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Research Article

Interval-Valued Intuitionistic Fuzzy Multiple Attribute Group Decision Making with Uncertain Weights

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The theory of interval-valued intuitionistic fuzzy sets (IVIFSs) has been an impactful and convenient tool in the construction of advanced multiple attribute group decision making (MAGDM) models to counter the uncertainty in the developing complex decision support system. To satisfy much more demands from fuzzy decision making problems, we propose a method to solve the MAGDM problem in which all the information supplied by the decision makers is expressed as interval-valued intuitionistic fuzzy decision matrices where each of the elements is characterized by an interval-valued intuitionistic fuzzy number, and the information about the weights of both decision makers and attributes may be completely unknown or partially known. Firstly, we introduce a consensus-based method to quantify the weights of all decision makers based on all interval-valued intuitionistic fuzzy decision matrices. Secondly, we utilize the interval-valued intuitionistic fuzzy weighted arithmetic (IVIFWA) operator to aggregate all interval-valued intuitionistic fuzzy decision matrices into the collective one. Thirdly, we establish an optimization model to determine the weights of attributes depending on the collective decision matrix and the given attribute weight information. Fourthly, we adopt the weighted correlation coefficient of IVIFSs to rank all the alternatives from the perspective of TOPSIS via the collective decision matrix and the obtained weights of attributes. Finally, some examples are used to illustrate the validity and feasibility of our proposed approach by comparison with some existing models.

1. Introduction

Atanassov [1] introduced intuitionistic fuzzy sets (IFSs) as an extension of conventional fuzzy set proposed by Zadeh in 1965 [2]. Atanassov and Gargov [3] further proposed interval-valued intuitionistic fuzzy sets (IVIFSs) on the basis of IFSs. After their pioneering work, both IFSs and IVIFSs are getting more and more attention and have been hot research issues in a number of fields, such as industrial control [4], pattern classification [5, 6], system modeling [7, 8], and decision making analysis [9–13]. It should be emphasized that multiple attribute decision making (MADM) and multiple attribute group decision making (MAGDM) on IFSs/IVIFSs have been two especially important branches of operations research. A MAGDM problem on IVIFSs can be regarded as a common human activity, which includes a group of experts (decision makers, DMs) to participate in the process of decision making so as to rank all the alternatives on given attributes through a number of decision making matrices provided by all DMs and the weights of both DMs and attributes.

In general, a MAGDM model involves five key parts: (1) quantization of all respective decision making matrices from every DMs, (2) assessing the weights of DMs, (3) aggregating all decision making matrices into a collective one, (4) determining the weights of attributes, and (5) ranking all the alternatives. Up to now, most of relevant studies have put emphasis on (3), (4), and (5). Concerning the topic of (3), a number of aggregation operators on IVIFSs have been successfully proposed in succession from different perspectives [14–19]. For (4), it is desirable first to consider the constraint condition of the given attribute weight information. In general, the given attribute weight information consists of three types, i.e., crisp values, partially known constraint condition, and completely unknown constraint condition. As described in [20, 21], the provided partially known constraint condition may be constructed with the following forms: a weak ranking, a strict ranking, a ranking with multiples, an interval form, a ranking of differences, and an interval-valued intuitionistic fuzzy numbers. Some models, such as multiple-objective programming model [20, 22], fractional programming method [23], nonlinear programming model [24], linear programming model [25], and grey relational analysis [21], have been successfully developed from different perspectives to determine the weight vector of attributes. For more relevant models, please refer to [15, 26, 27]. The primary methods of ranking alternatives include ranking functions [14], TOPSIS-based methods [24], and VIKOR-based methods [28]. On the topic of determining the weights of DMs, Ye [29] presented a method using the ranking functions on IFSs to determine the DMs' weights for MAGDM with completely unknown weight information on DMs. However, this method produces incorrect weight vector of DMs which may lead to unreasonable decision making results. Gupta et al. [30] developed an optimization model to determine DMs' weights where the weight information of DMs is expressed by IVIFNs. Compared with numerous methods on determining attributes weights, the research on assessing the DMs' weights in MAGDM is still in its infancy and remains to be developed.

In view of the above analysis, we shall focus on the issue of MAGDM under interval-valued intuitionistic fuzzy environment where all the information provided by the DMs is characterized by IVIFNs, the information about DMs is completely unknown, and the information about attributes is partially known. The main contributions of this work can be summarized as follows:

(i) A consensus-based method is developed to determine the weights of DMs.

(ii) A multiobjective optimization model is proposed to determine the weights of attributes.

(iii) A TOPSIS-based MAGDM model under interval-valued intuitionistic fuzzy environment is established via the aggregation operator, the weights of DMs, and the weights of attributes.

Overall, in light of the above three aspects, the proposed method delivers a new vision of modeling uncertain group decision making problems from application fields.

The remainder of this paper is organized as follows. In Section 2, we recall some basic concepts and operations. Section 3 proposes a method to solve those MAGDM problems under interval-valued intuitionistic fuzzy environment where all the information provided by the DMs is characterized by IVIFNs, the information about DMs is completely unknown, and the information about attributes is partially known. An example is employed in Section 4 to prove the performance of the proposed method by comparison with some existing algorithms. Section 5 draws a conclusion of this study.

2. Basic Concepts and Operations

2.1. Basic Concepts

Definition 1 (see [1, 3]). Let X be a set and $D[0,1]$ be the set of all closed subintervals of the interval; an IVIFS A on X has the form $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$, where $\mu_A : X \longrightarrow D[0,1], \nu_A : X \longrightarrow D[0,1]$ are two maps satisfying $\sup \mu_A(x) + \sup \nu_A(x) \le 1$ for all $x \in X$. For each IVIFS on X, $\pi_A(x) = [1 - \sup \mu_A(x) - \sup \nu_A(x), 1 - \inf \mu_A(x) - \inf \nu_A(x)]$ is an intuitionistic index of x in A. $\mu_A(x)$, $\nu_A(x)$, and $\pi_A(x)$ denote the membership degree, the nonmembership degree, and the hesitant degree, respectively.

Remark 2. When $\sup \mu_A(x) = \inf \mu_A(x)$ and $\sup \nu_A(x) = \inf \nu_A(x)$, the IVIFS A reduces to an IFS. If $\pi_A(x) = 0$, then an IFS becomes a fuzzy set. In the following part, we utilize (μ, ν, π) (or (μ, ν)) to denote an interval-valued intuitionistic fuzzy number (IVIFN) or an intuitionistic fuzzy number (IFN).

2.2. Operations

Definition 3 (see [31]). Let $X = \{x_1, x_2, \ldots, x_n\}$ be the finite universal set and $A, B \in \mathrm{IVIFS}(X)$ be given by $A = \{(x_i, \mu_A(x_i), \nu_A(x_i)) \mid x_i \in X\}$ and $B = \{(x_i, \mu_B(x_i), \nu_B(x_i)) \mid x_i \in X\}$ $(i = 1, 2, \ldots, n)$, where IVIFS(X) denotes all the IVIFSs on X. The correlation coefficient between A and B is defined by

$$c(A, B) = \frac{\gamma(A, B)}{(\gamma(A, A), \gamma(B, B))^{1/2}}, \tag{1}$$

where

$$\begin{aligned} \gamma(A, B) = \sum_{i=1}^{n} w_i \big(& \mu_{A_L}(x_i) \cdot \mu_{B_L}(x_i) + \mu_{A_U}(x_i) \\ & \cdot \mu_{B_U}(x_i) + \nu_{A_L}(x_i) \cdot \nu_{B_L}(x_i) + \nu_{A_U}(x_i) \cdot \nu_{B_U}(x_i) \\ & + \pi_{A_L}(x_i) \cdot \pi_{B_L}(x_i) + \pi_{A_U}(x_i) \cdot \pi_{B_U}(x_i) \big). \end{aligned} \tag{2}$$

The weight vector $w = [w_1, w_2, \ldots, w_n]^T$ of x_i $(i = 1, 2, \ldots, n)$ satisfies $w_i \ge 0$ $(i = 1, 2, \ldots, n)$ and $\sum_{i=1}^{n} w_i = 1$.

Remark 4. Note that the correlation coefficient satisfies the following conditions: (1) $c(A, B) = c(B, A)$; (2) $0 \le c(A, B) \le 1$; and (3) $A = B$ if and only if $c(A, B) = 1$. When A and B reduce to IFSs, the correlation coefficient can be described as

$$c(A, B) = \frac{\gamma(A, B)}{(\gamma(A, A) \cdot \gamma(B, B))^{1/2}}, \tag{3}$$

where $\gamma(A, B) = \sum_{i=1}^{n} w_i [\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i) + \pi_A(x_i)\pi_B(x_i)]$.

Following Definition 3, we introduce a correlation coefficient between two IVIF matrices.

Definition 5. Let $D_1 = [\alpha_{jk}]_{J\times K}$ and $D_2 = [\beta_{jk}]_{J\times K}$ be two interval-valued intuitionistic fuzzy matrices, where the elements of both D_1 and D_2 are expressed by IVIFNs. Then the correlation coefficient between D_1 and D_2 is defined by

$$C(D_1, D_2) = \frac{1}{JK}\sum_{j=1}^{J}\sum_{k=1}^{K} c(\alpha_{jk}, \beta_{jk}), \quad (4)$$

where $c(\alpha_{jk}, \beta_{jk})$ is the correlation coefficient between α_{jk} and β_{jk} (see Definition 3).

Clearly, the above correlation coefficient satisfies the following theorem.

Theorem 6. *For two interval-valued intuitionistic fuzzy matrices $D_1 = [\alpha_{jk}]_{J\times K}$ and $D_2 = [\beta_{jk}]_{J\times K}$, where the elements of both D_1 and D_2 are expressed by IVIFNs, $C(D_1, D_2)$ satisfies the three conditions:*

(i) $C(D_1, D_2) = C(D_2, D_1)$;

(ii) $0 \le C(D_1, D_2) \le 1$;

(iii) $D_1 = D_2$ *if and only if* $C(D_1, D_2) = 1$.

Definition 7 (see [14]). Let α_i $(i = 1, 2, \ldots, n)$ be n IVIFNs, where $\alpha_i = ([a_i, b_i], [c_i, d_i])$, $0 \le a_i, b_i, c_i, d_i \le 1$, $b_i + d_i \le 1$, and $1 \le i \le n$. Then the interval-valued intuitionistic fuzzy weighted arithmetic (IVIFWA) operator has the following form:

$$\begin{aligned} \mathrm{IVIFWA}_w(\alpha_1, \alpha_2, \ldots, \alpha_n) &= w_1\alpha_1 \oplus w_2\alpha_2 \oplus \cdots w_n\alpha_n \\ &= \left(\left[1 - \prod_{i=1}^{n}(1-a_i)^{w_i}, 1 - \prod_{i=1}^{n}(1-b_i)^{w_i}\right],\right. \\ &\quad \left.\left[\prod_{i=1}^{n} c_i^{w_i}, \prod_{i=1}^{n} d_i^{w_i}\right]\right), \end{aligned} \quad (5)$$

where w_i $(i = 1, 2, \ldots, n)$ is the weight of α_i satisfying $w_i \ge 0$ and $\sum_{i=1}^{n} w_i = 1$.

2.3. Review of TOPSIS. TOPSIS is a multicriteria decision analysis method, which was firstly introduced by Hwang and Yoon in 1981 [32] with further developments by Yoon in 1987 [33] and Hwang, Lai, and Liu in 1993 [34]. TOPSIS is based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution (PIS) and the longest geometric distance from the negative ideal solution (NIS) [33, 34]. After their pioneering work, TOPSIS has been extensively employed to establish various uncertain decision making models, especially in MADM on IFS/IVIFS.

Concerning TOPSIS within the framework of IVIFS, the maximal IVIFN and the minimum IVIFN are defined by $([1,1],[0,0])$ and $([0,0],[1,1])$, respectively.

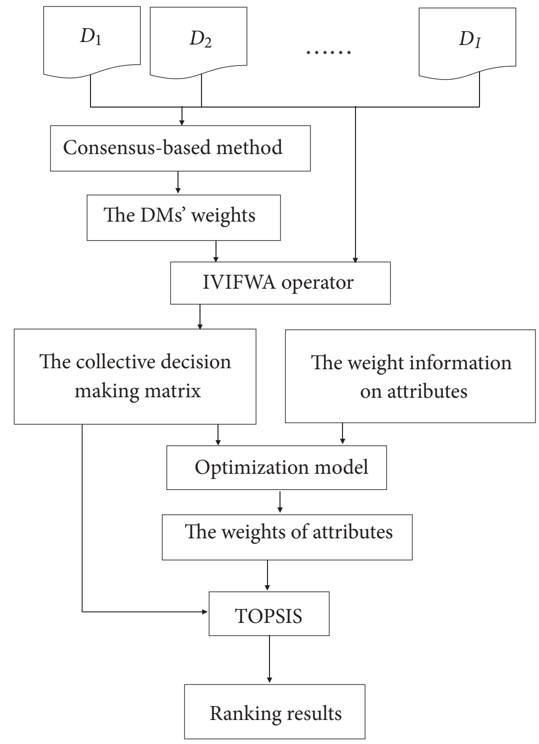

Figure 1: Block diagram of our MAGDM model

3. MAGDM under Interval-Valued Intuitionistic Fuzzy Environment

3.1. Our Proposed MAGDM Model. For a MAGDM problem under interval-valued intuitionistic fuzzy environment, every DM assesses all the alternatives A_j $(j = 1, 2, \ldots, J)$ on attributes x_k $(k = 1, 2, \ldots, K)$ through a decision making matrix D_i $(i = 1, 2, \ldots, I)$ as

$$D_i = \begin{pmatrix} \alpha_{11}^{(i)} & \alpha_{12}^{(i)} & \cdots & \alpha_{1K}^{(i)} \\ \alpha_{21}^{(i)} & \alpha_{22}^{(i)} & \cdots & \alpha_{2K}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{J1}^{(i)} & \alpha_{J2}^{(i)} & \cdots & \alpha_{JK}^{(i)} \end{pmatrix} \quad (6)$$

where $\alpha_{jk}^{(i)} = (\mu_{ij}^{(i)}, \nu_{ij}^{(i)})$ is an IVIFN. Assume that the weight vector of all I DMs is $\omega = [\omega_1, \omega_2, \ldots, \omega_I]$ and the weight vector of attributes is $w = [w_1, w_2, \ldots, w_K]$, where $\omega_i \ge 0$, $\sum_{i=1}^{I}\omega_i = 1$, $w_k \ge 0$, $\sum_{k=1}^{K} w_k = 1$, $i \in \{1, 2, \ldots, I\}$, and $k \in \{1, 2, \ldots, K\}$. In this paper, suppose that the information on DMs is completely unknown and the weight information on attributes has the form of linear constraint condition Λ. The block diagram of our MAGDM model is shown as Figure 1. The following part will clearly illustrate this model.

Step 1 (determine the weights of DMs). From the perspective of the majority criterion and consensus [27], those DMs whose decision making matrices have greater consensus with others should be given larger values. Thus, the weights of DMs can be defined as follows:

$$\omega_i = \frac{\theta_i}{\sum_{i=1}^{I} \theta_i}, \quad (7)$$

where θ_i has the form

$$\theta_i = \sum_{i'=1, i' \neq i}^{I} C\left(D_i, D_{i'}\right). \quad (8)$$

Remark 8. By comparison with the majority criterion, all DMs' importance levels have been fully considered and reflected through the obtained weights. This step lays good foundation for making reasonable decision.

Step 2 (calculate the collective decision making matrix D). Depending on all I decision making matrices D_i and their relative weights ω_i, we employ the IVIFWA aggregation operator to get the collective decision making matrix D.

Let

$$D = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1K} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{J1} & \alpha_{J2} & \cdots & \alpha_{JK} \end{pmatrix}. \quad (9)$$

Take α_{jk} $(j = 1, 2, \ldots, J;\ k = 1, 2, \ldots, K)$ as an example. α_{jk} is calculated by $\alpha_{jk} = \text{IVIFWA}_{\omega}(\alpha_{jk}^{(1)}, \alpha_{jk}^{(2)}, \ldots, \alpha_{jk}^{(I)})$.

Step 3 (determine the weight vector of attributes). From the standpoint of TOPSIS and the concept of IVIFS, the positive ideal solution (PIS) can be defined by $A^+ = \{\alpha_1^+, \ldots, \alpha_K^+\}$, where α_k equals $([1,1],[0,0])$ for $k \in \{1, 2, \ldots, K\}$. If we consider certain alternative A_j $(j = 1, 2, \ldots, J)$ with the highest priority, it is easy to establish the following optimization model:

$$\begin{aligned} \max \quad & c\left(\overline{A}_j, A^+\right), \\ \text{s.t.} \quad & w \in \Lambda, \\ & \sum_{k=1}^{K} w_k = 1, \\ & w_k \geq 0, \end{aligned} \quad (10)$$

where $\overline{A}_j$ is the collective value on X for A_j $(j = 1, 2, \ldots, J)$.

Clearly, the bigger the $c(\overline{A}_j, A^+)$, the better the alternative A_j. Solving this model, we can get the optimal solution $w^{(j)} = \left[w_1^{(j)}\ w_2^{(j)}\ \cdots\ w_K^{(j)}\right]^T$. This process is repeated until all the corresponding $w^{(j)}$ is determined. To fully consider all the alternatives as a whole, we define the weight matrix W as follows:

$$W = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(J)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(J)} \\ \vdots & \vdots & \ddots & \vdots \\ w_K^{(1)} & w_K^{(2)} & \cdots & w_K^{(J)} \end{pmatrix}. \quad (11)$$

Moreover, we calculate $\Gamma = (EW)^T(EW)$, where E is defined by

$$E = \begin{pmatrix} c\left(\alpha_{11}, \alpha^+\right) & c\left(\alpha_{12}, \alpha^+\right) & \cdots & c\left(\alpha_{1K}, \alpha^+\right) \\ c\left(\alpha_{21}, \alpha^+\right) & c\left(\alpha_{22}, \alpha^+\right) & \cdots & c\left(\alpha_{2K}, \alpha^+\right) \\ \vdots & \vdots & \ddots & \vdots \\ c\left(\alpha_{J1}, \alpha^+\right) & c\left(\alpha_{J2}, \alpha^+\right) & \cdots & c\left(\alpha_{JK}, \alpha^+\right) \end{pmatrix}, \quad (12)$$

$$\alpha^+ = \left([1,1], [0,0]\right). \quad (13)$$

Let ρ be the normalized eigenvector of Γ. Then w is determined by

$$w = W\rho. \quad (14)$$

Remark 9. As stated above, this optimization model has been established using TOPSIS from the perspective of the criterion of realism decision rule. Thus the attributes' effects have been fully considered and balanced.

Step 4 (calculate $c(\overline{A}_j, A^+)$ $(j = 1, 2, \ldots, J)$). Based on the obtained $\overline{A}_j$, A^+ and the weight vector of attributes w, calculate $c(\overline{A}_j, A^+)$ $(j = 1, 2, \ldots, J)$ via (1).

Step 5 (rank all the alternatives). Depending on the obtained $c(\overline{A}_j, A^+)$ $(j = 1, 2, \ldots, J)$, rank all the values in ascending order, which corresponds to the order of all the alternatives.

3.2. Comparison between Our Proposed Method and Ye's Method. Since the problem of determining the weights of MAGDM under interval-valued intuitionistic fuzzy environment with completely unknown weight information about DMs has been discussed and solved by Ye's method [29], we shall make a comparison between our proposed method and Ye's method.

Here we consider a MAGDM problem which includes three experts, who present their decisions of four alternatives A_j $(j = 1, 2, 3, 4)$ on five attributes x_k $(k = 1, 2, 3, 4, 5)$ through the following three interval-valued intuitionistic fuzzy decision making matrices: D_1, D_2, and D_3. Assume that the weights of three DMs are completely unknown for this decision making problem.

$$D_1 = \begin{pmatrix} ([0,0.2],[0.5,0.5]) & ([0.3,0.3],[0.7,0.7]) & ([0.4,0.6],[0.4,0.4]) & ([0.2,0.2],[0.8,0.8]) & ([0.4,0.4],[0.4,0.6]) \\ ([0.1,0.1],[0.7,0.9]) & ([0.2,0.2],[0.6,0.8]) & ([0,0.2],[0.5,0.7]) & ([0.3,0.3],[0.5,0.7]) & ([0.4,0.4],[0.4,0.6]) \\ ([0,0.2],[0.5,0.7]) & ([0.1,0.1],[0.9,0.9]) & ([0.1,0.3],[0.5,0.7]) & ([0.2,0.2],[0.8,0.8]) & ([0.4,0.4],[0.4,0.6]) \\ ([0.2,0.2],[0.7,0.7]) & ([0,0],[1,1]) & ([0.2,0.4],[0.4,0.6]) & ([0.1,0.3],[0.7,0.7]) & ([0.6,0.6],[0.4,0.4]) \end{pmatrix},$$

$$D_2 = \begin{pmatrix} ([0.5,0.7],[0,0.2]) & ([0.3,0.7],[0,0.2]) & ([0,0.8],[0,0]) & ([0.5,0.7],[0,0.2]) & ([0.6,0.8],[0,0.2]) \\ ([0.6,0.6],[0,0.2]) & ([0.6,0.8],[0.2,0.2]) & ([0.7,0.9],[0.1,0.1]) & ([0.6,0.8],[0,0]) & ([0.9,0.9],[0.1,0.1]) \\ ([0.8,0.8],[0.2,0.2]) & ([0.5,0.7],[0.3,0.3]) & ([0.4,0.8],[0,0.2]) & ([0.3,0.5],[0.3,0.5]) & ([0.2,0.6],[0.1,0.3]) \\ ([0.6,0.8],[0.1,0.1]) & ([0.7,0.9],[0,0]) & ([1,1],[0,0]) & ([0.5,0.7],[0.1,0.1]) & ([0.7,0.9],[0.1,0.1]) \end{pmatrix}, \quad (15)$$

$$D_3 = \begin{pmatrix} ([0.5,0.7],[0.2,0.2]) & ([0.4,0.6],[0.3,0.3]) & ([0.3,0.5],[0.4,0.4]) & ([0.4,0.6],[0.2,0.4]) & ([0.5,0.7],[0.1,0.3]) \\ ([0.5,0.5],[0,0.4]) & ([0.4,0.6],[0.3,0.3]) & ([0,0.6],[0.4,0.4]) & ([0.5,0.5],[0.2,0.4]) & ([0.6,0.6],[0.4,0.4]) \\ ([0.4,0.6],[0.3,0.3]) & ([0.6,0.6],[0,0.4]) & ([0.3,0.5],[0.2,0.4]) & ([0.2,0.4],[0.4,0.6]) & ([0.3,0.5],[0.4,0.4]) \\ ([0.4,0.6],[0.1,0.3]) & ([0.6,0.8],[0.2,0.2]) & ([0,0.6],[0.3,0.3]) & ([0.4,0.6],[0.1,0.3]) & ([0.5,0.7],[0.3,0.3]) \end{pmatrix}.$$

In what follows, we utilize Ye's method [29] and our proposed method to determine the weights of three DMs, respectively.

Case 1 (determine the weights of DMs via Ye's method [29]). Firstly, we get the score matrices S_i $(i = 1,2,3)$ of D_i as follows:

$$S_1 = \begin{pmatrix} -0.4 & -0.4 & 0.1 & -0.6 & -0.1 \\ -0.7 & -0.5 & -0.5 & -0.3 & -0.1 \\ -0.5 & -0.8 & -0.4 & -0.6 & -0.1 \\ -0.5 & -1.0 & -0.2 & -0.5 & 0.2 \end{pmatrix},$$

$$S_2 = \begin{pmatrix} 0.5 & 0.4 & 0.4 & 0.5 & 0.6 \\ 0.5 & 0.5 & 0.7 & 0.7 & 0.8 \\ 0.6 & 0.3 & 0.5 & 0 & 0.2 \\ 0.6 & 0.8 & 1.0 & 0.5 & 0.7 \end{pmatrix}, \quad (16)$$

$$S_3 = \begin{pmatrix} 0.4 & 0.2 & 0 & 0.2 & 0.4 \\ 0.3 & 0.2 & -0.1 & 0.2 & 0.2 \\ 0.2 & 0.4 & 0.1 & -0.2 & 0 \\ 0.3 & 0.5 & 0 & 0.3 & 0.3 \end{pmatrix}.$$

Secondly, we get the average score matrix S^* based on S_1, S_2, and S_3 as below:

$$S^* = \begin{pmatrix} 0.1667 & 0.0667 & 0.1667 & 0.0333 & 0.3000 \\ 0.0333 & 0.0667 & 0.0333 & 0.2000 & 0.3000 \\ 0.1000 & -0.0333 & 0.0667 & -0.2667 & 0.0333 \\ 0.1333 & 0.1000 & 0.2667 & 0.1000 & 0.4000 \end{pmatrix}. \quad (17)$$

Thirdly, we get the weights of three DMs

$$\omega = [-0.1677 \quad 0.6190 \quad 0.5487]. \quad (18)$$

Case 2 (determine the weights of DMs via our proposed method). By applying our proposed method to determine the weights of DMs, we get

$$\omega = [0.2908 \quad 0.3304 \quad 0.3787]. \quad (19)$$

Remark 10. As indicated in this example, ω_1 equals -0.1677 which completely contradicts the condition of Ye's approach, i.e., $\omega_i \geq 0$ $(i = 1,2,3)$ [29]. This example implies that our method can overcome the deficiencies from [29].

4. Illustrative Example

Here we consider a problem concerning a manufacturing company from [29]. The objective of this problem is to determine the best global supplier for one of its most critical parts used in assembling process. There are four alternatives A_j $(j = 1, 2, 3, 4)$ for choice. Four experts (DMs) provide their own decision making information of four alternatives on five attributes, namely, x_1 (cost), x_2 (quality), x_3 (service), x_4 (supplier's profile), and x_5 (risk factor), through the following four matrices: D_i $(i = 1, 2, 3, 4)$. The constraint conditions of attribute weights can be described by $\Lambda = \{0.1 \le w_1 \le 0.3,\ 0.1 \le w_2 \le 0.3,\ 0.2 \le w_3 \le 0.4,\ 0.2 \le w_4 \le 0.4,\ 0.1 \le w_5 \le 0.3\}$.

$$D_1 = \begin{pmatrix} ([0.5,0.6],[0.2,0.3]) & ([0.3,0.5],[0.4,0.5]) & ([0.6,0.7],[0.2,0.3]) & ([0.5,0.7],[0.1,0.2]) & ([0.1,0.4],[0.3,0.5]) \\ ([0.3,0.4],[0.4,0.6]) & ([0.1,0.3],[0.2,0.4]) & ([0.3,0.4],[0.4,0.5]) & ([0.2,0.4],[0.5,0.6]) & ([0.7,0.8],[0.1,0.2]) \\ ([0.4,0.5],[0.3,0.5]) & ([0.7,0.8],[0.1,0.2]) & ([0.5,0.8],[0.1,0.2]) & ([0.4,0.6],[0.2,0.3]) & ([0.5,0.6],[0.2,0.3]) \\ ([0.3,0.5],[0.4,0.5]) & ([0.1,0.2],[0.7,0.8]) & ([0.1,0.2],[0.5,0.8]) & ([0.2,0.3],[0.4,0.6]) & ([0.2,0.3],[0.5,0.6]) \end{pmatrix},$$

$$D_2 = \begin{pmatrix} ([0.4,0.5],[0.2,0.4]) & ([0.3,0.4],[0.4,0.6]) & ([0.6,0.7],[0.1,0.2]) & ([0.5,0.6],[0.1,0.3]) & ([0.1,0.3],[0.3,0.5]) \\ ([0.3,0.5],[0.4,0.5]) & ([0.1,0.3],[0.3,0.7]) & ([0.3,0.4],[0.4,0.5]) & ([0.2,0.3],[0.6,0.7]) & ([0.6,0.8],[0.1,0.2]) \\ ([0.4,0.6],[0.3,0.4]) & ([0.6,0.8],[0.1,0.2]) & ([0.7,0.8],[0.1,0.2]) & ([0.4,0.6],[0.3,0.4]) & ([0.5,0.6],[0.2,0.4]) \\ ([0.3,0.4],[0.4,0.6]) & ([0.1,0.2],[0.6,0.8]) & ([0.1,0.2],[0.7,0.8]) & ([0.3,0.4],[0.4,0.6]) & ([0.2,0.4],[0.5,0.6]) \end{pmatrix}, \tag{20}$$

$$D_3 = \begin{pmatrix} ([0.4,0.7],[0.1,0.2]) & ([0.3,0.5],[0.3,0.4]) & ([0.6,0.7],[0.1,0.2]) & ([0.5,0.6],[0.1,0.3]) & ([0.3,0.5],[0.4,0.5]) \\ ([0.4,0.5],[0.2,0.4]) & ([0.2,0.4],[0.4,0.5]) & ([0.4,0.5],[0.3,0.4]) & ([0.1,0.2],[0.7,0.8]) & ([0.6,0.7],[0.2,0.3]) \\ ([0.2,0.4],[0.3,0.4]) & ([0.6,0.8],[0.1,0.2]) & ([0.5,0.7],[0.1,0.3]) & ([0.5,0.7],[0.2,0.3]) & ([0.6,0.8],[0.1,0.2]) \\ ([0.3,0.4],[0.2,0.4]) & ([0.1,0.2],[0.6,0.8]) & ([0.1,0.3],[0.5,0.7]) & ([0.2,0.3],[0.5,0.7]) & ([0.1,0.2],[0.6,0.8]) \end{pmatrix},$$

$$D_4 = \begin{pmatrix} ([0.6,0.7],[0.2,0.3]) & ([0.3,0.4],[0.3,0.4]) & ([0.7,0.8],[0.1,0.2]) & ([0.5,0.6],[0.1,0.3]) & ([0.1,0.2],[0.5,0.7]) \\ ([0.4,0.5],[0.4,0.5]) & ([0.1,0.2],[0.2,0.3]) & ([0.3,0.4],[0.5,0.6]) & ([0.2,0.3],[0.4,0.6]) & ([0.6,0.7],[0.1,0.2]) \\ ([0.4,0.5],[0.3,0.4]) & ([0.6,0.7],[0.1,0.3]) & ([0.5,0.8],[0.1,0.2]) & ([0.4,0.5],[0.2,0.3]) & ([0.5,0.6],[0.3,0.4]) \\ ([0.3,0.4],[0.4,0.5]) & ([0.1,0.3],[0.6,0.7]) & ([0.1,0.2],[0.5,0.8]) & ([0.2,0.3],[0.4,0.5]) & ([0.3,0.4],[0.5,0.6]) \end{pmatrix}.$$

In what follows, we utilize the proposed method to solve this problem.

Step 1. Using (7), we get

$$\omega = [0.2523 \quad 0.2503 \quad 0.2478 \quad 0.2496]. \tag{21}$$

Step 2. On the basis of the known ω and four decision matrices D_i $(i = 1, 2, 3, 4)$, we get the aggregated decision making matrix D through IVIFWA operator as follows:

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ ([0.4821,0.6334],[0.1684,0.2916]) & ([0.3000,0.4523],[0.3467,0.4684]) & ([0.6277,0.7289],[0.1191,0.2215]) & ([0.5000,0.6280],[0.1000,0.2708]) & ([0.1543,0.3596],[0.3660,0.5438]) \\ ([0.3517,0.4765],[0.3369,0.4954]) & ([0.1259,0.3034],[0.2628,0.4526]) & ([0.3262,0.4265],[0.3938,0.4951]) & ([0.1763,0.3041],[0.5380,0.6697]) & ([0.6280,0.7553],[0.1187,0.2211]) \\ ([0.3557,0.5053],[0.3000,0.4232]) & ([0.6280,0.7787],[0.1000,0.2213]) & ([0.5600,0.7789],[0.1000,0.2211]) & ([0.4265,0.6062],[0.2214,0.3224]) & ([0.5269,0.6631],[0.1864,0.3133]) \\ ([0.3000,0.4270],[0.3369,0.4952]) & ([0.1000,0.2262],[0.6238,0.7738]) & ([0.1000,0.2260],[0.5439,0.7740]) & ([0.2263,0.3265],[0.4227,0.5956]) & ([0.2033,0.3301],[0.5231,0.6443]) \end{pmatrix}. \tag{22}$$

Step 3. According to the decision making matrix D, we get

$$E = \begin{pmatrix} 0.8434 & 0.6081 & 0.9339 & 0.8417 & 0.4008 \\ 0.6517 & 0.3362 & 0.6054 & 0.3496 & 0.9358 \\ 0.6917 & 0.9342 & 0.9048 & 0.8008 & 0.8712 \\ 0.5862 & 0.2174 & 0.2216 & 0.4315 & 0.3944 \end{pmatrix},$$
$$W = \begin{pmatrix} 0.2000 & 0.2000 & 0.1000 & 0.3000 \\ 0.1000 & 0.1000 & 0.3000 & 0.1000 \\ 0.4000 & 0.2000 & 0.3000 & 0.2000 \\ 0.2000 & 0.2000 & 0.2000 & 0.3000 \\ 0.1000 & 0.3000 & 0.1000 & 0.1000 \end{pmatrix}. \quad (23)$$

Moreover, we get

$$\Gamma = \begin{pmatrix} 1.8150 & 1.7727 & 1.7459 & 1.7819 \\ 1.7727 & 1.7474 & 1.7042 & 1.7423 \\ 1.7459 & 1.7042 & 1.6854 & 1.7120 \\ 1.7819 & 1.7423 & 1.7120 & 1.7539 \end{pmatrix}. \quad (24)$$

From Γ, we have $\rho = [0.0002 \;\; 0.0009 \;\; 0.0018 \;\; 0.9971]^T$. Finally, we get

$$w = [0.2995 \;\; 0.1004 \;\; 0.2002 \;\; 0.2997 \;\; 0.1002]^T. \quad (25)$$

Step 4. Based on the matrix E and the attribute weight vector w, we get

$$\begin{aligned} c\left(\overline{A}_1, A^+\right) &= 0.7931, \\ c\left(\overline{A}_2, A^+\right) &= 0.5487, \\ c\left(\overline{A}_3, A^+\right) &= 0.8094, \\ c\left(\overline{A}_4, A^+\right) &= 0.4106. \end{aligned} \quad (26)$$

Since $c(\overline{A}_3, A^+) > c(\overline{A}_1, A^+) > c(\overline{A}_2, A^+) > c(\overline{A}_4, A^+)$, the ranking order of four alternatives is $A_3 \succ A_1 \succ A_2 \succ A_4$ and the most desirable one is A_3.

By applying the methods from [18, 20, 29, 30, 35–37] to solve the above MAGDM problem, the decision results are shown as Table 1. (Note that determining the weights of both DMs and attributes has been partially or not been considered in [18, 30, 36, 37]; we employ the weights derived from our proposed method to these models for the above decision problem.)

Remark 11. As shown in Table 1, seven methods get the same decision results except for [18]. The reason is that this method utilizes a different aggregation operator which plays an important role in the process of decision making. What is more, the validity and feasibility of our proposed method have been verified by comparison with seven existing models.

Table 1: Decision results with different models.

Models	Decision results
[18]	$A_1 \succ A_3 \succ A_2 \succ A_4$
[20]	$A_3 \succ A_1 \succ A_2 \succ A_4$
[29]	$A_3 \succ A_1 \succ A_2 \succ A_4$
[30]	$A_3 \succ A_1 \succ A_2 \succ A_4$
[35]	$A_3 \succ A_1 \succ A_2 \succ A_4$
[36]	$A_3 \succ A_1 \succ A_2 \succ A_4$
[37]	$A_3 \succ A_1 \succ A_2 \succ A_4$
Our proposed method	$A_3 \succ A_1 \succ A_2 \succ A_4$

5. Conclusions and Discussions

In this paper, we have proposed a method to solve the MAGDM problem in which all the information supplied by the decision makers is expressed as interval-valued intuitionistic fuzzy decision matrices where each of the elements is characterized by an interval-valued intuitionistic fuzzy number, and the information about the weights of both decision makers and attributes may be completely unknown or partially known. The main merits of this method cover three aspects. Firstly, the problem of determining the weights of DMs and attributes has been solved by the proposed consensus-based method and the proposed multiobjective model, respectively. Secondly, a complete mathematical formulation of MAGDM has been established, and its advantages have been proved by two examples. In addition, we have defined the correlation coefficient between two interval-valued intuitionistic fuzzy matrices which develops basic theories on IVIFSs.

It should be noted that we just consider the situation where the information about DMs is completely unknown. In the future, we will consider the situations where the weights information about both DMs and attributes is expressed with various constraint conditions. Meanwhile, we will employ the proposed method to model some uncertain decision making problems from some concrete applied fields, such as medical decision making, social economic, and financial assessment.

Data Availability

All data generated or analyzed during this study are included in this published article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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