Research Article

Adaptive Chattering-Free Sliding Mode Control of Chaotic Systems with Unknown Input Nonlinearity via Smooth Hyperbolic Tangent Function

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Received 17 May 2019; Revised 17 July 2019; Accepted 1 August 2019; Published 7 October 2019

Academic Editor: Alessandro Lo Schiavo

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The design of adaptive chattering-free sliding mode controller (SMC) for chaotic systems with unknown input nonlinearities is studied in this paper. A smooth hyperbolic tangent function is utilized to replace the discontinuous sign function; therefore, the proposed adaptive SMC ensures that not only the chaos phenomenon can be suppressed effectively but also the chattering often appearing in the traditional discontinuous SMC with sign function is eliminated, even when the unknown input nonlinearity is present. A sufficient condition for stability of closed-loop system is acquired by Lyapunov theory. The numerical simulation results are illustrated to verify the proposed adaptive sliding mode control method.

1. Introduction

In recent decades, sliding mode control is one of the popular control theories due to its robustness and insensitivity to parameter uncertainty and external disturbance [1, 2]. The trajectories of SMC systems can be driven onto a specified sliding surface and slide toward desired stable condition, which is called sliding motion. There are many methodologies used to select sliding surfaces, while the different control law is obtained, and it is guaranteed that the controlled system is stable [2]. In general, the SMC and switched system control strategies are usually adopted to the practical application. For example, in [3, 4], the new SMC-based fuzzy controller and robust hybrid controller are applied to the 7-degree-of-freedom upper-limb exoskeleton robot such that the undesired external disturbance can be suppressed. Therefore, the SMC can be used to overcome undesired external disturbance successfully; in [5], a class of linear switched system is discussed for the cooperative stabilization problem and a novel class of switching signals is proposed. Based on above descriptions, the SMC is adopted to cope

with input nonlinearity considered in this paper due to its robustness.

In traditional SMC, the control force is difficult to achieve, which has high frequency chattering phenomenon due to the discontinuous sign function. Therefore, many quasisliding mode control techniques have been proposed [2, 6–8], and the chattering phenomenon is eliminated due to the controller designed by using a continuous function. On the other hand, chaotic systems have been proven the existence which is extensively and become an interesting topic in physical systems [9–11]. Also, the chaotic systems have special nonlinear dynamic behavior [7, 12]. In general, there are two sides to such physical systems [11]; the first side is the beneficial feature, such as chemical reactions and encrypted communication systems. On the other side, the chaos phenomenon in some engineering systems is highly unexpected for its applications, such as electronic systems, power converters, and high-precision mechanical systems. Therefore, many different methods have been proposed to solve the problems of chaos control and suppression for chaotic systems, such as adaptive feedback control [13–16],

sliding mode control [11, 17, 18], PID control [19–21], optimal control [22–26], and robust control [27–29] among many others [30, 31].

As described above, many modified methodologies have been developed to overcome the chattering phenomenon in traditional SMC, but they have not been well discussed when the system structure is subjected to input nonlinearity and even the input nonlinearity is unknown. It is well known that in practical systems, unavoidable external nonlinear perturbations do exist in control input, which not only affect system stability but also decrease the input performance and response. Furthermore, since the feature of chaotic system is very sensitive to parameter uncertainty and external perturbation [11], the stability analysis of controlled system with input nonlinearity is an important issue for chaotic systems. Recently, some papers [11, 32, 33] have researched SMC design for the chaotic systems with nonlinear input, but they only discussed the control design when input nonlinearity is well known. Hu et al. [34] proposed a SMC-based adaptive controller to deal with nonlinear input, but the adaptive law is high order. However, we present a low-order adaptive law to omit the harmful effects of nonlinear input. To our knowledge, the chattering-free SMC design with unknown input nonlinearity is still not well discussed.

In this paper, the adaptive continuous SMC design for chaotic systems with unknown input nonlinearity is studied. The main contribution is to introduce a new technique of adaptive sliding mode control with a smooth hyperbolic tangent function to avoid chattering. In addition, the undesired chaotic behavior of the considered chaotic systems can be fully suppressed even with unknown input nonlinearities. Finally, we present numerical simulation results to illustrate the effectiveness of the proposed adaptive continuous SMC scheme.

1.1. Notations. In this paper, R^n represents the n -dimensional Euclidean space, $R^{n \times m}$ denotes the set of all real n by m matrices, the superscript “+” denotes the matrix generalized inverse, and I_m stands for m by m identity matrices. $\|W\|$ denotes the Euclidean norm, when W is a vector, and the induced norm, when W is a matrix. $\lambda_{\max}(W)$ denotes the maximum eigenvalue of matrix W . $|W|$ represents the absolute value of W , and $\text{sign}(s)$ is the sign function of s ; if $s > 0$, $\text{sign}(s) = 1$; if $s = 0$, $\text{sign}(s) = 0$; and if $s < 0$, $\text{sign}(s) = -1$.

2. System Description and Problem Formulation

Consider a general class of chaotic systems given below:

$$\dot{x}(t) = Ax + f(x, t), \quad (1)$$

where $A \in R^{n \times n}$ is the system matrix and $x \in R^n$ is the system state vector. $f(x, t) \in R^{n \times 1}$ denotes the nonlinear vector of systems. Without loss of generality, we make the following assumption about system (1).

Assumption 1. The dynamic system (1) can be written as

$$\dot{x}(t) = Ax + Bg(x, t), \quad (2)$$

where $B \in R^{n \times m}$ and $g(x, t) \in R^{m \times 1}$. The pair A, B is controllable.

Remark 1. Assumption 1 is not restrictive. Many nonlinear chaotic systems described by (2) can be found. For example, the Matsumoto–Chua–Kobayashi circuit, Rössler system, modified Chua’s circuit, Lorenz system, Duffing–Holmes system, Lü system, and Chen chaotic dynamical system.

To suppress the chaos oscillation behavior, we introduce a control vector subjected to unknown nonlinearity described as

$$\dot{x}(t) = Ax + Bg(x, t) + B\varphi(u(t)), \quad (3)$$

where $\varphi(u(t)) = [\varphi_1(u_1) \cdots \varphi_m(u_m)]^T \in R^{m \times 1}$ is the control vector with nonlinearity. The continuous nonlinear function satisfies $\varphi(0) = 0$, where $\varphi_i(u_i(t)) \in R \rightarrow R$ with the law $u_i(t) \rightarrow \varphi_i(u_i(t))$, satisfying

$$\beta_{2,i}u_i^2 \geq u_i\varphi_i(u_i) \geq \beta_{1,i}u_i^2, \quad (4)$$

where $\beta_{1,i}$ and $\beta_{2,i}$ are unknown positive nonzero constants but bounded. Figure 1 shows a nonlinear function $\varphi_i(u_i(t))$ inside the sectors $\beta_{1,i}$ and $\beta_{2,i}$.

In SMC, the traditional controller is difficult to implement because there is an important adverse problem of high-frequency chattering phenomenon. In order to improve this issue, the following lemma with smooth hyperbolic tangent function is introduced.

Lemma 1. There always exists a constant $\gamma_i > 0$ for all $s_i \neq 0$ such that

$$s_i \cdot \tanh(s_i) > \gamma_i s_i \cdot \text{sign}(s_i). \quad (5)$$

Proof. Since $\tanh(s_i) = (e^{js_i} - e^{-js_i}) / (e^{js_i} + e^{-js_i})$, $j > 0$, obviously, $s_i \cdot \tanh(s_i) > 0$ and $s_i \cdot \text{sign}(s_i) = |s_i| > 0$ for all $s_i \neq 0$. In general, there always exists a positive constant $\varepsilon_i \in (0, 1)$ satisfying

$$s_i \text{sign}(s_i) < s_i \tanh(s_i) + \varepsilon_i s_i \text{sign}(s_i). \quad (6)$$

Also, it can be rewritten as

$$s_i \tanh(s_i) > (1 - \varepsilon_i) s_i \text{sign}(s_i) = \gamma_i s_i \text{sign}(s_i), \quad (7)$$

where $\gamma_i = (1 - \varepsilon_i)$. Therefore, there always exists a constant $\gamma_i > 0$ satisfying $s_i \cdot \tanh(s_i) > \gamma_i s_i \cdot \text{sign}(s_i)$ for all $s_i \neq 0$.

It is worthy to mention that the existing but unknown parameter γ_i does not appear in our proposed controller due to the adaptive control approach. By Lemma 1, a smooth continuous switching function is used to construct our control design such that the chattering phenomenon can be improved. As shown in Figure 2, the different parameters j are discussed for hyperbolic tangent function.

FIGURE 1: The scalar nonlinear function $\varphi_i(u_i(t))$.

According to Figure 2, the hyperbolic tangent function with a large value of j is close to the discontinuous sign function while the chattering phenomenon might appear. Therefore, the parameter j should be assigned by an appropriate value for avoiding the undesired chattering in SMC.

3. The Control Design Algorithm

In consequence, to complete the control objective mentioned above, there are two major steps. First, one needs to design an appropriate switching surface for the control system with input nonlinearity such that the stability of the dynamics on the sliding manifold defined later can be ensured. Second, one needs to propose a continuous adaptive SMC such that the existence of the sliding motion can be guaranteed without chattering.

First, the proportional-integral (PI) type sliding surface is first defined as follows:

$$S(t) = Cx(t) - \int_0^t (CA + K)x(\tau)d\tau, \quad (8)$$

where $S(t) \in R^m$, $C \in R^{m \times n}$, and $K \in R^{m \times n}$ and matrix $C = B^+ = (B^T B)^{-1} B^T$ results in $CB = I_m$. B^+ is the generalized inverse of B , and B is the full-column rank. $I_m \in R^m$ is the identity matrix. The control matrix K satisfies $\lambda_{\max}(A + BK) < 0$. As long as the system can operate in the sliding mode, the controlled dynamics will satisfy the following equation:

$$\begin{aligned} \dot{S}(t) &= 0, \\ S(t) &= 0. \end{aligned} \quad (9)$$

Therefore, when the system operates in the sliding mode, we can obtain the equivalent control $\varphi_{eq}(u(t))$ by differentiating (8) with respect to time and substituting from (3):

$$\dot{S}(t) = -Kx(t) + g(x, t) + \varphi_{eq}(u(t)) = 0, \quad (10)$$

where $CB = B^+ B = I_m$ has been introduced. Obviously, the equivalent control $\varphi_{eq}(u(t))$ in the sliding mode is obtained by

FIGURE 2: The $\tanh(s_i) = (e^{js_i} - e^{-js_i}) / (e^{js_i} + e^{-js_i})$ function with different j .

$$\varphi_{eq}(u(t)) = Kx(t) - g(x, t). \quad (11)$$

Substituting (11) into (3), we have

$$\dot{x}(t) = (A + BK)x(t). \quad (12)$$

According the above discussion and (12), we can conclude that when the system is in the sliding manifold, the controlled chaotic system in the sliding mode is stable if matrix K satisfies $\lambda_{\max}(A + BK) < 0$. Now, to guarantee the existence of the sliding mode, the continuous sliding controller and adaptive law are proposed, respectively, as

$$u(t) = -\alpha \hat{\eta}(t) (\|g(x, t) - Kx\| + \delta) \tanh(S), \quad (13)$$

$$\dot{\hat{\eta}}(t) = \alpha (\|g(x, t) - Kx\| + \delta) \cdot \|S\|. \quad (14)$$

From (13) and (14), we have

$$\begin{aligned} u_i(t) &= -\alpha \hat{\eta}(t) (\|g(x, t) - Kx\| + \delta) \tanh(s_i), \\ \dot{\hat{\eta}}(t) &= \alpha (\|g(x, t) - Kx\| + \delta) \cdot \|S\|, \end{aligned} \quad (15)$$

where $\alpha > 1$, $\delta > 0$ are constants and can be assigned. The factor $\hat{\eta}(t)$ is used to estimate the unknown input nonlinearity. The control block diagram is shown in Figure 3.

Theorem 1. Consider dynamic system (3) with unknown input nonlinearity. If the continuous adaptive SMC is properly designed as (13) and (14), then the system trajectory will be controlled to the sliding surface $S(t) = 0$ even with unknown input nonlinearities.

Proof. Let us consider a Lyapunov function for a closed-loop system as follows:

$$V(t) = \frac{1}{2} (S^T S + \eta e^2), \quad (16)$$

FIGURE 3: The control block diagram.

where $\eta = \gamma_1 \cdot \beta_1$, $\beta_1 = \min_{i \in [1, m]} \beta_{1,i}$, $\gamma_1 = \min_{i \in [1, m]} \gamma_i$, $e = \hat{\eta}(t) - \eta^{-1}$, and $\dot{e} = \dot{\hat{\eta}}(t)$ and η is an unknown constant. Then, the Lyapunov function (16) derivative with respect to time is obtained as

$$\begin{aligned} \dot{V}(t) &= S^T \dot{S} + e\eta \dot{e} = S^T (g(x, t) - Kx + \varphi(u)) + e\eta \dot{e} \\ &\leq \|S\| \cdot (\|g(x, t) - Kx\| + \delta) + S^T \varphi(u) + e\eta \dot{e}. \end{aligned} \quad (17)$$

Since $\beta_{2,i} u_i^2 \geq u_i \varphi_i(u_i) \geq \beta_{1,i} u_i^2$, one has

$$\begin{aligned} -\alpha \hat{\eta}(t) (\|g(x, t) - Kx\| + \delta) \tanh(s_i) \varphi(u_i) \\ \geq \beta_{1,i} \alpha^2 (\hat{\eta}(t))^2 (\|g(x, t) - Kx\| + \delta)^2 \tanh^2(s_i), \end{aligned} \quad (18)$$

and then

$$-\tanh(s_i) \varphi(u_i) \geq \alpha \beta_{1,i} \hat{\eta}(t) (\|g(x, t) - Kx\| + \delta) \tanh^2(s_i). \quad (19)$$

Since $s_i^2 \geq 0$, $\gamma_1 = \min_{i \in [1, m]} \gamma_i$, $\beta_1 = \min_{i \in [1, m]} \beta_{1,i}$, and $s_i \cdot \tanh(s_i) > \gamma_i s_i \cdot \text{sign}(s_i)$, for all $s_i \neq 0$,

$$s_i \cdot \tanh(s_i) > \gamma_i s_i \cdot \text{sign}(s_i), \quad \text{for all } s_i \neq 0, \quad (20)$$

by substituting (20) into (19), we get the following result:

$$\begin{aligned} -\tanh(s_i) s_i^2 \varphi(u_i) &\geq \alpha \beta_{1,i} \hat{\eta}(t) (\|g(x, t) - Kx\| + \delta) s_i^2 \tanh^2(s_i), \\ -s_i \varphi(u_i) &\geq \alpha \beta_{1,i} \hat{\eta}(t) (\|g(x, t) - Kx\| + \delta) s_i \tanh(s_i), \\ -s_i \varphi(u_i) &\geq \alpha \gamma_i \beta_{1,i} \hat{\eta}(t) (\|g(x, t) - Kx\| + \delta) s_i \cdot \text{sign}(s_i), \\ -s_i \varphi(u_i) &\geq \alpha \gamma_1 \beta_1 \hat{\eta}(t) (\|g(x, t) - Kx\| + \delta) s_i \cdot \text{sign}(s_i). \end{aligned} \quad (21)$$

Let $\eta = \beta_1 \gamma_1$ and $\sum_{i=1}^m |s_i| \geq \|S\|$; then, we have

$$-s_i \varphi(u_i) \geq \alpha \eta \hat{\eta}(t) (\|g(x, t) - Kx\| + \delta) \cdot |s_i|, \quad (22)$$

thus

$$-\sum_{i=1}^m s_i (\varphi u_i) \geq \alpha \eta \hat{\eta}(t) (\|g(x, t) - Kx\| + \delta) \cdot \sum_{i=1}^m |s_i| \quad (23)$$

$$\geq \alpha \eta \hat{\eta}(t) (\|g(x, t) - Kx\| + \delta) \cdot \|S\|.$$

Furthermore, $\sum_{i=1}^m s_i \varphi(u_i) = S^T \varphi(u)$, and one has

$$S^T \varphi(u) \leq -\alpha \eta \hat{\eta}(t) (\|g(x, t) - Kx\| + \delta) \cdot \|S\|. \quad (24)$$

By substituting (13), (14), and (24) into the derivative of Lyapunov function (16), one has

$$\begin{aligned} \dot{V}(t) &\leq \|S\| \cdot (\|g(x, t) - Kx\| + \delta) \\ &\quad - \alpha \eta \hat{\eta}(t) (\|g(x, t) - Kx\| + \delta) \cdot \|S\| + e\eta \dot{e} \\ &= (1 - \alpha) \|S\| \cdot (\|g(x, t) - Kx\| + \delta) \\ &\quad - \alpha (\hat{\eta}(t) - \eta^{-1}) \eta (\|g(x, t) - Kx\| + \delta) \cdot \|S\| + e\eta \dot{\hat{\eta}}(t) \\ &= (1 - \alpha) (\|g(x, t) - Kx\| + \delta) \cdot \|S\|. \end{aligned} \quad (25)$$

Let $w(t) = (1 - \alpha) (\|g(x, t) - Kx\| + \delta) \cdot \|S\|$; then, one has

$$\begin{aligned} V(t) &\leq V(0) - \int_0^t w(\lambda) d\lambda, \\ \implies V(0) &\geq V(t) + \int_0^t w(\lambda) d\lambda \geq \int_0^t w(\lambda) d\lambda. \end{aligned} \quad (26)$$

Thus, by using Barbalat lemma [31], we obtain $\lim_{t \rightarrow \infty} w(t) = 0$. Furthermore, since $\alpha > 1$, $\|g(x, t) - Kx\| + \delta > 0$, $S \rightarrow 0$ as $t \rightarrow \infty$. Hence, the proof is achieved completely.

Remark 2. Since a continuous adaptive SMC with a smooth hyperbolic tangent function is obtained, there is no high-frequency switching operation in sliding mode controller and the chattering is removed.

4. Numerical Simulations

In this section, to verify the proposed controller, we give two illustrative examples.

Example 1. Consider modified Chua's circuit, and the dynamic system can be described as [35]

$$\begin{aligned} \dot{x}_1(t) &= p x_2(t) + \frac{p}{7} x_1(t) + f(x_1, t), \\ \dot{x}_2(t) &= x_1(t) - x_2(t) + x_3(t), \\ \dot{x}_3(t) &= -q x_2(t), \end{aligned} \quad (27)$$

with the nonlinear function

$$f(x_1, t) = -2pm_0 x_1^3, \quad (28)$$

where $p > 0$, $q > 0$, m_0 are the system parameters and $x(t) = [x_1 \ x_2 \ x_3]^T$ is the state vector. The chaos response of Chua's system without control force is shown in Figure 4. It is easy to verify that system (27) with nonlinear control input $\phi(u(t))$ can be represented as

$$\dot{x} = Ax + B(g(x, t) + \phi(u(t))), \quad (29)$$

$$\text{where } A = \begin{bmatrix} p/7 & p & 0 \\ 1 & -1 & 1 \\ 0 & -q & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad g(x) = [-2pm_0 x_1^3],$$

$p = 10$, $q = 100/7$, and $m_0 = 1/7$.

The initial state condition is given by $x_0 = [-1.2 \ 0.2 \ -2]^T$. The controller parameters are given by

FIGURE 4: The chaos response of modified Chua's circuit without control force.

$j = 1$, $\delta = 8$, $\alpha = 1.2$, and $\hat{\eta}_0 = 0$. Let $K = [-6.4286 \ -1.7143 \ -4.58]$ such that the eigenvalues of the system are placed on $\{-2, -3, -1\}$. Also, for simulation, the unknown nonlinear function is given as

$$\varphi(u(t)) = (1 + 0.3 \sin(u(t)))u(t). \quad (30)$$

Under the above-mentioned input nonlinearity, the corresponding state responses, the proposed controller (13), the sliding surface (8), and adaptive parameter are shown in Figures 5–8, respectively. In order to compare the proposed approach with the traditional SMC, the tanh function is replaced by the traditional sign function in the controller (13). Based on traditional SMC, the corresponding state responses, the control input, the sliding surface, and adaptive parameter are shown in Figures 9–12, respectively. Comparing Figures 5–8 with Figures 9–12, it is shown that the undesired chattering phenomenon can be fully suppressed by the proposed adaptive chattering-free control law (13).

Example 2. Consider the chaotic Lorenz system [36] described as

$$\begin{aligned} \dot{x}_1(t) &= -m_1 x_1(t) + m_1 x_2(t), \\ \dot{x}_2(t) &= m_2 x_1(t) - x_2(t) + f_1(x, t), \\ \dot{x}_3(t) &= -m_3 x_2(t) + f_2(x, t), \end{aligned} \quad (31)$$

where $m_i, i = 1, 2, 3$ are the system parameters and $x(t) = [x_1 \ x_2 \ x_3]^T$ are the system states. The chaos response of the Lorenz system without control force is shown in Figure 13. Also, the state space with control input $\phi(u(t))$ can be rewritten as

$$\dot{x} = Ax + B(g(x, t) + \phi(u(t))), \quad (32)$$

where $A = \begin{bmatrix} -m_1 & m_1 & 0 \\ m_2 & -1 & 0 \\ 0 & 0 & -m_3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$, $g(x, t) = [f_1(x, t) \ f_2(x, t)]^T = [-x_1(t)x_3(t) \ x_1(t)x_2(t)]^T$, $m_1 = 10$, $m_2 = 28$, and $m_3 = 8/3$.

FIGURE 5: The system state response with the proposed continuous controller.

FIGURE 6: The control input response with the proposed continuous control input.

FIGURE 7: The switching function response with the continuous control input.

The initial state condition is given by $x_0 = [0 \ -1.5 \ 0.67]^T$. The controller parameters are given by $j = 8$, $\delta = 8$, $\alpha = 3$, and $\hat{\eta}_0 = 0$. Let $K = \begin{bmatrix} -34.3 & 7 & 0 \\ 0 & 0 & 0.6667 \end{bmatrix}$,

FIGURE 8: The time response for the adaptive parameter with the proposed continuous controller.

FIGURE 11: The switching function response with the traditional SMC controller.

FIGURE 9: The system state response with the traditional SMC controller.

FIGURE 12: The time response for the adaptive parameter with the traditional SMC controller.

FIGURE 10: The control input response with the traditional SMC controller.

and the eigenvalues of the system are placed on $\{-2, -3, -1\}$. Also, the nonlinear function is defined as

$$\varphi(u(t)) = (1 + 0.3 \sin(u_2(t)))u(t). \quad (33)$$

Under the input nonlinearity, the corresponding state responses, the proposed controller (13), the sliding surface

FIGURE 13: The chaos response of Lorenz system without control force.

(8), and adaptive parameter are shown in Figures 14–17, respectively. To compare the proposed approach with the traditional SMC, the tanh function is also replaced by the

FIGURE 14: The system state response with the proposed continuous controller.

FIGURE 17: The time response of Example 2 for the adaptive parameter with the proposed continuous controller.

FIGURE 15: The control input response with the proposed continuous control input.

FIGURE 18: The system state response with the traditional SMC controller.

FIGURE 16: The switching function response with the proposed switching function.

FIGURE 19: The control input response with the traditional SMC controller.

traditional sign function in the controller (13). Based on traditional SMC, the corresponding state responses, the control input, the sliding surface, and adaptive parameter are shown in Figures 18–21. Comparing Figures 14–17 with Figures 18–21 shows that the undesired chattering

phenomenon is suppressed by using the proposed control law (13); hence, the results demonstrate the validity of proposed method.

From the simulation results above, it is concluded that the proposed method is effective and the chattering can be eliminated due to the adaptive continuous SMC even when

FIGURE 20: The switching function response with the traditional SMC controller.

FIGURE 21: The time response for the adaptive parameter with the traditional SMC controller.

the controlled systems are subjected to unknown input nonlinearities.

5. Conclusions

This paper has proposed a continuous adaptive SMC design for chaos suppression of a general class of chaotic systems. In contrast to the previous works, the type of continuous adaptive SMC with smooth hyperbolic tangent function is newly introduced such that not only the chaos of systems can be suppressed but also the chattering in conventional SMC can be eliminated even with unknown input nonlinearity. Numerical simulations have verified the effectiveness of the proposed method.

Data Availability

The simulation data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This study was supported by the Ministry of Science and Technology of R.O.C (MOST-108-2221-E-006-213-MY3 and MOST-107-2221-E-366-002-MY2).

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