Research Article

Design, Analysis, and Experiment of Multiring Permanent Magnet Bearings by Means of Equally Distributed Sequences Based Monte Carlo Method

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Load-carrying capacity analysis is an important procedure for designing the permanent magnet bearing (PMB). Generally, the magnetic force exerted between the ring magnets of PMB can be modeled by means of the equivalent magnetic charge method. In this case, the analytical methods are always simply compared to numerical methods; however, they are restricted by their applicability. The model based on the equivalent magnetic charge method contains multiple integrals; it is difficult to get the simulation results quickly through self-contained function of MATLAB. The equally distributed sequences based Monte Carlo method is used to simplify the complicated mathematical derivation and calculus of bearing capacity characteristics of multiring PMBs, and it might improve the computational efficiency of PMB structure design. The results of the Monte Carlo model are compared with the results of finite element analysis (FEA) using ANSYS, and the error correction factor is presented. The theoretical model is verified by the finite element analysis. Finally, the bearing forces in radial and axial directions of the PMBs with 4 pairs of magnetic rings were tested experimentally; the experiment result is approximately agreed with the simulation analysis. This method will be perfect for the engineering application involving multiring structural design of PMBs.

1. Introduction

Permanent magnet bearing (PMB) levitate the rotor by magnetic field, which has the advantages of simple structure, negligible friction, and energy consumption. It is known as the most potential technology of magnetic levitation support. It has been widely used in engineering applications, such as heart pump, centrifugal pump, flywheel energy storage [1–3] and other small equipment, large volume such as wind turbine [4, 5], and long-distance transmission system [6].

PMBs were first studied by Yonnet J. P. team, who applied the virtual work principle to study the magnetic force, stiffness, and torque of PMBs and establish mathematical models of PMBs under axial and radial magnetization [7–9]. Since then, more scholars have begun to study PMBs. For example, Tian LL et al. [10] analyzed the magnetic force based on the principle of virtual work; to analyze changes of the magnetic force in various magnetization directions, they established the superposition theorem and a mathematical

model for axially stacked PMBs. Ravaut R. et al. [11, 12] applied the Coulomb method to study the bearing capacity of PMBs under axial and radial magnetization effects and established a semianalytical model to the bearing capacity of PMBs by taking the curvature factor of magnet into account. Tan et al. [13] analyzed the characteristics of degrees of freedom of radial PMBs by means of the Columbia method, and the comparison and optimization were performed to structure parameters of axially stacked radial permanent magnet by Moser et al. [14]. The multiobjective optimization was performed to radially stacked PMBs by means of the three-dimensional Coulomb method [15] to analyze the bearing capacity and stiffness of PMBs. In [16], the calculation equations of the force and stiffness in axially, radially, and perpendicularly polarized of multipaired permanent magnet rings were deduced, by means of the Coulomb model and the vector method, and structures were evaluated in MATLAB and the bearing characteristics were discussed. In [17], the axial-radial stiffness ratio of the geometric structure of PMBs

was analyzed by finite element method which is based on the magnetic charge method. And the results were applied to PMBs of magnetic suspension fan.

According to previous studies, researchers mainly use the equivalent magnetic charge method and molecular current method to establish the mathematical model of permanent magnet bearing. Because the bearing capacity of single pair of magnetic rings is limited in unit volume, many scholars propose to use multimagnetic ring magnetic bearings to improve the bearing capacity. The symbolic integral function in MATLAB is often used to solve the analytical solution of bearing capacity and stiffness of PMBs. But there are many multiple integrals in the mathematical model of equivalent magnetic charge method. Not only does this method require a lot of calculations but also often does not have a corresponding numerical solution. Therefore, many scholars used finite element simulation to analyze and design the structure of permanent magnet bearings [18–22]. Although the finite element simulation is more accurate than the theoretical calculation, it is difficult to carry out numerical simulation when there are no specific dimensions in the initial stage of PMB structure design. Therefore, theoretical calculation is needed to obtain PMB's structural parameters.

Due to the fact that the Monte Carlo method is independent of the integral multiplicity, the higher the dimension is, the more effective the Monte Carlo method is. Therefore, it is widely used in the calculation of important numerical integrals in practical engineering problems. Starting from the existing Monte Carlo method, reference [23] proposes a new method for estimating a physical quantity without additional random sampling. In [24], Monte Carlo method is applied to the study of slider air bearing. In [25], a workspace determination method for parallel robots was presented by the Monte Carlo simulation. An integral definition method of the axial stiffness of an axial permanent magnet bearing is based on molecular current model, and comparison was performed on results of the Monte Carlo and finite element methods in [26]. The magnetic force of axially stacked permanent magnet bearings was calculated by means of the Monte Carlo method to simplify the solution process for the magnetic force in [27]. However, the general Monte Carlo method has the disadvantage of calculating nonrepeatable numerical solutions. In contrast, after selecting the irrational number of equal sequence, the Monte Carlo method with equal distribution sequence can only determine the results. When high precision integral results are required, the Monte Carlo method with equal distribution sequence is more effective.

Based on previous research [10, 11, 13], this paper establishes the mathematical model of single-ring and multiring PMBs by using equivalent magnetic charge method.

Firstly, the bearing characteristics of single-pair permanent magnet bearing (the influence of air gap and axial length of magnetic ring on bearing capacity) are simulated and analyzed by MATLAB, and the structure size of magnetic ring is determined. Then the equally distributed sequence Monte Carlo method is used to optimize the MATLAB simulation code of multiring PMBs. The error correction factor is proposed by comparing the theoretical simulation and finite element simulation results of one pair, two pairs, and three pairs of PMBs; then, the theoretical simulation of four pairs of permanent magnet ring bearings is validated, which is consistent with the finite element simulation results. Finally, the load-bearing characteristics of permanent magnet bearings are experimented with both radial and axial loads. The relationship between radial loading and radial displacement is analyzed. The relationship between axial loading and axial displacement is analyzed based on two aspects of rigid connection and flexible connection. The experimental results verify the validity of the theoretical simulation.

2. Modeling of PMBs

2.1. Modeling of Bearing Capacity of Single Magnetic Ring Bearing. A mathematical model was established for PMBs by means of the equivalent magnetic charge analytical method. Based on equivalent magnetic charge theory, the positive and negative magnetic charges of radial permanent magnet bearing with axial magnetization are evenly distributed in the end faces of magnetic rings; force interactions between their end faces will emerge when there are any radial or axial offset between the inner and outer magnetic rings as shown in Figure 1. The symbolic meaning in Figure 1 is shown in Table 1.

In accordance with Coulomb's Law and the electro-magnetic theory, End Face 1 of the inner magnetic ring is magnetically acted by End Face 3 of the outer magnetic ring while they are made of RbFeB:

$$\vec{F}_{13} = \frac{B_{r1} \cdot B_{r2}}{4\pi\mu_0} \cdot \int_0^{2\pi} \int_0^{2\pi} \int_{R_1}^{R_2} \int_{R_3}^{R_4} \frac{r_1 \cdot r_3 \cdot \vec{r} d\alpha d\beta dr_1 dr_3}{|\vec{r}_{13}|^3} \quad (1)$$

where B_{r1} is the magnetic remanence of stator magnet and B_{r2} is of rotor magnet. The axial and radial components of $\vec{F}_{13}$ by its projection in Directions x, y, and z in accordance with the geometric relationship are expressed as follows:

$$\begin{aligned} F_{13}^x &= \frac{B_{r1} \cdot B_{r2}}{4\pi\mu_0} \int_0^{2\pi} \int_0^{2\pi} \int_{R_1}^{R_2} \int_{R_3}^{R_4} \frac{x_0 r_1 r_3 d\alpha d\beta dr_1 dr_3}{|\vec{r}_{13}|^3} \\ F_{13}^y &= \frac{B_{r1} \cdot B_{r2}}{4\pi\mu_0} \int_0^{2\pi} \int_0^{2\pi} \int_{R_1}^{R_2} \int_{R_3}^{R_4} \frac{(e \cdot \sin \theta + r_1 \sin \alpha - r_3 \sin \beta) r_1 r_3 d\alpha d\beta dr_1 dr_3}{|\vec{r}_{13}|^3} \end{aligned}$$

FIGURE 1: Model of axial magnetization of magnetic rings.

$$F_{13}^z = \frac{B_{r1} \cdot B_{r2}}{4\pi\mu_0} \int_0^{2\pi} \int_0^{2\pi} \int_{R_1}^{R_2} \int_{R_3}^{R_4} \frac{(e \cdot \cos \theta + r_1 \cos \alpha - r_3 \cos \beta) r_1 r_3 d\alpha d\beta dr_1 dr_3}{|\vec{r}_{13}|^3} \quad (2)$$

where

$$|\vec{r}_{13}| = \sqrt{x_0^2 + (e \cdot \sin \theta + r_1 \sin \alpha - r_3 \sin \beta)^2 + (e \cdot \cos \theta + r_1 \cos \alpha - r_3 \cos \beta)^2}; \quad (3)$$

Directions y and z represent the radial direction, where forces shall be symmetrical in accordance with the structural symmetry so that such forces may be solved in any direction based on the similar principle for solving forces, and Direction x represents the axial direction.

Similarly, End Face 2 of the inner magnetic ring is magnetically acted by End Face 3 of the outer magnetic ring:

$$\begin{aligned} F_{23}^x &= \frac{B_{r1} \cdot B_{r2}}{4\pi\mu_0} \int_0^{2\pi} \int_0^{2\pi} \int_{R_1}^{R_2} \int_{R_3}^{R_4} \frac{(l + x_0) r_2 r_3 d\alpha d\beta dr_2 dr_3}{|\vec{r}_{23}|^3} \\ F_{23}^y &= \frac{B_{r1} \cdot B_{r2}}{4\pi\mu_0} \int_0^{2\pi} \int_0^{2\pi} \int_{R_1}^{R_2} \int_{R_3}^{R_4} \frac{(e \cdot \sin \theta + r_2 \sin \alpha - r_3 \sin \beta) r_2 r_3 d\alpha d\beta dr_2 dr_3}{|\vec{r}_{23}|^3} \\ F_{23}^z &= \frac{B_{r1} \cdot B_{r2}}{4\pi\mu_0} \int_0^{2\pi} \int_0^{2\pi} \int_{R_1}^{R_2} \int_{R_3}^{R_4} \frac{(e \cdot \cos \theta + r_2 \cos \alpha - r_3 \cos \beta) r_2 r_3 d\alpha d\beta dr_2 dr_3}{|\vec{r}_{23}|^3} \end{aligned} \quad (4)$$

TABLE 1: Nomenclature.

Letter	Means	Letter	Means
$\vec{r}_1, \alpha$	Polar coordinates of magnetic charge A	D	n-dimension range
$\vec{r}_3, \beta$	Polar coordinates of magnetic charge B	I	The product of the measure (M_D) and the expectation of Function f
e, θ	Moving polar coordinates of the inner ring relative to the outer ring	M_D	The corresponding measure of D
R_1, R_2	Inner and outer radii of the inner magnet rings	M_C	The measure of a hypercube
R_3, R_4	Inner and outer radii of the outer magnet rings	N	The number of the uniformly distributed points
$\vec{r}$	Radius vector of the magnetic charge A and B	m	The number of points falling statistically in D
x_0	Axial displacement of rotor	$\bar{f}$	The expectation of Function f
l	Axial length of the inner magnet rings	u_s	The equidistant point series
L	Axial length of the outer magnet rings	∂_s	Irrational number
H	Radial thickness of the outer magnet rings	P_s	The uniform distribution of Point Sequences in an Ultra-cuboid Containing Integral Region D_s
h	Radial thickness of the inner magnet rings	F_y/F_z	The radial force of the PMB
g	Length of the air gap	F_x	The axial force of the PMB
B_r	The magnetic remanence of magnet	n	The number of pair(s) of magnetic rings
μ_0	Vacuum permeability	E^y	The introduced error factor

where

$$|\vec{r}_{23}| = \sqrt{(l + x_0)^2 + (e \cdot \sin \theta + r_2 \sin \alpha - r_3 \sin \beta)^2 + (e \cdot \cos \theta + r_2 \cos \alpha - r_3 \cos \beta)^2}; \quad (5)$$

The bearing capacity of the inner or outer magnetic ring shall be the resultant force of 4 force components on 4 end faces, which may be expressed in accordance with the principle (the force component shall be positive while magnetic charges are in the same sign in any end face; otherwise, it shall be negative while magnetic charges are in the different signs in any end face) as

$$\vec{F} = \vec{F}_{13} + \vec{F}_{24} - \vec{F}_{23} - \vec{F}_{14} \quad (6)$$

In case of the radial offset of e , the radial stiffness of the single paired magnetic rings is described as

$$k = \frac{d\vec{F}}{de} \quad (7)$$

2.2. Modeling of Bearing Capacity of Multimagnetic Ring Bearing. Due to size limitation of the roller, the bearing capacity of the single paired magnetic rings is relatively small while a

PMB is equipped in a fixed space. Multipaired magnetic rings are often superimposed in practical applications to improve the bearing capacity of a PMB. By applying the equivalent magnetic charge and vector summation methods, based on analysis of the force component in each end face for n pairs of magnetic rings, the radial and axial bearing capacities are deduced. The superimposed mode of the repulsive forces of n pairs of axially magnetized rings is shown in Figure 2; the arrow in the magnetic ring indicates the direction of the magnetic field line.

While repulsive forces of n pairs of permanent magnet rings ($i \in [1, n+1]$) and j ($j \in [1, n+1]$) represent the i^{th} of the inner magnetic ring and the j^{th} end face of the outer magnetic ring, respectively, are axially superimposed and the residual magnetic flux density of an end face of a single is B_r , the residual magnetic flux densities at the superimposed locations of S and N-poles of the adjacent magnetic rings are $-2B_r$ and $2B_r$, respectively.

The end face of the inner magnetic ring is radially forced by the end face of the outer one:

$$\begin{aligned} F_{ij}^y &= \frac{B_{ri} \cdot B_{rj}}{4\pi\mu_0} \int_0^{2\pi} \int_0^{2\pi} \int_{R_1}^{R_2} \int_{R_3}^{R_4} \frac{(e \sin \theta + r_i \sin \alpha - r_j \sin \beta) r_i \cdot r_j d\alpha d\beta dr_i dr_j}{[x_0^2 + (e \sin \theta + r_i \sin \alpha - r_j \sin \beta)^2 + (e \cos \theta + r_i \cos \alpha - r_j \cos \beta)^2]^{3/2}} \\ F_{ij}^z &= \frac{B_{ri} \cdot B_{rj}}{4\pi\mu_0} \int_0^{2\pi} \int_0^{2\pi} \int_{R_1}^{R_2} \int_{R_3}^{R_4} \frac{(e \cos \theta + r_i \cos \alpha - r_j \cos \beta) r_i \cdot r_j d\alpha d\beta dr_i dr_j}{[x_0^2 + (e \sin \theta + r_i \sin \alpha - r_j \sin \beta)^2 + (e \cos \theta + r_i \cos \alpha - r_j \cos \beta)^2]^{3/2}} \\ F_{ij}^x &= \frac{B_{ri} \cdot B_{rj}}{4\pi\mu_0} \int_0^{2\pi} \int_0^{2\pi} \int_{R_1}^{R_2} \int_{R_3}^{R_4} \frac{x_0 r_i \cdot r_j d\alpha d\beta dr_i dr_j}{[x_0^2 + (e \sin \theta + r_i \sin \alpha - r_j \sin \beta)^2 + (e \cos \theta + r_i \cos \alpha - r_j \cos \beta)^2]^{3/2}} \end{aligned} \quad (8)$$

where

$$\begin{aligned} i &= 1, 2, 3, \dots, n+1 \\ j &= 1, 2, 3, \dots, n+1 \\ k &\in \{1, 2, 3, \dots, n\} \\ B_{ri} &= B_{rj} = B_r \quad (\text{where } i = j = 1 \text{ or } i = j = n+1) \\ B_{ri} &= B_{rj} = 2B_r \quad (\text{when } n = 2k, i = j = \{2k, 2(k-1), 2(k-2) \dots\}, i > 0) \\ B_{rM} &= -2B_r \quad (\text{when } n = 2k+1, i = j = \{2k+1, 2(k-1)+1, 2(k-2)+1 \dots\}, i > 1) \\ x_0 &= a + |j - i - (n+1)|l \end{aligned} \quad (9)$$

FIGURE 2: Schematic diagram of the axial superposition of repulsive forces of n pairs of axially magnetized rings.

The radial resultant force and stiffness of the inner magnetic ring are as follows against the columned outer magnetic rings:

$$\begin{aligned} \vec{F} &= \sum_{i=1}^{n+1} \sum_{j=1}^{n+1} \vec{F}_{i/j} \\ K &= \frac{d\vec{F}}{de} \end{aligned} \quad (10)$$

3. Theoretical Simulation of Bearing Capacity Characteristics of PMB

3.1. Analytical Simulation of Bearing Capacity Characteristics of Single-Pair PMBs. Bearing capacity characteristics of any PMB are closely related to its structural sizes and mainly influencing factors are its air gap between magnetic rings, the axial and radial lengths of magnetic rings, relative displacement of the inner and outer magnetic rings, and the number of pair(s) of magnetic rings. The bearing capacity of PMB can be mathematically modeled in accordance with the equivalent magnetic charge method, and then the influence of structure parameters of permanent magnet bearing on bearing capacity of permanent magnet bearing is analyzed. By taking a single pair of magnetic rings as an example, various structure parameters are marked in Figure 1.

3.1.1. Effects of Air Gap on Bearing Capacity. The smaller the air gap and the stronger the magnetic induction at the air gap, the greater the bearing capacity generated and the more compact the permanent magnet bearing structure. Set the axial length of the inner magnetic ring and the outer magnetic ring as $L=l=8$ mm. The radial thickness of the inner and outer magnetic rings is preferably equal ($H=h=10$ mm), and the inner and outer radii of the inner magnetic ring remain unchanged ($R_1=15$ mm, $R_2=25$ mm). The structure types of magnetic rings are shown in Table 2.

In each case of the air gap ($g=1$ mm, 1.5 mm, 2 mm, and 2.5 mm), the radial magnetic force characteristics of a PMB including a single pair of magnetic rings are calculated, as shown in Figure 3. The smaller the air gap, the greater the radial bearing stiffness of the PMB, but the smaller

FIGURE 3: Radial bearing capacity vs. the air gap.

the maximum radial force and, meanwhile, the harder the processing difficulty. Therefore, it is necessary to comprehensively consider the influence of size, assembly, and working conditions, so that the PMB can have the greatest rigidity and bearing capacity.

3.1.2. Effects of Magnetic Ring Axial/Radial Length on Bearing Capacity. As for a radial PMB, it is better to have the radial thickness of the inner and outer magnetic rings equal or similar, and its radial bearing stiffness shall be optimal while its magnetic rings are square in cross sections [12]. Moreover, its bearing stiffness may rise while the axial heights of its inner and outer magnetic rings are closer ($L \approx l$). Thus, set $L=l$ and the axial length of the magnetic ring is taken as the independent variable and assigned 7 values among the range (6~12 mm), respectively. While the air gap is 2 mm ($R_1=15$ mm, $R_2=25$ mm, $R_3=27$ mm, and $R_4=37$ mm) and the radial offset (e) changes among the range (0.4 mm-1.6 mm), the simulated radial bearing capacities are shown in Figure 4.

Figure 4 indicates that the larger the radial offset (e), the greater the radial bearing capacity. When the radial offset (e) is constant and the axial length L of the magnetic ring is less than 8 mm, the radial force increases linearly with the increase of the axial length of the magnetic ring, and when

TABLE 2: Parameters of the radial magnetic rings.

Gap (mm)	R_1 (mm)	R_2 (mm)	R_3 (mm)	R_4 (mm)
1	15	25	26	36
1.5	15	25	26.5	36.5
2	15	25	27	37
2.5	15	25	27.5	37.5

FIGURE 4: Radial bearing capacity vs. the axial length of the magnetic ring.

L is greater than 8 mm, the radial force no longer increases linearly.

3.1.3. Comparison of Single Magnetic Ring Results between the Analytical and Finite Element Methods. According to theoretical analysis of the loading characteristics of a single magnetic ring, comparison with the finite element simulation results was performed to verify the rationality of the design of its structure parameters. The comparison results of the single paired magnetic rings by means of the equivalent magnetic charge and finite element methods are shown in Figure 5. This figure indicates that their errors rise along with the growth of the radial offset and changes linearly, which indicates there is a certain linear error between the theoretical and finite element analysis results.

In order to further analyze bearing characteristics of multipaired PMBs, pairs of radial permanent magnet rings are selected, whose structure parameters are shown in Table 3.

3.2. Analytical Simulation of Bearing Capacity Characteristics of Multiring PMBs

3.2.1. Effects of the Number of Pair(s) of Magnetic Rings on Bearing Capacity of PMBs. As the structure of a single pair of magnetic rings is determined, based on theoretical analysis, the radial bearing capacity of the PMB may be improved when the number of pairs of the magnetic rings increases

FIGURE 5: Radial force vs. the radial offset of a single pair of magnetic rings (comparison of the equivalent magnetic charge and finite element methods).

in the finite space. The previous theoretical analysis indicates that the quadruple integral is necessary for performance of simulation. While the number of pairs of magnetic rings increases, using integral function (nIntegrate) in MATLAB to get solution needs very long time and the correct numerical solution is often not obtained. Thus, the multiring permanent magnet bearing calculated based on the equivalent magnetic charge method may be solved by means of the equally distributed sequences based Monte Carlo method.

(1) Monte Carlo Method: it is also called the random sampling or statistical test method, where any probability phenomenon is taken as the research object for numerical simulation and the statistical value is solved by means of the sampling survey method for presumption of unknown characteristic quantities.

While D is assumed as one n -dimension range and $f(x) \in D \subset R^n \rightarrow R$ exists, its n -fold integration can be expressed by

$$I = \int_D f(p) dp \quad (11)$$

I can be regarded as the product of its measure (M_D) and the expectation of Function f . The basic Monte Carlo method is to find a range (D) contained in a hypercube whose measure is known as M_C . There are N uniformly distributed points which may be randomly generated in D . While it is

TABLE 3: Parameters of the radial permanent magnet bearing.

Parameter	Value
Inner diameter of static magnetic ring /mm	30
Outer diameter of static magnetic ring /mm	50
Inner diameter of dynamic magnetic ring /mm	54
Outer diameter of dynamic magnetic ring /mm	74
Axial length /mm	8
Air gap /mm	2

assumed that there are m points falling statistically in D , the corresponding measure of D is defined as

$$M_D \approx \frac{mM_C}{N} \quad (12)$$

The expectation of Function f is

$$\bar{f} \approx \frac{1}{m} \sum_{i \in D} f(p_i) \quad (13)$$

Then,

$$I \approx M_D \bar{f} = \frac{M_C}{N} \sum_{i \in D} f(p_i) \quad (14)$$

The Monte Carlo method has fewer constraints and flexible simulation processes. Its errors have the characteristics of probability, and errors are only related to its sample standard deviation and sample size rather than the space where the sample elements are located. However, it has the weakness of computational nonrepeatability. In contrast, the equally distributed sequences based Monte Carlo method has better orders of error than the random sequences based Monte Carlo method, whose calculation result is uniquely determined while the irrational number for generation of the equally distributed sequences has been selected.

(2) Equally distributed sequences based Monte Carlo method: as for a general interval $([a, b])$, a moderately distributed point sequence may be gained within $[a, b]$ by assuming $u_s = x_s(b - a) + a$. As for the s -fold integration ($I = \int_{D_s} f(p) dp$), select s irrational numbers $(\partial_1, \partial_2, \dots, \partial_s)$ which are linearly opposite to rational numbers linearly. The uniform distribution of Point Sequences in an Ultra-cuboid Containing Integral Region D_s is

$$P_s = ((n\partial_1)(b_1 - a_1) + a_1, (n\partial_2)(b_2 - a_2) + a_2, \dots, (n\partial_s)(b_s - a_s) + a_s) \quad (15)$$

where $[a_k, b_k]$ represents the length of the k^{th} ($k = 1, 2, 3, \dots, s$) dimension side of the hypercube containing the integral region, and the linear independence of irrational numbers against rational numbers means $a_j \neq 0$ ($a_k \in Q$, $k = 1, 2, 3, \dots, s$) existing at least at a time.

$$\alpha_1 \partial_1 + \alpha_2 \partial_2 + \dots + \alpha_s \partial_s \neq 0 \quad (16)$$

Thus, an approximation of the integral may be calculated out by means of the equally distributed sequences based Monte Carlo method:

$$I \approx M_{D_s} \bar{f} = \frac{M_C}{N} \sum_{i \in D_s} f(P_i) \quad (i = 1, 2, 3, \dots, N) \quad (17)$$

The basic steps for solving the equivalent magnetic charge model by means of the equally distributed sequences based Monte Carlo method are as follows.

(1) Determine the initial parameter values of the magnetic rings in accordance with the structural design and the value of the number (n , which is enough large and taken as 10000000 here) of random points.

(2) Convert an integral of the magnetic force model into the integrand where x represents a 4-column vector or matrix. Each column of x represents a point in the four-dimensional space which is an irrational number that is independent of any rational number selected from the corresponding $(\alpha, \beta, r_m, r_n)$ in the magnetic force model to generate the equally distributed sequences (sqrt(2); sqrt(3); sqrt(6)/3; sqrt(10)).

(3) Perform a transformation to convert the equally distributed sequences over the integration interval to various vectors over the integration interval (D).

(4) Determine those points falling into the integration interval (D).

(5) Calculate the approximate integral.

(6) Solve the resultant bearing force in accordance with the magnetic force equation ((6) and (10)).

Before using equally distributed sequence Monte Carlo method to simulate and analyze the load-bearing characteristics of multiring PMB, the load-bearing characteristic of single-pair PMB is simulated and compared based on the two simulation methods. Four groups of single magnetic ring PMB with different specifications are selected and named as Case 1, Case 2, Case 3, and Case 4. Among them, Case 1 is the size of this paper, and Cases 2, 3, and 4 are the size of permanent magnet ring in [10, 13, 28]. Comparing the radial forces of the four kinds of PMBs, the simulation results are shown in Figure 6.

In Figure 6, the simulation results show that the load-carrying characteristics of the two methods are similar in each structure; the difference between the maximum radial force values is only 0.88%. But the simulation efficiency of Monte Carlo method is much higher than that of MATLAB code, as shown in Table 4.

The comparison in Table 4 shows the deviation between the two methods being more than 10 times. As the pairs of the magnetic rings increase, the calculation time will increase exponentially, and even the solution may not be obtained

TABLE 4: Comparison of simulation efficiency between Monte Carlo method and MATLAB method.

No.	R1 (mm)	R2 (mm)	R3 (mm)	R4 (mm)	Gap (mm)	Axial Length (mm)	Time1 (Matlab) (s)	Time2 (M-C) (s)
Case1	15	25	27	37	2	8	861.53	60.95
Case2[10]	15	20	21	26	1	5	1082.59	61.64
Case3[28]	20	25	29	34	4	5	568.25	78.32
Case4[13]	10	19	21	30	2	7	589.03	76.12

FIGURE 6: Radial bearing capacity of four kinds of the PMB.

based on the MATLAB code. Therefore, the equally distributed sequences based Monte Carlo method can be used to analyze the load-bearing characteristics of multiring PMBs. Taking the structure of case 1 as an example, the bearing characteristics of multimagnetic rings will be analyzed.

Because the bearing capacity of repulsive PMB is superior to that of attractive, and the axial magnetization is easy to achieve, the axial magnetization and superposition of the repulsive force of axial permanent magnet rings are utilized for radial permanent magnet bearings to achieve suspension support between the inner and outer magnetic rings. The radial bearing capacity of the PMB with multipaired magnetic rings is simulated by means of the Monte Carlo method, whose radial offset changes in the range (0-1.8 mm). Compare the effects of the number of ring pairs of magnetic rings (1-4 pairs) on its radial bearing capacity and the corresponding results are shown in Figure 7.

Figure 7 indicates that the radial bearing capacity and stiffness of the PMB increase gradually along with the growth of the number of pairs of magnetic rings.

3.2.2. Comparison of Results of Multipaired Magnetic Rings between the Analytical and Finite Element Methods. While the number of pair(s) of magnetic rings is 1, 2, or 3, the comparison results based on the analytical and finite element methods are shown in Figure 8. While the number of pair(s) of magnetic rings grows, the larger the radial offset of the outer magnetic ring relative to the inner one, the more the error between the radial forces simulated by means of the above two methods.

The trends of error curves indicate that the errors simulated based on the analytical and finite element methods are linear, which means the linear correction can be performed to the analytical results. While the number of pair(s) of magnetic rings is n and the radial offset between the inner and outer magnetic ring is defined as e , the introduced error factor (E^y) for the radial bearing capacity of the PMB is expressed as

$$E^y = -17.538ne + 1.391n + 8.392e - 0.876 \quad (18)$$

FIGURE 7: Radial bearing capacity of the PMB vs. the number of pair(s) of magnetic rings.

FIGURE 8: Radial force vs. the radial offset of multipaired magnetic rings (comparison of the theoretical method (M-C) and finite element method (FEM)).

After the correction, comparison of simulated bearing capacities of the PMB including 4 pairs of magnetic rings was performed, whose results (Figure 9) indicate that the corrected theoretical simulation values are approximately close to those finite element results. Thus, the structure design may be preliminarily carried out first by means of the theoretical simulation while the structure of a permanent magnet roller is designed.

3.2.3. Effects of Axial Offset on Bearing Capacity Characteristics of PMB. In accordance with Eamshaw Theorem, stable suspension of the rotor would never come true while it was only under the action of the magnetic force from a permanent magnet; on the other hand, the axial mechanically auxiliary supports shall be necessary. Thus, analysis shall be performed to the axial force of the PMB and the axial position between

the inner and outer magnetic rings to reasonably adjust such axial position and improve the engineering practicability of the PMBs.

Considering a PMB of 4 pairs of magnetic rings, when the axial and radial offsets between the inner and outer permanent magnet rings are changed, the simulated radial bearing capacity is shown in Figure 10.

When the radial offset (e) is a certain value, the radial force and stiffness are the largest when the axial offset (x_0) is zero; the larger the axial offset, the smaller the radial force and stiffness. When the axial displacement reaches half of the width of the magnetic ring ($a = 4\text{mm}$), the radial force is approximately 0. If the axial offset is more than 4 mm, the direction of radial force will be reversed. That means the repulsion force between magnetic rings changes into attraction force, and the permanent magnet bearing

FIGURE 9: Radial force vs. the radial offset between 4 pairs of magnetic rings (comparison of analysis results based on the modified equivalent magnetic charge and finite element methods).

FIGURE 10: Axial offset vs. radial force.

becomes unstable. Thus, the axial offset between magnetic rings is necessarily under control to ensure it shall be no more than 50% of the axial length of a single pair of magnetic rings.

While there is any axial or radial offset between inner and outer magnetic rings of a permanent magnet bearing, its simulated axial force results are shown in Figure 11. The axial force correspondingly rises along with the growth of the axial offset. While the axial offset is up to 50% of the thickness of the magnetic rings (4 mm), the peak axial force is up to 1052 N which is very high. Thus, the axial position between the inner and outer magnetic rings of a PMB shall not be too large. Under the premise of ensuring its stable suspension, its axial force shall be minimized as possible.

FIGURE 11: Axial offset vs. axial force.

4. Experimental Investigation of Bearing Capacity Characteristics of Multiring PMBs

In order to verify the accuracy of theoretical analysis, the bearing characteristics of permanent magnet bearings are experimentally studied in both radial and axial directions.

4.1. Analysis of Radial Force. Two PMBs are used to support the permanent maglev roller in the experiment; the structure schematic diagram of the permanent maglev roller is shown in Figure 12. The PMB on both sides are the same including four pairs of permanent magnet rings. The permanent magnet rings are magnetized axially, and repulsive force is superimposed and axially superimposed. As a consequence of Earnshaw's theorem, it is not possible to levitate a body statically solely with permanent magnets in a static magnetic field. In the roller, the radial PMBs are unstable in the axial direction and the benefit of the thrust ball bearing is to keep the maglev roller stable. Permanent magnetic bearings near the thrust ball are defined as PMB1 and PMB2 on the other side.

Two PMBs are symmetrically mounted on both sides of the roller, where $a_1 = a_2$. Applying radial force in the middle of the idler, according to the principle of moment balance, the force applied on the PMB is half of the radial force. Under the action of radial force ($2F_y = 20\text{gN}, 40\text{gN}, 60\text{gN} \dots 200\text{gN}$), the radial displacement of idler is detected by displacement sensor. The load-bearing characteristic curve of permanent magnet bearing can be obtained, as shown in Figure 12.

In Figure 13, TEST1 is the experimental data of PMB1 and TEST 2 is the experimental data of PMB2. It shows that the radial force of PMBs increases as the radial displacement increases. Compared with the TEST2, TEST1 and the theoretical values and FEM simulation values are closer. TEST2 is slightly smaller than the theoretical values. On the one hand, this may be due to the tolerance of permanent magnet bearings installed in roller. On the other hand, there is friction

FIGURE 12: Measurements of radial force of Permanent Maglev Roller.

FIGURE 13: Measurement of radial force under different loads.

between thrust ball and PMB1, and the measured value of PMB1 is relatively larger than TEST2.

4.2. Analysis of Axial Force. In order to verify the variation of the axial position and force of inner and outer magnetic rings of a PMB, its bearing capacity characteristics were analyzed experimentally from two aspects: the rigid and flexible joints.

4.2.1. Analysis of Single-Sided Axial Force. The test schematic diagram and the experimental setups for analysis of the single-sided axial force are shown in Figures 14(a) and 14(b), respectively, as shown in the figure, the inner magnetic ring and the shaft are under pressure connection, and the outer magnetic ring is located inside the bearing housing. Rotate the adjusting centre to push the shaft horizontally move to the right direction, thus the relative position of the inner and outer magnetic rings can be changed. The axial offset (x) is measured by displacement sensor, and the axial force (F_x) of the shaft against the outer magnetic ring is detected with a tension-compression sensor.

Based on our experimental analysis, when the axial offset (x) is more than 50% of the width of the magnetic rings (4 mm), the shaft will not remain suspended and the inner magnetic rings will be attracted by the outer magnetic ring. While its tip is rotated, the inner and outer magnetic rings are rubbed and the axial force is relatively large.

Comparison of five groups of measured data and the corresponding simulation results are shown in Figure 15, which indicates that the axial force (F_x) is changing trends along the axial offset (x), which agrees with the simulated results. When the axial offset is up to 4 mm, the axial force is the largest; at the same time, the shaft is suspended while the axial offset is within the range (0-4 mm). In the experiment, the measured axial forces are lower than the simulated ones maybe due to the effects of magnetization and the residual magnetism of magnetic rings. When the axial offset is more than 4 mm, the shaft cannot be suspended and the inner and outer magnetic rings are in contact. The friction between the inner and outer magnetic rings leads to such fact that the measured axial forces are more than the simulated ones. The measured maximum axial force is about 1117.78 N which is slightly larger than the theoretical maximum simulation value (1052.25 N) and their maximum error is about 6.2%.

4.2.2. Analysis of the Axial Force of a Rigid Joint. While the tip is replaced with an adjusting screw and the screw and the shaft are rigidly connected, the test schematic diagram and the experimental setups are shown in Figures 16(a) and 16(b), respectively. The screw is rotated to horizontally push or pull the shaft and the inner magnetic ring to the left and right and measure the relationship between the axial displacement (x) and force (F_x) of the PMB.

Start measurement when the inner and outer magnetic rings are aligned. The shaft is pulled by the adjusting screw (feed rate: 0.2 mm/time) to the left or right side. The axial force and displacement are synchronously displayed by the tension and displacement sensors, respectively. The experimental data is compared with the simulation values as shown in Figure 17. The error between the measured and simulated values is very small. When the axial offset reaches about 4 mm, the error reaches the maximum value. According to the experimental phenomena, when the axle

FIGURE 14: Measurements of single-sided axial force of PMB.

FIGURE 15: Measurements of the single-sided axial force.

moves between -4 mm and 4 mm, the axle is suspended and frictionless. While the axis moves beyond this range, the inner and outer magnetic rings begin to contact with each other. Therefore, the axial displacement between -4 mm and 4 mm is continuously measured for several times, and three groups of measured values are compared with the simulation values, as shown in Figure 17.

As shown in Figure 18, the three groups of measured data are basically coincident, which are basically consistent with the simulation results but slightly smaller. While the axial offset is less than 1 mm, the axial force is no more than 300 N. In case that the axial offset is 4 mm, the maximum axial force is about 955.4 N, which is less by about 9.2% than the theoretical value (1052.25 N), possibly because of certain errors such as the sizes of magnetic rings or the installation process. It is difficult to make the two adjacent magnetic rings fully fit, so when the axial offset is 0 mm, the measured values are smaller than theoretical value.

4.2.3. Analysis of the Axial Force of a Flexible Joint. While the dynamometer and the shaft are connected through a wire rope to form a flexible joint and both sides of magnetic rings are radially forced, effects of the radial load on the suspension force of the PMB are analyzed. At this time,

the shaft cannot be pushed to the horizontal right side by rotating the adjusting screw; on the other hand, the shaft can only be pulled with the wire rope to the horizontal left side. The test schematic diagram is shown in Figure 19(a), where a load (m_0) is hung at the shaft at each side of the PMB so that the shaft shall be acted under the load ($m=2m_0$). The experimental setups are shown in Figure 19(b). The counterweights (0 kg, 4 kg, 8 kg, and 12 kg) are applied sequentially during the tests, and the corresponding axial forces are measured.

While the radial load is 0 , the shaft is pulled by the adjusting screw to the left side (feed rate: 0.2 mm/time) until the shaft has been ejected. The axial force and displacement are synchronously displayed by the tension and displacement sensors, respectively. While the shaft is pulled horizontally by the wire rope to the left side within 50% of the thickness of magnetic rings (4 mm), the axial force falls along with the decrease of the axial offset. While the axial offset is less than 1 mm, the shaft is ejected and the tests end.

Effects of the radial offset due to different loads on the axial force (Figure 20(a)) are simulated theoretically. In case that the loads (0 kg, 4 kg, 8 kg, and 12 kg) are applied during the tests, the corresponding axial forces are shown in Figure 20(b) which indicates that the axial force falls along with the growth of the axial offset while the load rises. Comparing the measured and simulated results under loads, it shows that their axial forces change consistently and their axial forces are affected by the radial loads slightly.

4.2.4. Comparison of Axial Force Measurements. In the case of no load, comparison of the measured and simulated data for the rigid and flexible single-sided and fully rigid joints is shown in Figure 21, which indicates that the rigid full-section axial forces have larger measurement errors but the axial forces at both sides of Point 0 are smaller on the whole. In contrast, the measured and simulated results for the rigid and flexible single-sided joints are basically the same.

5. Conclusions

The mathematical models of single-pair and multipair permanent magnet bearings are established based on equivalent magnetic charge method in this paper, and the equal

FIGURE 16: Measurements of the axial force of a rigid joint.

FIGURE 17: Measurements of the axial force of a rigid joint (full segment).

FIGURE 18: Measurements of the axial force of a rigid joint (suspension).

distribution sequence Monte Carlo method is used to optimize the MATLAB simulation code. By comparing theoretical simulation results and finite element results, error correction coefficient is proposed. With this, the error between analytical method results and finite element simulation

results approximate zero, and the theory results and the FEM results are consistent. Finally, the bearing characteristics of permanent magnet bearings are tested by radial force and axial force. The experimental results are consistent with the simulation analysis. The simulation method will be of benefit

FIGURE 19: Schematic diagram of measurement of the axial force of a flexible joint.

FIGURE 20: Comparison of measurements of the axial force of a flexible joint.

FIGURE 21: Comparison of axial force measurements (no-load).

for the design of permanent magnet bearing in engineering applications.

Data Availability

All the data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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