

Research Article

Study of the Modified Logistic Model of Chinese Electricity Consumption Based on the Change of the GDP Growth Rate under the Economic New Normal

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In the context of the new normal, the global economy is entering a deep adjustment period, and the driving forces of development are also constantly changing. As a result, China's economy has also entered a "new normal" phase in which it is growing in a manageable and relatively balanced manner. In addition, the new normal characteristics of the power industry's development in China are also very significant, and they affect the adaptability of traditional power forecasting methods. By analyzing the new characteristics of China's economic development and the changing electricity demand in recent years, this paper quantitatively studied the effect of the national economy on the consumption of electricity. Meanwhile, a modified logistic model based on the change of the gross domestic product (GDP) growth rate is constructed to make a reasonable prediction of the future power consumption in China. Subsequently, by calculating the elasticity of electricity consumption in the future, it is found that the coefficient decreases each year, which indicates that electricity consumption in China is following a new trend. Based on the research results, this paper proposes rational suggestions for China's power development, and they are expected to provide references for the power planning and power industry layout in China.

1. Introduction

During the "12th Five-Year Plan" period, influenced by the international economic situation and the adjustment of its domestic industrial structure, China's economy began to enter a "new normal". In the context of the new normal economic development, the economy has shifted from the previous high-speed growth to medium-to-high-speed growth. The economic structure is constantly being improved and upgraded. The economy is increasingly driven by innovation instead of input and investment. Economic transformation and supply-side structural reforms have reduced the proportion of electricity consumption in high-energy-consuming industries. Together, these observations indicate that China's electric power has begun to enter a new normal [1]. The new normal state of electric power is mainly reflected in the deceleration in the growth

of electricity consumption in throughout Chinese society, the negative growth in electricity consumption in secondary industry for the first time, and the rapid optimization and upgrading of the power consumption structure [2]. The traditional forecasting method is mainly based on the laws of electricity power demand during the highspeed development of power in the past [3]. Under the new normal state, the precision and accuracy of the traditional method will inevitably be affected to a certain extent, resulting in a decrease in its applicability. To compensate for the shortcomings of traditional methods, this paper reanalyzes the historical characteristics of China's electric power consumption based on China's current economic development, and it studies the degree of influence of the national economy on power consumption, which will allow us to make a reasonable prediction of the power consumption.

2. Brief Literature Review

2.1. Electricity Consumption and Economic Development in China. After the reform and opening up in 1978, China began the processes of urbanization and industrialization, and the economy began to rapidly develop. In recent decades, China's economy has maintained rapid growth with an average annual GDP growth rate of approximately 10% [4]. China has become the world's second largest economy with an overall economic output that is second only to that of the United States. With the rapid economic growth, the energy demand in China has also rapidly grown. The British Petroleum (BP) World Energy Statistics Yearbook (2014) [5] showed that China has become the major contributor to the global energy demand growth. The energy demand of China is rapidly growing, and it surpassed that of the EU in 2007, the United States in 2010, and all of North America in 2013. Electricity is a kind of secondary energy, and it is converted from primary energy by processing. As energy is closely related to the Chinese national economy and people's livelihoods, demand is also rapidly growing. In 2011, China's total electricity consumption reached 4692.8 billion kWh, surpassing that of the United States, and China became the country with the highest demand for electricity in the world [6].

It can be seen that China occupies an increasingly important position on the international stage. China's economic development, energy security and electricity shortages have begun to receive attention from scholars at home and abroad.

Shiu and Lam [7], Yuan et al. [8], and Chi et al. [6] have proved that there is causality from electricity consumption to economic growth. The results of Rong et al. [9] show that there was bidirectional long-run causality between electricity consumption and China's economic growth. For different regions, the results of Liu et al. [10] show that there is a oneway causal relationship between regional economic growth and electricity consumption in different regions of China. Li and Sun [11] proved that the electricity consumption in Yunnan province is a one-way Granger cause of real GDP, and there is a long-term cointegration relationship between GDP and electricity consumption. Lin and Liu [12] argued that there is a relatively stable and positive correlation between electricity consumption and economic growth and there should not be a large deviation between them.

According to the Mid-year Report on the Analysis and Forecast of China's Macroeconomic Situation in 2018 that was issued by the Institute for Advanced Research of SHUFE, since 2018, the international situation has become increasingly uncertain. China's economy has begun to decline and the growth rate of international trade has slowed down. The Chinese economy has entered a three-year adjustment period and will continue to face new challenges. Facing the new economic situation, many scholars have reexamined the relationship between China's economy and electricity consumption. In reference [13], the cointegration analysis shows that there is a long-term equilibrium relationship between economic growth and electricity consumption in Hubei Province, and the results of a Granger causality test show that there is one-way Granger causality

between electricity consumption and GDP in Hubei Province. Yang et al. [14] used Anhui Province as an example to analyze the underlying reasons for the inconsistent relationship between economic growth and electricity consumption under the new normal. It is found that the acceleration of industrial restructuring, the optimization and upgrading of product structures, and the upgrading of energy consumption standards are the main reasons for the deviation between the economic and electricity growth trend since 2015. Long [15] finds that China's economic output is the long-term Granger cause of electricity consumption, the two are short-term Granger causes of each other, and their impacts are positive. The results of Liu and Lu [16] showed that China's overall power output was positively correlated with economic growth, but the power price distortions in some provinces and regions have caused the power sector to contribute little to the region's economic growth.

In summary, we find that even if the deviation between electricity consumption and economic growth appeared in China during the global economic crisis, it sparked an economic adjustment period. Once the economy returns to stable growth, the deviation between electricity consumption and economic growth will quickly shrink and disappear.

2.2. Forecast Method for Electricity Consumption. The accurate prediction of power consumption cannot be separated from reasonable prediction methods and models. At present, the research on power demand forecasting is relatively mature. Domestic and foreign scholars have summarized a series of effective forecasting models and methods. Among those models and methods, the classical prediction methods are as follows: regression analysis [17, 18], the autoregressive integrated moving average model [19, 20], the electric elastic coefficient method [21, 22], and input-output analysis [23]. The regression analysis method and inputoutput method need large amounts of data to obtain more accurate prediction accuracies, which will take substantial time. In addition, some data are difficult to obtain, which also increases the prediction difficulty. The elasticity of electricity method and the time series method are very common and simple methods for power forecasting. These methods are suitable for rapid calculations and approximate estimations with low precision. Although the economic significance is clear, many factors affecting electricity consumption are not taken into account.

Due to the nonlinearity, time-varying and uncertainty of, in recent years, the focus of electricity consumption forecasting research has shifted from the traditional methods that were mentioned above to emerging nonlinear methods. The common nonlinear methods that are used by scholars include the gray-forecasting model [13, 24], the genetic algorithm [25, 26], and the neural network model [27, 28], in addition to more complex probability forecasting model [29, 30] and the combination method [31, 32]. In gray theory, we need to address the unknown gray information. The factors affecting the power system are time-varying, so the accuracy of long-term predictions will decrease. Because the parameter setting of genetic algorithms mainly depends on the experience of modelers, it has certain subjectivity, and different genetic algorithms have different parameters, which are complex and are not universal. The amount of data in the power sector is constantly increasing, and future values tend to exceed the maximum value of the learning sample, which make neural network algorithms unsuitable for long-term predictions. The prediction accuracy and degrees of freedom of the probability forecasting model and combination method are improved, but they are complicated to use. These models are relatively accurate and reliable for quick power demand forecasting. However, if the growth of the power demand slows down and saturation occurs, this will lead to a large prediction error.

In contrast, the logistic model is a better choice for the saturation prediction of the power demand, since it can describe the process of an object growing in an "S" shape under resource constraints. The logistic saturated load time series forecasting model was established by literature [25] based on the optimization algorithm. The logistic time series analysis was carried out on the basis of the historical data of a regional power grid. The simulation results show that it is feasible to apply the improved algorithm to saturated load forecasting. Bao et al. [33] proposed the amended solution to the exponential function to avoid the dependency of logistic model on the saturated value. The simulation results show that the fitting error of the improved logistic model is smaller than that of existing methods. Zhang et al. [34] employed the logistic curve to explore the turning points of electricity consumption. In the literature, the logistic model was used to forecast the long-term electricity consumption. Finally, these authors prove that it is feasible to apply the model to load forecasting.

Based on the above analysis, it is necessary to improve the logistic model by combining the main features of the new normal. Therefore, in our article, we consider the impact of the change of the GDP growth rate on electricity consumption under the economic new normal and construct the modified logistic model with the change of the GDP growth rate as one of the variables. Then, we use the analytical method, the dynamic method, and the static difference method to solve the model. Finally, our empirical analysis of China's electricity consumption from 2000 to 2018 verified that the modified models have good fitting effects and predictive abilities.

3. Establishment and Solution of the Modified Logistic Model

3.1. Construction of the Modified Logistic Model. This paper assumes that R_{GDP} is the growth rate of China's gross domestic product (GDP). The Equation $\Delta R_{GDP}(t) = R_{GDP}(t) - R_{GDP}(t-1)$ calculates the change of the GDP growth rate. Furthermore, it is assumed that the change in the GDP will affect the total electricity consumption of the whole society to some extent, and the degree of the influence is expressed by the conversion coefficient α . Therefore, in unit time, the change in the total electricity consumption of the whole society that is caused by the change in the GDP growth rate can be expressed by the following equation: $\alpha N (dR_{GDP}/dt)$ [1].

Based on the classical logistic model of the total annual electricity consumption, the modified logistic model considering the change of the GDP growth rate can be expressed as follows:

$$\begin{cases} \frac{\mathrm{d}N}{\mathrm{d}t} = rN\left(1 - \frac{N}{K}\right) + \alpha N \frac{\mathrm{d}R_{\mathrm{GDP}}}{\mathrm{d}t},\\ N|_{t=0} = N_0, \end{cases}$$
(1)

where N is the dependent variable, indicating the total electricity consumption of the whole society; t is the independent variable, indicating the year; and K is the constant, indicating the saturation value of the total electricity consumption of the whole society.

3.2. Analytical Method to Solve the Modified Model

3.2.1. Solution Process. Through the mathematical transformation of Equation (1), the following equations are obtained:

$$\frac{\mathrm{d}N}{N\mathrm{d}t} = r\left(1 - \frac{N}{K}\right) + \alpha \frac{\mathrm{d}R_{\mathrm{GDP}}}{\mathrm{d}t},\tag{2}$$

$$\frac{d\ln N}{dt} = r\left(1 - \frac{N}{K}\right) + \alpha \frac{dR_{\rm GDP}}{dt}.$$
(3)

By integrating the two sides of Equation (3), the following result can be obtained:

$$\ln N = \int \left[r \left(1 - \frac{N}{K} \right) \right] dt + \alpha \Delta R_{\rm GDP}.$$
(4)

Through the above mathematical deduction process, the general solution of differential equations can be obtained as follows:

$$N = N_0 \exp\left\{\int \left[r\left(1 - \frac{N}{K}\right)\right] dt + \alpha \Delta R_{\rm GDP}\right\}$$

$$= N_0 \exp\left\{\int \left[r\left(1 - \frac{N}{K}\right)\right] dt\right\} \cdot \exp\{\alpha \Delta R_{\rm GDP}\}.$$
(5)

According to the economic development and historical data of each country, the change in the GDP growth rate of each country varies between -0.1 and 0.1, and the conversion coefficient α is between 0 and 1. Then, through Taylor's Series Expansion Theorem, it can be known that $\exp(\alpha \Delta R_{GDP})$ meets the convergence condition of the series. Therefore, the Taylor expansion of $\exp(\alpha \Delta R_{GDP})$ is expressed as follows:

$$\exp\left(\alpha\Delta R_{\rm GDP}\right) = 1 + \alpha\Delta R_{\rm GDP} + \left(\alpha\Delta R_{\rm GDP}\right). \tag{6}$$

By omitting the high-order infinitesimal in Equation (6), we can obtain the following:

$$\exp\left(\alpha\Delta R_{\rm GDP}\right)\approx 1+\alpha\Delta R_{\rm GDP}.\tag{7}$$

Substituting Equation (7) into Equation (5) yields the following:

$$N = N_0 \exp\left\{\int \left[r\left(1 - \frac{N}{K}\right)\right] dt\right\} \cdot \left(1 + \alpha \Delta R_{\rm GDP}\right),\tag{8}$$

where $N_0 \exp\{\int [r(1 - (N/K))]dt\}$ is the analytic function solution of the classical logistic model [2]. After substituting this solution into Equation (8), the expression equation of the national electricity consumption is obtained as follows:

$$\begin{cases} N = \frac{K}{1 + ((K/N_0) - 1)e^{-rt}} \cdot (1 + \alpha \Delta R_{\rm GDP}), \\ N|_{t=0} = N_0. \end{cases}$$
(9)

Through the above mathematical derivation, the modified logistic analytical function based on the change in the GDP growth rate can be obtained. Then, the medium and long-term forecasts of the power consumption can be made by using the future economic planning under the new normal situation in China, which can guide the operational planning and expansion of the power grid.

3.2.2. Determination of the Relevant Parameters and the Results of the Model. Under the background of the new normal economy, in order to better predict the total national electricity consumption, this paper selects the total electricity consumption data of China from 2000 to 2018 for the empirical research, and the relevant data are illustrated in Figure 1. It can be seen from Figure 1 that the growth trend of China's total electricity consumption was S-type, and the logistic model is usually used to predict such data.

In the modified logistic model that is constructed in this paper, the variation of the GDP growth rate causes only a micro modulation in the total electricity consumption. In addition, the impact on the fluctuation of the power consumption is not large and does not affect its overall trend. Therefore, we can use the inherent growth rate r and the saturation prediction value K that are obtained by the fitted curve using the classical logistic model.

Setting parameter $\alpha = 0$ in the model makes the model degenerate into a classical logistic model. Subsequently, this paper uses the least squares method to fit and optimize the total electricity consumption data forecast curve in China and to obtain the corresponding parameter values.

The general solution of the logistic model is as follows:

$$\begin{cases} N = \frac{K}{1 + ((K/N_0) - 1)e^{-rt}}, \\ N|_{t=0} = N_0. \end{cases}$$
(10)

Based on Equation (10), the expansion parameters [1] of the model are set as follows:

$$\begin{cases} Y = \ln \frac{K - N}{N}, \\ \frac{K}{N_0} - 1 = e^a. \end{cases}$$
(11)



FIGURE 1: Historical power consumption statistics in China.

By substituting the extended parameters into the logical general solution, we can obtain a linear equation: Y = a - rt.

The sample data of national electricity consumption $\{N_t, t = 1, 2, ..., n\}$ are transformed into the extended parameter value $\{Y_t, t = 1, 2, ..., n\}$ of a specific K value by Equation (11). Then, the least squares method is used to linearly fit $\{Y_t\}$ to obtain the parameter values for a and r. Finally, the predicted power consumption value sequence is obtained by calculation. Finally, the forecasted electricity consumption sequence $\{\widehat{Y}_t\}$ can be calculated using the model.

The determination coefficient max R^2 is used as the test criterion to determine the optimal electricity consumption saturation value *K* [35]:

$$\max_{K} R^{2} = \max_{K} \frac{\sum \left(\widehat{Y}_{t} - \overline{Y}\right)^{2}}{\sum \left(Y_{t} - \overline{Y}\right)^{2}},$$
(12)

where $\overline{Y} = \sum Y_t / n$.

According to Figure 1, referring to the historical data of China's total electric power consumption in recent years, an effective range of initial values of K can be given. K_{\min} should not be less than the historical data of electric power consumption $\{N_t, t = 0, 1, 2, ..., n\}$. Here, T is a given year, and it is determined that the approximate range of K is [70000, 90000]. Then, the following regression analysis results are obtained by using different K's in Equation (12).

From the calculation results in Table 1, we can see that when K = 8700 (billion kWh), the judgment coefficient $R^2 = 0.9968$ reaches its maximum, corresponding to a = 1.8651 and r = 0.1757. Based on the obtained parameter values, the modified analytical equation for the predicted power consumption is obtained:

$$N(t) = \frac{8700}{1 + e^{1.8651 - 0.1757t}} = \frac{8700}{1 + 6.4566 \cdot e^{-0.1757t}}.$$
 (13)

3.3. Difference Method to Solve the Modified Model. As the variation of the GDP growth rate causes minor changes in the total electricity consumption, it has little effect on the fluctuation of electricity consumption and does not affect its overall trend. Similarly, the intrinsic growth rate r and

TABLE 1: Logistic extended parameter regression analysis.

K (billion kWh)	R^2
7	0.8793
7.1	0.9495
7.2	0.9684
7.3	0.9780
7.4	0.9838
7.5	0.9877
7.6	0.9903
7.7	0.9923
7.8	0.9936
7.9	0.9947
8	0.9954
8.1	0.9959
8.2	0.9963
8.3	0.9965
8.4	0.9966
8.5	0.9967
8.6	0.9967
8.7	0.9968
8.8	0.9966
8.9	0.9965
9	0.9962
9.1	0.9961
9.2	0.9959
9.3	0.9957
9.4	0.9955
9.5	0.9952
9.6	0.9950
9.7	0.9948
9.8	0.9945
9.9	0.9943

saturation prediction value K that are obtained by the fitted curve in the classical logistic model can be used to discretize Equation (10), and the equation is obtained is as follows:

$$N(i) - N(i-1) = rN(i-1) \left[1 - \frac{N(i-1)}{K} \right] + \alpha N(i-1) \cdot \left[R_{\text{GDP}}(i) - R_{\text{GDP}}(i-1) \right].$$
(14)

The annual electricity consumption can be obtained by iterative calculations. If the first year of the research period is used as the base year, the total electricity consumption in the following years can be recursively derived using Equation (14). The mathematical model that is obtained by this calculation method is called the modified logistic dynamic difference model, which is suitable for the medium and longterm prediction of electricity consumption. Correspondingly, if the actual sample value of the previous year is used to calculate the predicted power consumption of the current year, the model that is obtained is called the modified logistic static difference model. By comparing the two forecasting models, the static difference method uses more actual values for its prediction, and so its accuracy and effectiveness are better than those of the dynamic difference model.

3.4. Optimal Solution for the Conversion Coefficient. For the parameters in the revised model, we can use the genetic

algorithm to estimate their reasonable values. The pattern search method is a direct search optimization method [26]. In the calculation process, we only need to calculate the objective function to realize the global optimal search with a large probability and fast speed. The steps of the pattern search method are as follows:

(1) Construct the objective function:

$$R_{\rm SS} = \sqrt{\sum_{t=1}^{n} (N(t) - N_t)^2}.$$
 (15)

- (2) Set the initial parameter value to α = 0 and substitute the calculation result of Equation (13) into Equation (15). Then, we can get the initial value of R_{SS}.
- (3) Repeatedly or alternately change the parameters with a certain step size (this paper selects 0.1 as the change step of parameter α) to observe the directional change of the objective function and move it in the direction where it is decreasing until it is no longer decreasing.
- (4) Select the statistic that describes the search accuracy.

 R^2 is selected as the statistic to describe the search accuracy. A value closer to 1 represents that the model is more accurate.

$$R^{2} = \frac{\sum \left[N(t) - \overline{N}\right]^{2}}{\sum (N_{t} - \overline{N})^{2}},$$
(16)

where $\overline{N} = \sum N_t / n$.

4. Empirical Analysis

4.1. Data Collection and Preprocessing. The time interval of the study is from 2000 to 2018. According to the convention, this paper selects the GDP to measure the level of economic development. To eliminate the impact of different price levels, the real GDP data that are used in this paper use 2000 as the base year for prices. We use the total electricity consumption of the whole society to measure China's electricity consumption. The data in this paper come from the China Statistical Yearbook (2000~2018) [36] that was issued by the National Bureau of statistics and the Statistical Data of the National Electric Power Industry (2000~2018) [37] that was issued by the National Energy Administration.

Table 2 is a statistical table of China's electricity consumption growth rate and GDP growth rate from 2000 to 2018. It should be noted that the GDP data are transformed and converted using the base year prices in 2000. This paper uses this method to eliminate the impacts of price changes in different years on the GDP and calculate the real GDP. According to the GDP growth rate, China's economy has maintained large growth rates in recent years. Due to the impact of the international financial crisis and internal economic restructuring, the Chinese economy has experienced two major turning points. The first turning point

TABLE 2: Power consumption growth rate and GDP growth rate of China from 2000 to 2018.

Year	GDP growth rate (%)	Power consumption growth rate (%)
2000	8.5	9.7
2001	8.3	8
2002	9.1	10.3
2003	10	15.4
2004	10.1	14.9
2005	11.4	13.45
2006	12.7	14
2007	14.2	14.42
2008	9.7	5.6
2009	9.4	6.4
2010	10.6	14.56
2011	9.6	11.7
2012	7.9	5.5
2013	7.8	7.5
2014	7.3	3.8
2015	6.9	0.5
2016	6.7	4.9
2017	6.9	6.6
2018	6.8	8.5

occurred in 2008. Due to the outbreak of the international financial crisis, China's GDP growth rate fell sharply. After the second turning point in 2012, China entered the "new normal" of economic development. As a result of internal economic restructuring, GDP growth began to slow each year but still maintained a stable growth state. China's electricity consumption quickly grew from 2000 to 2007 [38]. The large impact of the 2008 financial crisis on the real economy led to large fluctuations in power consumption, which shifted from high-speed growth to low-speed growth. Then, with the economic recovery, power consumption rebounded in 2009 and 2010. After 2012, as the Chinese economy entered the "new normal," the growth rate of electricity consumption shifted to low-speed growth again and has continued in this way to this day [39].

In Figure 2, the historical data of China's GDP growth rate and electricity consumption growth rate are drawn as curves. By observing Figure 2, it can be found that China's electricity consumption has mainly high-speed growth, GDP has mainly stable growth, and the change cycles are highly synergistic from a time perspective, which shows there a positive correlation between electricity consumption changes and economic changes. As far as the fluctuation of the curve is concerned, the fluctuations of the power consumption growth in the high-speed growth stage and the low-speed growth stage are obviously higher than those of GDP growth.

4.2. Error Analysis of Models

4.2.1. Modified Analytical Models. The accuracy of the model can be estimated by calculating the relative errors and mean squared errors of the modified models for different α 's. The equation for calculating the relative error between the actual and predicted values of the power consumption at a given time is as follows:



FIGURE 2: Growth rate of GDP and power consumption in China.

$$\delta(t) = \frac{N(t) - N_t}{N_t}.$$
(17)

The corresponding mean squared error can be used to measure the overall prediction accuracy of the model, and its calculation equation is as follows:

$$\sigma = \sqrt{\frac{\sum \delta(t)^2}{n}}.$$
 (18)

According to the two equations, the corresponding errors of the modified models under different α 's are recorded in Table 3.

In Figure 3, we draw the actual data of China's electricity consumption, the predicted results of the classical logistic model, and the results that are obtained by the modified analytical method together. It can be seen from Figure 3 that the modified analytical method simulates the change of China's total electricity consumption well, and the trend curve that is fit by the model is generally consistent with the actual electricity consumption trend. From Table 3, we can see that the relative error of the modified analytical model will be lower when the economic development is relatively stable. Only when the conversion coefficient α is reasonable will the overall accuracy of the model increase. In summary, compared with the classical logistic model, the modified analytic model compensates for the shortcomings of the classical logistic model that has low degrees of freedom and can only fit a smooth curve so that its predictions are more scientific. By introducing the change of the GDP growth rate as a variable, the modified logistic model can fit the fluctuation curve and accurately assess the fluctuation mechanism, which will improve its strong ability to reproduce historical sample data.

In addition, two very obvious anomalies can also be observed in Figure 3 and Table 3. One anomaly is that compared with the sample values of historical statistics, the fit curves of the modified analytical model fluctuated much more in 2008 and 2012, resulting in large errors in the prediction results. The other anomaly is that the fitting data

TABLE 3: Comparison table of the errors under different conversion coefficients.

Year	Logistic	$\alpha = 0.3$	$\alpha = 0.6$	$\alpha = 0.9$
2000	0	0	0	0
2001	-0.0738	-0.0749	-0.0761	-0.0772
2002	-0.0312	-0.0266	-0.0219	-0.0173
2003	-0.0438	-0.0386	-0.0335	-0.0283
2004	-0.0463	-0.0458	-0.0452	-0.0446
2005	-0.0430	-0.0355	-0.0280	-0.0206
2006	-0.0523	-0.0449	-0.0375	-0.0301
2007	-0.0717	-0.0633	-0.0549	-0.0466
2008	-0.0204	-0.0468	-0.0733	-0.0997
2009	0.0212	0.0193	0.0175	0.0156
2010	-0.0184	-0.0113	-0.0043	0.0028
2011	-0.0417	-0.0475	-0.0532	-0.0590
2012	-0.0166	-0.0266	-0.0367	-0.0467
2013	-0.0139	-0.0145	-0.0151	-0.0157
2014	0.0151	0.0121	0.0090	0.0060
2015	0.0716	0.0691	0.0665	0.0639
2016	0.0587	0.0574	0.0561	0.0548
2017	0.0404	0.0417	0.0429	0.0442
2018	-0.0017	-0.0023	-0.0029	-0.0035
Mean squared error	0.0423	0.0417	0.0425	0.0446



FIGURE 3: Curves of the classical logistic model, the modified analytic method, and historical statistics.

in two fluctuating years follow the opposite trend as the historical data. According to our analysis, the formation mechanisms of these two phenomena are consistent, which are mainly related to the GDP change rate in the past three years. China's GDP has experienced deep V-shaped fluctuations in the two fluctuating years, and the GDP changes reached 4.6% and 1.8%. The violent fluctuations in GDP are bound to cause significant changes in the demand for electricity.

There are two main sources of the deviation in the analytical model. First, from the mathematical equation, the higher-order minor terms are omitted in Equation (8), which leads to the calculations' deviations. However, the high-order terms are very small in the calculation, so their effect on the result is not large. Second, from the perspective of the market, electricity supply and demand are a dynamic balancing process. The allocation of market resources is an extremely complicated process that involves multiple factors, and it is regulated and controlled by many kinds of factors. Therefore, changes in market resources tend to lag changes in the economy.

However, the mathematical model directly uses the conversion coefficient to characterize the impact of the GDP on power consumption. Compared with the market with lags, the adjustment of the national economy in the model is more direct, so the fluctuation of the forecast results will be more intense. In the years when the GDP growth rate is greatly reduced, the power consumption that is predicted by the modified analytical model is lower than the historical data, and the downward trend of the curve is more obvious. According to the error data in Table 3, in the following years, the "intense" and "ahead" characteristics of the mathematical model will inevitably reconcile the previous overfit values with more significant reductions and deceleration, thus maintaining the overall optimization of the fitting accuracy.

4.2.2. Influence of the Conversion Coefficient α . In recent years, with the depletion of fossil energy and the aggravation of environmental pollution, the relationship between energy and the economy has gradually become one of the hotspots in energy research. Because electricity is at the center of energy, many scholars at home and abroad have studied the relationship between electricity consumption and economic development using various methods. Due to the differences between the regions, variables, research years, and research methods that are selected by different scholars, the results are not nearly the same. The research in this paper is based on the fact that economic changes have a certain impact on power consumption. Referring to the existing literature, this paper assumes that there is a positive correlation between China's power consumption changes and economic changes. Thus, we choose the conversion coefficient α to characterize the impact of economic changes on electricity consumption.

Figure 4 shows the predicted power consumption curves that are obtained from the modified model when α is set at different values. The modified analytical model degenerates into the classical logistic model when $\alpha = 0$; thus, here, economic changes have no effect on power consumption. There is a positive correlation between economic changes and electricity consumption changes when $\alpha \in (0, 1)$. The curve that is fit by the modified analytical model based on the change of the GDP growth rate fluctuates, and the fluctuation trend is consistent with the fluctuation direction of the actual electricity consumption data. With the increase in α , the influence of economic changes on electricity consumption changes becomes greater and the fluctuation of the forecast curve becomes increasingly more obvious.

We obtain Table 4 using the Pattern Search Method. Then, by comparing the mean squared differences under the different conversion coefficients α in Table 4, we can draw the following conclusions.



FIGURE 4: Curves of the modified analytic method's predicted results and the historical statistics.

(1) The gaps between the mean squared errors corresponding to different α values are small, and their fitting results are basically equivalent. (2) When $\alpha \in (0.1, 0.6)$, the mean squared error of the modified analytical model is smaller than that of the classical logistic model, which indicates that the fitting accuracy of the modified analytical model is improved compared with the classical logistic model. When $\alpha = 0.3$, the mean squared error of the modified analytical model is the best. (3) When $\alpha = 0.6$, the fitting accuracy of the modified model begins to decrease while the curve fluctuation trend continues to increase, which better reproduces the fluctuation of the actual power consumption curve.

The results show that there is a positive correlation between economic change and electricity consumption. Therefore, the fluctuation trend of actual electric power consumption can be better reproduced by introducing changes in the GDP growth rate as one of the model variables, and the predicted results will be helpful to the future expansion and operational planning of electric power systems. In the application process of the modified analytical model, the conversion coefficient is generally in between 0.2 and 0.6, and it needs to be adjusted according to the actual situation. α should be adjusted as follows. When economic development is relatively stable, α should be intermediate. When economic development shows a sharp and rapid decline, a larger α is more conducive to fitting the "V" shape of the curve. When the economy improves, α should be smaller.

4.2.3. Difference Method. From the above analysis of the conversion coefficient α , it can be seen that when $\alpha = 0.3$, the error of the modified model is the smallest. Therefore, the following analysis is performed based on the conversion coefficient $\alpha = 0.3$.

(1) Dynamic Difference Method. To facilitate the analysis and comparison, the actual curve of China's total electricity

TABLE 4: Comparison table of the mean squared errors underdifferent conversion coefficients.

α	Mean squared error
0	0.0423
0.1	0.0420
0.2	0.0418
0.3	0.0417
0.4	0.0418
0.5	0.0420
0.6	0.0425
0.7	0.0430
0.8	0.0437
0.9	0.0446
1	0.0455

consumption from 2000 to 2018, the fitting curve of the classical logistic model and the modified analytical model are shown in Figure 5. First, by observing Figure 5, we can see that the trends of the data that are predicted by the dynamic difference method and the modified analytical method are generally similar. Second, it can be seen that the fitting results of the dynamic difference method are very close to the actual electricity consumption data only in some years, such as 2001, 2010~2013, and 2016, but its overall accuracy is lower than those of the classic logic model and the modified analysis model.

(2) Static Difference Method. It can be seen from Figure 5 that the static difference method has higher fitting accuracy than the dynamic difference method. In addition, in the year when the actual power consumption greatly fluctuated, the fitting trend of the curve that is obtained by the static difference method is better matched with the actual power consumption trend, which means that the static difference method has a strong short-term prediction ability. As shown in Table 5, the overall accuracy of the predicted data of the static difference model is better than those of the classical logistic model and the modified analytical model, and its error is limited to 6%.

4.3. Analysis of Prediction Results

4.3.1. Long-Term and Short-Term Predictions. Based on the error analysis of the three models that are mentioned above, we can see that the static difference method has a strong short-term prediction ability and the modified analytic model has a strong long-term prediction ability, when the conversion coefficient α is reasonably selected.

According to the forecasted future economic development data of China by the OECD in Figure 6, it can be seen that China's economy will be stable in a reasonable range with a generally slow growth trend in the next 20 years. Combined with the above analysis of the conversion coefficient α , the static difference model, and the modified analytical model should set α as 0.3 when making predictions. The forecast value of electricity consumption in 2019 based on the static difference method is as follows:



FIGURE 5: Curves of the dynamic difference, correction analysis, and classic models' predicted results.

TABLE 5: Error comparison of the Chinese power consumption forecasts using different models.

Year	Logistic	$\alpha = 0.3$	Static difference	Dynamic difference
2000	0	0	0	0
2001	-0.0738	-0.0749	0.0596	0.0596
2002	-0.0312	-0.0266	0.0493	0.1101
2003	-0.0438	-0.0386	-0.0064	0.0995
2004	-0.0463	-0.0458	-0.0087	0.0863
2005	-0.0430	-0.0355	0.0177	0.1019
2006	-0.0523	-0.0449	0.0052	0.1028
2007	-0.0717	-0.0633	-0.0021	0.0949
2008	-0.0204	-0.0468	-0.0311	0.0543
2009	0.0212	0.0193	0.0340	0.0865
2010	-0.0184	-0.0113	-0.0187	0.0601
2011	-0.0417	-0.0475	-0.0420	0.0108
2012	-0.0166	-0.0266	-0.0071	0.0026
2013	-0.0139	-0.0145	0.0004	0.0028
2014	0.0151	0.0121	0.0204	0.0229
2015	0.0716	0.0691	0.0516	0.0731
2016	0.0587	0.0574	-0.0063	0.0581
2017	0.0404	0.0417	-0.0053	0.0456
2018	-0.0017	-0.0023	-0.0357	0.0027
Mean squared error	0.0423	0.0417	0.0282	0.0682

$$N(2019) = N(2018) + rN(2018) \left[1 - \frac{N(2018)}{K} \right] + 0.3 * N(2018) \left[R_{\text{GDP}}(2019) - R_{\text{GDP}}(2018) \right] = 7055.$$
(19)

Based on the modified analytical method, the long-term electricity consumption forecasted values are shown in Table 6.

$$N(t) = \frac{8700}{1 + e^{1.8651 - 0.1757t}} \cdot (1 + 0.3 \times \Delta R_{\rm GDP})$$

$$= \frac{8700}{1 + 6.4566 \cdot e^{-0.1757t}} \cdot (1 + 0.3 \times \Delta R_{\rm GDP}).$$
(20)

4.3.2. Saturated Electricity Consumption Forecasting. "The growth rate of electricity consumption is less than 2%" is a commonly used criterion for determining the saturation of electricity consumption [40]. However, when the average annual growth rate is approximately 2%, the total electricity consumption still has certain space to increase. That is, there is a certain time span between "entering the saturation stage" and "reaching the saturation stage." To better describe the changing trend of electricity consumption when it approaches the saturation value, the "growth rate of electricity consumption is less than 2%" is used as the basis for the power demand to enter the saturation stage, and "the predicted value reaches 95% of the saturation value of power consumption" is used as the basis for the saturation of electricity demand.

TABLE 6: Long-term predicted values of the modified analytic method.

Year	Predicted value (billion kWh)
2019	7068
2020	7269
2021	7476
2022	7653
2023	7805
2024	7937
2025	8051
2026	8148
2027	8231
2028	8303
2029	8364
2030	8416
2031	8460
2032	8497
2033	8528
2034	8555
2035	8577
2036	8596
2037	8612
2038	8626
2039	8637
2040	8647

TABLE 7: Predicted electricity consumption growth rate based on the modified analytic method.

Year	Growth rate of electricity consumption
2019	3.42
2020	2.73
2021	2.94
2022	2.41
2023	2.00
2024	1.70
2025	1.44
2026	1.21
2027	1.03
2028	0.87
2029	0.74
2030	0.62
2031	0.52
2032	0.44
2033	0.37
2034	0.31
2035	0.26
2036	0.22
2037	0.19
2038	0.16
2039	0.14
2040	0.11

According to the results of China's power consumption forecast, the growth rate of China's electricity consumption can be calculated (see Table 7). As shown in Table 7, China's





FIGURE 7: Elasticity of electricity consumption in the research period.

power consumption growth rate will fall below 2% in 2024. Combined with the above judgment conditions, 2024 is the year when China's electricity demand is "into the saturation stage." In 2028, China's electricity consumption will reach 95% of the saturated value *K* that is predicted by the logistic model. At this time, the total electricity consumption of the whole society will be 8303 billion kWh. Therefore, it is estimated that the year in which China's electricity demand is saturated will be between 2024 and 2028, and the time span from "entering the saturation phase" to "achieving saturation" will be approximately 3 years.

4.3.3. Elasticity of Electricity Consumption. The elasticity of electricity consumption coefficient is an important indicator reflecting the proportional relationship and change between the growth rate of power consumption and the growth rate of the national economy [21]. It is generally the ratio of the power consumption growth rate to the GDP growth rate of a country or region. The calculation equation is as follows:

elasticity of electricity consumption coefficient =	_ average annual growth rate of electricity consumption	(21)
	average annual growth rate of GDP	(21)

Figure 7 shows the elasticity of electricity consumption coefficients in different years, including the measured values that were calculated using the historical data and the predicated values that were calculated by the modified logistic model. As seen from Figure 7, due to the impact of the financial crises between 2007 and 2009 and 2013~2016, the elasticity coefficient of power consumption significantly decreased. Then, the elasticity coefficient of power consumption will be less than 0.5 starting in 2023, and it will gradually decline. On one hand, this observation shows that the national control measures, such as energy savings and emission reductions, the elimination of backward production capacity, and the reduction of power consumption per unit of GDP, have played positive roles. On the other hand, this observation also shows that China's power supply has begun to gradually restrict the development of China's national economy, and the competition in power consumption has become increasingly fierce.

5. Conclusion

In the context of the new economic normal, in order to more accurately predict power consumption, this paper introduced the change of the GDP growth rate as one of the influencing factors based on the classical logistic prediction model, and it constructed a modified logistic model based on the change of the GDP growth rate. Because we adopt different solution methods, the power consumption modified logistic model mainly includes three mathematical models: the mathematical analytic model, the dynamic difference model, and the static difference model. The prediction accuracy is evaluated using empirical analyses that include China's actual electricity consumption data from 2000 to 2018. The conclusions are as follows:

- The growth of electricity consumption has its own development law, which mainly depends on its inherent growth rate, the saturation value that the market can absorb, and the market environment at that time.
- (2) The modified logistic analytical model based on the change of the GDP growth rate has higher prediction accuracy and historical reproducibility than the classical logistic model. The static difference model has a lower error than the classical logistic model, and the dynamic difference model has a larger error than the classical logistic model. The above models are superior to the classical logistic model at reproducing historical data fluctuations.
- (3) The fluctuation trend of the fitted curve that is obtained by the modified logistic model is consistent with that of the historical data, which proves that there is a positive correlation between national economic development and power consumption.
- (4) The value of the conversion coefficient α in the modified logistic analytical model is between 0.2 and 0.6. When economic development is relatively stable, α should be intermediate. When economic development

experiences a sharp and rapid decline, α should be larger. When the economy improves, it should be smaller. Through the reasonable selection of the conversion coefficient α , we can use the future economic plan under the new normal to accurately predict future power consumption.

Economic development is inseparable from electricity consumption. In the process of economic transformation and industrial restructuring, the mode of power development must also be transformed. First, we should seize the opportunity for economic transformation, strengthen investments in technology research and development, develop more green energy, and eliminate low efficiency and highly polluting thermal power plants. By increasing investments in scientific and technological innovation, we will make clean energy account for a larger proportion of energy consumption, thus maintaining strong energy competitiveness. Second, we should balance the supply and demand of electricity well. Electric power companies should not only fully support economic construction but also rationally arrange the supply of electricity. Only in this way can we succeed in power planning during the 13th Five-Year Plan period.

Data Availability

1. The gross domestic product and GDP growth rate used to support the findings of this study have been deposited in the China Statistical Yearbook issued by National Bureau of Statistics of the People's Republic of China. http://data.stats. gov.cn/easyquery.htm?cn=C01&zb=A0201&sj=2018 http:// data.stats.gov.cn/easyquery.htm?cn=C01&zb=A0208&sj= 2018. 2. The electricity consumption and its growth rate used to support the findings of this study have been deposited in the China Statistical Data of National Electric Power Industry issued by National Energy Administration. http://www.nea.gov.cn/. 3. The real GDP long-term forecast data used to help make reasonable electricity consumption predictions have been published by the OECD on the following website: https://data.oecd.org/gdp/real-gdp-longterm-forecast.htm#indicator-chart.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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