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Research Article

Impulse Elimination and Fault-Tolerant Preview Controller Design for a Class of Descriptor Systems

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In this paper, a fault-tolerant preview controller is designed for a class of impulse controllable continuous time descriptor systems with sensor faults. Firstly, the impulse is eliminated by introducing state prefeedback; then an algebraic equation and a normal control system are obtained by restricted equivalent transformation for the descriptor system after impulse elimination. Next, the model following problem in fault-tolerant control is transformed into the optimal regulation problem of the augmented system which is constructed by a general method. And the final augmented system and its corresponding performance index function are obtained by state feedback for the augmented system constructed above. The controller with preview effect for the final augmented system is attained based on the existing conclusions of optimal preview control; then, the fault-tolerant preview controller for the original system is obtained through integral and backstepping. The relationships between the stabilisability and detectability of the final augmented system and the corresponding characteristics of the original descriptor system are also strictly discussed. The effectiveness of the proposed method is verified by numerical simulation.

1. Introduction

A descriptor system is also called a singular system, differential-algebraic system, semistate system, implicit system, etc. An important characteristic distinguishing a descriptor system from a normal system is that impulse behavior may exist in the system response. The emergence of impulse often causes the system to fail to function properly or damage, so we must find ways to eliminate impulse.

Cobb [1] first obtained the sufficient and necessary condition for the existence of state feedback to eliminate impulse in a regular descriptor system if the system is impulse controllable. In reference [2], the impulse was eliminated by state feedback, and a design method of state feedback gain matrix was proposed. Lovass-Nagy et al. [3] pointed out that it is feasible to eliminate impulse by using output feedback if and only if the system is impulse controllable and impulse observable, and the output feedback gain matrix was designed based on dynamic decomposition. Dai [4] and Chu and Cai [5] considered the problem of impulse elimination for descriptor linear systems based on P-D state feedback and P-D output feedback, respectively. The problem of robust impulse elimination for uncertain systems was considered in references [6, 7].

Preview control is a control method that utilises the future information of the desired signal or disturbance signal to improve the transient response of the system, suppress external disturbance, and improve the tracking performance of the system. After decades of development, preview control has basically formed a complete theoretical system [8–12], and some research results [13–16] have been published by combining with the theory of descriptor systems.

In reference [13], the concept of preview control for descriptor systems was proposed, and preliminary studies were carried out. Liao et al. [14] converted a class of linear continuous time descriptor impulse-free systems into a normal system and an algebraic equation through restricted equivalent transformation, and the controller was designed according to the conclusion of reference [10]. Cao and Liao [15] transformed a class of linear discrete time descriptor systems with state delay into delay-free descriptor systems by using discrete lifting

technique, and the design method of optimal preview controller was given based on reference [13]. The preview control of descriptor systems was extended to multiagent generalized systems, and a new simulation idea was presented in reference [16].

In the actual control system, when the actuator, sensor, or other original components of the system fails, the traditional feedback controller may lead to unsatisfactory performance or even lose stability, so the research of fault-tolerant control has been widely valued. Model following control is an important theoretical method in fault-tolerant control and does not need fault diagnosis unit. Its main idea is to introduce a reference model in which the desired signal is the output of the reference model and the input of the reference model (called the reference input) is often known and then design the controller of the fault system to realize the output tracking for the reference model. In recent years, there have been some achievements in fault-tolerant control of descriptor systems [17–21], but most of them were based on robust fault-tolerant control. At present, the research results of model following control for descriptor systems are relatively few.

Although the preview control theory and the fault-tolerant control theory have made many achievements, it needs to be pointed out that there is seldom a result about the combination of the preview control and the fault-tolerant control. In order to expand the research direction of preview control theory, we consider applying it to fault-tolerant control, especially when the fault information is known to respond for the system in advance, eliminate the impact of the fault signal on the system as soon as possible, and realize the tracking of the reference output.

The paper is organized as follows: The problem formulation is presented in Section 2. Section 3 introduces the impulse elimination and the restricted equivalent transformation. Section 4 details the construction of augmented system. Section 5 shows the conditions for the existence of the controller. In Section 6, two simulation examples are given to confirm the validity of the theoretical result, and Section 7 provides the conclusion.

2. Problem Formulation

Consider a class of continuous time descriptor systems with impulse modes

$$\begin{cases} E\dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t) + Df_s(t), \end{cases} \tag{1}$$

where $x(t)$ is the state vector of n dimensions, $u(t)$ is the input vector of r dimensions, $y(t)$ is the output vector of m dimensions, and $f_s(t)$ is the known sensor fault signal of h dimensions. E, A, B, C, D are known constant matrices with appropriate dimensions, respectively. $E$is a singular matrix and satisfies $\operatorname{rank}(E) = q < n$.

The fault-free reference model is described by

$$\begin{cases} \dot{x}_m(t) = A_m x_m(t) + B_m u_m(t), \\ y_m(t) = C_m x_m(t), \end{cases} \tag{2}$$

where $x_m(t)$ is the state vector of n_m dimensions, $u_m(t)$ is the known bounded input vector of r_m dimensions, $y_m(t)$ is the output vector of m dimensions. A_m, B_m, C_m are known constant matrices with appropriate dimensions, respectively. The reference model describes the ideal dynamic performance of the closed-loop system, in which the output vector of the reference model corresponds to the desired signal in the traditional control theory. Notice that the dimensions of $x_m(t)$ and $x(t)$ can be different and the dimensions of $u_m(t)$ and $u(t)$ can also be different.

The tracking error signal is defined as the following:

$$e(t) = y(t) - y_m(t). \tag{3}$$

The objective of this paper is to design a fault-tolerant controller with preview function for system (1) to eliminate the influence of fault signal $f_s(t)$, so that the output $y(t)$ of system (1) can track the output $y_m(t)$ of reference model (2) asymptotically, namely,

$$\lim_{t\longrightarrow\infty} e(t) = \lim_{t\longrightarrow\infty} \left[y(t) - y_m(t)\right] = 0. \tag{4}$$

For this purpose, we construct the quadratic performance index function

$$J = \int_0^\infty \left[e^T(t)Q_e e(t) + \dot{x}^T(t)Q_x\dot{x}(t) + \dot{u}^T(t)R\dot{u}(t)\right]\mathrm{d}t, \tag{5}$$

where Q_e, Q_x, and R are symmetric positive definite matrices.

Remark 1. It is noted that bringing $\dot{u}(t)$ into the performance index function can make the closed-loop system contain an integrator, which is helpful to eliminate static error [10]; introducing $\dot{x}(t)$ can shorten the time required for the closed-loop system to stabilise. The weight matrices in (5) are mutually restricted and need to be selected according to the actual needs. If it is necessary to improve the response speed of the system, the corresponding elements in Q_e can be increased or the corresponding elements in Q_x can be reduced. However, if the amount of change is too large, the number of system oscillations will be increased, the adjustment time will be prolonged, or even the closed-loop system will diverge. If it is necessary to effectively suppress the overshoot and the energy consumption caused, the corresponding elements in R can be increased, but this will make the system respond slower.

The following assumptions are further made for systems (1) and (2).

A1. Suppose (E, A) is regular, (E, A, B) is impulse controllable.

Remark 2. Impulse controllability is the important condition for eliminating impulse in descriptor systems. According to reference [22], we know the following rank criterions:

(i) (E, A) is impulse-free if and only if

$$\operatorname{rank}\begin{bmatrix} E & 0 \\ A & E \end{bmatrix} = n + \operatorname{rank}(E). \tag{6}$$

(ii) (E, A, B) is impulse controllable if and only if

$$\operatorname{rank}\begin{bmatrix} E & 0 & 0 \\ A & E & B \end{bmatrix} = n + \operatorname{rank}(E). \tag{7}$$

A2. Suppose (E, A, B) is stabilisable, and the matrix $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$ is of full row rank.

A3. Suppose the coefficient matrix A_m of system (2) is stable, namely, the eigenvalues of the A_m have a negative real part [23].

A4. Suppose the reference input signal $u_m(t)$ is a piecewise continuously differentiable function satisfying

$$\begin{aligned} &\lim_{t\longrightarrow\infty} u_m(t) = \overline{u}_m, \\ &\lim_{t\longrightarrow\infty} \dot{u}_m(t) = 0, \end{aligned} \tag{8}$$

where $\overline{u}_m$ is a constant vector. Moreover, $u_m(\tau)(t \le \tau \le t + l_r)$ is previewable at any moment t, l_r is the preview length of $u_m(t)$.

A5. Suppose the fault signal $f_s(t)$ is a piecewise continuously differentiable function satisfying

$$\begin{aligned} &\lim_{t\longrightarrow\infty} f_s(t) = \overline{f}_s, \\ &\lim_{t\longrightarrow\infty} \dot{f}_s(t) = 0, \end{aligned} \tag{9}$$

where $\overline{f}_s$ is a constant vector. Moreover, $f_s(\tau)(t \le \tau \le t + l_f)$ is previewable at any moment t and l_f is the preview length of $f_s(t)$.

Remark 3. A2, A4, and A5 are the basic assumptions in the preview control theory [10, 23, 24]. A2 is one of the conditions to ensure the existence of the controller; A4 and A5 are the assumptions of previewable signals, a situation in which the signal is unpreviewable if the preview length is zero.

Remark 4. Consider the rolling system (as an automatic control system). When the sensor detects a fault (for example, the distance between rolls deviates from the normal set value) on a roll ahead, the accumulation of the billet in the fault position can be prevented by adjusting the conveying speed of the billet in rolling [8, 25]. This can be regarded as a control system with previewable fault information, in which the fault information ahead detected by the sensor is the previewable fault information.

3. Impulse Elimination and Restricted Equivalent Transformation

Firstly, the impulse is eliminated by introducing state prefeedback based on the impulse controllability of the original system, and then the restricted equivalent transformation of the descriptor system after eliminating impulse is carried out to obtain an algebraic equation and a normal control system.

According to Theorem 6.2.1 in reference [22], a state prefeedback can be found to make the closed-loop system (1) under its action impulse-free when A1 holds. Let such state prefeedback as

$$u(t) = Kx(t) + v(t), \tag{10}$$

where $v(t) \in R^r$ is the auxiliary input signal, $K \in R^{r\times n}$ is the state prefeedback gain matrix.

Under the action of (10), system (1) becomes

$$\begin{cases} E\dot{x}(t) = (A + BK)x(t) + Bv(t), \\ y(t) = Cx(t) + Df_s(t), \end{cases} \tag{11}$$

and system (11) is impulse-free [22]. Obviously, the input $u(t)$ in (1) can be derived if only the input $v(t)$ in (11) is obtained.

Since system (11) is impulse-free and $\operatorname{rank}(E) = q$, there exist nonsingular matrices Q_1 and P_1, such that [16]

$$\begin{aligned} Q_1EP_1 &= \begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix}, \\ Q_1(A + BK)P_1 &= \begin{bmatrix} \overline{A}_1 & 0 \\ 0 & I_{n-q} \end{bmatrix}. \end{aligned} \tag{12}$$

Making nonsingular linear transform to system (11) by using matrix P_1, that is,

$$x(t) = P_1\overline{x}(t), \tag{13}$$

we get

$$\begin{cases} EP_1\dot{\overline{x}}(t) = (A + BK)P_1\overline{x}(t) + Bv(t), \\ y(t) = CP_1\overline{x}(t) + Df_s(t), \end{cases} \tag{14}$$

taking the left transformation matrix Q_1 over both sides of system (14), we can obtain

$$\begin{cases} \dot{\overline{x}}_1(t) = \overline{A}_1\overline{x}_1(t) + \overline{B}_1v(t), \\ 0 = \overline{x}_2(t) + \overline{B}_2v(t), \\ y(t) = \overline{C}_1\overline{x}_1(t) + \overline{C}_2\overline{x}_2(t) + Df_s(t), \end{cases} \tag{15}$$

which is the restricted equivalence type of system (11), where $\overline{x}(t) = \begin{bmatrix} \overline{x}_1(t) \\ \overline{x}_2(t) \end{bmatrix}$, $\overline{x}_1(t) \in R^q$, $\overline{x}_2(t) \in R^{n-q}$, $Q_1B = \begin{bmatrix} \overline{B}_1 \\ \overline{B}_2 \end{bmatrix}$, $CP_1 = \begin{bmatrix} \overline{C}_1 & \overline{C}_2 \end{bmatrix}$.

Solve $\overline{x}_2(t)$ from the second form in (15) and substitute it into the third form in (15) to get a normal control system:

$$\begin{cases} \dot{\overline{x}}_1(t) = \overline{A}_1\overline{x}_1(t) + \overline{B}_1v(t), \\ y(t) = \overline{C}_1\overline{x}_1(t) - \overline{C}_2\overline{B}_2v(t) + Df_s(t), \end{cases} \tag{16}$$

and the next step is to study system (16).

Remark 5. The restricted equivalent transformation does not change the dynamic characteristics of the system, including regularity, stabilisability, detectability, impulsiveness, etc. Without losing generality, we can study system (11) through system (16).

4. Construction of Augmented System

According to the method of preview control theory, an augmented system including the output error, the state equation of the normal control system, and the state equation of the reference model can be obtained by deriving both sides of formula (3), the state equation of system (16) and (2), respectively, in addition, taking $e(t)$ as the output vector [10, 23],

$$\begin{cases} \dot{X}(t) = \widehat{A}X(t) + \widehat{B}\dot{v}(t) + \widehat{B}_m\dot{u}_m(t) + \widehat{D}\dot{f}_s(t), \\ e(t) = \widehat{C}X(t), \end{cases} \tag{17}$$

where

$$\begin{aligned} X(t) &= \begin{bmatrix} e(t) \\ \dot{\bar{x}}_1(t) \\ \dot{x}_m(t) \end{bmatrix}, \\ \widehat{A} &= \begin{bmatrix} 0 & \overline{C}_1 & -C_m \\ 0 & \overline{A}_1 & 0 \\ 0 & 0 & A_m \end{bmatrix}, \\ \widehat{B} &= \begin{bmatrix} -\overline{C}_2\overline{B}_2 \\ \overline{B}_1 \\ 0 \end{bmatrix}, \\ \widehat{B}_m &= \begin{bmatrix} 0 \\ 0 \\ B_m \end{bmatrix}, \\ \widehat{D} &= \begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix}, \\ \widehat{C} &= \begin{bmatrix} I & 0 & 0 \end{bmatrix}. \end{aligned} \tag{18}$$

Remark 6. The output of system (1) is $y(t)$, and the output of system (2) is $y_m(t)$; then, we can get $e(t) = y(t) - y_m(t)$ at the current time t. Therefore, it is reasonable to use the output error $e(t)$ as the output of the augmented system.

Next, the state vector and input vector of the augmented system (17) are used to represent the performance index function (5).

In the light of formula (10), namely,

$$\begin{bmatrix} \dot{x}(t) \\ \dot{u}(t) \end{bmatrix} = \begin{bmatrix} \dot{x}(t) \\ K\dot{x}(t) + \dot{v}(t) \end{bmatrix} = \begin{bmatrix} I_n & 0 \\ K & I_r \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix}, \tag{19}$$

and from the above process of restricted equivalent transformation we can get

$$\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} P_1 & 0 \\ 0 & I_r \end{bmatrix} \begin{bmatrix} \dot{\bar{x}}(t) \\ \dot{v}(t) \end{bmatrix} = \left[\begin{array}{c|c} P_1 & 0 \\ 0 & I_r \end{array}\right] \left[\begin{array}{c} \dot{\bar{x}}_1(t) \\ -\overline{B}_2\dot{v}(t) \\ \hline \dot{v}(t) \end{array}\right] = \begin{bmatrix} P_1 & 0 \\ 0 & I_r \end{bmatrix} \left[\begin{array}{c|c} I_q & 0 \\ 0 & -\overline{B}_2 \\ 0 & I_r \end{array}\right] \left[\begin{array}{c} \dot{\bar{x}}_1(t) \\ \hline \dot{v}(t) \end{array}\right]. \tag{20}$$

Combining (19) and (20) we can obtain

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{u}(t) \end{bmatrix} &= \begin{bmatrix} I_n & 0 \\ K & I_r \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} I_n & 0 \\ K & I_r \end{bmatrix} \begin{bmatrix} P_1 & 0 \\ 0 & I_r \end{bmatrix} \left[\begin{array}{c|c} I_q & 0 \\ 0 & -\overline{B}_2 \\ 0 & I_r \end{array}\right] \begin{bmatrix} \dot{\bar{x}}_1(t) \\ \dot{v}(t) \end{bmatrix} \\ &= \begin{bmatrix} P_1 & 0 \\ KP_1 & I_r \end{bmatrix} \left[\begin{array}{c|c} I_q & 0 \\ 0 & -\overline{B}_2 \\ 0 & I_r \end{array}\right] \begin{bmatrix} \dot{\bar{x}}_1(t) \\ \dot{v}(t) \end{bmatrix}. \end{aligned} \tag{21}$$

Substituting (21) into the performance index function (5),

$$\begin{aligned} J &= \int_0^\infty \left[e^T(t)Q_e e(t) + \dot{x}^T(t)Q_x\dot{x}(t) + \dot{u}^T(t)R\dot{u}(t)\right]\mathrm{d}t \\ &= \int_0^\infty \left[e^T(t)Q_e e(t) + \begin{bmatrix}\dot{x}(t)\\ \dot{u}(t)\end{bmatrix}^T \begin{bmatrix}Q_x & 0\\ 0 & R\end{bmatrix}\begin{bmatrix}\dot{x}(t)\\ \dot{u}(t)\end{bmatrix}\right]\mathrm{d}t \\ &= \int_0^\infty \left[e^T(t)Q_e e(t)\right]\mathrm{d}t \\ &\quad + \int_0^\infty \left(\begin{bmatrix}\dot{\bar{x}}_1(t)\\ \dot{v}(t)\end{bmatrix}^T \begin{bmatrix}I_q & 0\\ 0 & -\overline{B}_2\\ 0 & I_r\end{bmatrix}^T \begin{bmatrix}P_1 & 0\\ KP_1 & I_r\end{bmatrix}^T \begin{bmatrix}Q_x & 0\\ 0 & R\end{bmatrix}\right. \\ &\quad \left. \cdot \begin{bmatrix}P_1 & 0\\ KP_1 & I_r\end{bmatrix}\begin{bmatrix}I_q & 0\\ 0 & -\overline{B}_2\\ 0 & I_r\end{bmatrix}\begin{bmatrix}\dot{\bar{x}}_1(t)\\ \dot{v}(t)\end{bmatrix}\right)\mathrm{d}t. \end{aligned} \tag{22}$$

Defining

$$\begin{bmatrix}\widetilde{Q} & \Phi\\ \Phi^T & \widetilde{R}\end{bmatrix} = \begin{bmatrix}I_q & 0\\ 0 & -\overline{B}_2\\ 0 & I_r\end{bmatrix}^T \begin{bmatrix}P_1 & 0\\ KP_1 & I_r\end{bmatrix}^T \begin{bmatrix}Q_x & 0\\ 0 & R\end{bmatrix} \cdot \begin{bmatrix}P_1 & 0\\ KP_1 & I_r\end{bmatrix}\begin{bmatrix}I_q & 0\\ 0 & -\overline{B}_2\\ 0 & I_r\end{bmatrix}, \tag{23}$$

because $\begin{bmatrix}P_1 & 0\\ KP_1 & I_r\end{bmatrix}$ is nonsingular and $\begin{bmatrix}I_q & 0\\ 0 & -\overline{B}_2\\ 0 & I_r\end{bmatrix}$ is of full column rank, so $\begin{bmatrix}\widetilde{Q} & \Phi\\ \Phi^T & \widetilde{R}\end{bmatrix}$ is symmetric positive definite, then, $\widetilde{Q}$ and $\widetilde{R}$ are symmetric positive definite [26].

Modifying (22) further yields

$$\begin{aligned} J &= \int_0^\infty \left[e^T(t)Q_e e(t) + \begin{bmatrix}\dot{\bar{x}}_1(t)\\ \dot{v}(t)\end{bmatrix}^T \begin{bmatrix}\widetilde{Q} & \Phi\\ \Phi^T & \widetilde{R}\end{bmatrix}\begin{bmatrix}\dot{\bar{x}}_1(t)\\ \dot{v}(t)\end{bmatrix}\right]\mathrm{d}t \\ &= \int_0^\infty \left[e^T(t)Q_e e(t) + \left\{\begin{bmatrix}\dot{\bar{x}}_1(t)\\ \dot{v}(t)\end{bmatrix}^T \begin{bmatrix}I & 0\\ \widetilde{R}^{-1}\Phi^T & I\end{bmatrix}^T\right\}\begin{bmatrix}\widetilde{Q}-\Phi\widetilde{R}^{-1}\Phi^T & 0\\ 0 & \widetilde{R}\end{bmatrix}\left\{\begin{bmatrix}I & 0\\ \widetilde{R}^{-1}\Phi^T & I\end{bmatrix}\begin{bmatrix}\dot{\bar{x}}_1(t)\\ \dot{v}(t)\end{bmatrix}\right\}\right]\mathrm{d}t \\ &= \int_0^\infty \left[e^T(t)Q_e e(t) + \begin{bmatrix}\dot{\bar{x}}_1(t)\\ \dot{v}(t)+\widetilde{R}^{-1}\Phi^T\dot{\bar{x}}_1(t)\end{bmatrix}^T \begin{bmatrix}\widetilde{Q}-\Phi\widetilde{R}^{-1}\Phi^T & 0\\ 0 & \widetilde{R}\end{bmatrix}\begin{bmatrix}\dot{\bar{x}}_1(t)\\ \dot{v}(t)+\widetilde{R}^{-1}\Phi^T\dot{\bar{x}}_1(t)\end{bmatrix}\right]\mathrm{d}t \\ &= \int_0^\infty \left[e^T(t)Q_e e(t) + \dot{\bar{x}}_1^T(t)\left(\widetilde{Q}-\Phi\widetilde{R}^{-1}\Phi^T\right)\dot{\bar{x}}_1(t) + \left(\dot{v}(t)+\widetilde{R}^{-1}\Phi^T\dot{\bar{x}}_1(t)\right)^T\widetilde{R}\left(\dot{v}(t)+\widetilde{R}^{-1}\Phi^T\dot{\bar{x}}_1(t)\right)\right]\mathrm{d}t \\ &= \int_0^\infty \left\{X^T(t)\begin{bmatrix}Q_e & & \\ & \widetilde{Q}-\Phi\widetilde{R}^{-1}\Phi^T & \\ & & 0\end{bmatrix}X(t) + \left(\dot{v}(t)+\widetilde{R}^{-1}\Phi^T\dot{\bar{x}}_1(t)\right)^T\widetilde{R}\left(\dot{v}(t)+\widetilde{R}^{-1}\Phi^T\dot{\bar{x}}_1(t)\right)\right\}\mathrm{d}t \\ &= \int_0^\infty \left[X^T(t)\widehat{Q}X(t) + \dot{w}^T(t)\widetilde{R}\dot{w}(t)\right]\mathrm{d}t, \end{aligned} \tag{24}$$

where

$$\widehat{Q} = \begin{bmatrix}Q_e & & \\ & \widetilde{Q}-\Phi\widetilde{R}^{-1}\Phi^T & \\ & & 0\end{bmatrix}, \qquad \dot{w}(t) = \dot{v}(t) + \widetilde{R}^{-1}\Phi^T\dot{\bar{x}}_1(t). \tag{25}$$

Remark 7. Due to the matrix $\begin{bmatrix}\widetilde{Q} & \Phi\\ \Phi^T & \widetilde{R}\end{bmatrix}$ is symmetric positive definite and the congruent transformation does not change the positive definiteness of matrices, we can know that $\widetilde{Q}-\Phi\widetilde{R}^{-1}\Phi^T$ is positive definite and $\widehat{Q}$ is semipositive definite.

Formula (25) gives $\dot{v}(t) = \dot{w}(t) - \widetilde{R}^{-1}\Phi^T\dot{\bar{x}}_1(t)$, and then substituting it into system (17) gives

$$\begin{cases} \dot{X}(t) &= \widehat{\widehat{A}}X(t) + \widehat{B}\dot{w}(t) + \widehat{B}_m\dot{u}_m(t) + \widehat{D}\dot{f}_s(t), \\ e(t) &= \widehat{C}X(t), \end{cases} \tag{26}$$

where

$$\widehat{\widehat{A}} = \begin{bmatrix}0 & \overline{C}_1 + \overline{C}_2\overline{B}_2\widetilde{R}^{-1}\Phi^T & -C_m\\ 0 & \overline{A}_1 - \overline{B}_1\widetilde{R}^{-1}\Phi^T & 0\\ 0 & 0 & A_m\end{bmatrix}. \tag{27}$$

At this point, the problem is turned into an optimal regulation problem for system (26) and the performance index function (23). Similar to reference [10], we can obtain the following theorem.

Theorem 1. *If A4 and A5 hold, then when $(\widehat{\widehat{A}}, \widehat{B})$ is stabilisable and $(\widehat{Q}^{1/2}, \widehat{\widehat{A}})$ is detectable, the optimal preview control*

input of the system (26) which minimizes the performance index function (23) can be expressed as

$$\dot{w}(t) = -\widetilde{R}^{-1}\widehat{B}^T PX(t) - \widetilde{R}^{-1}\widehat{B}^T g(t), \tag{28}$$

where P is a semipositive definite matrix satisfying the algebraic Riccati equation

$$\widehat{\widehat{A}}^T P + P\widehat{\widehat{A}} - P\widehat{B}\widetilde{R}^{-1}\widehat{B}^T P + \widehat{Q} = 0, \tag{29}$$

$$\begin{aligned} g(t) = &\int_0^{l_f}\left[\exp\left(\sigma\widehat{A}_c^T\right)P\widehat{D}\dot{f}_s(t+\sigma)\right]d\sigma \\ &+\int_0^{l_r}\left[\exp\left(\sigma\widehat{A}_c^T\right)P\widehat{B}_m\dot{u}_m(t+\sigma)\right]d\sigma, \end{aligned} \tag{30}$$

and the matrix in the following form

$$\widehat{A}_c = \widehat{\widehat{A}} - \widehat{B}\widetilde{R}^{-1}\widehat{B}^T P, \tag{31}$$

is stable.

Owing to that the system (26) and the performance index function (23) have the same form with the corresponding parts in reference [10], the derivation is completely similar. It is omitted here.

5. Conditions for the Existence of the Controller

Theorem 1 requires $(\widehat{\widehat{A}}, \widehat{B})$ is stabilisable and $(\widehat{Q}^{1/2}, \widehat{\widehat{A}})$ is detectable; next, we discuss the circumstances which the original system (1) needs to satisfy so that the above conditions are established.

Lemma 1. *Under the assumption of A3, $(\widehat{\widehat{A}}, \widehat{B})$ is stabilisable if and only if $(\overline{A}_1 - \overline{B}_1\widetilde{R}^{-1}\Phi^T, \overline{B}_1)$ is stabilisable and the matrix $\begin{bmatrix} \overline{A}_1 & \overline{B}_1 \\ \overline{C}_1 & -\overline{C}_2\overline{B}_2 \end{bmatrix}$ is of full row rank.*

Proof. On the basis of the PBH criterion [27], $(\widehat{\widehat{A}}, \widehat{B})$ is stabilisable if and only if for any complex number s satisfying $\mathrm{Re}(s) \geq 0$, the matrix

$$U_c = \begin{bmatrix} sI_{m+q+n_m} - \hat{\hat{A}} & \hat{B} \end{bmatrix} = \left[\begin{array}{ccc|c} sI_m & -(\overline{C}_1 + \overline{C}_2\overline{B}_2\widetilde{R}^{-1}\Phi^T) & C_m & -\overline{C}_2\overline{B}_2 \\ 0 & sI_q - (\overline{A}_1 - \overline{B}_1\widetilde{R}^{-1}\Phi^T) & 0 & \overline{B}_1 \\ 0 & 0 & sI_{n_m} - A_m & 0 \end{array}\right], \tag{32}$$

is of full row rank. The matrix $sI - A_m$ is nonsingular when $\mathrm{Re}(s) \geq 0$ is known from the stability of A_m established by A3. Therefore, based on the structure of U_c, we can see that U_c is of full row rank if and only if

$$\Psi = \begin{bmatrix} sI_m & -\left(\overline{C}_1 + \overline{C}_2\overline{B}_2\widetilde{R}^{-1}\Phi^T\right) & -\overline{C}_2\overline{B}_2 \\ 0 & sI_q - \left(\overline{A}_1 - \overline{B}_1\widetilde{R}^{-1}\Phi^T\right) & \overline{B}_1 \end{bmatrix}, \tag{33}$$

is of full row rank. Two cases of the matrix Ψ are discussed.

(i) When $s = 0$,

$$\begin{aligned} \mathrm{rank}(\Psi) &= \mathrm{rank}\begin{bmatrix} \overline{C}_1 + \overline{C}_2\overline{B}_2\widetilde{R}^{-1}\Phi^T & \overline{C}_2\overline{B}_2 \\ \overline{A}_1 - \overline{B}_1\widetilde{R}^{-1}\Phi^T & -\overline{B}_1 \end{bmatrix} \\ &= \mathrm{rank}\begin{bmatrix} 0 & I_m \\ I_q & 0 \end{bmatrix}\begin{bmatrix} \overline{A}_1 & \overline{B}_1 \\ \overline{C}_1 & -\overline{C}_2\overline{B}_2 \end{bmatrix}\begin{bmatrix} I_q & 0 \\ -\widetilde{R}^{-1}\Phi^T & -I_r \end{bmatrix} \\ &= \mathrm{rank}\begin{bmatrix} \overline{A}_1 & \overline{B}_1 \\ \overline{C}_1 & -\overline{C}_2\overline{B}_2 \end{bmatrix}. \end{aligned} \tag{34}$$

(ii) When $\mathrm{Re}(s) \geq 0$ and $s \neq 0$,

$$\mathrm{rank}(\Psi) = m + \mathrm{rank}\left[\, sI_q - \left(\overline{A}_1 - \overline{B}_1\widetilde{R}^{-1}\Phi^T\right) \;\; \overline{B}_1 \,\right]. \tag{35}$$

Above all, if $(\overline{A}_1 - \overline{B}_1\widetilde{R}^{-1}\Phi^T, \overline{B}_1)$ is stabilisable and the matrix $\begin{bmatrix} \overline{A}_1 & \overline{B}_1 \\ \overline{C}_1 & -\overline{C}_2\overline{B}_2 \end{bmatrix}$ is of full row rank, then the matrix Ψ is of full row rank for any complex number s satisfying $\mathrm{Re}(s) \geq 0$. Conversely, if the matrix Ψ is of full row rank for any complex number s satisfying $\mathrm{Re}(s) \geq 0$, then the matrixes in (i) and (ii) are of full row rank, and then $\left[\, sI_q - (\overline{A}_1 - \overline{B}_1\widetilde{R}^{-1}\Phi^T) \;\; \overline{B}_1 \,\right]\Big|_{s=0} = \left[\, -(\overline{A}_1 - \overline{B}_1\widetilde{R}^{-1}\Phi^T) \;\; \overline{B}_1 \,\right]$ is of full row rank from the case (ii); thus, we can get the conclusion that $(\overline{A}_1 - \overline{B}_1\widetilde{R}^{-1}\Phi^T, \overline{B}_1)$ is stabilisable by combining it with the case (ii). This accomplishes the proof of the Lemma 1. □

Lemma 2. *$(\overline{A}_1 - \overline{B}_1\widetilde{R}^{-1}\Phi^T, \overline{B}_1)$ is stabilisable if and only if (E, A, B) is stabilisable.*

Proof. According to that the state feedback does not change the stability of the control system [22] and Lemma 2 in [17], this lemma holds. □

Lemma 3. *The matrix $\begin{bmatrix} \overline{A}_1 & \overline{B}_1 \\ \overline{C}_1 & -\overline{C}_2\overline{B}_2 \end{bmatrix}$ is of full row rank if and only if the matrix $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$ is of full row rank.*

Proof. Due to

$$\begin{bmatrix} Q_1 & 0 \\ 0 & I_m \end{bmatrix}\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}\begin{bmatrix} I_n & 0 \\ K & I_r \end{bmatrix}\begin{bmatrix} P_1 & 0 \\ 0 & I_r \end{bmatrix} = \begin{bmatrix} \overline{A}_1 & 0 & \overline{B}_1 \\ 0 & I_{n-q} & \overline{B}_2 \\ \overline{C}_1 & \overline{C}_2 & 0 \end{bmatrix}, \tag{36}$$

in which the matrices $\begin{bmatrix} Q_1 & 0 \\ 0 & I_m \end{bmatrix}$, $\begin{bmatrix} I_n & 0 \\ K & I_r \end{bmatrix}$, $\begin{bmatrix} P_1 & 0 \\ 0 & I_r \end{bmatrix}$ are nonsingular; hence,

$$\operatorname{rank}\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = \operatorname{rank}\begin{bmatrix} \overline{A}_1 & 0 & \overline{B}_1 \\ 0 & I_{n-q} & \overline{B}_2 \\ \overline{C}_1 & \overline{C}_2 & 0 \end{bmatrix}, \tag{37}$$

so then $\operatorname{rank}\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = \operatorname{rank}\begin{bmatrix} \overline{A}_1 & \overline{B}_1 \\ \overline{C}_1 & -\overline{C}_2\overline{B}_2 \end{bmatrix} + n - q$. This accomplishes the proof of Lemma 3.

Synthesizing Lemma 1 to Lemma 3, we can get the following theorem: □

Theorem 2. *Under the assumption of A3, $(\hat{\hat{A}}, \hat{B})$ is stabilisable if and only if (E, A, B) is stabilisable and the matrix $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$ is of full row rank.*

Theorem 3. *If A3 holds and Q_e, Q_x, and R are symmetric positive definite matrices, then $(\hat{Q}^{1/2}, \hat{\hat{A}})$ is detectable.*

Proof. Based on the PBH criterion [27], $(\hat{Q}^{1/2}, \hat{\hat{A}})$ is detectable if and only if for any complex number s satisfying $\operatorname{Re}(s) \geq 0$, the matrix $U_o = \begin{bmatrix} sI_{m+q+n_m} - \hat{\hat{A}} \\ \hat{Q}^{1/2} \end{bmatrix}$ is of full column rank.

Note that

$$U_o = \left[\begin{array}{ccc} sI_m & -(\overline{C}_1 + \overline{C}_2\overline{B}_2\tilde{R}^{-1}\Phi^T) & C_m \\ 0 & sI_q - (\overline{A}_1 - \overline{B}_1\tilde{R}^{-1}\Phi^T) & 0 \\ 0 & 0 & sI_{n_m} - A_m \\ \hdashline Q_e^{1/2} & 0 & 0 \\ 0 & (\tilde{Q} - \Phi\tilde{R}^{-1}\Phi^T)^{1/2} & 0 \\ 0 & 0 & 0 \end{array}\right]. \tag{38}$$

From the structure of U_o and the nonsingularity of $sI_{n_m} - A_m$, $Q_e^{1/2}$, $(\tilde{Q} - \Phi\tilde{R}^{-1}\Phi^T)^{1/2}$ (Remark 7 shows that $(\tilde{Q} - \Phi\tilde{R}^{-1}\Phi^T)^{1/2}$ is positive definite), we can obtain

$$\begin{aligned} \operatorname{rank}(U_o) &= \operatorname{rank}\left[\begin{array}{ccc} sI_m & -(\overline{C}_1 + \overline{C}_2\overline{B}_2\tilde{R}^{-1}\Phi^T) & C_m \\ 0 & sI_q - (\overline{A}_1 - \overline{B}_1\tilde{R}^{-1}\Phi^T) & 0 \\ 0 & 0 & sI_{n_m} - A_m \\ \hdashline Q_e^{1/2} & 0 & 0 \\ 0 & (\tilde{Q} - \Phi\tilde{R}^{-1}\Phi^T)^{1/2} & 0 \end{array}\right] \\ &= \operatorname{rank}(Q_e^{1/2}) + \operatorname{rank}((\tilde{Q} - \Phi\tilde{R}^{-1}\Phi^T)^{1/2}) + \operatorname{rank}(sI_{n_m} - A_m) \\ &= m + q + n_m. \end{aligned} \tag{39}$$

Theorem 3 is proved.

Applying Theorems 2 and 3 in Theorem 1 and solving $u(t)$, we get the final result of this paper. □

Theorem 4. *If A1 to A5 hold, Q_e, Q_x, and R are symmetric positive definite matrices, then the algebraic Riccati equation (29) has a unique symmetric semipositive definite solution P, and the optimal preview control input of the system (1) which minimizes the performance index function (5) can be expressed as*

$$\begin{aligned} u(t) &= u(0) + (K - K_x)x(t) - K_{x_m}[x_m(t) - x_m(0)] \\ &\quad - K_e\int_0^t e(\sigma)\mathrm{d}\sigma - f_1(t) - f_2(t), \end{aligned} \tag{40}$$

where $f_1(t)$, $f_2(t)$, K_x, K_{x_m}, and K_e are

$$\begin{aligned} f_1(t) &= \tilde{R}^{-1}\hat{B}^T\int_0^{l_r}\exp\left(\sigma\hat{A}_c^T\right)P\hat{B}_m[u_m(t+\sigma) - u_m(\sigma)]\mathrm{d}\sigma, \\ f_2(t) &= \tilde{R}^{-1}\hat{B}^T\int_0^{l_f}\exp\left(\sigma\hat{A}_c^T\right)P\hat{D}[f_s(t+\sigma) - f_s(\sigma)]\mathrm{d}\sigma, \\ \tilde{R}^{-1}\hat{B}^T P &= \left[K_e \;\; K_{\overline{x}_1} \;\; K_{x_m}\right], \\ K_x &= \left[\tilde{R}^{-1}\Phi^T + K_{\overline{x}_1} \;\; 0\right]P_1^{-1}, \end{aligned} \tag{41}$$

the stable matrix $\hat{A}_c$ is shown as (31); $u(0)$ and $x_m(0)$ are the initial values that can be arbitrarily taken.

Proof. According to Theorems 2 and 3, the conditions of Theorem 4 guarantee the validity of Theorem 1. Therefore, the optimal preview control input of the system (26) which minimizes the performance index function (23) is expressed as (28).

By partitioning the gain matrix $\widetilde{R}^{-1}\widehat{B}^T P$ in (28) as (41), in which $K_e \in R^{r\times r}$, $K_{\overline{x}_1} \in R^{r\times q}$, and $K_{x_m} \in R^{r\times n_m}$, (28) can be written as

$$\dot{w}(t) = -K_e e(t) - K_{\overline{x}_1}\dot{\overline{x}}_1(t) - K_{x_m}\dot{x}_m(t) - \widetilde{R}^{-1}\widehat{B}^T g(t), \tag{42}$$

then integrating the both sides on the interval of $[0,t]$, we get

$$\begin{aligned} w(t) = w(0) &- K_e\int_0^t e(\sigma)\mathrm{d}\sigma - K_{\overline{x}_1}[\overline{x}_1(t) - \overline{x}_1(0)] \\ &- K_{x_m}[x_m(t) - x_m(0)] - \widetilde{R}^{-1}\widehat{B}^T\int_0^t g(s)\mathrm{d}s. \end{aligned} \tag{43}$$

It is easy to know that $v(t) = w(t) - \widetilde{R}^{-1}\Phi^T\overline{x}_1(t)$ and $v(0) = w(0) - \widetilde{R}^{-1}\Phi^T\overline{x}_1(0)$ from (25), putting them into (43) yields

$$\begin{aligned} v(t) = v(0) &- K_e\int_0^t e(\sigma)\mathrm{d}\sigma - \left(\widetilde{R}^{-1}\Phi^T + K_{\overline{x}_1}\right)[\overline{x}_1(t) - \overline{x}_1(0)] \\ &- K_{x_m}[x_m(t) - x_m(0)] - \widetilde{R}^{-1}\widehat{B}^T\int_0^t g(s)\mathrm{d}s. \end{aligned} \tag{44}$$

Noting $K_x = \begin{bmatrix} \widetilde{R}^{-1}\Phi^T + K_{\overline{x}_1} & 0 \end{bmatrix} P_1^{-1}$, we get the below result because of $x(t) = P_1\overline{x}(t) = P_1\begin{bmatrix} \overline{x}_1(t) \\ \overline{x}_2(t) \end{bmatrix}$:

$$\begin{aligned} K_x x(t) &= \begin{bmatrix} \widetilde{R}^{-1}\Phi^T + K_{\overline{x}_1} & 0 \end{bmatrix} P_1^{-1} x(t) \\ &= \begin{bmatrix} \widetilde{R}^{-1}\Phi^T + K_{\overline{x}_1} & 0 \end{bmatrix}\begin{bmatrix} \overline{x}_1(t) \\ \overline{x}_2(t) \end{bmatrix} \\ &= \left(\widetilde{R}^{-1}\Phi^T + K_{\overline{x}_1}\right)\overline{x}_1(t), \end{aligned} \tag{45}$$

then $K_x x(0) = (\widetilde{R}^{-1}\Phi^T + K_{\overline{x}_1})\overline{x}_1(0)$ when taking $t = 0$ in the above formula and substituting it in (44), we can obtain

$$\begin{aligned} v(t) = v(0) &- K_e\int_0^t e(\sigma)\mathrm{d}\sigma - K_x[x(t) - x(0)] \\ &- K_{x_m}[x_m(t) - x_m(0)] - \widetilde{R}^{-1}\widehat{B}^T\int_0^t g(s)\mathrm{d}s. \end{aligned} \tag{46}$$

The state prefeedback (10) gives $u(0) = Kx(0) + v(0)$ by taking $t = 0$ and substituting it and (46) in (10) yields

$$\begin{aligned} u(t) = u(0) &- K_e\int_0^t e(\sigma)\mathrm{d}\sigma + (K - K_x)[x(t) - x(0)] \\ &- K_{x_m}[x_m(t) - x_m(0)] - \widetilde{R}^{-1}\widehat{B}^T\int_0^t g(s)\mathrm{d}s. \end{aligned} \tag{47}$$

Moreover,

$$\begin{aligned} \int_0^t g(s)\mathrm{d}s &= \int_0^t\int_0^{l_f}\left[\exp\left(\sigma\widehat{A}_c^T\right)P\widehat{D}\dot{f}_s(s+\sigma)\right]\mathrm{d}\sigma\,\mathrm{d}s \\ &\quad + \int_0^t\int_0^{l_r}\left[\exp\left(\sigma\widehat{A}_c^T\right)P\widehat{B}_m\dot{u}_m(s+\sigma)\right]\mathrm{d}\sigma\,\mathrm{d}s \\ &= \int_0^{l_f}\left\{\exp\left(\sigma\widehat{A}_c^T\right)P\widehat{D}\left[\int_0^t\dot{f}_s(s+\sigma)\mathrm{d}s\right]\right\}d\sigma \\ &\quad + \int_0^{l_r}\left\{\exp\left(\sigma\widehat{A}_c^T\right)P\widehat{B}_m\left[\int_0^t\dot{u}_m(s+\sigma)\mathrm{d}s\right]\right\}\mathrm{d}\sigma \\ &= \int_0^{l_f}\left[\exp\left(\sigma\widehat{A}_c^T\right)P\widehat{D}[f_s(t+\sigma) - f_s(\sigma)]\right]d\sigma \\ &\quad + \int_0^{l_r}\left[\exp\left(\sigma\widehat{A}_c^T\right)P\widehat{B}_m[u_m(t+\sigma) - u_m(\sigma)]\right]\mathrm{d}\sigma, \end{aligned} \tag{48}$$

substituting it in (47) obtains the desired conclusion. □

Remark 8. In (40), $-K_x x(t)$ is the state feedback which is used to guarantee the closed-loop system of (1); $-K_{x_m}x_m(t)$ acts on the closed-loop system as feedforward compensation term; $-K_e\int_0^t e(\sigma)\mathrm{d}\sigma$ is the integral of the output tracking error (i.e., integrator) which is applied to eliminate the steady-state tracking error. $f_1(t)$ and $f_2(t)$ are the preview compensation for the fault signal and the reference input signal, respectively, which are served to improve the transient response of system.

The controller given by (40) is called a fault-tolerant preview controller.

6. Numerical Simulation

Example 1. Consider the descriptor system such as (1), where

$$\begin{aligned} E &= \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 2 \\ 0 & 2 & 0 \end{bmatrix}, \\ A &= \begin{bmatrix} -2 & -1 & 1 \\ -3 & 2 & 5 \\ 1 & 0.7 & 3 \end{bmatrix}, \\ B &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \\ C &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \\ D &= 1, \end{aligned} \tag{49}$$

the fault signal $f_s(t)$ is

$$f_s(t) = \begin{cases} 0, & 0 \le t \le 15, \\ 10(t-15), & 15 < t \le 15.1, \\ 1, & t > 15.1. \end{cases} \tag{50}$$

For this system, $n = 3$, $\operatorname{rank}(E) = 2$, and

$$\begin{aligned} \operatorname{rank}\begin{bmatrix} E & 0 \\ A & E \end{bmatrix} &= 4 \neq n + \operatorname{rank}(E), \\ \operatorname{rank}\begin{bmatrix} E & 0 & 0 \\ A & E & B \end{bmatrix} &= 5 = n + \operatorname{rank}(E). \end{aligned} \tag{51}$$

Thus, impulse terms are certain to appear in the state response, and it is verified that the system is impulse controllable. Therefore, there are state feedback controllers which can eliminate the system impulse.

Suppose in the reference model (2)

$$\begin{aligned} A_m &= \begin{bmatrix} -1 & 0.2 \\ 0 & -0.7 \end{bmatrix}, \\ B_m &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\ C_m &= \begin{bmatrix} 1 & 1 \end{bmatrix}, \end{aligned} \tag{52}$$

the reference input $u_m(t)$ is

$$u_m(t) = \begin{cases} 0, & 0 \le t \le 2, \\ 5(t-2), & 2 < t \le 4, \\ 10, & t > 4. \end{cases} \tag{53}$$

Note that the output vector $y_m(t)$ is the ideal value of system (2).

The calculation shows that the above system satisfies these assumptions of A1 to A5 in this paper. Taking the state prefeedback gain matrix $K = \begin{bmatrix} 0.5 & 1 & 7 \end{bmatrix}$ in (10) makes system (11) impulse-free. Selecting nonsingular matrices Q_1 and P_1, respectively,

$$\begin{aligned} Q_1 &= \begin{bmatrix} 0.0769 & 0.4615 & -0.4615 \\ -1.3077 & 0.6538 & -0.1538 \\ 0.3077 & -0.1538 & 0.1538 \end{bmatrix}, \\ P_1 &= \begin{bmatrix} 1.0769 & -1.4308 & -1 \\ 0 & 1 & 0 \\ -0.0769 & 0.4308 & 1 \end{bmatrix}, \end{aligned} \tag{54}$$

and the restricted equivalent transformation of the impulse-free descriptor system (11) is carried out. Then, the corresponding matrices in (15) are obtained, respectively.

$$\begin{aligned} \overline{A}_1 &= \begin{bmatrix} -2.2308 & 2.2923 \\ 0.4231 & 1.9308 \end{bmatrix}, \\ \overline{B}_1 &= \begin{bmatrix} -0.4615 \\ -0.1538 \end{bmatrix}, \\ \overline{B}_2 &= 0.1538, \\ \overline{C}_1 &= \begin{bmatrix} 1.0769 & -1.4308 \end{bmatrix}, \\ \overline{C}_2 &= -1. \end{aligned} \tag{55}$$

Choosing the weight matrix of performance index function (5) as $Q_e = 100$, $Q_x = 1$, and $R = 0.1$, the matrices defined in (23) can be calculated as

$$\begin{aligned} \widetilde{Q} &= \begin{bmatrix} 1.1657 & -1.5740 \\ -1.5740 & 4.3217 \end{bmatrix}, \\ \widetilde{R} &= 0.0473, \\ \Phi &= \begin{bmatrix} 0.1775 \\ -0.2864 \end{bmatrix}, \end{aligned} \tag{56}$$

then

$$\widehat{Q} = \begin{bmatrix} 100 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & -0.5 & 0 & 0 \\ 0 & -0.5 & 2.5890 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{57}$$

and we can get the augmented system (26), where

$$\begin{aligned} \widehat{\widehat{A}} &= \begin{bmatrix} 0 & 0.5 & -0.5 & -1 & -1 \\ 0 & -0.5 & -0.5 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0.2 \\ 0 & 0 & 0 & 0 & -0.7 \end{bmatrix}, \\ \widehat{B} &= \begin{bmatrix} 0.1538 \\ -0.4615 \\ -0.1538 \\ 0 \\ 0 \end{bmatrix}, \\ \widehat{B}_m &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \\ \widehat{D} &= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \end{aligned} \tag{58}$$

So far, it is easy to verify that the conditions of Theorem 4 are all satisfied. Therefore, the solution of Riccati equation (29) and the correlation matrices of fault-tolerant preview controller (40) can be obtained, respectively:

$$P = \begin{bmatrix} 105.3344 & 36.1561 & -17.2761 & -47.4797 & -54.0968 \\ 36.1561 & 14.4391 & -9.6098 & -19.8172 & -23.1086 \\ -17.2761 & -9.6098 & 24.0013 & 20.1537 & 26.1131 \\ -47.4797 & -19.8172 & 20.1537 & 30.7438 & 37.1758 \\ -54.0968 & -23.1086 & 26.1131 & 37.1758 & 45.5940 \end{bmatrix},$$

$$\widehat{A}_c = \begin{bmatrix} -7.0711 & -0.7242 & 5.7240 & 3.0909 & 4.4421 \\ 21.2132 & 3.1727 & -19.1721 & -12.2728 & -16.3262 \\ 7.0711 & 2.2242 & -5.2240 & -4.0909 & -5.4421 \\ 0 & 0 & 0 & -1 & 0.2 \\ 0 & 0 & 0 & 0 & -0.7 \end{bmatrix},$$

$$\begin{aligned} K_e &= 45.9619, \\ K_{\overline{x}_1} &= \begin{bmatrix} 7.9574 & -40.4563 \end{bmatrix}, \\ K_x &= \begin{bmatrix} 11.7074 & -34.7989 & 11.7074 \end{bmatrix}, \\ K_{x_m} &= \begin{bmatrix} -26.5912 & -35.3735 \end{bmatrix}, \end{aligned} \tag{59}$$

where P is symmetric positive definite and $\widehat{A}_c$ is stable.

The initial conditions are as follows:

$$\begin{aligned} u(0) &= 0, \\ x(0) &= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T, \\ x_m(0) &= \begin{bmatrix} 0 & 0 \end{bmatrix}^T, \end{aligned} \tag{60}$$

and selecting the sampling period to be $T = 0.1$, the method in reference [18] is used for simulation. Figures 1 and 2 show the output response and tracking error when both the reference input signal and the fault signal are previewable, respectively.

The simulation results show that the effect of output tracking is noticeable under the preview compensations. By adopting the fault-tolerant preview control, the adverse effects caused by the fault signal can be effectively suppressed, and the response speed of the system output to the ideal value is accelerated.

If the fault signal is zero vector (i.e., no fault occurs), the controller designed here is still valid. Figures 3 and 4 reveal the output response and tracking error of the system in this case, respectively.

The simulation results illustrate that the tracking effect of the output vector of the original system to the ideal output vectors of the reference system is still noticeable under the action of reference input preview.

Example 2. Consider a fault-free system in the form of (1) with the following parameters [28, 29]:

$$\begin{aligned} E &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ A &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \\ B &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}, \\ C &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \end{aligned} \tag{61}$$

The system has impulse behavior and is impulse controllable.

Suppose the parameters in the reference model (2) are the same as Example 1 except

$$C_m = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \tag{62}$$

and we can turn the model following problem into a stability problem by choosing $x_m(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$, $u_m(t) \equiv 0$.

Let

$$\begin{aligned} u(0) &= \begin{bmatrix} 0 & 0 \end{bmatrix}^T, \\ x(0) &= \begin{bmatrix} 5 & 5 & 5 & 5 & 5 & 5 \end{bmatrix}^T, \\ K &= \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \\ Q_e &= 10, \\ Q_x &= 10, \\ R &= 1, \\ l_r &= 0. \end{aligned} \tag{63}$$

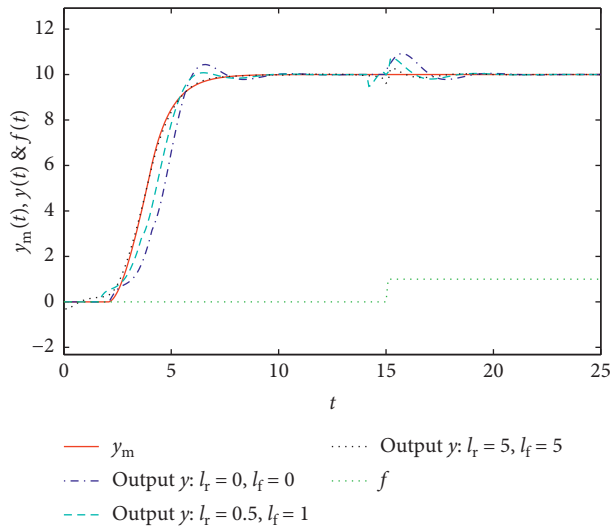

Figure 1: Closed-loop output response with reference input preview and fault signal preview.

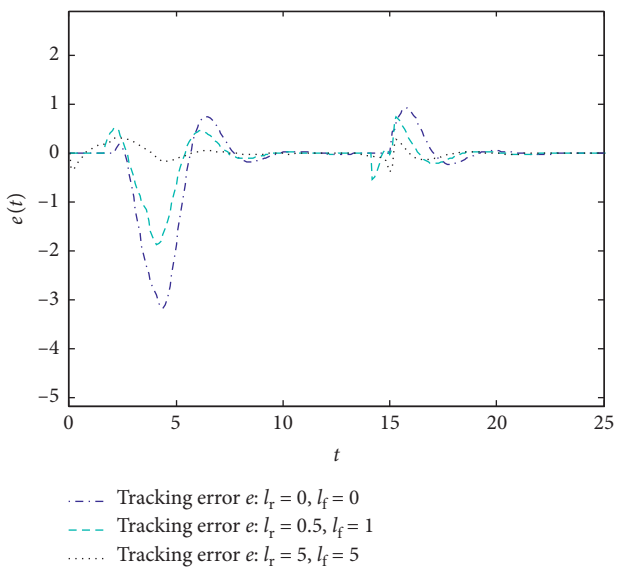

Figure 2: Output tracking error with reference input preview and fault signal preview.

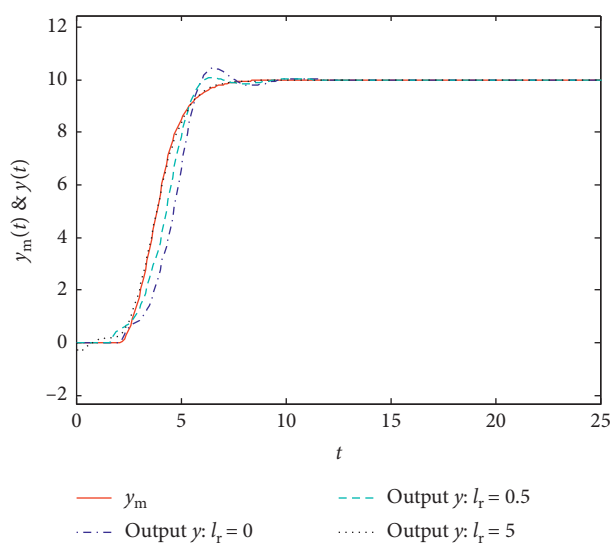

Figure 3: Closed-loop output response with reference input preview and no fault occurs.

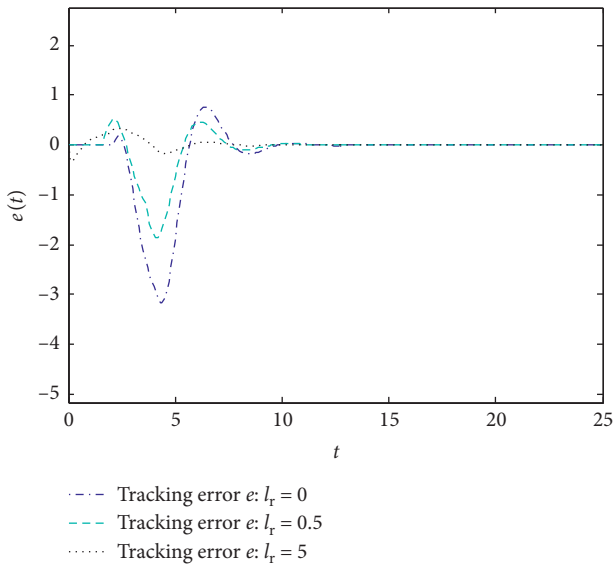

Figure 4: Output tracking error with reference input preview and no fault occurs.

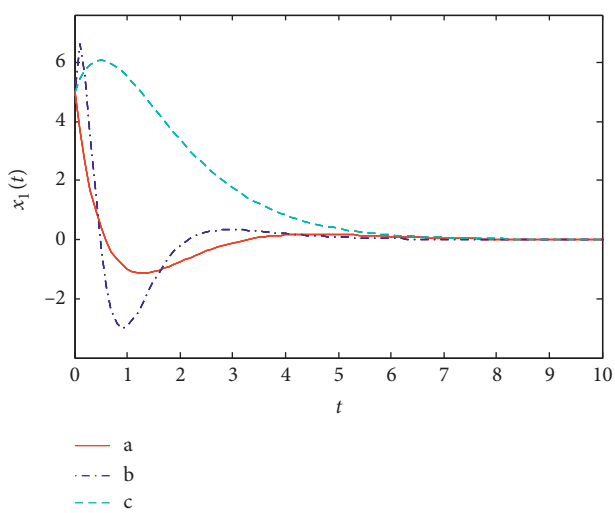

Figure 5: Response of the vector component $x_1(t)$ under different controllers: a. this work; b. reference [28]; c. reference [29].

Figure 5 shows the vector component $x_1(t)$ drawn based on the methods of this paper and references [28, 29]. It can be seen that the control method in this paper can reduce overshoot and accelerate the system response.

7. Conclusion

In this paper, the controller design problem of impulse controllable descriptor systems with sensor failure is studied through the combination of model following control theory and preview control theory. Key features of the proposed method are summarized as follows: (i) the preview control theory is applied to the fault-tolerant control, and the fault-tolerant preview controller improves the transient response of the closed-loop system; (ii) the impulse elimination and the transformation for the quadratic performance index function are carried out for the more general descriptor systems based on reference [23], which is an extension of reference [10]. Our future research topics will focus on the fault-tolerant adaptive preview controller design for the hybrid systems based on references [30, 31].

Data Availability

Data sharing is not applicable to this article as no data sets were generated or analysed during the current study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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