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Research Article

An Approach for Achieving Consistency for Symmetric Trapezoidal Interval Type-2 Fuzzy Sets

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Interval type-2 fuzzy sets (IT2 FSs) are powerful tools for dealing with linguistic information in decision making. However, there is a dearth of research regarding the consistency of preference relations based on IT2 FSs. In this paper, symmetric IT2 FSs and IT2 additive preference relations are defined, whilst at the same time a mapping method is proposed to convert IT2 numbers into the corresponding linguistic terms based on the ranking values for IT2 FSs, and some properties for symmetric IT2 FSs are proved. Then, we discuss the process for achieving consistency for IT2 additive preference relations. An algorithm is developed for the IT2 additive preference relation process for achieving consistency, and some desired algorithmic properties are proved. Finally, an actual case study is used in order to demonstrate the effectiveness of the proposed approach.

1. Introduction

The type-2 fuzzy sets (T2 FSs), developed by Zadeh [1] in 1975, can overcome the limitations of type-1 fuzzy sets (T1 FSs), because T2 FSs have both the primary and secondary memberships to provide greater freedom and flexibility. However, due to their greater computational complexity, it is very difficult to apply T2 FSs in practice [2–4]. This led to the development of IT2 FSs, which can be regarded as a special type of T2 FS. IT2 FSs can effectively handle the vagueness and uncertainty [5–7], when all secondary membership values are equal to 1 [8]. In recent years, some useful results have been obtained using IT2 FSs [9–15]. For example, Mendel and his collaborators [16–22], Greenfield and Chiclala [23–26], contributed enormously to the theory of T2 FSs. In addition to the aforementioned studies, many researchers have recently focused on IT2 FSs and their applications [27–34]. For example, Qin et al. [31] studied green supplier selection in an IT2 fuzzy environment; Raju and Pillai [32] studied an application of T2 FSs for DFIG (doubly fed induction generator) based wind power plants; Sumati and Patvardhan [33] proposed an IT2 mutual subsethood fuzzy neural inference system; and Park and Shin [30] studied the robust stability conditions required to stabilize the T2 Takagi-Sugeno (T-S) fuzzy systems.

However, there is a dearth of research regarding IT2 FSs, which is called a symmetric trapezoidal IT2 FS in this paper. Symmetric trapezoidal IT2 FSs are a very special and interesting type of FS, but as far as we know, nobody has specifically focused on the symmetric trapezoidal IT2 FS or the corresponding process for achieving consistency. Therefore, this is the focus of this paper. Since the multiplication operation of T2 FSs is too complicated to be applied, an additional operation is applied in this paper.

In this paper, symmetric IT2 FSs and IT2 additive preference relations are defined, whilst a mapping method is proposed to convert IT2 numbers into the corresponding linguistic terms based on the ranking values for IT2 FSs, and some properties for symmetric IT2 FSs are proved. Then, we discuss the process for achieving consistency for IT2 additive preference relations. An algorithm is developed for the IT2 additive preference relation process for achieving consistency, and some desired algorithmic properties are proved. Finally, an example is provided to illustrate the algorithm.

The rest of this paper is organized as follows. In Section 2, we briefly review the concepts and properties for IT2 FSs. In

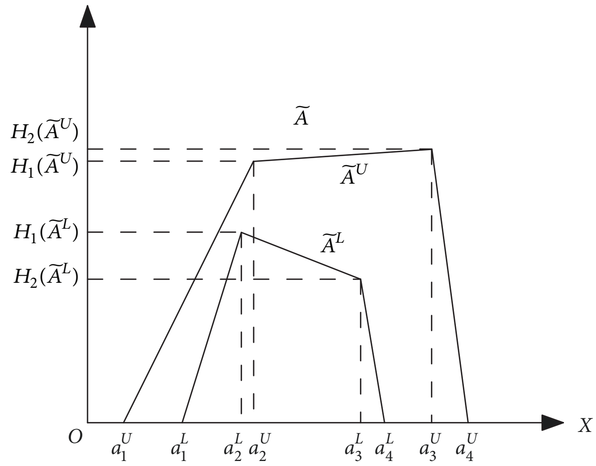

Figure 1: Trapezoidal IT2 FS $\widetilde{A} = (\widetilde{A}^U, \widetilde{A}^L)$.

Section 3, we define symmetric trapezoidal IT2 FSs and IT2 additive preference relation, meanwhile propose a mapping method to convert IT2 FSs into the corresponding linguistic terms based on the ranking values for IT2 FSs, and then discuss some of the properties of symmetric trapezoidal IT2 FSs and common IT2 FSs. In Section 4, we handle GDM problems, such as the process for achieving consistency for IT2 additive preference relation. We then reference conventional method and generalize it to IT2 FSs, develop an algorithm for the IT2 additive preference relation consistency reaching process, and present some properties of the algorithm. Finally, an illustrative example is used to verify the algorithm.

2. Preliminaries

A T2 FS $\widetilde{A}$ [15, 35, 36] in the universe of discourse X can be represented by a type-2 membership function $\mu_{\widetilde{A}}$, where

$$\widetilde{A} = \{((x,u), \mu_{\widetilde{A}}(x,u)) \mid x \in X, u \in [0,1]\} \tag{1}$$

Let $\widetilde{A}$ be a T2 FS represented by a T2 membership function $\mu_{\widetilde{A}}$. If $\mu_{\widetilde{A}}(x,u) = 1$ for arbitrary $x \in X$ and arbitrary $u \in [0,1]$, then $\widetilde{A}$ is called an IT2 FS [36]. It should be noted that the upper membership function and the lower membership function in an IT2 FS are both type-1 membership functions, respectively [36].

An IT2 FS is called trapezoidal IT2 FS where the upper membership function and lower membership function are both trapezoidal fuzzy numbers [37]. Figure 1 shows a trapezoidal IT2 FS $\widetilde{A}$ in the universe of discourse X represented by

$$\begin{aligned}\widetilde{A} &= \left(\widetilde{A}^U, \widetilde{A}^L\right) = \left(\left(a_1^U, a_2^U, a_3^U, a_{4;}^U H_1\left(\widetilde{A}^U\right), H_2\left(\widetilde{A}^U\right)\right),\right.\\ &\left.\left(a_1^L, a_2^L, a_3^L, a_{4;}^L H_1\left(\widetilde{A}^L\right), H_2\left(\widetilde{A}^L\right)\right)\right),\end{aligned} \tag{2}$$

where both $\widetilde{A}^U$ and $\widetilde{A}^L$ are type-1 FSs, $a_1^U, a_2^U, a_3^U, a_{4;}^U$ and $a_1^L, a_2^L, a_3^L, a_{4;}^L$ are the reference points of the trapezoidal IT2 FS $\widetilde{A}$, $H_j(\widetilde{A}^U)$ denotes the membership value of the element a_{j+1}^U in the upper trapezoidal membership function $\widetilde{A}^U$, $j = 1, 2$, $H_j(\widetilde{A}^L)$ denotes the membership value of the element a_{j+1}^U in the lower trapezoidal membership function $\widetilde{A}^L$, $j = 1, 2$, $H_1(\widetilde{A}^U) \in [0,1], H_2(\widetilde{A}^U) \in [0,1], H_1(\widetilde{A}^L) \in [0,1], H_2(\widetilde{A}^L) \in [0,1]$ [10].

Suppose that there are two trapezoidal IT2 FSs, denoted as

$$\begin{aligned}\widetilde{A}_1 &= \left(\widetilde{A}_1^U, \widetilde{A}_1^L\right)\\ &= \left(\left(a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1\left(\widetilde{A}_1^U\right), H_2\left(\widetilde{A}_1^U\right)\right),\right.\\ &\left.\left(a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1\left(\widetilde{A}_1^L\right), H_2\left(\widetilde{A}_1^L\right)\right)\right)\\ \widetilde{A}_2 &= \left(\widetilde{A}_2^U, \widetilde{A}_2^L\right)\\ &= \left(\left(a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1\left(\widetilde{A}_2^U\right), H_2\left(\widetilde{A}_2^U\right)\right),\right.\\ &\left.\left(a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1\left(\widetilde{A}_2^L\right), H_2\left(\widetilde{A}_2^L\right)\right)\right)\end{aligned} \tag{3}$$

Several defects have been found in the traditional IT2 FSs operations; for example, it is not feasible to only consider the minimum membership of upper and lower membership functions, because this loses sight of the influence of the larger membership function. Then operational rules are defined as follows [8]:

$$\begin{aligned}\widetilde{A}_1 + \widetilde{A}_2 &= \left(\widetilde{A}_1^U, \widetilde{A}_1^L\right) + \left(\widetilde{A}_2^U, \widetilde{A}_2^L\right) = \left(\left(a_{11}^U + a_{21}^U, a_{12}^U\right.\right.\\ &+ a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; H_1\left(\widetilde{A}_1^U\right) + H_1\left(\widetilde{A}_2^U\right)\\ &- H_1\left(\widetilde{A}_1^U\right) \times H_1\left(\widetilde{A}_2^U\right), H_2\left(\widetilde{A}_1^U\right) + H_2\left(\widetilde{A}_2^U\right)\\ &\left.- H_2\left(\widetilde{A}_1^U\right) \times H_2\left(\widetilde{A}_2^U\right)\right), \left(a_{11}^L + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L\right.\\ &+ a_{23}^L, a_{14}^L + a_{24}^L; H_1\left(\widetilde{A}_1^L\right) + H_1\left(\widetilde{A}_2^L\right) - H_1\left(\widetilde{A}_1^L\right)\\ &\times H_1\left(\widetilde{A}_2^L\right), H_2\left(\widetilde{A}_1^L\right) + H_2\left(\widetilde{A}_2^L\right) - H_2\left(\widetilde{A}_1^L\right)\\ &\left.\left.\times H_2\left(\widetilde{A}_2^L\right)\right)\right),\\ \widetilde{A}_1 - \widetilde{A}_2 &= \left(\widetilde{A}_1^U, \widetilde{A}_1^L\right) - \left(\widetilde{A}_2^U, \widetilde{A}_2^L\right) = \left(\left(a_{11}^U - a_{24}^U, a_{12}^U\right.\right.\\ &- a_{23}^U, a_{13}^U - a_{22}^U, a_{14}^U - a_{21}^U; H_1\left(\widetilde{A}_1^U\right) + H_1\left(\widetilde{A}_2^U\right)\\ &- H_1\left(\widetilde{A}_1^U\right) \times H_1\left(\widetilde{A}_2^U\right), H_2\left(\widetilde{A}_1^U\right) + H_2\left(\widetilde{A}_2^U\right)\\ &\left.- H_2\left(\widetilde{A}_1^U\right) \times H_2\left(\widetilde{A}_2^U\right)\right), \left(a_{11}^L - a_{24}^L, a_{12}^L - a_{23}^L, a_{13}^L\right.\\ &- a_{22}^L, a_{14}^L - a_{21}^L; H_1\left(\widetilde{A}_1^L\right) + H_1\left(\widetilde{A}_2^L\right) - H_1\left(\widetilde{A}_1^L\right)\\ &\times H_1\left(\widetilde{A}_2^L\right), H_2\left(\widetilde{A}_1^L\right) + H_2\left(\widetilde{A}_2^L\right) - H_2\left(\widetilde{A}_1^L\right)\\ &\left.\left.\times H_2\left(\widetilde{A}_2^L\right)\right)\right),\end{aligned}$$

$$\lambda\widetilde{A}_1 = \left(\left(\lambda a_{11}^U, \lambda a_{12}^U, \lambda a_{13}^U, \lambda a_{14}^U; 1 - \left(1 - H_1\left(\widetilde{A}_1^U\right)\right)^\lambda, 1 - \left(1 - H_2\left(\widetilde{A}_1^U\right)\right)^\lambda\right), \left(\lambda a_{11}^L, \lambda a_{12}^L, \lambda a_{13}^L, \lambda a_{14}^L; 1 - \left(1 - H_1\left(\widetilde{A}_1^L\right)\right)^\lambda, 1 - \left(1 - H_2\left(\widetilde{A}_1^L\right)\right)^\lambda\right)\right), \quad \lambda \in R \tag{4}$$

It should be pointed out that in this paper only trapezoidal IT2 FSs, also called trapezoidal IT2 fuzzy numbers, are discussed.

3. Symmetric IT2 Fuzzy Numbers and IT2 Additive Preference Relations

In this section, we present a definition for symmetric IT2 fuzzy numbers and IT2 additive preference relations and then discuss some properties of the symmetric IT2 fuzzy numbers and common IT2 fuzzy numbers.

3.1. Definition for Symmetric IT2 Fuzzy Numbers and IT2 Additive Preference Relations. In the following, we consider the concept of fuzzy preference relations based on IT2 FSs which can be depicted by a linguistic judgment matrix $M = (\widetilde{A}_{ij})_{n\times n}, i, j = 1, 2, \ldots, n$, where $\widetilde{A}_{ij}$ is a trapezoidal IT2 FS in the universe of discourse X.

In general, if $((a_1^U, a_2^U, a_3^U, a_4^U; H_1(\widetilde{A}^U), H_2(\widetilde{A}^U)), (a_1^L, a_2^L, a_3^L, a_4^L; H_1(\widetilde{A}^L), H_2(\widetilde{A}^L)))$ represents a linguistic term, then $((1 - a_4^U, 1 - a_3^U, 1 - a_2^U, 1 - a_1^U; H_1(\widetilde{A}^U), H_2(\widetilde{A}^U)), (1 - a_4^L, 1 - a_3^L, 1 - a_2^L, 1 - a_1^L; H_1(\widetilde{A}^L), H_2(\widetilde{A}^L)))$ can represent the opposite linguistic term.

For example, for the linguistic term "very low," the corresponding trapezoidal IT2 FS is $((0, 0, 0, 0.1; 1, 1), (0, 0, 0, 0.05; 0.9, 0.9))$.

Moreover, for the linguistic term "very high," the corresponding trapezoidal IT2 FS is $((0.9, 1, 1, 1; 1, 1), (0.95, 1, 1, 1; 1, 1))$.

For the linguistic term "low," the corresponding trapezoidal IT2 FS is $((0, 0.1, 0.1, 0.3; 1, 1), (0.05, 0.1, 0.1, 0.2; 0.9, 0.9))$.

For the linguistic term "high," the corresponding trapezoidal IT2 FS is $((0.7, 0.9, 0.9, 1; 1, 1), (0.8, 0.9, 0.9, 0.95; 0.9, 0.9))$, etc. [38, 39].

In this paper we take a subscript-symmetric linguistic evaluation scale [40]:

$$S = \{s_\alpha \mid \alpha = -\tau, \cdots, -1, 0, 1, \cdots, \tau\} \tag{5}$$

Here we take the cardinality value of S as 9; namely $\tau = 4$ [41]. Since additive preference relations are considered in this paper, for the linguistic term s_α, the opposite linguistic term is $s_{-\alpha}$ [42].

Table 1: Linguistic terms and their corresponding trapezoidal IT2 FSs.

Linguistic Terms	Corresponding Trapezoidal IT2 FSs
Very Low	((0, 0, 0, 0.1; 1, 1), (0, 0, 0, 0.05; 0.9, 0.9))
Low	((0, 0.1, 0.1, 0.3; 1, 1), (0.05, 0.1, 0.1, 0.2; 0.9, 0.9))
Medium Low	((0.1, 0.3, 0.3, 0.5; 1, 1), (0.2, 0.3, 0.3, 0.4; 0.9, 0.9))
Medium	((0.3, 0.5, 0.5, 0.7; 1, 1), (0.4, 0.5, 0.5, 0.6; 0.9, 0.9))
Medium High	((0.5, 0.7, 0.7, 0.9; 1, 1), (0.6, 0.7, 0.7, 0.8; 0.9, 0.9))
High	((0.7, 0.9, 0.9, 1; 1, 1), (0.8, 0.9, 0.9, 0.95; 0.9, 0.9))
Very High	((0.9, 1, 1, 1; 1, 1), (0.95, 1, 1, 1; 0.9, 0.9))

Generally, if we denote an IT2 fuzzy number $\widetilde{A}$ in the form of the linguistic term:

$$\widetilde{A} = \left(\left(a_1^U, a_2^U, a_3^U, a_4^U; H_1\left(\widetilde{A}^U\right), H_2\left(\widetilde{A}^U\right)\right), \left(a_1^L, a_2^L, a_3^L, a_4^L; H_1\left(\widetilde{A}^L\right), H_2\left(\widetilde{A}^L\right)\right)\right) = s_\alpha \quad (-4 \le \alpha \le 4), \tag{6}$$

then we can denote

$$\left(\left(1 - a_4^U, 1 - a_3^U, 1 - a_2^U, 1 - a_1^U; H_1\left(\widetilde{A}^U\right), H_2\left(\widetilde{A}^U\right)\right), \left(1 - a_4^L, 1 - a_3^L, 1 - a_2^L, 1 - a_1^L; H_1\left(\widetilde{A}^L\right), H_2\left(\widetilde{A}^L\right)\right)\right) = s_{-\alpha}. \tag{7}$$

For convenience, we call $s_{-\alpha}, s_\alpha$ $(-4 \le \alpha \le 4)$ a couple of symmetric IT2 fuzzy numbers, or one is a symmetric IT2 fuzzy number of the other.

In this paper, we generally take $-4 < \alpha < 4$ in addition to some specified situations where $\alpha = 4$ or $\alpha = -4$. As a special case, we also call $s_{-4} = ((0, 0, 0, 0, ; 0, 0), (0, 0, 0, 0; 0, 0))$ and $s_4 = ((1, 1, 1, 1; 1, 1), (1, 1, 1, 1; 1, 1))$ are a couple of symmetric IT2 fuzzy numbers, or one is a symmetric IT2 fuzzy number of the other. The symmetric IT2 fuzzy number of $s_0 = ((0.3, 0.5, 0.5, 0.7; 1, 1), (0.4, 0.5, 0.5, 0.6; 0.9, 0.9))$ is itself.

In this paper, all linguistic terms $s_\alpha, s_\beta, s_\gamma$, etc., denote IT2 fuzzy numbers.

Example 1. Table 1 shows the linguistic terms and their corresponding IT2 FSs [10, 38, 39, 43, 44].

If the lower boundary $((0, 0, 0, 0; 0, 0), (0, 0, 0, 0; 0, 0))$ and the upper boundary $((1, 1, 1, 1; 1, 1), (1, 1, 1, 1; 1, 1))$ are added to them, they also can be denoted with a set as follows:

$$S_1 = \{s_{-4} = ((0, 0, 0, 0; 0, 0), (0, 0, 0, 0; 0, 0)), s_{-3} = ((0, 0, 0, 0.1; 1, 1), (0, 0, 0, 0.05; 0.9, 0.9)), s_{-2} = ((0, 0.1, 0.1, 0.3; 1, 1), (0.05, 0.1, 0.1, 0.2; 0.9, 0.9)), s_{-1} = ((0.1, 0.3, 0.3, 0.5; 1, 1), (0.2, 0.3, 0.3, 0.4; 0.9, 0.9)), s_0 = ((0.3, 0.5, 0.5, 0.7; 1, 1),$$

$$(0.4, 0.5, 0.5, 0.6; 0.9, 0.9)), s_1 = ((0.5, 0.7, 0.7, 0.9; 1, 1), (0.6, 0.7, 0.7, 0.8; 0.9, 0.9)), s_2 = ((0.7, 0.9, 0.9, 1; 1, 1), (0.8, 0.9, 0.9, 0.95; 0.9, 0.9)), s_3 = ((0.9, 1, 1, 1; 1, 1), (0.95, 1, 1, 1; 0.9, 0.9)), s_4 = ((1, 1, 1, 1; 1, 1), (1, 1, 1, 1; 1, 1))\} \quad (8)$$

Definition 2. The linguistic judgment matrix for an IT2 FS $M = (\widetilde{A}_{ij})_{n\times n}, i, j = 1, 2, \ldots, n$ is called an IT2 additive judgement matrix, or an IT2 FS symmetric judgement matrix, if $\widetilde{A}_{ij}$ and $\widetilde{A}_{ji}$ are a couple of symmetric IT2 fuzzy numbers for any given $i, j = 1, 2, \ldots, n$. The corresponding preference relation is called an IT2 additive preference relation, or an IT2 symmetric preference relation.

For example, consider an IT2 judgement matrix as follows:

$$B = \begin{bmatrix} s_0 & s_2 & s_{-1} & s_2 \\ s_{-2} & s_0 & s_3 & s_2 \\ s_1 & s_{-3} & s_0 & s_3 \\ s_{-2} & s_{-2} & s_{-3} & s_0 \end{bmatrix} \quad (9)$$

In the matrix B, $\widetilde{A}_{ii} = s_0$, if $\widetilde{A}_{ij} = s_\alpha$, then $\widetilde{A}_{ji} = s_{-\alpha}, i, j = 1, 2, \ldots, 5$. The matrix B is an IT2 FS symmetric judgement matrix.

3.2. The Proposed Mapping Method Based on the Ranking Value for IT2 Fuzzy Sets. In this section, the three ranking formulas based on the average operator for ranking the IT2 FSs are first reviewed. In the previous section, all the linguistic labels s_α ($\alpha = -4, \cdots, -1, 0, 1, \cdots, 4$) denote specific IT2 numbers which are assigned by the authors. However, for any common IT2 fuzzy number, such as

$$\widetilde{A}_e = ((0.45, 0.55, 0.85, 0.95; 1, 1), (0.55, 0.65, 0.85, 0.95; 0.9, 0.9)), \quad (10)$$

how can such a number be converted into the corresponding form of a linguistic term? In order to resolve this problem, the following mapping method is proposed, which is based on the ranking value method.

Definition 3. Let

$$\widetilde{A} = (\widetilde{A}^U, \widetilde{A}^L) = ((a_1^U, a_2^U, a_3^U, a_4^U; H_1(\widetilde{A}^U), H_2(\widetilde{A}^U)), (a_1^L, a_2^L, a_3^L, a_4^L; H_1(\widetilde{A}^L), H_2(\widetilde{A}^L))) \quad (11)$$

be an IT2 FS; then the ranking value of A can be defined as follows [45]:

$$R_{(1)}(\widetilde{A}) = \left(\frac{a_1^U + a_4^U}{2} + \frac{\sum_{k=L}^{U}(H_1(\widetilde{A}^k) + H_2(\widetilde{A}^k))}{4}\right) \times \frac{\sum_{i=1}^4 (a_i^U + a_i^L)}{8}$$

$$R_{(2)}(\widetilde{A}) = \left(\sqrt{a_1^U a_4^U} + \left(\prod_{k=L}^{U} H_1(\widetilde{A}^k) H_2(\widetilde{A}^k)\right)^{1/4}\right) \times \sqrt[8]{\prod_{i=1}^{4} a_i^U a_i^L} \quad (12)$$

$$R_{(3)}(\widetilde{A}) = \left(\frac{2a_1^U a_4^U}{a_1^U + a_4^U} + \frac{4}{\sum_{k=L}^{U}((H_1(\widetilde{A}^k) + H_2(\widetilde{A}^k))/H_1(\widetilde{A}^k) H_2(\widetilde{A}^k))}\right) \times \frac{8}{\sum_{i=1}^4 (1/a_i^U + 1/a_i^L)}$$

For convenience, $R_{(1)}(\widetilde{A})$ is called an arithmetic average ranking value of IT2 FS A; $R_{(2)}(\widetilde{A})$ is called a geometric average ranking value of IT2 FS A; and $R_{(3)}(\widetilde{A})$ is referred to as a harmonic average ranking value of IT2 FS A, respectively.

In some special cases, the denominator in $R_{(3)}(\widetilde{A})$ may be zero, and $R_{(3)}(\widetilde{A})$ can be modified as follows [45]:

$$R_{(3)}(\widetilde{A}) = \lim_{\varepsilon \to 0}\left(\frac{2(a_1^U + \varepsilon)(a_4^U + \varepsilon)}{a_1^U + a_4^U + 2\varepsilon} + \frac{4}{\sum_{k=L}^{U}((H_1(\widetilde{A}^k) + H_2(\widetilde{A}^k) + 2\varepsilon)/(H_1(\widetilde{A}^k) + \varepsilon)(H_2(\widetilde{A}^k) + \varepsilon))}\right) \times \frac{8}{\sum_{i=1}^4 (1/(a_i^U + \varepsilon) + 1/(a_i^L + \varepsilon))} \quad (13)$$

where ε is an arbitrarily small positive number.

As shown in Table 2, if the universe of discourse X is the interval $[0, 1]$, then $R_1, R_2, R_3 \subseteq [0, 2]$ and $R_2(s_{-4}) = R_2(s_{-3}) = R_2(s_{-2}) = 0$, $R_3(s_{-4}) = R_3(s_{-3}) = R_3(s_{-2}) = R_3(s_{-1}) = 0$; by contrast R_1 is better able to distinguish s_{-4}, s_{-3}, s_{-2}; therefore R_1 is used as the criterion.

Table 2: IT2 FSs and their ranking value [10, 38, 39, 42–45].

linguistic terms	IT2 FSs	$R_{(1)}$	$R_{(2)}$	$R_{(3)}$
s_{-4}	((0,0,0,0;0,0),(0,0,0,0;0,0))	0	0	0
s_{-3}	((0,0,0,0.1;1,1),(0,0,0,0.05;0.9,0.9))	0.0188	0	0
s_{-2}	((0,0.1,0.1,0.3;1,1),(0.05,0.1,0.1,0.2;0.9,0.9))	0.1306	0	0
s_{-1}	((0.1,0.3,0.3,0.5;1,1),(0.2,0.3,0.3,0.4;0.9,0.9))	0.375	0.3220	0
s_0	((0.3,0.5,0.5,0.7;1,1),(0.4,0.5,0.5,0.6;0.9,0.9))	0.725	0.6848	0.6462
s_1	((0.5,0.7,0.7,0.9;1,1),(0.6,0.7,0.7,0.8;0.9,0.9))	1.155	1.1188	1.083
s_2	((0.7,0.9,0.9,1;1,1),(0.8,0.9,0.9,0.95;0.9,0.9))	1.586	1.565	1.544
s_3	((0.9,1,1,1;1,1),(0.95,1,1,1;0.9,0.9))	1.864	1.861	1.857
s_4	((1,1,1,1;1,1),(1,1,1,1;1,1))	2	2	2

We call $\widetilde{A}_s \geq \widetilde{A}_t$, if and only if $R_1(\widetilde{A}_s) \geq R_1(\widetilde{A}_t)$, and $\widetilde{A}_s > \widetilde{A}_t$ if and only if $R_1(\widetilde{A}_s) > R_1(\widetilde{A}_t)$.

For any other IT2 fuzzy number, such as

$$\widetilde{A}_e = ((0.45, 0.55, 0.85, 0.95; 1, 1), (0.55, 0.65, 0.85, 0.95; 0.9, 0.9)), \tag{14}$$

its ranking value can be calculated: $R_1(\widetilde{A}_e) = 1.196$; then it can be found that $R_1(s_1) \leq R_1(\widetilde{A}_e) \leq R_1(s_2)$, because $R_1(s_1) = 1.155$, $R_1(s_2) = 1.586$, a mapping can be constructed:

$$\begin{gathered}\Omega : R_1(\widetilde{A}) \longrightarrow \beta(\widetilde{A}), \\ R_1(\widetilde{A}) \subseteq [1.155, 1.586], \\ \beta(\widetilde{A}) \subseteq [1, 2]\end{gathered} \tag{15}$$

where $\beta(\widetilde{A})$ is the subscript of the corresponding linguistic label of $\widetilde{A}$, which can be denoted simply as β if it does not cause any confusion.

For example, by using a linear function, we can in fact look on the graph of the function as a straight line passing through $(1.155, 1)$ and $(1.586, 2)$, and the horizontal axis is $R_1(\widetilde{A})$, the vertical axis is $\beta(\widetilde{A})$, $\beta(\widetilde{A})$ is the subscript of the corresponding linguistic label of $\widetilde{A}$, and subsequently a linear equation is obtained:

$$\beta(\widetilde{A}) = \frac{R_1(\widetilde{A}) - 1.155}{1.586 - 1.155} + 1 \tag{16}$$

By substituting the value of $R_1(\widetilde{A}_e)$ into the equation, we have

$$\beta(\widetilde{A}) = \frac{1.196 - 1.155}{1.586 - 1.155} + 1 = 1.095, \tag{17}$$

i.e., $\widetilde{A}_e = s_{1.095}$.

Definition 4. Generally, if

$$R_1(s_\alpha) \leq R_1(\widetilde{A}) \leq R_1(s_{\alpha+1}) \tag{18}$$

where α is an integer, then, a linear mapping is constructed:

$$\begin{gathered}\Omega : R_1(\widetilde{A}) \longrightarrow \beta(\widetilde{A}) \\ R_1(\widetilde{A}) \in [0, 2], \\ \beta(\widetilde{A}) \in [-4, 4]\end{gathered} \tag{19}$$

$\beta(\widetilde{A})$ is the subscript of the corresponding linguistic label of IT2 FS $\widetilde{A}$. We can in fact look on the graph of the function as a straight line passing through $(R_1(s_\alpha), \alpha)$ and $(R_1(s_{\alpha+1}), \alpha+1)$, and the horizontal axis is $R_1(\widetilde{A})$; the vertical axis is $\beta(\widetilde{A})$. Then a linear equation is obtained:

$$\beta(\widetilde{A}) = \frac{R_1(\widetilde{A}) - R_1(s_\alpha)}{R_1(s_{\alpha+1}) - R_1(s_\alpha)} + \alpha \tag{20}$$

By substituting the value of $R_1(\widetilde{A})$ into the equation, the value of $\beta(\widetilde{A})$ is obtained.

The above method is called the linear mapping method.

Finally, when the element $\widetilde{A}_{ij}, i \leq j$ in an IT2 judgement matrix $(\widetilde{A}_{ij})_{n\times n}$ is changed to S_α, in order to maintain complementarity, $\widetilde{A}_{ji}$ will be recommended to be changed to $S_{-\alpha}$. Indeed, it should be pointed out that the mapping method is simply an approximate calculation method.

3.3. Some Properties of Symmetric IT2 Fuzzy Numbers. In this section we discuss some properties of symmetric IT2 fuzzy numbers and common IT2 fuzzy numbers.

Proposition 5 (law of commutation). *$s_\alpha + s_\beta = s_\beta + s_\alpha$, where s_α and s_β are arbitrary IT2 fuzzy numbers.*

Proof. According to the definition of the additive IT2 fuzzy numbers operation can be directly determined. □

Proposition 6 (law of association). *$(s_\alpha + s_\beta) + s_\gamma = s_\alpha + (s_\beta + s_\gamma)$, where s_α, s_β, and s_γ are arbitrary IT2 fuzzy numbers.*

Proof. Suppose that

$$\begin{aligned} s_\alpha &= \left(\left(a_1^U, a_2^U, a_3^U, a_4^U; H_1\left(s_\alpha^U\right), H_2\left(s_\alpha^U\right)\right),\right. \\ &\quad \left.\left(a_1^L, a_2^L, a_3^L, a_4^L; H_1\left(s_\alpha^L\right), H_2\left(s_\alpha^L\right)\right)\right), \\ s_\beta &= \left(\left(b_1^U, b_2^U, b_3^U, b_4^U; H_1\left(s_\beta^U\right), H_2\left(s_\beta^U\right)\right),\right. \\ &\quad \left.\left(b_1^L, b_2^L, b_3^L, b_4^L; H_1\left(s_\beta^L\right), H_2\left(s_\beta^L\right)\right)\right), \\ s_\gamma &= \left(\left(c_1^U, c_2^U, c_3^U, c_4^U; H_1\left(s_\gamma^U\right), H_2\left(s_\gamma^U\right)\right),\right. \\ &\quad \left.\left(b_1^L, b_2^L, b_3^L, b_4^L; H_1\left(s_\gamma^L\right), H_2\left(s_\gamma^L\right)\right)\right) \end{aligned} \tag{21}$$

due to

$$\begin{aligned} &\left[H_1\left(s_\alpha^U\right) + H_1\left(s_\beta^U\right) - H_1\left(s_\alpha^U\right) \times H_1\left(s_\beta^U\right)\right] + H_1\left(s_\gamma^U\right) \\ &\quad - \left[H_1\left(s_\alpha^U\right) + H_1\left(s_\beta^U\right) - H_1\left(s_\alpha^U\right) \times H_1\left(s_\beta^U\right)\right] \\ &\quad \times H_1\left(s_\gamma^U\right) \\ &= H_1\left(s_\alpha^U\right) + H_1\left(s_\beta^U\right) + H_1\left(s_\gamma^U\right) - H_1\left(s_\alpha^U\right) \\ &\quad \times H_1\left(s_\beta^U\right) - H_1\left(s_\alpha^U\right) \times H_1\left(s_\gamma^U\right) - H_1\left(s_\beta^U\right) \\ &\quad \times H_1\left(s_\gamma^U\right) + H_1\left(s_\alpha^U\right) \times H_1\left(s_\beta^U\right) \times H_1\left(s_\gamma^U\right) \end{aligned} \tag{22}$$

and

$$\begin{aligned} &H_1\left(s_\alpha^U\right) + \left[H_1\left(s_\beta^U\right) + H_1\left(s_\gamma^U\right)\right] - H_1\left(s_\beta^U\right) \\ &\quad \times H_1\left(s_\gamma^U\right)\right] - H_1\left(s_\alpha^U\right) \times \left[H_1\left(s_\beta^U\right) + H_1\left(s_\gamma^U\right)\right] \\ &\quad - H_1\left(s_\beta^U\right) \times H_1\left(s_\gamma^U\right)\right] \\ &= H_1\left(s_\alpha^U\right) + H_1\left(s_\beta^U\right) + H_1\left(s_\gamma^U\right) - H_1\left(s_\alpha^U\right) \\ &\quad \times H_1\left(s_\beta^U\right) - H_1\left(s_\alpha^U\right) \times H_1\left(s_\gamma^U\right) - H_1\left(s_\beta^U\right) \\ &\quad \times H_1\left(s_\gamma^U\right) + H_1\left(s_\alpha^U\right) \times H_1\left(s_\beta^U\right) \times H_1\left(s_\gamma^U\right) \end{aligned} \tag{23}$$

In the same way, we can derive the remaining; i.e., $(s_\alpha + s_\beta) + s_\gamma$ and $s_\alpha + (s_\beta + s_\gamma)$ have the same height values.

Consequently,

$$\begin{aligned} &\left(s_\alpha + s_\beta\right) + s_\gamma = \left(\left(a_1^U + b_1^U + c_1^U, a_2^U + b_2^U + c_2^U, a_3^U\right.\right. \\ &\quad + b_3^U + c_3^U, a_4^U + b_4^U + c_4^U; H_1\left(s_\alpha^U\right) + H_1\left(s_\beta^U\right) \\ &\quad + H_1\left(s_\gamma^U\right) - H_1\left(s_\alpha^U\right) \times H_1\left(s_\beta^U\right) - H_1\left(s_\alpha^U\right) \\ &\quad \times H_1\left(s_\gamma^U\right) - H_1\left(s_\beta^U\right) \times H_1\left(s_\gamma^U\right) + H_1\left(s_\alpha^U\right) \\ &\quad \times H_1\left(s_\beta^U\right) \times H_1\left(s_\gamma^U\right); H_2\left(s_\alpha^U\right) + H_2\left(s_\beta^U\right) \\ &\quad + H_2\left(s_\gamma^U\right) - H_2\left(s_\alpha^U\right) \times H_2\left(s_\beta^U\right) - H_2\left(s_\alpha^U\right) \\ &\quad \times H_2\left(s_\gamma^U\right) - H_2\left(s_\beta^U\right) \times H_2\left(s_\gamma^U\right) + H_2\left(s_\alpha^U\right) \\ &\quad \times H_2\left(s_\beta^U\right) \times H_2\left(s_\gamma^U\right); \left(a_1^U + b_1^U + c_1^U, a_2^U + b_2^U\right. \\ &\quad + c_2^U, a_3^U + b_3^U + c_3^U, a_4^U + b_4^U + c_4^U; H_1\left(s_\alpha^U\right) \\ &\quad + H_1\left(s_\beta^U\right) + H_1\left(s_\gamma^U\right) - H_1\left(s_\alpha^U\right) \times H_1\left(s_\beta^U\right) \\ &\quad - H_1\left(s_\alpha^U\right) \times H_1\left(s_\gamma^U\right) - H_1\left(s_\beta^U\right) \times H_1\left(s_\gamma^U\right) \\ &\quad + H_1\left(s_\alpha^U\right) \times H_1\left(s_\beta^U\right) \times H_1\left(s_\gamma^U\right); H_2\left(s_\alpha^U\right) \\ &\quad + H_2\left(s_\beta^U\right) + H_2\left(s_\gamma^U\right) - H_2\left(s_\alpha^U\right) \times H_2\left(s_\beta^U\right) \\ &\quad - H_2\left(s_\alpha^U\right) \times H_2\left(s_\gamma^U\right) - H_2\left(s_\beta^U\right) \times H_2\left(s_\gamma^U\right) \\ &\quad \left.\left.+ H_2\left(s_\alpha^U\right) \times H_2\left(s_\beta^U\right) \times H_2\left(s_\gamma^U\right)\right)\right); = s_\alpha + \left(s_\beta\right. \\ &\quad \left.+ s_\gamma\right). \end{aligned} \tag{24}$$

This completes the proof for Proposition 6. □

Proposition 7. *$s_0 - s_\alpha = s_{-\alpha} - s_0$, where s_α and $s_{-\alpha}$ denote a couple of symmetric IT2 fuzzy numbers, s_0 =* ((0.3, 0.5, 0.5, 0.7; 1, 1), (0.4, 0.5, 0.5, 0.6; 0.9, 0.9)).

Proof. Suppose that

$$\begin{aligned} s_\alpha &= \left(\left(a_1^U, a_2^U, a_3^U, a_4^U; H_1\left(s_\alpha^U\right), H_2\left(s_\alpha^U\right)\right),\right. \\ &\quad \left.\left(a_1^L, a_2^L, a_3^L, a_4^L; H_1\left(s_\alpha^L\right), H_2\left(s_\alpha^L\right)\right)\right). \end{aligned} \tag{25}$$

Then

$$\begin{aligned} s_{-\alpha} &= \left(\left(1 - a_4^U, 1 - a_3^U, 1 - a_2^U, 1\right.\right. \\ &\quad \left. - a_1^U; H_1\left(s_\alpha^U\right), H_2\left(s_\alpha^U\right)\right), \left(1 - a_4^L, 1 - a_3^L, 1 - a_2^L, 1\right. \\ &\quad \left.\left. - a_1^L; H_1\left(s_\alpha^L\right), H_2\left(s_\alpha^L\right)\right)\right), \end{aligned} \tag{26}$$

due to s_0 = ((0.3, 0.5, 0.5, 0.7; 1, 1), (0.4, 0.5, 0.5, 0.6; 0.9, 0.9)); therefore

$$\begin{aligned} s_{-\alpha} - s_0 &= \left(\left(0.3 - a_4^U, 0.5 - a_3^U, 0.5 - a_2^U, 0.7\right.\right. \\ &\quad \left. - a_1^U; 1, 1\right), \left(0.4 - a_4^L, 0.5 - a_3^L, 0.5 - a_2^L, 0.6\right. \\ &\quad \left.\left. - a_1^L; 0.1 \times H_1\left(s_\alpha^L\right) + 0.9, 0.1 \times H_2\left(s_\alpha^L\right) + 0.9\right)\right), \\ &= s_0 - s_\alpha, \end{aligned} \tag{27}$$

which completes the proof. □

Proposition 8. *$s_{-\alpha} - s_{-\beta} = s_\beta - s_\alpha$, where s_α and $s_{-\alpha}$, s_β and $s_{-\beta}$ are, respectively, symmetric IT2 fuzzy numbers.*

Proof. Suppose that

$$\begin{aligned} s_\alpha &= \left(\left(a_1^U, a_2^U, a_3^U, a_4^U; H_1\left(s_\alpha^U\right), H_2\left(s_\alpha^U\right)\right),\right. \\ &\quad \left.\left(a_1^L, a_2^L, a_3^L, a_4^L; H_1\left(s_\alpha^L\right), H_2\left(s_\alpha^L\right)\right)\right). \\ s_\beta &= \left(\left(b_1^U, b_2^U, b_3^U, b_4^U; H_1\left(s_\beta^U\right), H_2\left(s_\beta^U\right)\right),\right. \\ &\quad \left.\left(b_1^L, b_2^L, b_3^L, b_4^L; H_1\left(s_\beta^L\right), H_2\left(s_\beta^L\right)\right)\right), \end{aligned} \tag{28}$$

Then

$$\begin{aligned} s_{-\alpha} &= \left(\left(1-a_4^U, 1-a_3^U, 1-a_2^U, 1\right.\right. \\ &\quad \left. - a_1^U; H_1\left(s_\alpha^U\right), H_2\left(s_\alpha^U\right)\right), \left(1-a_4^L, 1-a_3^L, 1-a_2^L, 1\right. \\ &\quad \left.\left. - a_1^L; H_1\left(s_\alpha^L\right), H_2\left(s_\alpha^L\right)\right)\right), \\ s_{-\beta} &= \left(\left(1-b_4^U, 1-b_3^U, 1-b_2^U, 1\right.\right. \\ &\quad \left. - b_1^U; H_1\left(s_\beta^U\right), H_2\left(s_\beta^U\right)\right), \left(1-b_4^L, 1-b_3^L, 1-b_2^L, 1\right. \\ &\quad \left.\left. - b_1^L; H_1\left(s_\beta^L\right), H_2\left(s_\beta^L\right)\right)\right). \end{aligned} \tag{29}$$

Therefore

$$\begin{aligned} s_{-\alpha} - s_{-\beta} &= \left(\left(b_1^U - a_4^U, b_2^U - a_3^U, b_3^U - a_2^U, b_4^U\right.\right. \\ &\quad - a_1^U; H_1\left(s_\alpha^U\right) + H_1\left(s_\beta^U\right) - H_1\left(s_\alpha^U\right) \\ &\quad \times H_1\left(s_\beta^U\right), H_2\left(s_\alpha^U\right) + H_2\left(s_\beta^U\right) - H_2\left(s_\alpha^U\right) \\ &\quad \left.\times H_2\left(s_\beta^U\right)\right), \left(b_1^L - a_4^L, b_2^L - a_3^L, b_3^L - a_2^L, b_4^L\right. \\ &\quad - a_1^L; H_1\left(s_\alpha^L\right) + H_1\left(s_\beta^L\right) - H_1\left(s_\alpha^L\right) \\ &\quad \times H_1\left(s_\beta^L\right), H_2\left(s_\alpha^L\right) + H_2\left(s_\beta^L\right) - H_2\left(s_\alpha^L\right) \\ &\quad \left.\left.\times H_2\left(s_\beta^L\right)\right)\right) = s_\beta - s_\alpha. \end{aligned} \tag{30}$$

This completes the proof. □

Proposition 9. *Suppose that $s_\alpha + s_\beta - s_0 = s_\gamma$; then we get $s_{-\alpha} + s_{-\beta} - s_0 = s_{-\gamma}$, where s_α and $s_{-\alpha}$, s_β and $s_{-\beta}$, s_γ and $s_{-\gamma}$, respectively, denote symmetric IT2 fuzzy numbers,*

$$\begin{aligned} &s_0 \\ &= ((0.3, 0.5, 0.5, 0.7; 1, 1), (0.4, 0.5, 0.5, 0.6; 0.9, 0.9)). \end{aligned} \tag{31}$$

Proof. Suppose that

$$\begin{aligned} s_\alpha &= \left(\left(a_1^U, a_2^U, a_3^U, a_4^U; H_1\left(s_\alpha^U\right), H_2\left(s_\alpha^U\right)\right),\right. \\ &\quad \left.\left(a_1^L, a_2^L, a_3^L, a_4^L; H_1\left(s_\alpha^L\right), H_2\left(s_\alpha^L\right)\right)\right), \\ s_\beta &= \left(\left(b_1^U, b_2^U, b_3^U, b_4^U; H_1\left(s_\beta^U\right), H_2\left(s_\beta^U\right)\right),\right. \\ &\quad \left.\left(b_1^L, b_2^L, b_3^L, b_4^L; H_1\left(s_\beta^L\right), H_2\left(s_\beta^L\right)\right)\right). \end{aligned} \tag{32}$$

Then

$$\begin{aligned} s_{-\alpha} &= \left(\left(1-a_4^U, 1-a_3^U, 1-a_2^U, 1\right.\right. \\ &\quad \left. - a_1^U; H_1\left(s_\alpha^U\right), H_2\left(s_\alpha^U\right)\right), \left(1-a_4^L, 1-a_3^L, 1-a_2^L, 1\right. \\ &\quad \left.\left. - a_1^L; H_1\left(s_\alpha^L\right), H_2\left(s_\alpha^L\right)\right)\right), \\ s_{-\beta} &= \left(\left(1-b_4^U, 1-b_3^U, 1-b_2^U, 1\right.\right. \\ &\quad \left. - b_1^U; H_1\left(s_\beta^U\right), H_2\left(s_\beta^U\right)\right), \left(1-b_4^L, 1-b_3^L, 1-b_2^L, 1\right. \\ &\quad \left.\left. - b_1^L; H_1\left(s_\beta^L\right), H_2\left(s_\beta^L\right)\right)\right) \end{aligned} \tag{33}$$

and

$$\begin{aligned} s_\alpha + s_\beta &= \left(\left(a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U\right.\right. \\ &\quad + b_4^U; H_1\left(s_\alpha^U\right) + H_1\left(s_\beta^U\right) - H_1\left(s_\alpha^U\right) \\ &\quad \times H_1\left(s_\beta^U\right), H_2\left(s_\alpha^U\right) + H_2\left(s_\beta^U\right) - H_2\left(s_\alpha^U\right) \\ &\quad \left.\times H_2\left(s_\beta^U\right)\right), \left(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L\right. \\ &\quad + b_4^L; H_1\left(s_\alpha^L\right) + H_1\left(s_\beta^L\right) - H_1\left(s_\alpha^L\right) \\ &\quad \times H_1\left(s_\beta^L\right), H_2\left(s_\alpha^L\right) + H_2\left(s_\beta^L\right) - H_2\left(s_\alpha^L\right) \\ &\quad \left.\left.\times H_2\left(s_\beta^L\right)\right)\right) \end{aligned} \tag{34}$$

Due to $s_0 = ((0.3, 0.5, 0.5, 0.7; 1, 1), (0.4, 0.5, 0.5, 0.6; 0.9, 0.9))$, therefore

$$\begin{aligned} s_\alpha + s_\beta - s_0 &= \left(\left(a_1^U + b_1^U - 0.7, a_2^U + b_2^U - 0.5, a_3^U\right.\right. \\ &\quad \left. + b_3^U - 0.5, a_4^U + b_4^U - 0.3; 1, 1\right), \left(a_1^L + b_1^L - 0.6, a_2^L\right. \\ &\quad + b_2^L - 0.5, a_3^L + b_3^L - 0.5, a_4^L + b_4^L - 0.4; 0.1 \\ &\quad \times \left(H_1\left(s_\alpha^L\right) + H_1\left(s_\beta^L\right) - H_1\left(s_\alpha^L\right) \times H_1\left(s_\beta^L\right)\right) \\ &\quad + 0.9, 0.1 \\ &\quad \times \left(H_2\left(s_\alpha^L\right) + H_2\left(s_\beta^L\right) - H_2\left(s_\alpha^L\right) \times H_2\left(s_\beta^L\right)\right) \\ &\quad \left.\left. + 0.9\right)\right) \end{aligned} \tag{35}$$

At the same time,

$$\begin{aligned} s_{-\alpha} + s_{-\beta} &= \left(\left(2 - a_4^U - b_4^U, 2 - a_3^U - b_3^U, 2 - a_2^U\right.\right. \\ &\quad - b_2^U, 2 - a_1^U - b_1^U; H_1\left(s_\alpha^U\right) + H_1\left(s_\beta^U\right) - H_1\left(s_\alpha^U\right) \\ &\quad \times H_1\left(s_\beta^U\right), H_2\left(s_\alpha^U\right) + H_2\left(s_\beta^U\right) - H_2\left(s_\alpha^U\right) \\ &\quad \left.\times H_2\left(s_\beta^U\right)\right), \left(2 - a_4^L - b_4^L, 2 - a_3^L - b_3^L, 2 - a_2^L\right. \\ &\quad - b_2^L, 2 - a_1^L - b_1^L; H_1\left(s_\alpha^L\right) + H_1\left(s_\beta^L\right) - H_1\left(s_\alpha^L\right) \end{aligned}$$

$$\times H_1\left(s_\beta^L\right), H_2\left(s_\alpha^L\right)+H_2\left(s_\beta^L\right)-H_2\left(s_\alpha^L\right) \times H_2\left(s_\beta^L\right)\Big)\Big)$$

$$\begin{aligned} s_{-\alpha}+s_{-\beta}-s_0 = \Big(&\big(1.3-a_4^U-b_4^U, 1.5-a_3^U-b_3^U, 1.5 - a_2^U-b_2^U, 1.7-a_1^U-b_1^U; 1, 1\big), \big(1.4-a_4^L-b_4^L, 1.5 \\ &- a_3^L-b_3^L, 1.5-a_2^L-b_2^L, 1.6-a_1^L-b_1^L; 0.1 \\ &\times \left(H_1\left(s_\alpha^L\right)+H_1\left(s_\beta^L\right)-H_1\left(s_\alpha^L\right)\times H_1\left(s_\beta^L\right)\right) + 0.9, 0.1 \\ &\times \left(H_2\left(s_\alpha^L\right)+H_2\left(s_\beta^L\right)-H_2\left(s_\alpha^L\right)\times H_2\left(s_\beta^L\right)\right) + 0.9\big)\Big) \end{aligned} \tag{36}$$

Hence, if we denote $s_\alpha + s_\beta - s_0 = s_\gamma$, we get $s_{-\alpha}+s_{-\beta}-s_0 = s_{-\gamma}$. □

Here we let $s_0 = ((0.3, 0.5, 0.5, 0.7; 1, 1), (0.4, 0.5, 0.5, 0.6; 0.9, 0.9))$; note that from the proof procedure it can be found that if we let $s_0 = ((0.5, 0.5, 0.5, 0.5; 1, 1), (0.5, 0.5, 0.5, 0.5; 0.9, 0.9))$, or $s_0 = ((0.3, 0.5, 0.5, 0.7; 1, 1), (0.4, 0.5, 0.5, 0.6; 0.8, 0.8))$, or taking some other values, the same results also can be obtained; Propositions 7 and 9 also hold. In other words, the conclusions are universal. The situations are similar for the rest of the relevant conclusions involving s_0 in this article.

Proposition 10. *If we denote*

$$\frac{s_{\gamma_1}+s_{\gamma_2}+\cdots s_{\gamma_n}}{n} = s_\gamma, \tag{37}$$

n is a finite positive integer, then we get

$$\frac{s_{-\gamma_1}+s_{-\gamma_2}+\cdots s_{-\gamma_n}}{n} = s_{-\gamma}, \tag{38}$$

where s_{γ_1} and $s_{-\gamma_1}$, s_{γ_2} and $s_{-\gamma_2}$,$\cdots$,s_{γ_n} and $s_{-\gamma_n}$, s_γ and $s_{-\gamma}$, respectively, denote symmetric IT2 fuzzy numbers. That is to say, the average value of the symmetric IT2 fuzzy numbers and the average value of the original IT2 fuzzy numbers are still a couple of symmetric IT2 fuzzy numbers.

Proof. (1) When $n = 1$, it is obvious that the proposition holds.

(2) When $n = 2$, suppose that

$$\begin{aligned} s_{\gamma_1} &= \Big(\big(a_1^U, a_2^U, a_3^U, a_4^U; H_1\left(s_{\gamma_1}^U\right), H_2\left(s_{\gamma_1}^U\right)\big), \big(a_1^L, a_2^L, a_3^L, a_4^L; H_1\left(s_{\gamma_1}^L\right), H_2\left(s_{\gamma_1}^L\right)\big)\Big), \\ s_{\gamma_2} &= \Big(\big(b_1^U, b_2^U, b_3^U, b_4^U; H_1\left(s_{\gamma_2}^U\right), H_2\left(s_{\gamma_2}^U\right)\big), \big(b_1^L, b_2^L, b_3^L, b_4^L; H_1\left(s_{\gamma_2}^L\right), H_2\left(s_{\gamma_2}^L\right)\big)\Big). \end{aligned} \tag{39}$$

Then

$$\begin{aligned} s_{-\gamma_1} &= \Big(\big(1-a_4^U, 1-a_3^U, 1-a_2^U, 1 - a_1^U; H_1\left(s_{\gamma_1}^U\right), H_2\left(s_{\gamma_1}^U\right)\big), \big(1-a_4^L, 1-a_3^L, 1-a_2^L, 1 - a_1^L; H_1\left(s_{\gamma_1}^L\right), H_2\left(s_{\gamma_1}^L\right)\big)\Big), \\ s_{-\gamma_2} &= \Big(\big(1-b_4^U, 1-b_3^U, 1-b_2^U, 1 - b_1^U; H_1\left(s_{\gamma_2}^U\right), H_2\left(s_{\gamma_2}^U\right)\big), \big(1-b_4^L, 1-b_3^L, 1-b_2^L, 1 - b_1^L; H_1\left(s_{\gamma_2}^L\right), H_2\left(s_{\gamma_2}^L\right)\big)\Big), \\ s_{-\gamma_1}+s_{-\gamma_2} &= \Big(\big(2-a_4^U-b_4^U, 2-a_3^U-b_3^U, 2-a_2^U - b_2^U, 2-a_1^U-b_1^U; H_1\left(s_{\gamma_1}^U\right)+H_1\left(s_{\gamma_2}^U\right)-H_1\left(s_{\gamma_1}^U\right) \times H_1\left(s_{\gamma_2}^U\right), H_2\left(s_{\gamma_1}^U\right)+H_2\left(s_{\gamma_2}^U\right)-H_2\left(s_{\gamma_1}^U\right) \times H_2\left(s_{\gamma_2}^U\right)\big), \big(2-a_4^L-b_4^L, 2-a_3^L-b_3^L, 2-a_2^L-b_2^L, \\ &\quad 2-a_1^L-b_1^L; H_1\left(s_{\gamma_1}^U\right)+H_1\left(s_{\gamma_2}^U\right)-H_1\left(s_{\gamma_1}^U\right) \times H_1\left(s_{\gamma_2}^U\right), H_2\left(s_{\gamma_1}^U\right)+H_2\left(s_{\gamma_2}^U\right)-H_2\left(s_{\gamma_1}^U\right) \times H_2\left(s_{\gamma_2}^U\right)\big), \end{aligned} \tag{40}$$

and

$$\begin{aligned} s_{\gamma_1}+s_{\gamma_2} = \Big(&\big(a_1^U+b_1^U, a_2^U+b_2^U, a_3^U+b_3^U, a_4^U + b_4^U; H_1\left(s_{\gamma_1}^U\right)+H_1\left(s_{\gamma_2}^U\right)-H_1\left(s_{\gamma_1}^U\right) \times H_1\left(s_{\gamma_2}^U\right), H_2\left(s_{\gamma_1}^U\right)+H_2\left(s_{\gamma_2}^U\right)-H_2\left(s_{\gamma_1}^U\right) \times H_2\left(s_{\gamma_2}^U\right)\big), \big(a_1^L+b_1^L, a_2^L+b_2^L, a_3^L+b_3^L, a_4^L \\ &+ b_4^L; H_1\left(s_{\gamma_1}^L\right)+H_1\left(s_{\gamma_2}^L\right)-H_1\left(s_{\gamma_1}^L\right) \times H_1\left(s_{\gamma_2}^L\right), H_2\left(s_{\gamma_1}^L\right)+H_2\left(s_{\gamma_2}^L\right)-H_2\left(s_{\gamma_1}^L\right) \times H_2\left(s_{\gamma_2}^L\right)\big)\Big) \end{aligned} \tag{41}$$

Hence

$$\begin{aligned} \frac{s_{\gamma_1}+s_{\gamma_2}}{2} = \Bigg(\bigg(&\frac{a_1^U+b_1^U}{2}, \frac{a_2^U+b_2^U}{2}, \frac{a_3^U+b_3^U}{2}, \frac{a_4^U+b_4^U}{2}; 1-\left(1-\left(H_1\left(s_{\gamma_1}^U\right)+H_1\left(s_{\gamma_2}^U\right)-H_1\left(s_{\gamma_1}^U\right) \times H_1\left(s_{\gamma_2}^U\right)\right)\right)^{1/2}, 1-\Big(1-\left(H_2\left(s_{\gamma_1}^U\right)+H_2\left(s_{\gamma_2}^U\right) - H_2\left(s_{\gamma_1}^U\right)\times H_2\left(s_{\gamma_2}^U\right)\right)^{1/2}\Big), \bigg(\frac{a_1^L+b_1^L}{2}, \frac{a_2^L+b_2^L}{2}, \\ &\frac{a_3^L+b_3^L}{2}, \frac{a_4^L+b_4^L}{2}; 1-\left(1-\left(H_1\left(s_{\gamma_1}^U\right)+H_1\left(s_{\gamma_2}^U\right)\right.\right.\right. \end{aligned}$$

$$- H_1\left(s^U_{\gamma_1}\right) \times H_1\left(s^U_{\gamma_2}\right)\right)\right)^{1/2}, 1 - \left(1 - \left(H_2\left(s^U_{\gamma_1}\right)\right.\right.$$

$$\left.\left.+ H_2\left(s^U_{\gamma_2}\right) - H_2\left(s^U_{\gamma_1}\right) \times H_2\left(s^U_{\gamma_2}\right)\right)^{1/2}\right)\right)$$

$$\frac{s_{-\gamma_1} + s_{-\gamma_2}}{2} = \left(\left(1 - \frac{a^U_4 + b^U_4}{2}, 1 - \frac{a^U_3 + b^U_3}{2}, 1 - \frac{a^U_2 + b^U_2}{2}, 1 - \frac{a^U_1 + b^U_1}{2}; 1 - \left(1 - \left(H_1\left(s^U_{\gamma_1}\right) + H_1\left(s^U_{\gamma_2}\right) - H_1\left(s^U_{\gamma_1}\right) \times H_1\left(s^U_{\gamma_2}\right)\right)\right)^{1/2}, 1 - \left(1 - \left(H_2\left(s^U_{\gamma_1}\right) + H_2\left(s^U_{\gamma_2}\right) - H_2\left(s^U_{\gamma_1}\right) \times H_2\left(s^U_{\gamma_2}\right)\right)^{1/2}\right),\right.\right.$$
$$\left(1 - \frac{a^L_4 + b^L_4}{2}, 1 - \frac{a^L_3 + b^L_3}{2}, 1 - \frac{a^L_2 + b^L_2}{2}, 1 - \frac{a^L_1 + b^L_1}{2}; 1 - \left(1 - \left(H_1\left(s^U_{\gamma_1}\right) + H_1\left(s^U_{\gamma_2}\right) - H_1\left(s^U_{\gamma_1}\right) \times H_1\left(s^U_{\gamma_2}\right)\right)\right)^{1/2}, 1 - \left(1 - \left(H_2\left(s^U_{\gamma_1}\right) + H_2\left(s^U_{\gamma_2}\right) - H_2\left(s^U_{\gamma_1}\right) \times H_2\left(s^U_{\gamma_2}\right)\right)^{1/2}\right)\right) \quad (42)$$

If we denote $(s_{\gamma_1} + s_{\gamma_2})/2 = s_{\mu_2}$, then we get $(s_{-\gamma_1} + s_{-\gamma_2})/2 = s_{-\mu_2}$. So in fact, we prove that when $n = 2$, the proposition holds.

(3) When $n = k$ ($k \geq 3$, k is an integer), suppose that

$$\begin{aligned} s_{\gamma_1} &= \left(\left(a^U_{11}, a^U_{12}, a^U_{13}, a^U_{14}; H_1\left(s^U_{\gamma_1}\right), H_2\left(s^U_{\gamma_1}\right)\right), \left(a^L_{11}, a^L_{12}, a^L_{13}, a^L_{14}; H_1\left(s^L_{\gamma_1}\right), H_2\left(s^L_{\gamma_1}\right)\right)\right), \\ s_{\gamma_2} &= \left(\left(a^U_{21}, a^U_{22}, a^U_{23}, a^U_{24}; H_1\left(s^U_{\gamma_2}\right), H_2\left(s^U_{\gamma_2}\right)\right), \left(a^L_{21}, a^L_{22}, a^L_{23}, a^L_{24}; H_1\left(s^L_{\gamma_2}\right), H_2\left(s^L_{\gamma_2}\right)\right)\right), \\ &\cdots\cdots \\ s_{\gamma_n} &= \left(\left(a^U_{n1}, a^U_{n2}, a^U_{n3}, a^U_{n4}; H_1\left(s^U_{\gamma_n}\right), H_2\left(s^U_{\gamma_n}\right)\right), \left(a^L_{n1}, a^L_{n2}, a^L_{n3}, a^L_{n4}; H_1\left(s^L_{\gamma_n}\right), H_2\left(s^L_{\gamma_n}\right)\right)\right) \end{aligned} \quad (43)$$

Then

$$\begin{aligned} s_{-\gamma_1} &= \left(\left(1 - a^U_{14}, 1 - a^U_{13}, 1 - a^U_{12}, 1 - a^U_{11}; H_1\left(s^U_{\gamma_1}\right), H_2\left(s^U_{\gamma_1}\right)\right), \left(1 - a^L_{14}, 1 - a^L_{13}, 1 - a^L_{12}, 1 - a^L_{11}; H_1\left(s^L_{\gamma_1}\right), H_2\left(s^L_{\gamma_1}\right)\right)\right), \\ s_{-\gamma_2} &= \left(\left(1 - a^U_{24}, 1 - a^U_{23}, 1 - a^U_{22}, 1 - a^U_{21}; H_1\left(s^U_{\gamma_2}\right), H_2\left(s^U_{\gamma_2}\right)\right), \left(1 - a^L_{24}, 1 - a^L_{23}, 1 - a^L_{22}, 1 - a^L_{21}; H_1\left(s^L_{\gamma_2}\right), H_2\left(s^L_{\gamma_2}\right)\right)\right), \\ &\cdots\cdots \\ s_{-\gamma_n} &= \left(\left(1 - a^U_{n4}, 1 - a^U_{n3}, 1 - a^U_{n2}, 1 - a^U_{n1}; H_1\left(s^U_{\gamma_n}\right), H_2\left(s^U_{\gamma_n}\right)\right), \left(1 - a^L_{n4}, 1 - a^L_{n3}, 1 - a^L_{n2}, 1 - a^L_{n1}; H_1\left(s^L_{\gamma_n}\right), H_2\left(s^L_{\gamma_n}\right)\right)\right). \end{aligned} \quad (44)$$

Similar to (48) where $n = 2$, because s_{γ_i} and $s_{-\gamma_i}$ ($i = 1, 2, 3, \ldots,$) have the same height value $H_1(s^U_{\gamma_i}), H_2(s^U_{\gamma_i})$, $H_1(s^L_{\gamma_i}), H_2(s^L_{\gamma_i})$, respectively, therefore $(s_{\gamma_1} + s_{\gamma_2} + \cdots s_{\gamma_n})/n$ and $(s_{-\gamma_1} + s_{-\gamma_2} + \cdots s_{-\gamma_n})/n$ have the same height value, and they are denoted as $H_1(s^U_\gamma), H_2(s^U_\gamma), H_1(s^L_\gamma), H_2(s^L_\gamma)$.

Then we get

$$\frac{s_{\gamma_1} + s_{\gamma_2} + \cdots s_{\gamma_n}}{n} = \left(\left(\frac{1}{n}\sum_{i=1}^n a^U_{i1}, \frac{1}{n}\sum_{i=1}^n a^U_{i2}, \frac{1}{n}\sum_{i=1}^n a^U_{i3}, \frac{1}{n} \cdot \sum_{i=1}^n a^U_{i4}; H_1\left(s^U_\gamma\right), H_2\left(s^U_\gamma\right)\right), \frac{1}{n}\sum_{i=1}^n a^L_{i1}, \frac{1}{n}\sum_{i=1}^n a^L_{i2}, \frac{1}{n} \cdot \sum_{i=1}^n a^L_{i3}, \frac{1}{n}\sum_{i=1}^n a^L_{i4}; H_1\left(s^L_\gamma\right), H_2\left(s^L_\gamma\right)\right)$$
$$\frac{s_{-\gamma_1} + s_{-\gamma_2} + \cdots s_{-\gamma_n}}{n} = \left(\left(1 - \frac{1}{n}\sum_{i=1}^n a^U_{i4}, 1 - \frac{1}{n}\sum_{i=1}^n a^U_{i3}, 1 - \frac{1}{n}\sum_{i=1}^n a^U_{i2}, 1 - \frac{1}{n}\sum_{i=1}^n a^U_{i1}; H_1\left(s^U_\gamma\right), H_2\left(s^U_\gamma\right)\right), \left(1 - \frac{1}{n} \cdot \sum_{i=1}^n a^L_{i4}, 1 - \frac{1}{n}\sum_{i=1}^n a^L_{i3}, 1 - \frac{1}{n}\sum_{i=1}^n a^L_{i2}, 1 - \frac{1}{n} \cdot \sum_{i=1}^n a^L_{i1}; H_1\left(s^L_\gamma\right), H_2\left(s^L_\gamma\right)\right)\right) \quad (45)$$

Hence if we denote

$$\frac{s_{\gamma_1} + s_{\gamma_2} + \cdots s_{\gamma_n}}{n} = s_\gamma \quad (46)$$

then

$$\frac{s_{-\gamma_1} + s_{-\gamma_2} + \cdots s_{-\gamma_n}}{n} = s_{-\gamma} \quad (47)$$

So, we have proven that for an arbitrary finite positive integer n, the proposition holds. □

4. IT2 Additive Preference Relation Process for Achieving Consistency

Cardinal consistency is a stronger concept than ordinal consistency. In Analytic Hierarchy Process, Saaty [46] first addressed the issue of consistency and developed the notions of perfect consistency and acceptable consistency. Ordinal consistency is based on the notion of transitivity [47–51].

We use the conventional additive consistency method for reference and then generalize it to IT2 FSs. We handle the GDM problems, such as the process for achieving consistency for IT2 additive preference relations, using a generalized method to calculate and improve the consistency degrees.

For fuzzy preference relations, transitivity has been modelled in many different ways depending on the role the preference intensities have. In this paper, we use the conventional additive transitivity property [52] and extend it to the following definition:

Definition 11. Let $P = (p_{ij})_{n\times n}$ be a fuzzy preference relation for an IT2; P is the "additive consistent" when for every three alternatives in the problem $x_i, x_j, x_k \in X$ their associated preference degrees p_{ij}, p_{jk}, p_{ik}, the following holds:

$$p_{ik} = p_{ij} + p_{jk} - s_0 \quad \forall i, j, k \in \{1, 2, \ldots, n\}, \tag{48}$$

where p_{ik}, p_{ij}, p_{jk} and s_0 denote IT2 fuzzy numbers. As mentioned above, here we take $s_0 = ((0.3, 0.5, 0.5, 0.7; 1, 1), (0.4, 0.5, 0.5, 0.6; 0.9, 0.9))$.

4.1. Algorithm for the Calculation and Improvement of the Consistency Degrees. Here, the conventional additive consistency method [39, 52–56] is referenced and generalized to IT2 FSs to calculate and improve the consistency degrees. To calculate and improve the consistency degrees for IT2 additive preference relations, the procedure is as follows:

Step 1 (calculate the estimated value). Using an intermediate alternative x_j, the following estimated value for p_{ik} $(i \neq k)$ is obtained:

$$ep_{ik}^{j} = p_{ij} + p_{jk} - s_0 \quad \forall i, j, k \in \{1, 2, \ldots, n\}, \tag{49}$$

where ep_{ik}^{j} denotes the estimated value of p_{ik} $(i \neq k)$ using an intermediate alternative x_j.

The overall estimated value is

$$ep_{ik} = \sum_{j=1}^{n} \frac{ep_{ik}^{j}}{n-2} \quad (j \neq i, k) \tag{50}$$

Step 2. To calculate the error between a preference value and its overall estimated value.

The error between a preference value p_{ik} and its final estimated value cp_{ik} is

$$\varepsilon p_{ik} = \begin{cases} cp_{ik} - p_{ik}, & \text{if } cp_{ik} - p_{ik} \geq p_{ik} - cp_{ik} \\ p_{ik} - cp_{ik}, & \text{if } cp_{ik} - p_{ik} < p_{ik} - cp_{ik} \end{cases} \tag{51}$$

Step 3 (calculate the additive consistency degrees). Under an IT2 fuzzy environment, the consistency degree associated with a pair of alternatives p_{ik} is $cd_{ik} = s_4 - \varepsilon p_{ik}$.

The consistency degree associated with an alternative x_i is $cd_i = \sum_{k=1}^{n} (cd_{ik}/(n-1))$ $(k \neq i)$.

The consistency degree of the symmetric fuzzy preference relation is $cd = \sum_{i=1}^{n} (cd_i/n)$.

Step 4. Improve the additive consistency of the IT2 additive preference relation.

If the global consistency value of the preference matrix is less than the threshold value $\xi' = k \times s_4$ $(0 < k \leq 1$, k is a constant), it is necessary to amend the preference matrix to improve the global consistency. If $cd_{ij} < \xi$, to reach the threshold value, $p_{ij} (i < j)$ should be changed to

$$\overline{p}_{ij} = \begin{cases} p_{ij} + \left(\xi' - cd_{ij}\right), & \text{if } cp_{ij} \geq p_{ij} \\ p_{ij} - \left(\xi' - cd_{ij}\right), & \text{if } cp_{ij} < p_{ij} \end{cases} \tag{52}$$

Finally, to maintain complementarity, the value p_{ji} should be changed to the symmetric IT2 fuzzy number of p_{ij}.

4.2. Algorithmic Properties. In this section, we discuss some of the algorithmic properties.

Proposition 12. *Let ep_{ik}, ep_{ki} be as before; suppose that $ep_{ik} = s_\gamma$ $(i \neq k, i, k = 1, 2, \cdots, n)$; then $ep_{ki} = s_{-\gamma}$; i.e., they are a couple of symmetric IT2 fuzzy numbers.*

Proof. Suppose that

$$ep_{ik}^{j} = p_{ij} + p_{jk} - s_0 \quad (j \neq i, k) \tag{53}$$

Then $ep_{ki}^{j} = p_{kj} + p_{ji} - s_0$ $(j \neq i, k)$.

So if we denote $p_{ij} = s_\alpha, p_{jk} = s_\beta$, according to the reciprocity, we get

$$p_{ji} = s_{-\alpha}, \; p_{kj} = s_{-\beta} \tag{54}$$

Hence

$$\begin{aligned} ep_{ik}^{j} &= s_\alpha + s_\beta - s_0 \quad (j \neq i, k) \\ ep_{ki}^{j} &= s_{-\beta} + s_{-\alpha} - s_0 \quad (j \neq i, k) \end{aligned} \tag{55}$$

According to Proposition 10, ep_{ik}^{j} and ep_{ki}^{j} are symmetric IT2 fuzzy numbers; namely if we denote

$$ep_{ik}^{j} = s_{\gamma_{ik}^{j}} \tag{56}$$

then $ep_{ki}^{j} = s_{-\gamma_{ik}^{j}}$ and

$$\begin{aligned} \sum_{j=1}^{n} \frac{ep_{ik}^{j}}{n-2} &= \sum_{j=1}^{n} \frac{s_{\gamma_{ik}^{j}}}{n-2} \quad (j \neq i, k) \\ \sum_{j=1}^{n} \frac{ep_{ki}^{j}}{n-2} &= \sum_{j=1}^{n} \frac{s_{-\gamma_{ik}^{j}}}{n-2} \quad (j \neq i, k) \end{aligned} \tag{57}$$

According to Proposition 12, if we denote

$$\sum_{j=1}^{n} \frac{ep_{ik}^{j}}{n-2} = \sum_{j=1}^{n} \frac{s_{\gamma_{ik}^{j}}}{n-2} \quad (j \neq i, k) = s_\gamma \tag{58}$$

then

$$\sum_{j=1}^{n}\frac{ep_{ki}^{j}}{n-2}=\sum_{j=1}^{n}\frac{s_{-\gamma_{ik}^{j}}}{n-2}\quad (j\neq i,k)=s_{-\gamma} \tag{59}$$

That is to say, if we denote

$$ep_{ik}=s_{\gamma}, \tag{60}$$

then $ep_{ki}=s_{-\gamma}$, which completes the proof. □

Proposition 13. *Let εp_{ik}, εp_{ki} be as before; then $\varepsilon p_{ik}=\varepsilon p_{ki}$, $(i\neq k, i,k=1,2,\cdots,n)$.*

Proof. According to the complementarity, we get $p_{ki}=s_{-\varsigma}$ and $ep_{ki}=s_{-\eta}$. It is then easy to derive $s_{-\varsigma}\geq s_{-\eta}$; therefore

$$\varepsilon p_{ki}=p_{ki}-ep_{ki}=s_{-\varsigma}-s_{-\eta} \tag{61}$$

Furthermore, according to Proposition 10, we get

$$s_{-\varsigma}-s_{-\eta}=s_{\eta}-s_{\varsigma}, \tag{62}$$

i.e., $\varepsilon p_{ik}=\varepsilon p_{ki}$. □

5. An Actual Case Study and Discussion

In this section, an actual case study is used for demonstrating the effectiveness of the proposed approach.

5.1. An Actual Case Study. Many companies have recently tried to select suitable suppliers in order to improve product quality. Therefore, supplier selection has become a key issue that both enterprises and scholars are paying attention to. Supplier selection is particularly important for seafood companies. Accordingly, in this case study a seafood company needs to select a supplier. There are five companies, x_1, x_2, x_3, x_4, and x_5, which can be selected (the companies' names will not be released due to confidentiality).

To select the best supplier, the seafood company employs a consultancy organization to evaluate the five competing suppliers. Due to uncertainties and the complexity of the situation, the experts can use T2 FSs and provide their preference information regarding alternatives in the form of IT2 FSs and an IT2 additive preference relation as follows:

$$P_0=(\mathrm{p}_{ij})_{5\times5}=(\widetilde{A}_{ij})_{5\times5}$$
$$=\begin{bmatrix}
\begin{pmatrix}(0.3,0.5,0.5,0.7;1,1),\\(0.4,0.5,0.5,0.6;0.9,0.9)\end{pmatrix} & \cdots & \cdots & \cdots & \cdots\\
\begin{pmatrix}(0,0.1,0.1,0.3;1,1),\\(0.05,0.1,0.1,0.2;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0.3,0.5,0.5,0.7;1,1),\\(0.4,0.5,0.5,0.6;0.9,0.9)\end{pmatrix} & \cdots & \cdots & \cdots\\
\begin{pmatrix}(0.5,0.7,0.7,0.9;1,1),\\(0.6,0.7,0.7,0.8;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0,0,0,0.1;1,1),\\(0,0,0,0.05;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0.3,0.5,0.5,0.7;1,1),\\(0.4,0.5,0.5,0.6;0.9,0.9)\end{pmatrix} & \cdots & \cdots\\
\begin{pmatrix}(0.5,0.7,0.7,0.9;1,1),\\(0.6,0.7,0.7,0.8;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0.7,0.9,0.9,1;1,1),\\(0.8,0.9,0.9,0.95;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0.9,1,1,1;1,1),\\(0.95,1,1,1;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0.3,0.5,0.5,0.7;1,1),\\(0.4,0.5,0.5,0.6;0.9,0.9)\end{pmatrix} & \cdots\\
\begin{pmatrix}(0,0.1,0.1,0.3;1,1),\\(0.05,0.1,0.1,0.2;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0,0.1,0.1,0.3;1,1),\\(0.05,0.1,0.1,0.2;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0.1,0.3,0.3,0.5;1,1),\\(0.2,0.3,0.3,0.4;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0,0,0,0.1;1,1),\\(0,0,0,0.05;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0.3,0.5,0.5,0.7;1,1),\\(0.4,0.5,0.5,0.6;0.9,0.9)\end{pmatrix}
\end{bmatrix} \tag{63}$$

The elements in P_0 can be explained as follows: take the element $\widetilde{A}_{21}$ as an example; since the experts deem that the quality level of the supplier x_2 is "lower" than that of the supplier x_1, and due to uncertainties and the complexity of the situation, they use the IT2 fuzzy number $((0,0.1,0.1,0.3;1,1),(0.05,0.1,0.1,0.2;0.9,0.9))$ to explain their preference. In a similar way, the rest of the elements in P_0 can be obtained.

From P_0, it can be ascertained that x_5 is inferior to x_1, x_2, x_3, and x_4 simultaneously; hence we will only consider the following submatrix:

$$P=(\mathrm{p}_{ij})_{4\times4}=(\widetilde{A}_{ij})_{4\times4}$$
$$=\begin{bmatrix}
\begin{pmatrix}(0.3,0.5,0.5,0.7;1,1),\\(0.4,0.5,0.5,0.6;0.9,0.9)\end{pmatrix} & \cdots & \cdots & \cdots\\
\begin{pmatrix}(0,0.1,0.1,0.3;1,1),\\(0.05,0.1,0.1,0.2;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0.3,0.5,0.5,0.7;1,1),\\(0.4,0.5,0.5,0.6;0.9,0.9)\end{pmatrix} & \cdots & \cdots\\
\begin{pmatrix}(0.5,0.7,0.7,0.9;1,1),\\(0.6,0.7,0.7,0.8;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0,0,0,0.1;1,1),\\(0,0,0,0.05;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0.3,0.5,0.5,0.7;1,1),\\(0.4,0.5,0.5,0.6;0.9,0.9)\end{pmatrix} & \cdots\\
\begin{pmatrix}(0.5,0.7,0.7,0.9;1,1),\\(0.6,0.7,0.7,0.8;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0.7,0.9,0.9,1;1,1),\\(0.8,0.9,0.9,0.95;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0.9,1,1,1;1,1),\\(0.95,1,1,1;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0.3,0.5,0.5,0.7;1,1),\\(0.4,0.5,0.5,0.6;0.9,0.9)\end{pmatrix}
\end{bmatrix} \tag{64}$$

According to the complementarity, the rest of the elements are obtained, namely,

$$P=\left(\mathrm{p}_{ij}\right)_{4\times 4}=\left(\widetilde{A}_{ij}\right)_{4\times 4}$$
$$=\begin{bmatrix}\begin{pmatrix}(0.3,0.5,0.5,0.7;1,1),\\(0.4,0.5,0.5,0.6;0.9,0.9)\end{pmatrix} & \cdots & \cdots & \cdots\\ \begin{pmatrix}(0,0.1,0.1,0.3;1,1),\\(0.05,0.1,0.1,0.2;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0.3,0.5,0.5,0.7;1,1),\\(0.4,0.5,0.5,0.6;0.9,0.9)\end{pmatrix} & \cdots & \cdots\\ \begin{pmatrix}(0.5,0.7,0.7,0.9;1,1),\\(0.6,0.7,0.7,0.8;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0,0,0,0.1;1,1),\\(0,0,0,0.05;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0.3,0.5,0.5,0.7;1,1),\\(0.4,0.5,0.5,0.6;0.9,0.9)\end{pmatrix} & \cdots\\ \begin{pmatrix}(0.5,0.7,0.7,0.9;1,1),\\(0.6,0.7,0.7,0.8;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0.7,0.9,0.9,1;1,1),\\(0.8,0.9,0.9,0.95;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0.9,1,1,1;1,1),\\(0.95,1,1,1;0.9,0.9)\end{pmatrix} & \begin{pmatrix}(0.3,0.5,0.5,0.7;1,1),\\(0.4,0.5,0.5,0.6;0.9,0.9)\end{pmatrix}\end{bmatrix} \tag{65}$$

We calculate and improve the consistency degree following the aforementioned steps. The calculation process is as follows:

$$P\Longrightarrow EP=\left(ep_{ik}\right)_{4\times 4} \tag{66}$$

Due to

$$ep_{12}^{3}=p_{13}+p_{32}-s_0=\widetilde{A}_{13}+\widetilde{A}_{32}-s_0$$
$$=((-0.6,-0.2,-0.2,0.3;1,1),$$
$$(-0.4,-0.2,-0.2,0.05;0.999,0.999))$$

$$ep_{12}^{4}=p_{14}+p_{42}-s_0=\widetilde{A}_{14}+\widetilde{A}_{42}-s_0$$
$$=((-0.1,0.7,0.7,1.2;1,1),$$
$$(0.4,0.7,0.7,0.95;0.999,0.999)) \tag{67}$$

We have

$$ep_{12}=\frac{ep_{12}^{3}+ep_{12}^{4}}{2}=((-0.25,0.25,0.25,0.75;1,1),$$
$$(0,0.25,0.25,0.5;1,1)) \tag{68}$$

In a similar way,

$$EP=\left(ep_{ik}\right)_{4\times 4}$$
$$=\begin{bmatrix} - & \begin{pmatrix}(-0.25,0.25,0.25,0.75;1,1),\\(0,0.25,0.25,0.5;1,1)\end{pmatrix} & \begin{pmatrix}(0.6,1.1,1.1,1.45,1.1;1,1),\\(0.85,1.1,1.1,1.275;1,1)\end{pmatrix} & \begin{pmatrix}(-0.3,0.15,0.15,0.65;1,1),\\(-0.075,0.15,0.15,0.4;1,1)\end{pmatrix}\\ \begin{pmatrix}(0.25,0.75,0.75,1.25;1,1),\\(0.5,0.75,0.75,1;1,1)\end{pmatrix} & - & \begin{pmatrix}(-0.2,0.25,0.25,0.75;1,1),\\(0.025,0.25,0.25,0.5;1,1)\end{pmatrix} & \begin{pmatrix}(-0.2,0.2,0.2,0.65;1,1),\\(0,0.2,0.2,0.425;1,1)\end{pmatrix}\\ \begin{pmatrix}(-0.45,-0.1,-0.1,0.4;1,1),\\(-0.275,-0.1,-0.1,0.15;1,1)\end{pmatrix} & \begin{pmatrix}(0.25,0.75,0.75,1.2;1,1),\\(0.5,0.75,0.75,0.975;1,1)\end{pmatrix} & - & \begin{pmatrix}(-0.4,0.05,0.05,0.6;1,1),\\(-0.175,0.05,0.05,0.325;1,1)\end{pmatrix}\\ \begin{pmatrix}(0.35,0.85,0.85,1.3;1,1),\\(0.6,0.85,0.85,1.075;1,1)\end{pmatrix} & \begin{pmatrix}(0.35,0.8,0.8,1.2;1,1),\\(0.575,0.8,0.8,1;1,1)\end{pmatrix} & \begin{pmatrix}(0.4,0.95,0.95,1.4;1,1),\\(0.675,0.95,0.95,1.175;1,1)\end{pmatrix} & -\end{bmatrix} \tag{69}$$

can be obtained.

Suppose that $\varepsilon p=\left(\varepsilon p_{ik}\right)_{4\times 4}$; by Eq. (51), we get the error values as follows:

$$\varepsilon p_{12}=\varepsilon p_{21}=((-0.05,0.65,0.65,1.25;1,1),$$
$$(0.3,0.65,0.65,0.95;1,1))$$
$$\varepsilon p_{13}=\varepsilon p_{31}=((0.1,0.8,0.8,1.35;1,1),$$
$$(0.45,0.8,0.8,1.075;1,1))$$
$$\varepsilon p_{14}=\varepsilon p_{41}=((-0.55,0.15,0.15,0.8;1,1),$$
$$(-0.2,0.15,0.15,0.475;1,1))$$
$$\varepsilon p_{23}=\varepsilon p_{32}=((0.15,0.75,0.75,1.2;1,1),$$
$$(0.45,0.75,0.75,0.975;1,1))$$
$$\varepsilon p_{24}=\varepsilon p_{42}=((-0.5,0.1,0.1,0.65;1,1),$$
$$(-0.2,0.1,0.1,0.375;1,1))$$
$$\varepsilon p_{34}=\varepsilon p_{43}=((-0.5,0.05,0.05,0.6;1,1),$$
$$(-0.225,0.05,0.05,0.325;1,1)) \tag{70}$$

Suppose that $CD = (cd_{ik})_{4\times4}$; by applying the equation $cd_{ik} = s_4 - \varepsilon p_{ik}$, we get the consistency degrees:

$$\begin{aligned} cd_{12} = cd_{21} &= ((-0.25, 0.35, 0.35, 1.05; 1, 1), \\ &\quad (0.05, 0.35, 0.35, 0.7; 1, 1)) \\ cd_{13} = cd_{31} &= ((-0.35, 0.2, 0.2, 0.9; 1, 1), \\ &\quad (-0.075, 0.2, 0.2, 0.55; 1, 1)) \\ cd_{14} = cd_{41} &= ((0.2, 0.85, 0.85, 1.55; 1, 1), \\ &\quad (0.525, 0.85, 0.85, 1.2; 1, 1)) \\ cd_{23} = cd_{32} &= ((-0.2, 0.25, 0.25, 0.85; 1, 1), \\ &\quad (0.025, 0.25, 0.25, 0.55; 1, 1)) \\ cd_{24} = cd_{42} &= ((0.35, 0.9, 0.9, 1.5; 1, 1), \\ &\quad (0.625, 0.9, 0.9, 1.2; 1, 1)) \\ cd_{34} = cd_{43} &= ((0.4, 0.95, 0.95, 1.5; 1, 1), \\ &\quad (0.675, 0.95, 0.95, 1.225; 1, 1)) \end{aligned} \tag{71}$$

Here we take $\xi' = 0.25 \times s_4$, so, after the calculation, we get

$$\begin{aligned} cd_{12} &= cd_{21} > 0.25 \times s_4, \\ cd_{13} &= cd_{31} > 0.25 \times s_4, \\ cd_{14} &= cd_{41} > 0.25 \times s_4, \\ cd_{23} &= cd_{32} > 0.25 \times s_4, \\ cd_{24} &= cd_{42} > 0.25 \times s_4, \\ cd_{34} &= cd_{43} > 0.25 \times s_4 \end{aligned} \tag{72}$$

They do not need to be improved. But for the rest of cd_{ik} $(i \neq k)$,

$$cd_{ik} \quad (i \neq k) < 0.25 \times s_4, \tag{73}$$

i.e., $cd_{13} = cd_{31} < 0.25 \times s_4$, they need to be improved. We can find that

$$\begin{aligned} p_{12} &> p_{21}, \\ p_{23} &> p_{32}, \\ p_{41} &> p_{14}, \\ p_{42} &> p_{24}, \\ p_{43} &> p_{34}. \end{aligned} \tag{74}$$

Hence,

$$\begin{aligned} x_1 &\succ x_2, \\ x_2 &\succ x_3, \\ x_4 &\succ x_1, \\ x_4 &\succ x_2, \\ x_4 &\succ x_3. \end{aligned} \tag{75}$$

At the same time, $p_{31} > p_{13}$, but $p_{31} > p_{13}$ should be adjusted owing to their weakest consistency degrees. Therefore we have $p_{13} > p_{31}$, i.e., $x_1 \succ x_3$. Finally, it is concluded that

$$x_4 \succ x_1 \succ x_2 \succ x_3, \tag{76}$$

and then

$$x_4 \succ x_1 \succ x_2 \succ x_3 \succ x_5, \tag{77}$$

which indicates that the fourth supplier is the most desirable according to the opinion of the consultancy firm.

5.2. Discussion and Comparison. In the above example, if the IT2 preference information submatrix $P = (\mathrm{p}_{ij})_{4\times4}$ is aggregated firstly at the beginning of the calculation, i.e., using Definition 4 and the formula

$$\begin{aligned} R_{(1)}\left(\widetilde{A}\right) &= \left(\frac{a_1^U + a_4^U}{2} + \frac{\sum_{k=L}^{U}\left(H_1\left(\widetilde{A}^k\right) + H_2\left(\widetilde{A}^k\right)\right)}{4}\right) \\ &\quad \times \frac{\sum_{i=1}^{4}\left(a_i^U + a_i^L\right)}{8} \end{aligned} \tag{78}$$

and let

$$P' = \left(\mathrm{p}'_{ij}\right)_{4\times4} = \left(R_{(1)}\left(\widetilde{A}_{ij}\right)\right)_{4\times4} \tag{79}$$

then we have

$$P' = \begin{bmatrix} 0.725 & 1.586 & 0.375 & 0.375 \\ 0.131 & 0.725 & 1.864 & 0.131 \\ 1.155 & 0.019 & 0.725 & 0.019 \\ 1.155 & 1.586 & 1.864 & 0.725 \end{bmatrix} \tag{80}$$

Let $P'' = (p''_{ij})_{4\times4}$, $p''_{ij} = (p'_{ij}/(p'_{ij} + p'_{ji}))(i, j = 1, 2, 3, 4)$; then the following symmetric matrix is obtained:

$$\begin{bmatrix} 0.5 & 0.924 & 0.245 & 0.245 \\ 0.076 & 0.5 & 0.990 & 0.076 \\ 0.755 & 0.010 & 0.5 & 0.010 \\ 0.755 & 0.924 & 0.990 & 0.5 \end{bmatrix} \tag{81}$$

indicating that

$$\begin{aligned} x_4 &\succ x_1, \\ x_4 &\succ x_2, \\ x_4 &\succ x_3, \\ x_1 &\succ x_2, \\ x_2 &\succ x_3, \\ x_3 &\succ x_1, \end{aligned} \tag{82}$$

which is inconsistent. Because the inconsistency occurs only in x_1, x_2, x_3, only the submatrix

$$\widetilde{P} = \left(\widetilde{p}_{ij}\right)_{3\times 3} = \begin{bmatrix} 0.5 & 0.924 & 0.245 \\ 0.076 & 0.5 & 0.990 \\ 0.755 & 0.010 & 0.5 \end{bmatrix} \quad (83)$$

can be considered. Only when we let critical value $\lambda > 0.755$, e.g., $\lambda = 0.8$, can a consistent result be obtained. Let

$$\widetilde{p}_{ij} = \begin{cases} 1, & if \ p_{ij} \geq \lambda, \\ 0, & if \ p_{ij} < \lambda \end{cases} \quad (84)$$

At this time,

$$\widetilde{P}' = \left(\widetilde{p}'_{ij}\right)_{3\times 3} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad (85)$$

which indicates that $x_1 \succ x_2$ and $x_2 \succ x_3$, namely,

$$x_1 \succ x_2 \succ x_3 \quad (86)$$

Because we have obtained

$$\begin{aligned} &x_4 \succ x_1, \\ &x_4 \succ x_2, \\ &x_4 \succ x_3, \end{aligned} \quad (87)$$

finally, we have

$$x_4 \succ x_1 \succ x_2 \succ x_3 \quad (88)$$

and then

$$x_4 \succ x_1 \succ x_2 \succ x_3 \succ x_5 \quad (89)$$

5.3. The Differences and Superiorities of the Proposed Approach. From the results of the calculations, one can ascertain that the two methods have the same results. The second method confirms the first method; however, the process for achieving consistency for additive preference relations with symmetric IT2 FSs proposed in this paper has the following main superiorities and differences compared to other relevant theories:

(1) In the second method, such an aggregation actually amounts to implementing a transformation of IT2 FSs into real numbers. As a result, it leads to the loss of information, which may affect the final ranking results. Therefore, the first method, i.e., the proposed approach, is more reliable.

(2) The proposed approach allows for the development of symmetric IT2 FSs to represent the uncertainty or fuzziness in the consistency reaching process for additive preference relations and models uncertainty more accurately than T1 FSs and real numbers.

(3) The proposed approach enriches and develops IT FSs theory and consistency reaching theory. There is a dearth of research regarding the consistency of preference relations based on IT2 FSs. As a special case of IT2 FSs, symmetric IT2 FSs have been presented in this paper. The process for achieving consistency for additive preference relations with symmetric IT2 FSs was shown to have some desired properties. We discussed these properties and then used these properties to deal with the process for achieving consistency.

6. Conclusion

In this paper, we defined symmetric IT2 FSs and IT2 additive preference relations and presented some properties for symmetric IT2 FSs. We proved these properties, and then we applied them to a process for achieving consistency for IT2 additive preference relations. We used conventional method for reference and generalized it to IT2 FSs, and then we developed an algorithm to calculate and improve the consistency degrees for IT2 additive preference relations. Finally, an actual case study is used in order to demonstrate the effectiveness of the proposed approach. Through a comparison, we have found that in fact the two methods are as follows: a method which calculates the IT2 FSs directly and a transformation method that came to the same or similar conclusions. Our method was therefore verified from different perspectives. In future work, we will analyze the IT2 additive preference relations, which involve the additive consistency and ordinal consistency, study the consensus process for IT2 additive preference relations, and discuss the applications in decision making problems.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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