Research Article

Stable Switching Rule Design for Buck Converter via Affine Switched Systems

Jian Li , Jingjing Wang, Quankang Wang, and Yingzhou Wang

School of Automation Engineering, Northeast Electric Power University, Jilin, Jilin 132012, China

Correspondence should be addressed to Yingzhou Wang; wangyingzhou@neepu.edu.cn

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This paper presents design and analysis of the switching rule based on affine switched systems, and simulation and experimental verifications are carried out for the Buck converter. Firstly, the switched system is modeled according to the dynamic characteristics of the Buck converter, and the affine switched systems are introduced. Secondly, a switching rule based on affine switched systems is introduced which takes the maximum Lyapunov function between the two sets of Lyapunov functions. Finally, the experiment verifies the closed loop control based on the Buck converter after the simulation. The real-time switching signal of the model is the real-time PWM waveform in the experiment. The experimental results show that the switching rule makes the system asymptotically stable.

1. Introduction

In recent years, DC-DC converters have become more complex and have been widely used in many fields, especially in the fields of new energy power generation, electric vehicle, communication equipment, medical equipment, industrial instrument, and military instrument [1–5]. It is of great significance in security monitoring, energy saving, and environmental protection. Electric energy is applied in all aspects of life, which needs to be transformed into appropriate forms of electricity to serve people's life perfectly. With the development of DC-DC converters, various control technologies are developing rapidly.

DC-DC converter is a typical nonlinear time-varying system. Its output ripple voltage is large. The circuit is sensitive to parameter changes and easy to be affected by the environment. It is known that the anti-interference ability of DC-DC converter is poor. The improvement and optimization of topology can only improve the performance of the system to a small extent, but using advanced control theory to control the converter can make up for the

deficiency of topology and make the whole system stable quickly.

Compared with the traditional linear control strategy, various nonlinear control strategies are widely used in the field of control, such as fuzzy control [6], adaptive control [7], sliding mode control [8], predictive control [9], and so on. Many methods based on the Lyapunov function are applied in the control system [10–12]. However, few studies have focused on the maximum Lyapunov function [13, 14]. It is significant that the maximum Lyapunov function control rule is used in the experimental verification of the affine switched system. Relevant experimental verification is still in a blank state.

In this paper, we use the Buck converter to prove the feasibility of the switching rule based on affine switched systems and complete the experimental verification. This experiment proves that the two states of the system are asymptotically stable. Firstly, the dynamic characteristics of the model are analyzed, and the model is built. Then, the method of Lyapunov function comparison is adopted to ensure the system asymptotically stable. Finally, the closed loop experiment is verified. The main contribution

is to complete the experimental closed loop verification of the Buck converter by the switching rule.

The remainder of the paper is described as follows. Section 2 establishes the mathematical model. Section 3 describes the control scheme of the switched system. The simulation results and the experimental verification are presented in Section 4. Section 5 concludes the paper.

2. Preliminary

2.1. Circuit and Control Platform of Buck Converter. Figure 1 shows the closed loop system based on the Buck converter. It includes two parts: circuit and control platform of the Buck converter. The sensing devices obtain current from the inductor branch and voltage from both ends of the capacitor. The switching rule is written in the controller, and the real-time inductive current and capacitor voltage of the circuit are read from the sensing devices. The controller generates real-time variable switching signals according to the switching rule, and controls the driver to generate actions for T .

2.2. Modeling of Buck Converter. The mathematical model is described in this section for the Buck converter. The output voltage of the Buck converter is less than or equal to the input voltage. When the switch is switched on, the power supply supplies the load, and the inductor stores energy and inductor current increases. When the switch is off, the diode continues to flow, the inductor current decreases, and the inductor keeps the load running by releasing energy.

According to Kirchhoff's law of circuit and equivalent circuits of the Buck converter in Figure 2, the following equation is obtained:

$$\begin{bmatrix} \dot{i}_L(t) \\ \dot{v}_C(t) \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} + \begin{bmatrix} \frac{s_k}{L} \\ 0 \end{bmatrix} [V_i(t)], \quad (1)$$

where $i_L(t)$ is the value of inductive current, $v_C(t)$ is the value of capacitor voltage, and $V_i(t)$ is the supply voltage.

The switched system is described as follows:

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), \\ y(t) &= Cx(t), \end{aligned} \quad (2)$$

where $x(t)$ is the state vector, $y(t)$ is the output vector, $A_{\sigma(t)}$ is the system matrix, $B_{\sigma(t)}$ is the input matrix, $u(t)$ is the input vector, C is the output matrix, and $\sigma(t)$ is the switching signal (see Table 1). The linear-switched state-space model of the Buck converter in the continuous conduction mode is given by:

$$\begin{aligned} x(t) &= \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix}, \\ u(t) &= [V_i(t)], \end{aligned}$$

FIGURE 1: Closed loop system based on the Buck converter.

$$\begin{aligned} A_{\sigma(t)} &= \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}, \\ B_{\sigma(t)} &= \begin{bmatrix} \frac{s_k}{L} \\ 0 \end{bmatrix}, \\ C &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \end{aligned} \quad (3)$$

The Buck converter has two modes of working as shown in Figure 2. Table 1 shows the switching signal $\sigma(t) = 1$, where $s_k = 1$ indicates open T and close D . Its working mode is shown in Figure 2(a). The second mode is the switching signal $\sigma(t) = 2$, where $s_k = 0$ indicates open D and close T in Figure 2(b).

The switched system finally reaches the equilibrium point $\bar{x}$ by choosing the appropriate switching rule. Thus, the error system $e(t) = x(t) - \bar{x}$ is studied.

The system is composed of two affine subsystems. The state-space expression for affine switched systems [15, 16] is shown as follows:

$$\begin{aligned} \dot{e}(t) &= A_{\sigma(t)}e(t) + k_{\sigma(t)}, \\ k_{\sigma(t)} &= A_{\sigma(t)}\bar{x} + B_{\sigma(t)}u(t), \\ y_{e(t)} &= Ce(t), \end{aligned} \quad (4)$$

where $k_{\sigma(t)}$ is the affine item. As $e(t) = 0$, affine switched system achieves asymptotic stability. The objective of this paper is to make the system (3) asymptotically stable under

FIGURE 2: Equivalent circuits of the Buck converter.

TABLE 1: Possible open/close switch positions for (T) and (D).

$\sigma(t)$	1	2
T	1	0
D	0	1

the action of the control signal and to adopt closed loop experiment to verify the feasibility of the theory.

3. Control Scheme

Inspired by [17], the switching signal is obtained by the maximum value of multiple Lyapunov functions for the affine switched systems. The switching rule is described as

$$\sigma(e(t)) = \arg \max \{v_\sigma(e(t))\}, \quad (5)$$

where $\sigma(e(t))$ is two cases, so the switching signal can be calculated by comparing the largest Lyapunov function. The Lyapunov function is shown below:

$$v_\sigma(e(t)) = e(t)^T P_\sigma e(t) + 2e(t)^T S_\sigma, \quad (6)$$

where $P_\sigma \in R^{2 \times 2}$ is a positive definite matrix and $S_\sigma \in R^{2 \times 1}$ is a matrix.

Denote $V(e(t)) = \max \{v_\sigma(e(t))\}$ and $\dot{V}(e(t)) = \dot{e}(t)^T P_\sigma e(t) + e(t)^T P_\sigma \dot{e}(t) + 2\dot{e}(t)^T S_\sigma$. In order to make the system (3) asymptotically stable, we have $\dot{V}(e(t)) < 0$. Now, applying the S-procedure, we get

$$\dot{V}(e(t)) + 2a_{\theta(t)}(V(e(t)) - e(t)^T \bar{P}e(t)) < 0, \quad (7)$$

taking into account that $a_\theta = a_1\theta_1 + a_2\theta_2 > 0$ and $\theta_1 + \theta_2 = 1$, where $\theta_1 \geq 0$, $\theta_2 \geq 0$ and $0 \leq a_1 \leq \alpha_1$, $0 \leq a_2 \leq \alpha_2$, α_1 , and α_2 are the absolute values of the real part of the eigenvalues of the imaginary axes closest to A_1 and A_2 , respectively. In addition, $P_\theta = \theta_1 P_1 + \theta_2 P_2$ and $\bar{P} = P_\theta(\bar{\theta})$. A_θ , K_θ , and S_θ can follow the same principle from P_θ . In order to calculate easily, we convert (8) as

$$\begin{bmatrix} e(t) \\ 1 \end{bmatrix}^T \begin{bmatrix} A_\theta^T P_\theta + P_\theta A_\theta + 2a_\theta(P_\theta - \bar{P}) & * \\ K_\theta^T P_\theta + S_\theta^T A_\theta + 2S_\theta^T a_\theta & 2K_\theta^T S_\theta \end{bmatrix} \begin{bmatrix} e(t) \\ 1 \end{bmatrix} < 0. \quad (8)$$

Therefore, (6) can be realized if $\begin{bmatrix} A_\theta^T P_\theta + P_\theta A_\theta + 2a_\theta(P_\theta - \bar{P}) & * \\ K_\theta^T P_\theta + S_\theta^T A_\theta + 2S_\theta^T a_\theta & 2K_\theta^T S_\theta \end{bmatrix} < 0$, where P_θ and S_θ are obtained by linear matrix inequality (6) through simulation.

The switching signal of the Buck converter can be obtained according to the following switching rule. If $v_1(e(t))$

TABLE 2: Simulation parameters.

Inductance	4.5 mh
Resistance	12 ohm
Capacitance	100 μ f
Power supply	30 V

is greater than $v_2(e(t))$, $\sigma(e(t))$ is equal to 1, as shown in Figure 2(a). Otherwise, $\sigma(e(t))$ is equal to 2, as shown in Figure 2(b). The real-time PWM waveform is generated to control the Buck converter.

4. Simulation and Experimental Verification

In this section, the feasibility of the switching rule is verified by simulation and experiment. With Buck circuit as the model and DSP as the digital signal processor, we complete the closed loop control.

4.1. Simulation Results and Analysis. The switching rule achieves the effect that the state of Buck converter is asymptotically stable, and the switching frequency changes in real time according to the collected voltage and current. Inductance is 4.5 mh, when a certain margin is satisfied in the experimental design. Capacitance is 100 μ f that related to inductance and frequency, and the power resistance is 12 ohm (see Table 2). The initial frequency is low and changes in real time. The frequency gradually increases when the state reaches a gradual stability.

In order to verify the feasibility of the control scheme, we describe and analyze two cases. The objective of the first case is to achieve an effect where the output voltage is 12 V and the output power is 12 W, when the input voltage is 30 V. P_θ and S_θ are obtained by simulation according to the control scheme. They are described as follows:

$$\begin{aligned} P_1 &= \begin{bmatrix} 0.0390 & 0.0004 \\ 0.0004 & 0.0011 \end{bmatrix}, \\ S_1 &= \begin{bmatrix} -0.0138 \\ -0.0088 \end{bmatrix}, \\ P_2 &= \begin{bmatrix} 0.0644 & -0.0025 \\ -0.0025 & 0.0018 \end{bmatrix}, \\ S_2 &= \begin{bmatrix} 0.0328 \\ 0.0049 \end{bmatrix}. \end{aligned} \quad (9)$$

FIGURE 3: Two states of the Buck converter. (a) The inductive current stabilizes at 1 A in 2.6 ms. (b) The capacitor voltage stabilizes at 12 V in 2.9 ms.

FIGURE 4: Two states of the Buck converter. (a) The inductive current stabilizes at 1.3 A in 3.3 ms. (b) The capacitor voltage stabilizes at 15.6 V in 4.5 ms.

The objective of the second case is to achieve an effect where the output voltage is 15.6 V and the output power is 20.28 W, when the input voltage is 30 V. P_θ and S_θ are shown as follows:

$$\begin{aligned}
 P_1 &= \begin{bmatrix} 0.0173 & -0.0005 \\ -0.0005 & 0.0005 \end{bmatrix}, \\
 S_1 &= \begin{bmatrix} -0.0091 \\ -0.0011 \end{bmatrix}, \\
 P_2 &= \begin{bmatrix} 0.0163 & -0.0004 \\ -0.0004 & 0.0004 \end{bmatrix}, \\
 S_2 &= \begin{bmatrix} 0.0084 \\ 0.0013 \end{bmatrix}.
 \end{aligned} \tag{10}$$

The simulation results of the first case show that the inductive current tends to be stable at about 2.6 ms in Figure 3(a), and the capacitor voltage tends to be stable at 2.9 ms in Figure 3(b). The speed is fast, and the adjustment time is much less than 1 s. Finally, the inductive current is stable around 1 A and the waveform is triangular wave, while the capacitor voltage is stable at 12 V and the waveform is a straight line. The simulation results of the second case are shown in Figures 4(a) and 4(b). The inductive current reaches a stable value of 1.3 A at 3.3 ms, while the capacitor voltage is 15.6 V at 4.5 ms.

4.2. Experimental Verification. The structure of the system is shown in Figure 5, including four parts: control unit, Buck converter, driver, and sampling module. Control unit is realized by DSP28335. Buck converter, driver, and sampling module are

FIGURE 5: Experimental verification platform.

introduced in the circuit model. In this experiment, DSP is used as the digital signal processor, TLP250 is used as the driver, and hall element is used as the sampling module. The relevant parameters of the Buck converter are shown in Table 3. The complete experimental testbed setup is shown in Figure 5.

4.2.1. Control Unit. DSP28335 has played an important role in the fields of industrial control because of its high processing speed and abundant peripheral interfaces. The voltage and current of the Buck converter are collected through AD sampling pins. Closed loop real-time control can be realized programmatically by comparing $v(1)$ with $v(2)$. The control logic for programming is shown in Figure 6. The closed loop experimental verification combines the conversion between analog and digital quantities.

4.2.2. Circuit Model. MOSFET can work to hundreds of KHZ, and its driver is simple; so, we choose IRF540N to switch the Buck converter. The driver adopts TLP250 to drive MOSFET. Schottky diode SB3100 is selected to meet the design requirements that the maximum current is 3 A with a withstand voltage of 100 V.

DSP is used to collect inductor current and capacitor voltage. Hall voltage sensing LV25-P is used to collect the voltage of the capacitor. The input resistance is 3 kohm, and the output resistance is 120 ohm. The capacitor voltage can be reduced by 10 times, and the AD pin in DSP can be collected. The inductive current adopts LA55-P/SP50, and the output resistance is 1 kohm. The current can be reproduced into voltage and collected to another pin of DSP.

4.2.3. Experimental Results and Analysis. The experimental results are proposed for the switching rule to ensure the stability of the system. We select the power resistance of 12 ohms and select two sets of images for discussion and analysis (see Figures 7 and 8). The average inductive current is 0.923 A in Figure 7(a) and the capacitor voltage is equal to

TABLE 3: Experimental parameters.

Inductance	4.5 mh
Capacitance	100 μ f
Diode	SB3100
Controller	DSP
Drive chip	TLP250
Mos	IRF540 N
Sampling chip1	LV25-P
Sampling chip2	LA55-P/SP50

FIGURE 6: DSP control logic diagram.

the load voltage with an average value of 11.6 V in Figure 7(b), while the target value of current is 1 A and the target value of voltage is 12 V. The other set of images are

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