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Research Article

Existence of Solutions for Sublinear Kirchhoff Problems with Sublinear Growth

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In this paper, we consider the following sublinear Kirchhoff problems $-(a+b\int_{\mathbb{R}^N}|\nabla u|^2dx)\Delta u+V(x)u=f(x,u)$, in $\mathbb{R}^N$, where $a>0$ and $b\geq 0$ with $N\geq 3$. A new sublinear growth condition is given. When $f(x,u)$ is not odd in u and not integrable in x, we obtain the existence of solutions for the above problem.

1. Introduction and Main Results

In this paper, we consider the following nonlinear Kirchhoff type problems:

$$-\left(a+b\int_{\mathbb{R}^N}|\nabla u|^2\,dx\right)\Delta u+V(x)\,u=f(x,u),\quad \text{in } \mathbb{R}^N, \tag{1}$$

where $a,b\geq 0$, $f(x,u)\in C(\mathbb{R}^N\times\mathbb{R},\mathbb{R})$. The Kirchhoff type problems with general potentials on a bounded domain are introduced by

$$\begin{gathered}-\left(a+b\int_{\Omega}|\nabla u|^2\,dx\right)\Delta u=f(x,u),\quad \text{in } \Omega\\ u=0,\quad \text{on } \partial\Omega\end{gathered} \tag{2}$$

which is related to the stationary analogue of the Kirchhoff equation

$$u_{tt}-\left(a+b\int_{\mathbb{R}^N}|\nabla u|^2\,dx\right)\Delta u=g(x,u). \tag{3}$$

With the development of the variational methods in the last decades, many mathematicians tried to use these methods to search for the existence and multiplicity of solutions for differential equations (see [1–30]). After the work of Lions [14], the Kirchhoff problems have been studied by many mathematicians using the functional analysis methods.

When $f(x,t)$ is superlinear but subcritical with respect to t, many mathematicians obtained the existence and multiplicity of solutions for problem (1) (see [4, 9, 15–18, 20, 22–24, 30]). But there are only a few papers concerning the sublinear Kirchhoff type problems. In 2013, Ye and Tang [30] obtained the existence of infinitely many solutions for (1) by symmetrical mountain pass theorem with $f(x,t)=\nu\xi(x)|t|^{\nu-2}t$, where $\nu\in(1,2)$ and $\xi\in L^{2/(2-\nu)}(\mathbb{R}^N,\mathbb{R}^+)$. In [2], Bahrouni obtained the infinitely many solutions for sublinear Kirchhoff equations with sign-changing potentials. In 2014, Duan and Huang [6] obtained the existence and multiplicity of solutions for problem (1) with more general sublinear nonlinearities. In a recent paper [19], Li and Zhong showed the existence of infinitely many solutions for problem (1) with local sublinear nonlinearities. However, in the above papers, the nonlinear term $f(x,t)$ is assumed to belong to some $L^p(\mathbb{R}^N)$ with respect to x for some $p>1$. Under a coercive condition, Wang and Han considered a class of sublinear nonlinearities for problem (1) and showed the existence of infinitely many solutions when $f(x,t)$ is odd in t which is the following theorem.

Theorem 1 (see [25]). *Suppose that the following conditions are satisfied:*

(V) $\inf_{x\in\mathbb{R}^N} V(x) > 0$ *and there are constants* $\varrho > 0$ *and* $\alpha > N$ *such that, for any* $b > 0$, $\lim_{|y|\longrightarrow+\infty} meas(\{x \in B_\varrho(y) : V(x)/|x|^\alpha \le b\}) = 0$, *where* $meas(\cdot)$ *denotes the Lebesgue measure;*

$(s1)$ *there exists a constant* $\delta > 0$ *such that* $f \in C(\mathbb{R}^N \times [-\delta,\delta],\mathbb{R})$ *and* $f(x,-t) = -f(x,t)$ *for all* $|t| \le \delta$ *and* $x \in \mathbb{R}^N$;

$(s2)$ *There is a ball* $B_{r_0}(x_0)$ *such that*

$$\liminf_{t\longrightarrow 0}\left(\inf_{x\in B_{r_0}(x_0)} \frac{F(x,t)}{t^2}\right) > -\infty \tag{4}$$

and

$$\limsup_{t\longrightarrow 0}\left(\inf_{x\in B_{r_0}(x_0)} \frac{F(x,t)}{t^2}\right) = +\infty, \tag{5}$$

where $F(x,t) = \int_0^t f(x,s)ds$;

$(s3)$ *There is a constant* $\tau > 0$ *and a function* $g \in C([-\tau,\tau],\mathbb{R}^+)$ *such that* $|f(x,t)| \le g(t)$ *for all* $|t| \le \tau$ *and* $x \in \mathbb{R}^N$.

Then Equation (1) possesses a sequence of weak solutions u_n *in* $X \cap L^\infty(\mathbb{R}^N)$ *such that* $u_n \longrightarrow 0$ *in* $L^\infty(\mathbb{R}^N)$ *as* $n \longrightarrow \infty$.

With coercive condition (V), the authors obtained a new compact embedding theorem, stated as follows.

Lemma 2 (see [25]). *Suppose that* V *satisfies conditions* (V). *Then* E *is compact embedded into* $L^p(\mathbb{R}^N)$ *for any* $p \in [1, 2^*)$.

Remark 3. From Lemma 2, for any $p \in [1,2^*)$, there exists a constant $\eta_p > 0$ such that

$$\|u\|_p \le \eta_p \|u\| \quad \textit{for all } u \in E. \tag{6}$$

A natural question is whether there exists solution for problem (1) if there is no symmetrical condition on $f(x,t)$ and $f(x,t)$ is not integrable in x. In this paper, we try to give an existence theorem on this problem. Motivated by the above papers, we consider problem (1) with some new sublinear nonlinearities and, before we state our result, we assume that Γ is a continuous function space such that, for any $\delta(s) \in \Gamma$, there exists a constant $l_0 > 0$ such that

(i) $\delta(s) > 0$ for all $s > 0$; (ii) $\int_{l_0}^{l} (1/s\delta(s))ds \longrightarrow +\infty$ as $l \longrightarrow +\infty$.

In order to obtain the critical points of the corresponding functional, we consider the following function space in this paper:

$$E := \left\{u \in H^1(\mathbb{R}^N) : \int_{\mathbb{R}^N} \left(|\nabla u|^2 + V(x)u^2\right)dx < \infty\right\} \tag{7}$$

with the inner product

$$(u,v)_E = \int_{\mathbb{R}^N} (\nabla u \cdot \nabla v + V(x)uv)\,dx. \tag{8}$$

For any $u \in E$, it follows from (6) that

$$\widetilde{c} = \inf_{u\in E, \|u\|_2=1} \|u\| > 0. \tag{9}$$

In order to show that the infimum can be achieved, consider a minimizing sequence $\{u_m\} \subset E$ such that

$$\begin{gathered}\|u_m\|_2 = 1,\\ \|u_m\| \longrightarrow \widetilde{c}\\ \text{as } m \longrightarrow \infty.\end{gathered} \tag{10}$$

It is easy to see that there exists $\widetilde{u} \in E$ such that $u_m \rightharpoonup \widetilde{u}$. From Brézis-Lieb Lemma and Lemma 2, we obtain $\|\widetilde{u}\|_2 = 1$, $\|\widetilde{u}\| = \widetilde{c}$ and

$$\|u\|_2 \le \frac{1}{\widetilde{c}}\|u\| \quad \text{for all } u \in E. \tag{11}$$

Then we state our main result.

Theorem 4. *Suppose that* V *and* f *satisfy* (V) *and the following conditions:*

$(f1)$ *letting* $F(x,t) = \int_0^t f(x,v)dv$, *there exist* $\delta(s) \in \Gamma$ *with* $0 < 1/\delta(|s|) < 2$ *and* $l_\infty > 0$ *such that*

$$\begin{gathered}0 \le tf(x,t) \le \left(2 - \frac{1}{\delta(|t|)}\right)F(x,t)\\ \text{for all } x \in \mathbb{R}^N \text{ and } |t| \ge l_\infty,\end{gathered} \tag{12}$$

$(f2)$ *there exists* $\widetilde{d} > (\widetilde{c}^2/2)(a+1)$, $d_1 > 0$, *and* $\mu \in [1,2^*)$ *such that*

$$F(x,t) \ge \widetilde{d}t^2 - d_1|t|^\mu \quad \forall (x,t) \in \mathbb{R}^N \times \mathbb{R}; \tag{13}$$

$(f3)$ $\lim_{t\longrightarrow 0} f(x,t) = 0$ *uniformly in* x;
$(f4)$ *there is* $S > 0$ *such that*

$$\sup_{x\in\mathbb{R}^N, |t|=l_\infty} F(x,t) \le S. \tag{14}$$

Then there exists at least one nontrivial solution for problem (1).

Remark 5. Let

$$F(x,t) = \begin{cases} \widetilde{c}^2(a+1)t^2 & \textit{for } |t| \le 1\\ P(t) & \textit{for } 1 \le |t| \le 2\\ \dfrac{t^2+2}{\ln(t^2+2)} & \textit{for } |t| \ge 2,\end{cases} \tag{15}$$

where $P(t) \in C^1(\mathbb{R})$ such that $F(x,t)$ is a C^1 class function. It is easy to see that $F(x,t)$ satisfies $(f1)$–$(f4)$. However, since $\lim_{t\longrightarrow 0}(F(x,t)/t^2) = \widetilde{c}^2(a+1)$, we see that $F(x,t)$ does not satisfy $(s2)$

Remark 6. Condition $(f1)$ is introduced by Wang and Xiao in [26] to prove the existence of periodic solutions for a class of subquadratic Hamiltonian systems. As we know, this is the first time to use such condition on the existence of solutions for sublinear Kirchhoff equations.

2. Preliminaries

Lemma 7. *Suppose that $(f1)$ holds; then there exists $d_2 > 0$ such that*

$$|F(x,t)| \le d_2 Q(|t|)\, t^2 \quad \text{for all } x \in \mathbb{R}^N \text{ and } |t| \ge l_\infty, \text{ where } Q(|t|) = \exp\left(-\int_{l_\infty}^{|t|} \frac{1}{s\delta(s)}\, ds\right). \tag{16}$$

Proof. For any $x \in \mathbb{R}^N$ and $s \ge l_\infty/|t|$, let $w(s) = F(x, st)$. It follows from condition $(f1)$ that

$$w'(s) \le \frac{1}{s}\left(2 - \frac{1}{\delta(s|t|)}\right) w(s) \tag{17}$$

for all $s \ge l_\infty/|t|$. Set

$$T(s) = w'(s) - \frac{1}{s}\left(2 - \frac{1}{\delta(s|t|)}\right) w(s). \tag{18}$$

Then $T(s) = 0$ yields that

$$w(s) = \left(\int_{l_\infty/|t|}^{s} \frac{w(r)}{r^2\delta(r|t|)}\, dr + \frac{w(l_\infty/|t|)\, t^2}{l_\infty^2}\right) s^2 Q(s|t|). \tag{19}$$

For any $s \ge l_\infty/|t|$, (17) shows that $T(s) \le 0$, which implies that

$$w(s) \le \frac{w(l_\infty/|t|)\, t^2}{l_\infty^2} s^2 Q(s|t|). \tag{20}$$

Then we can obtain

$$\begin{aligned} F(x,t) = w(1) &\le \frac{w(l_\infty/|t|)\, t^2}{l_\infty^2} \eta(|t|) \\ &= \frac{F(x, (t/|t|)\, l_\infty)}{l_\infty^2} Q(|t|)\, t^2 \quad \text{for } |t| \ge l_\infty. \end{aligned} \tag{21}$$

From (21) and $(f4)$, we can obtain our conclusion. □

Remark 8. By the definition of Q, it follows from the properties of η that $Q(|t|) \longrightarrow 0$ as $|t| \longrightarrow +\infty$.

Lemma 9. *Suppose that V satisfies conditions $(f1)$, $(f3)$, and $(f4)$; then, for any $\varepsilon > 0$, there exists $d_\varepsilon > 0$ such that*

$$|f(x,t)| \le (\varepsilon + d_\varepsilon |t|) \quad \text{for all } x \in \mathbb{R}^N \text{ and } t \in \mathbb{R} \tag{22}$$

and

$$|F(x,t)| \le \frac{1}{2}\left(\varepsilon|t| + d_\varepsilon t^2\right) \quad \text{for all } x \in \mathbb{R}^N \text{ and } t \in \mathbb{R}. \tag{23}$$

Proof. From Lemma 7, there exists $d_3 > 0$ such that

$$F(x,t) \le d_3 t^2 \quad \text{for } |t| \ge l_\infty. \tag{24}$$

For $|t| \ge l_\infty$, it follows from $(f1)$ that

$$|f(x,t)| \le 2\frac{F(x,t)}{|t|} \le 2d_3|t|. \tag{25}$$

We can easily obtain (22) from $(f3)$ and (25). (23) can be deduced from (22) directly. □

3. Proof of Theorem 1

By standard arguments, we know that the functional $I : E \longrightarrow \mathbb{R}$, defined by

$$\begin{aligned} I(u) = {} & \frac{a}{2}\int_{\mathbb{R}^N} |\nabla u|^2\, dx + \frac{b}{4}\left(\int_{\mathbb{R}^N} |\nabla u|^2\, dx\right)^2 \\ & + \frac{1}{2}\int_{\mathbb{R}^N} V(x)\, u^2\, dx - \int_{\mathbb{R}^N} F(x,u)\, dx, \end{aligned} \tag{26}$$

is well defined and the critical points of I are the solutions for problem (1).

Lemma 10. *Suppose that (V), $(f1)$–$(f4)$ hold; then I satisfies the (PS) condition.*

Proof. From Remark 8 and (V_1), there exists $r_\infty \ge l_\infty$ such that

$$Q(|t|) \le \frac{\inf_{x\in\mathbb{R}^N} V(x)}{8d_2} \quad \text{for any } |t| \ge r_\infty. \tag{27}$$

First, we show that $\{u_n\}$ is bounded in E. It follows from the definition of I, (16) and (27), that

$$\begin{aligned} I(u_n) = {} & \frac{a}{2}\int_{\mathbb{R}^N} |\nabla u_n|^2\, dx + \frac{b}{4}\left(\int_{\mathbb{R}^N} |\nabla u_n|^2\, dx\right)^2 \\ & + \frac{1}{2}\int_{\mathbb{R}^N} V(x)\, u_n^2\, dx - \int_{\mathbb{R}^N} F(x,u_n)\, dx \\ \ge {} & \frac{a}{2}\int_{\mathbb{R}^N} |\nabla u_n|^2\, dx + \frac{1}{2}\int_{\mathbb{R}^N} V(x)\, u_n^2 dx \end{aligned}$$

$$
\begin{aligned}
&-\int_{|u_n|\geq r_\infty} F(x,u_n)\,dx \\
&-\int_{|u_n|\leq r_\infty} F(x,u_n)\,dx \\
&\geq \frac{a}{2}\int_{\mathbb{R}^N} |\nabla u_n|^2\,dx + \frac{1}{2}\int_{\mathbb{R}^N} V(x)\,u_n^2\,dx \\
&-d_2\int_{|u_n|\geq r_\infty} Q(|u_n|)\,u_n^2\,dx \\
&-\frac{1}{2}\int_{|u_n|\leq r_\infty}\left(\varepsilon|u_n| + d_\varepsilon u_n^2\right)dx \\
&\geq \frac{a}{2}\int_{\mathbb{R}^N} |\nabla u_n|^2\,dx + \frac{3}{8}\int_{\mathbb{R}^N} V(x)\,u_n^2 dx \\
&-\frac{1}{2}\left(\varepsilon + d_\varepsilon r_\infty\right)\int_{|u_n|\leq r_\infty} |u_n|\,dx \\
&\geq \min\left\{\frac{a}{2},\frac{3}{8}\right\}\|u_n\|^2 - \frac{\eta_1}{2}\left(\varepsilon + d_\varepsilon r_\infty\right)\|u_n\|,
\end{aligned}
\tag{28}
$$

which implies that I is bounded from below on E. Since we have Lemma 2, it follows from a standard argument, we obtain that $\|u_n - u_0\| \longrightarrow 0$ as $n \longrightarrow \infty$. Hence I satisfies the (PS) condition. □

Proof of Theorem 1. By above discussions, we can see that I is of C^1 class and satisfies the (PS) condition. Similar to (28), we obtain that I is bounded from below. Then $c = \inf_E I(u)$ is a critical value of I and there exists $\overline{u}$ such that $I(\overline{u}) = c$. Finally, we show that $\overline{u} \not\equiv 0$. With $\theta > 0$ being small enough, it follows from $(f2)$ and (11) that

$$
\begin{aligned}
I(\theta\overline{u}) &= \frac{a\theta^2}{2}\int_{\mathbb{R}^N} |\nabla\overline{u}|^2\,dx + \frac{b\theta^4}{4}\left(\int_{\mathbb{R}^N} |\nabla\overline{u}|^2\,dx\right)^2 \\
&+\frac{\theta^2}{2}\int_{\mathbb{R}^N} V(x)\,\overline{u}^2 dx - \int_{\mathbb{R}^N} F(x,\theta\overline{u})\,dx \\
&\leq \frac{a\theta^2}{2}\int_{\mathbb{R}^N} |\nabla\overline{u}|^2\,dx + \frac{b\theta^4}{4}\left(\int_{\mathbb{R}^N} |\nabla\overline{u}|^2\,dx\right)^2 \\
&+\frac{\theta^2}{2}\int_{\mathbb{R}^N} V(x)\,\overline{u}^2 dx - \widetilde{d}\theta^2\int_{\mathbb{R}^N} \overline{u}^2 dx \\
&+d_1\theta^\mu\int_{\mathbb{R}^N} \overline{u}^\mu dx \\
&\leq \left(\frac{1}{2}\widetilde{c}^2(a+1) - \widetilde{d}\right)\theta^2 \\
&+\frac{b\theta^4}{4}\left(\int_{\mathbb{R}^N} |\nabla\overline{u}|^2\,dx\right)^2 + d_1\theta^\mu\eta_\mu^\mu\widetilde{c}^\mu < 0.
\end{aligned}
\tag{29}
$$

Then we can deduce $\inf_E I(u) < 0$, which implies that $\overline{u} \not\equiv 0$. □

Abbreviations

(PS) condition: Palais-Smale condition.

Data Availability

Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

Conflicts of Interest

The authors declare that they have no competing interests.

Authors' Contributions

The research and writing of this manuscript were a collaborative effort from both of the authors. Zhuo Yao and Wei Yang discussed many details of the problems together. Yao managed this manuscript and Yang revised it. Both of the authors read and approved the final version of the manuscript.

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