assumption 5. There exist constants $a, b > 0$, such that $g_a(\|y\|) = a$, $g_r(\|y\|) \leq b/\|y\|$ for any $y \in R^n$. That is, the attraction function is fixed linear and the repulsion function is bounded.

Lemma 6 (see [11]). *Let $x \in R^n$ and $y \in R^n$, then for any positive definite symmetric matrix $P \in R^{n\times n}$ such that*

$$\pm 2x^T y \leq x^T P x + y^T P^{-1} y \tag{9}$$

Lemma 7 (see [3]). *Given a matrix $W = [\omega_{ij}] \subset R^{M\times M}$, suppose that $\omega_{ij} \geq 0$ for $i \neq j$ and $\omega_{ii} = -\sum_{j=1, j\neq i}^{M} \omega_{ij} < 0$ for $i = 1, 2, \ldots, M$. If W is irreducible, then $W + W^T$ and $EW + W^T E$ are both irreducible, where $E = \mathrm{diag}(\xi_1, \xi_2, \ldots, \xi_M)$ and $\xi = (\xi_1, \xi_2, \ldots, \xi_M)^T$ are the left eigenvector of W corresponding to eigenvalue zero.*

Lemma 8 (see [25]). *Let $w : [\mu - \tau, +\infty) \longrightarrow [0, +\infty)$ be a continuous function such that*

$$\dot{w}(t) \leq -a w(t) + b \max w_t \tag{10}$$

is satisfied for $t \geq \mu$. *If* $a > b > 0$, *then*

$$w(t) \leq \left[\max w_\mu\right] e^{-\varepsilon(t-\mu)}, \quad t \geq \mu, \tag{11}$$

where $\max w_t = \sup_{t-\tau\leq s\leq t} w(s)$, *and* $\varepsilon > 0$ *is the smallest real root of the equation*

$$\varepsilon - a + be^{\varepsilon\tau} = 0. \tag{12}$$

Lemma 9 (see [18]). *Let* $w : [\mu - \tau, +\infty) \longrightarrow [0, +\infty)$ *be a continuous function such that*

$$\dot{w}(t) \leq aw(t) + b\max w_t \tag{13}$$

is satisfied for $t \geq \mu$. *If* $a > 0, b > 0$, *then*

$$w(t) \leq \max w_t \leq \left[\max w_\mu\right] e^{(a+b)(t-\mu)}, \quad t \geq \mu, \tag{14}$$

where $\max w_t = \sup_{t-\tau\leq s\leq t} w(s)$.

3. Main Result

In this section, we propose some criteria of swarming behaviors for multiagent systems with nonlinear dynamics and aperiodically intermittent communication.

Theorem 10. *Let Assumptions 3–5 hold,* $\tau \leq s_i - t_i$ *and* $\tau \leq t_{i+1} - s_i$. *If the following conditions are satisfied:*

(1) there exist positive constants α, λ, *such that* $l + a\lambda_2/2\widehat{\xi} + \alpha \leq 0$ *and* $\lambda - 2\alpha + 2\beta e^{\lambda\tau} = 0$ *hold, where* $l = \varepsilon/2 + l_1^2/2\varepsilon$, $\beta = l_2^2/2\varepsilon$, $\widehat{\xi} = \max\{\xi_i \mid i = 1, 2, \ldots, N\}$, $\xi = (\xi_1, \xi_2, \ldots, \xi_N)^T$ *is the left eigenvector corresponding to eigenvalues* 0 *of* A, *and* $\sum_{i=1}^{N} \xi_i = 1$;

(2) for any $k \in Z^+$, $\sum_{i=0}^{k}\{-\lambda(s_i - t_i) + \gamma(t_{i+1} - s_i)\} \leq -\eta(t_{k+1} - t_0)$, *where* $\gamma = 2(l + \beta e^{\lambda\tau})$ *and* $\eta > 0$;

then, all the agents of multiagent systems (3) will converge to a hyperball B_ϵ *centered at* $\overline{x}(t)$,

$$B_\epsilon = \left\{(x_1, \ldots, x_N) \mid \sum_{i=1}^{N} \xi_i \left\|x_i(t) - \overline{x}(t)\right\|^2 \leq \epsilon\right\}, \tag{15}$$

where $\epsilon = (bd_A)^2/(l + a\lambda_2/2\widehat{\xi} + \alpha)^2$. *Furthermore, all agents will move into the hyperball* B_ϵ *in a finite time specified by*

$$t = t_0 - \frac{1}{\eta}\ln\left(\frac{\epsilon}{\sup_{t_0-\tau\leq\theta\leq t_0} V(\theta)}\right). \tag{16}$$

Proof. Choose a nonnegative Lyapunov function as follows:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} \xi_i e_i^T(t)\, e_i(t)\,). \tag{17}$$

Then the derivative of $V(t)$ with respect to time t along the solutions of error system (6) can be calculated as follows.

When $t \in [t_k, s_k)$, $k = 0, 1, 2, \ldots$, we get

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{N} \xi_i e_i^T(t) \Bigg\{\widetilde{f}(t, e_i(t), e_i(t-\tau)) \\ &\quad + \sum_{j=1, j\neq i}^{N} a_{ij} g\left(e_i(t) - e_j(t)\right) + J \\ &\quad - \sum_{s=1}^{N}\sum_{j=1}^{N} \xi_s a_{sj} g\left(e_s(t) - e_j(t)\right)\Bigg\} = \sum_{i=1}^{N} \xi_i e_i^T(t) \\ &\quad \cdot \widetilde{f}(t, e_i(t), e_i(t-\tau)) + \sum_{i=1}^{N} \xi_i e_i^T(t)\, J \\ &\quad - \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} \xi_i a_{ij} e_i^T(t)\left(e_i(t) - e_j(t)\right) \\ &\quad \cdot \left[g_a\left(\left\|e_i(t) - e_j(t)\right\|\right) - g_r\left(\left\|e_i(t) - e_j(t)\right\|\right)\right] \\ &\quad + \sum_{i=1}^{N} \xi_i e_i^T(t) \left(\sum_{s=1}^{N}\sum_{j=1}^{N} \xi_s a_{sj}\left(e_s(t) - e_j(t)\right)\right. \\ &\quad \left.\cdot \left[g_a\left(\left\|e_s(t) - e_j(t)\right\|\right) - g_r\left(\left\|e_s(t) - e_j(t)\right\|\right)\right]\right).\end{aligned} \tag{18}$$

Note that $\sum_{i=1}^{N} \xi_i e_i^T(t) = 0$; we have the following:

$$\sum_{i=1}^{N} \xi_i e_i^T(t)\, J = 0, \tag{19}$$

and

$$\begin{aligned}&\sum_{i=1}^{N} \xi_i e_i^T(t) \left(\sum_{s=1}^{N}\sum_{j=1}^{N} \xi_s a_{sj}\left(e_s(t) - e_j(t)\right)\right. \\ &\quad \left.\cdot \left[g_a\left(\left\|e_s(t) - e_j(t)\right\|\right) - g_r\left(\left\|e_s(t) - e_j(t)\right\|\right)\right]\right) \\ &\quad = 0.\end{aligned} \tag{20}$$

By Assumption 3 and Lemma 6, one has

$$\begin{aligned}&\sum_{i=1}^{N} \xi_i e_i^T(t)\, \widetilde{f}(t, e_i(t), e_i(t-\tau)) \leq \sum_{i=1}^{N} \xi_i \left\{\frac{\varepsilon}{2} e_i^T(t)\, e_i(t)\right. \\ &\quad \left. + \frac{1}{2\varepsilon}\widetilde{f}^T(t, e_i(t), e_i(t-\tau))\, \widetilde{f}(t, e_i(t), e_i(t-\tau))\right\} \\ &\quad \leq \left(\frac{\varepsilon}{2} + \frac{l_1^2}{2\varepsilon}\right)\sum_{i=1}^{N} \xi_i e_i^T(t)\, e_i(t) + \frac{l_2^2}{2\varepsilon}\sum_{i=1}^{N} \xi_i e_i^T(t-\tau) \\ &\quad \cdot e_i(t-\tau)\,.\end{aligned} \tag{21}$$

According to Assumption 4 and $\sum_{j=1}^{N} a_{ij} = 0$, we can obtain

$$\begin{aligned} & -\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} \xi_i a_{ij} e_i^T(t)\left(e_i(t)-e_j(t)\right) \\ & \cdot g_a\left(\left\|e_i(t)-e_j(t)\right\|\right) \\ & = -a\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} \xi_i a_{ij}\left(e_i^T(t)e_i(t)-e_i^T(t)e_j(t)\right) \\ & = -a\sum_{i=1}^{N}\xi_i\left\{\left(\sum_{j=1,j\neq i}^{N} a_{ij}\right)e_i^T(t)e_i(t)\right. \\ & \left. -\sum_{j=1,j\neq i}^{N} a_{ij}e_i^T(t)e_j(t)\right\} = a\sum_{i=1}^{N}\sum_{j=1}^{N}\xi_i a_{ij}e_i^T(t)e_j(t). \end{aligned} \tag{22}$$

$$\begin{aligned} & \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} \xi_i a_{ij} e_i^T(t)\left(e_i(t)-e_j(t)\right) g_r\left(\left\|e_s(t)-e_j(t)\right\|\right) \\ & \leq -\sum_{i=1}^{N} b a_{ii}\xi_i\left\|e_i(t)\right\|. \end{aligned} \tag{23}$$

Substituting (19)-(23) into (18) yields

$$\begin{aligned} \dot{V}(t) \leq & \left(\frac{\varepsilon}{2}+\frac{l_1^2}{2\varepsilon}\right)\sum_{i=1}^{N}\xi_i e_i^T(t)e_i(t) \\ & + \frac{l_2^2}{2\varepsilon}\sum_{i=1}^{N}\xi_i e_i^T(t-\tau)e_i(t-\tau) \\ & + a\sum_{i=1}^{N}\sum_{j=1}^{N}\xi_i a_{ij}e_i^T(t)e_j(t) - \sum_{i=1}^{N} b a_{ii}\xi_i\left\|e_i(t)\right\|. \end{aligned} \tag{24}$$

Exchanging rows and columns, it is not difficult to verify that

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\xi_i a_{ij}e_i^T(t)e_j(t) = \frac{1}{2}\sum_{j=1}^{n}\tilde{e}_j^T(t)\left(EA+A^TE\right)\tilde{e}_j(t), \tag{25}$$

where $\tilde{e}_j(t) = [e_{1j}(t), e_{2j}(t), \ldots, e_{Nj}(t)]^T$, $E = \mathrm{diag}(\xi_1, \xi_2, \ldots, \xi_N)$.

Consider that the matrix A is irreducible, then $EA + A^TE$ is symmetric irreducible (see Lemma 7) and the sum of rows is equal to zero. There exists an unitary matrix $P = (p_1, p_2, \ldots, p_n)$ such that $P^T(EA + A^TE)P = \Lambda$, where $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$ and $0 = \lambda_1 > \lambda_2 \geq \cdots \geq \lambda_N$. Let $y_j = P^T\tilde{e}_j(t) = (y_{j1}, y_{j2}, \ldots, y_{jN})^T$. Since $p_1 = (1/\sqrt{N})[1, 1, \ldots, 1]^T$, one has $y_{j1} = (p_1)^T\tilde{e}_j(t) = (1/\sqrt{N})\sum_{i=1}^{n} e_{ij}(t) = 0$. Then,

$$\begin{aligned} \sum_{i=1}^{N}\sum_{j=1}^{N}\xi_i a_{ij}e_i^T(t)e_j(t) & \leq \frac{\lambda_2}{2}\sum_{j=1}^{n} y_j^T(t)y_j(t) \\ & = \frac{\lambda_2}{2}\sum_{j=1}^{n}\tilde{e}_j^T(t)\tilde{e}_j(t) \\ & = \frac{\lambda_2}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t). \end{aligned} \tag{26}$$

Let $\hat{\xi} = \max\{\xi_i \mid i = 1, 2, \ldots, N\}$, $d_A = \max\{-a_{ii} \mid i = 1, 2, \ldots, N\}$, $l = \varepsilon/2 + l_1^2/2\varepsilon$ and $\beta = l_2^2/2\varepsilon$. Note that

$$\begin{aligned} \sum_{i=1}^{N}\xi_i\left\|e_i(t)\right\| & = \sum_{i=1}^{N}\sqrt{\xi_i}\left\|\sqrt{\xi_i}e_i(t)\right\| \\ & \leq \left(\sum_{i=1}^{N}\xi_i\right)^{1/2}\left(\sum_{i=1}^{N}\left\|\sqrt{\xi_i}e_i(t)\right\|^2\right)^{1/2} \\ & = \sqrt{\sum_{i=1}^{N}\xi_i e_i^T(t)e_i(t)}. \end{aligned} \tag{27}$$

Then, taking (26) and (27) into (24), we have

$$\begin{aligned} \dot{V}(t) \leq & \left(l+\frac{a\lambda_2}{2\hat{\xi}}\right)\sum_{i=1}^{N}\xi_i e_i^T(t)e_i(t) \\ & + bd_A\sqrt{\sum_{i=1}^{N}\xi_i e_i^T(t)e_i(t)} + \beta\sum_{i=1}^{N}\xi_i e_i^T(t-\tau)e_i(t-\tau) \\ = & -\alpha\sum_{i=1}^{N}\xi_i e_i^T(t)e_i(t) + \left(1+\frac{a\lambda_2}{2\hat{\xi}}+\alpha\right) \\ & \cdot\sqrt{\sum_{i=1}^{N}\xi_i e_i^T(t)e_i(t)}\left(\sqrt{\sum_{i=1}^{N}\xi_i e_i^T(t)e_i(t)}\right. \\ & \left. + \frac{bd_A}{l+a\lambda_2/2\hat{\xi}+\alpha}\right) + \beta\sum_{i=1}^{N}\xi_i e_i^T(t-\tau)e_i(t-\tau). \end{aligned} \tag{28}$$

If $\sum_{i=1}^{N}\xi_i e_i^T(t)e_i(t) \geq (bd_A)^2/(l + a\lambda_2/2\hat{\xi} + \alpha)^2$, then one has

$$\dot{V}(t) \leq -2\alpha V(t) + 2\beta\sup_{t-\tau\leq s\leq t} V(s). \tag{29}$$

Based on Lemma 8 and condition (1) of Theorem 10, there exists $\lambda > 0$, such that $\lambda - 2\alpha + 2\beta e^{\lambda\tau} = 0$ and

$$V(t) \leq \sup_{t_k-\tau\leq s\leq t_k} V(s)e^{-\lambda(t-t_k)}. \tag{30}$$

Similarly, when $t \in [s_k, t_{k+1})$, we have

$$\begin{aligned} \dot{V}(t) & \leq l\sum_{i=1}^{N}\xi_i e_i^T(t)e_i(t) + \beta\sum_{i=1}^{N}\xi_i e_i^T(t-\tau)e_i(t-\tau) \\ & \leq 2lV(t) + 2\beta\sup_{t-\tau\leq s\leq t} V(s). \end{aligned} \tag{31}$$

Define $W(t) = e^{\lambda t}V(t)$, based on (30) and (31), one has

$$\dot{W}(t) \le \lambda W(t) + 2lW(t) + 2\beta e^{\lambda\tau} \sup_{t-\tau\le s\le t} W(s), \quad t \in [s_k, t_{k+1}). \tag{32}$$

According to Lemma 9 and (30), we have

$$\begin{aligned} W(t) &\le \sup_{s_k-\tau\le s\le s_k} W(s)\, e^{(\lambda+\gamma)(t-s_k)} \\ &= \sup_{s_k-\tau\le s\le s_k} V(s)\, e^{\lambda s} e^{(\lambda+\gamma)(t-s_k)} \\ &\le \sup_{s_k-\tau\le s\le s_k} \left\{ \sup_{t_k-\tau\le\theta\le t_k} V(\theta)\, e^{-\lambda(s-t_k)} e^{\lambda s} \right\} e^{(\lambda+\gamma)(t-s_k)} \\ &= \sup_{t_k-\tau\le\theta\le t_k} V(\theta)\, e^{\lambda t_k+(\lambda+\gamma)(t-s_k)}, \end{aligned} \tag{33}$$

where $\gamma = 2(l + \beta e^{\lambda\tau})$.

Hence,

$$V(t) \le \sup_{t_k-\tau\le s\le t_k} V(s)\, e^{-\lambda(s_k-t_k)+\gamma(t-s_k)}, \quad t \in [s_k, t_{k+1}). \tag{34}$$

By induction, we have

$$\begin{aligned} V(t) &\le \sup_{t_k-\tau\le s\le t_k} \left\{ \sup_{t_{k-1}-\tau\le\theta\le t_{k-1}} V(\theta)\, e^{-\lambda(s_{k-1}-t_{k-1})+\gamma(s-s_{k-1})} \right\} \\ &\quad \cdot e^{-\lambda(s_k-t_k)+\gamma(t_{k+1}-s_k)} \\ &\le \sup_{t_{k-1}-\tau\le\theta\le t_{k-1}} V(\theta)\, e^{-\lambda(s_{k-1}-t_{k-1}+s_k-t_k)+\gamma(t_k-s_{k-1}+t_{k+1}-s_k)} \\ &\le \cdots \le \sup_{t_0-\tau\le\theta\le t_0} V(\theta)\, e^{-\lambda\sum_{i=1}^{k}(s_i-t_i)+\gamma\sum_{i=1}^{k}(t_{i+1}-s_i)}. \end{aligned} \tag{35}$$

According to condition (2) in Theorem 10, we can get

$$V(t) \le \sup_{t_0-\tau\le\theta\le t_0} V(\theta)\, e^{-\eta(t-t_0)}, \quad \forall t \in [t_k, t_{k+1}). \tag{36}$$

This implies that the swarm will globally and exponentially converge to the hyperellipsoid B_ϵ. Furthermore, all agents will move into the hyperball B_ϵ in a finite time specified by

$$t \le t_0 - \frac{1}{\eta}\ln\left(\frac{\epsilon}{\sup_{t_0-\tau\le\theta\le t_0} V(\theta)}\right). \tag{37}$$

The proof is complete. □

Remark 11. In Theorem 10, we have given a sufficient condition to guarantee multiagent systems to achieve swarming behaviors under intermittent communication. It needs to be pointed out that under the premise that the guarantee condition 2 is established, communication or noncommunication is allowed in some communication period. It means that $-\lambda(s_i - t_i) + \gamma(t_{i+1} - s_i) = \gamma(t_{i+1} - t_i)$ or $-\lambda(s_i - t_i) + \gamma(t_{i+1} - s_i) = -\lambda(t_{i+1} - t_i)$ holds in some period. In addition, it can be seen from the proof of Theorem 10 that in the time period $[t_i, s_i)$, there is communication between agents which is beneficial to system stability. In the time period $[s_i, t_{i+1})$, there is no communication between agents, which is not beneficial (possibly harmful) to system stability. Therefore, in order to ensure the swarm, the communication time between agents should be as long as possible, but the noncommunication time should be as short as possible. On the other hand, condition (2) in Theorem 10 is easy to achieve when $-\lambda(s_i - t_i) + \gamma(t_{i+1} - s_i) \le -\eta(t_{k+1} - t_i)$ holds in all periods. For any $i = 0, 1, 2, \ldots$, if $s_i - t_i \equiv T_c$, $t_{i+1} - s_i \equiv T_{uc}$, then aperiodically intermittent communication becomes the case of periodically intermittent communication.

In the following, we will verify (35) for some special cases.

Corollary 12. *Let Assumptions 3–5 hold, $\tau \le s_i - t_i$ and $\tau \le t_{i+1} - s_i$. If the following conditions are satisfied:*

(1) there exist positive constants α, λ, such that $l + a\lambda_2/2\hat{\xi} + \alpha \le 0$ and $\lambda - 2\alpha + 2\beta e^{\lambda\tau} = 0$ hold, where $l = \varepsilon/2 + l_1^2/2\varepsilon$, $\beta = l_2^2/2\varepsilon$, $\hat{\xi} = \max\{\xi_i \mid i = 1, 2, \ldots, N\}$, $\xi = (\xi_1, \xi_2, \ldots, \xi_N)^T$ is the left eigenvector corresponding to eigenvalues 0 of A, and $\sum_{i=1}^{N}\xi_i = 1$;

(2) for any $k \in Z^+$, $-\lambda(s_k - t_k) + \gamma(t_{k+1} - s_k) \le -\eta(t_{k+1} - t_k)$, where $\gamma = 2(l + \beta e^{\lambda\tau})$ and $\eta > 0$;

then, all the agents of multiagent systems (3) will converge to a hyperball B_ϵ centered at $\overline{x}(t)$,

$$B_\epsilon = \left\{ (x_1, \ldots, x_N) \mid \sum_{i=1}^{N} \xi_i \left\| x_i(t) - \overline{x}(t) \right\|^2 \le \epsilon, \right\} \tag{38}$$

where $\epsilon = (bd_A)^2/(l + a\lambda_2/2\hat{\xi} + \alpha)^2$. Furthermore, all agents will move into the hyperball B_ϵ in a finite time specified by

$$t = t_0 - \frac{1}{\eta}\ln\left(\frac{\epsilon}{\sup_{t_0-\tau\le\theta\le t_0} V(\theta)}\right). \tag{39}$$

For any $i = 0, 1, 2, \ldots$, if $s_i - t_i \equiv T_c$, $t_{i+1} - s_i \equiv T_{uc}$, where T_c and T_{uc} are both positive constants. We have the following result.

Corollary 13. *Let Assumptions 3–5 hold, $\tau \le T_c$ and $\tau \le T_{uc}$. If the following conditions are satisfied:*

(1) there exist positive constants α, λ, such that $l + a\lambda_2/2\hat{\xi} + \alpha \le 0$ and $\lambda - 2\alpha + 2\beta e^{\lambda\tau} = 0$ hold, where $l = \varepsilon/2 + l_1^2/2\varepsilon$, $\beta = l_2^2/2\varepsilon$, $\hat{\xi} = \max\{\xi_i \mid i = 1, 2, \ldots, N\}$, $\xi = (\xi_1, \xi_2, \ldots, \xi_N)^T$ is the left eigenvector corresponding to eigenvalues 0 of A, and $\sum_{i=1}^{N}\xi_i = 1$;

(2) for any $k \in Z^+$, $-\lambda T_c + \gamma T_{uc} \le -\eta(T_u + T_{uc})$, where $\gamma = 2(l + \beta e^{\lambda\tau})$ and $\eta > 0$;

then, all the agents of multiagent systems (3) will converge to a hyperball B_ϵ centered at $\overline{x}(t)$,

$$B_\epsilon = \left\{(x_1,\ldots,x_N) \mid \sum_{i=1}^{N} \xi_i \left\|x_i(t) - \overline{x}(t)\right\|^2 \le \epsilon, \right\} \quad (40)$$

where $\epsilon = (bd_A)^2/(l + a\lambda_2/2\widehat{\xi} + \alpha)^2$. Furthermore, all agents will move into the hyperball B_ϵ in a finite time specified by

$$t = t_0 - \frac{1}{\eta}\ln\left(\frac{\epsilon}{\sup_{t_0-\tau\le\theta\le t_0} V(\theta)}\right). \quad (41)$$

Next, we use the special class of attraction and repulsion function to analyze the swarming behaviors of multiagents systems with aperiodically intermittent communication. It takes the form

$$g(y) = -y(a - b\exp\left(-\frac{\|y\|^2}{c}\right), \quad (42)$$

where $b > a > 0$ and $c > 0$ are constants. We can derive the following result.

Theorem 14. *Let Assumptions 3-4 hold, $\tau \le T_c$ and $\tau \le T_{uc}$. If the following conditions are satisfied:*

(1) there exist positive constants α, λ, such that $l + a\lambda_2/2\widehat{\xi} + \alpha \le 0$ and $\lambda - 2\alpha + 2\beta e^{\lambda\tau} = 0$ hold, where $l = \varepsilon/2 + l_1^2/2\varepsilon$, $\beta = l_2^2/2\varepsilon$, $\widehat{\xi} = \max\{\xi_i \mid i = 1,2,\ldots,N\}$, $\xi = (\xi_1,\xi_2,\ldots,\xi_N)^T$ is the left eigenvector corresponding to eigenvalues 0 of A, and $\sum_{i=1}^{N}\xi_i = 1$;

(2) for any $k \in Z^+$, $-\lambda T_c + \gamma T_{uc} \le -\eta(T_u + T_{uc})$, where $\gamma = 2(l + \beta e^{\lambda\tau})$ and $\eta > 0$;

then, all the agents of multiagents systems (3) will converge to a hyperball B_ϵ centered at $\overline{x}(t)$,

$$B_\epsilon = \left\{(x_1,\ldots,x_N) \mid \sum_{i=1}^{N} \xi_i \left\|x_i(t) - \overline{x}(t)\right\|^2 \le \epsilon, \right\} \quad (43)$$

where $\epsilon = (bd_A\sqrt{c})^2/2e(l + a\lambda_2/2\widehat{\xi} + \alpha)^2$. Furthermore, all agents will move into the hyperball B_ϵ in a finite time specified by

$$t = t_0 - \frac{1}{\eta}\ln\left(\frac{\epsilon}{\sup_{t_0-\tau\le\theta\le t_0} V(\theta)}\right). \quad (44)$$

Proof. It is easy to see that $g_a(\|y\|) = a$.

Define $F(x) = bxe^{-x^2/c}$, then

$$F'(x) = b\left(1 - \frac{2x^2}{c}\right)e^{-x^2/c} = \begin{cases} > 0, & x < \sqrt{\dfrac{c}{2}}, \\ < 0, & x > \sqrt{\dfrac{c}{2}}. \end{cases} \quad (45)$$

Hence,

$$F(x) \le F\left(\sqrt{\frac{c}{2}}\right) = \frac{b\sqrt{c}}{2}e^{-1/2}. \quad (46)$$

So,

$$\|y\|\, g_r(\|y\|) \le \frac{b\sqrt{c}}{2}e^{-1/2}. \quad (47)$$

Therefore, Assumption 5 holds, when $g(y)$ is the same as (42).

It is known from Corollary 13 that Theorem 14 is established.

The proof is complete. □

4. Numerical Simulation

In this section, we will use a numerical simulation to illustrate the feasibility and effectiveness of our results.

Consider the swarm model consisting of six agents, which is described as follows:

$$\dot{x}_i(t) = \begin{cases} f(t, x_i(t), x_i(t-\tau)) + \displaystyle\sum_{j=1, j\ne i}^{N} a_{ij}\, g\left(x_i(t) - x_j(t)\right), & t \in [t_k, s_k), \\ f(t, x_i(t), x_i(t-\tau)), & t \in [s_k, t_{k+1}), \end{cases} \quad (48)$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t)]^T$, $f(\cdot,\cdot,\cdot) = \tanh(x_i(t)) + 0.2\tanh(x_i(t-\tau))$, $g(x_i - x_j) = -(x_i - x_j)(a - be^{-\|x_i - x_j\|^2/c})$, and $\tau = 1$, $a = 1$, $b = 20$, $c = 0.2$, and the coupled matric is chosen as

$$A = \begin{bmatrix} -5 & 1 & 1 & 1 & 1 & 1 \\ 1 & -5 & 1 & 1 & 1 & 1 \\ 1 & 1 & -5 & 1 & 1 & 1 \\ 1 & 1 & 1 & -5 & 1 & 1 \\ 1 & 1 & 1 & 1 & -5 & 1 \\ 1 & 1 & 1 & 1 & 1 & -5 \end{bmatrix}. \quad (49)$$

The nonlinear function $f(\cdot,\cdot,\cdot)$ satisfies the Lipschitz condition with $l_1 = 1$, $l_2 = 0.2$. It is easy to know that $\lambda_2 = -5$ and $\widehat{\xi} = 1/6$, according to the coupled matrix A. Choose $\varepsilon = 1$ and $\alpha = 4$, then we get $\lambda = 4.4779$, $\gamma = 5.5221$. Let the ratio of communication time to noncommunication time be greater than 1.3376, then all conditions of Theorem 14 are satisfied, and $\eta > 0.2$. Let the initial states of the agents be chosen randomly from $[-10, 10]$. Moreover, the communication period $T = 3s$, and for the communication width, see Figure 1 ($y = 1$). The results of simulation are given in Figure 2. It is clear that under the aperiodically intermittent communication, all agents can achieve swarming behavior.

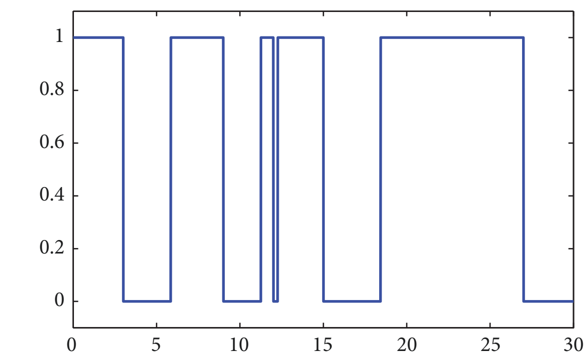

Figure 1: Communication ($y = 1$) and noncommunication ($y = 0$) time of the system (48).

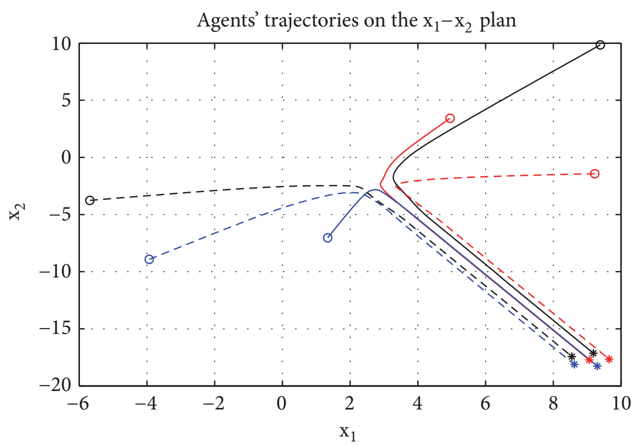

Figure 2: The trajectories of agents in x_1-x_2 plan.

5. Conclusions

In this paper, the swarm stability of nonlinear multiagent systems with aperiodically intermittent communication is studied. Each agent gets information from its neighbors in a series of aperiodically intervals. In addition, each agent has nonlinear dynamics and time delay. It is shown in this paper that under aperiodic discontinuous communication, all agents in a swarm can reach cohesion within a finite time, and the upper bounds of cohesion depend on the parameters of the swarm model, the second largest eigenvalue of the coupling matrix and communication time. Furthermore, an example is given to verify the correctness of the theoretical results.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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