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Research Article

A Simplified Fractional Order Equivalent Circuit Model and Adaptive Online Parameter Identification Method for Lithium-Ion Batteries

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In order to improve the battery management system performance and enhance the adaptability of the system, a fractional order equivalent circuit model of lithium-ion battery based on electrochemical test was established. The parameters of the fractional order equivalent circuit model are identified by the least squares parameter identification method. The least squares parameter identification method needs to rely on the harsh test conditions of the laboratory, and the parameter identification result is static; it cannot adapt to the characteristics of the lithium battery under dynamic conditions. Taking into account the dynamic changes of lithium batteries, a parameter adaptive online estimation algorithm for fractional equivalent circuit model is proposed. Based on the theory of fractional order calculus and indirect Lyapunov method, the stability and convergence of the estimator are analyzed. Finally, simulation experiments show that this method can continuously estimate the parameters of the fractional order equivalent circuit under UDDS conditions.

1. Introduction

The battery management system (BMS) is one of the most important technologies in the electric vehicle systems. A good battery management system can fully utilize the battery energy, which can not only prolong the mileage but also improve the performance of the electric vehicle [1–3]. The current electric vehicle's power system is mainly based on lithium batteries, because of their high safety, long life, and high energy density. But the electrochemical reaction of lithium-ion battery is highly nonlinear and uncertain, and conversion of electric energy, chemical energy, and thermal energy is very complex [4, 5]. Therefore, it is impossible to build an accurate, intuitive, and efficient battery model for EVs. As a result, it is difficult to design a battery management system.

According to different modeling mechanism, the commonly used battery models at present are mainly including electrochemical model and equivalent circuit model. The electrochemical model describes the battery characteristics with high accuracy, but its calculation process is complicated and often contains many complex partial differential equations [6]. The equivalent circuit model is not accurate enough, but it is simple, intuitive, and suitable for electrical design and simulation [7–9]. In [10], GaoPeng and his partners designed an equivalent circuit model for lithium batteries based on electrochemical impedance spectroscopy, which is more accurate but still not simple enough.

In the traditional equivalent circuit model, the integer-order RC networks are used to describe the battery concentration effects based on the electrochemical reaction of lithium-ion catteries. First-order RC equivalent circuit model is the simplest model, and it is suitable for engineering application. However the first-order RC equivalent circuit model cannot accurately simulate the static and dynamic characteristics of the battery in the begining and the end of charge and discharge. Although the accuracy of the battery model can be improved by increasing the series number of RC networks, it will become difficult to get the model parameters and greatly increase computing tasks. Therefore, satisfying the engineering requirements and improving the accuracy of the model is mutually restrictive, and we should try to improve the accuracy of the model as much as possible when the RC network is limited [11, 12].

A capacitor is a fractional reactance component with fractional characteristics, and fractional derivatives and integrals can better describe the dynamic behavior of RC networks in a circuit system [13, 14]. Some researchers have established fractional equivalent circuits for the electrochemical performance of lithium batteries, but they mainly consider model accuracy and neglect engineering applications. Such as in [15], the author designed a fractional equivalent circuit model containing 2-RC networks, and there are seven parameters in the circuit that need to be identified so that it is difficult to realize online identification.

In order to balance the accuracy of the fractional equivalent circuit model and the practical application of engineering, a simplified fractional equivalent circuit model is proposed by electrochemical test of lithium battery and combined with equivalent circuit modeling method. This simplified fractional equivalent circuit model contains only one RC network, except that the capacitance is described by a fractional order.

Parameter identification is very important in the modeling and analysis of lithium batteries. The general parameter identification method is based on collecting a large amount of laboratory data and using the fitting method to find the optimal parameter value in an offline situation [16–18]. In practical engineering applications, this offline parameter identification method cannot reflect the influence of operating current, state of charge, temperature, and self-discharge on the battery internal characteristics. Although algorithms such as sliding mode and Kalman filter can compensate a certain model system error by feedback, they do not eliminate the state-of-charge estimation error from the root cause [19, 20].

Therefore, in order to improve the battery management system performance and enhance the adaptability of the system, the battery model parameters need to be estimated online to update the battery models. In [21], the authors propose an adaptive battery model based on online identification of battery parameters to account for changes in the internal characteristics of the battery under variable conditions. The first-order RC integer-order dynamic equivalent model established by this method has good results. In addition, many researchers have given some methods in the field of online identification of fractional parameters, such as [22]. However, online parameter identification of fractional equivalent circuit models is rarely reported.

In this paper, a simplified fractional order equivalent circuit model for lithium-ion batteries and adaptive online parameter identification method are presented. The rest of the paper is organized as follows: Section 2 introduces the electrochemical impedance spectroscopy (EIS) and hybrid pulse power characteristic (HPPC) test and the fractional order equivalent circuit model. Section 3 discusses the parameter identification of fractional equivalent circuit model by least squares method under offline condition. Section 4 deals with the design of an adaptive online parameter identification method for fractional order equivalent circuit model. Section 5 shows simulation results. Section 6 states some conclusions and guidelines for further works.

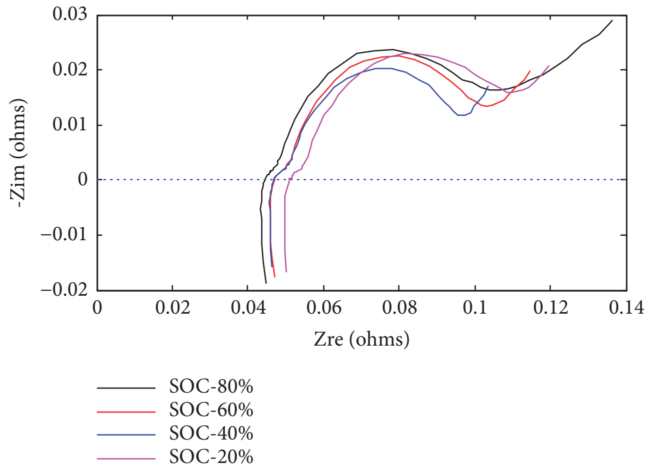

Figure 1: EIS curves of lithium-ion batteries with different SOC [15].

2. Problem Statement and Fractional Impedance Model

In this study, the NCR18650 lithium-ion battery was applied and tested. It has a rated voltage of 3.7V and a cutoff voltage of 2.8V and 2.9Ah capacity. In order to establish a precise battery model, the battery characterization tests include electrochemical impedance spectroscopy (EIS) and mixed pulse power characteristics (HPPC) [23–25].

The state of health (SOH) of the test battery is 98%. In this case, the EIS test of battery was measured using a Princeton electrochemical workstation, and the battery was under different state of charge (SOC). In the EIS test, the test signal amplitude was set to 5 mV and the frequency range was set to 0.005 Hz to 5000 Hz. Results of the EIS test are shown in Figure 1, and the four curves represent the different test results when the SOC of battery is 20%, 40%, 60%, and 80%, respectively.

Considering the characteristics of ternary lithium-ion battery, the pulse charging current of battery was set to 2C, the discharge current of battery was set to 2.5C, and the sample time of battery current and voltage was set to 0.1S in the hybrid pulse power characteristic (HPPC) test. In order to measure the open circuit voltage of the battery, the battery was left standing for one hour after each pulse test, and the voltage after standing is considered as the open circuit voltage of the battery. Results of the HPPC test are shown in Figure 2.

It can be seen from Figure 1 that the four curves of EIS test have similar shapes when the SOC of battery is different, and the Zre and –Zim values also approximately unchanged at the same frequency. When the test frequency exceeds 0.05HZ, the impedance spectrum shows an irregular semicircle that can be modeled by several parallel combinations of a resistor and a CPE. The parallel combination may represent the

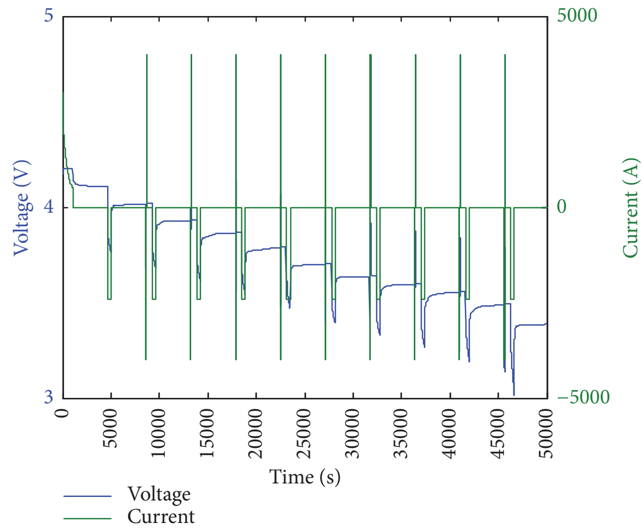

FIGURE 2: HPPC test of lithium-ion battery.

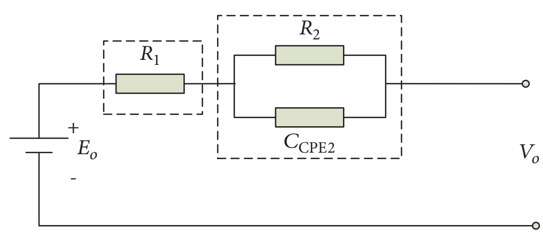

FIGURE 3: Simplified fractional order equivalent circuit model of lithium-ion battery.

charge-transfer reaction on the solid electrolyte interphase layer described by the Butler-Volmer equation.

It can be seen from Figure 2 that the HPPC test includes a rapid voltage rise phase and a voltage ramp stabilization phase. Regarding the battery in the impulse response test voltage rise phase and electrochemical impedance spectroscopy of the high-frequency phase combination analysis, at this stage, the battery voltage is rapidly rising; the main reason for this phenomenon is the battery internal ohmic polarization. In other words, the battery is in the ohmic polarization stage at this stage. When building the battery equivalent model, a resistance element can be used to simulate the battery response to this stage. Combining the cell's ramp stabilization phase of the impulse response test to the mid-frequency and low-frequency sections of the electrochemical impedance spectroscopy, several parallel combinations can be simplified as a parallel representation of one resistor and one CPE element.

On the basis of the EIS analysis and HPPC test, the fractional order equivalent circuit model of lithium-ion battery could be simplified as shown in Figure 3. V_1 denotes the voltage for R_1, which represents the ohmic voltage. V_2 denotes the voltage for R_2, which represents the concentration polarization voltage. E_O is open circuit voltage of the battery and C_{CPE2} is a constant phase element.

The C_{CPE2}in Figure 3 could be deciphered by fractional order elements

$$Z_{CPE2}(s) = \frac{1}{C_2 \cdot s^{\alpha}} \tag{1}$$

where $\alpha \in R$ and $0 < \alpha < 1$, $C_2 \in R$ and is coefficient as same as capacitance. When $\alpha = 1$, C_{CPE2} correspond to capacitor with capacitance C_2; (1) can be written as

$$Z_{CPE2}(s)\big|_{\alpha=1} = \frac{1}{C_2 \cdot s} \tag{2}$$

The total impedance of the fractional equivalent circuit model is

$$Z(s) = R_1 + \frac{R_2}{1 + R_2 C_2 \cdot s^{\alpha}} \tag{3}$$

3. Parameter Identification of Least Squares for Fractional Order Equivalent Circuit Model

In the field of parameter identification, since the principle of least squares (LS) is simple and the estimation performance is good, it is considered to be the most basic estimation method.

Table 1: Fitting data of every SOC case in fractional equivalent circuit model.

SOC	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
R_1/Ω	0.0495	0.0498	0.0489	0.0494	0.0491	0.0489	0.0498	0.0512	0.0501
R_2/Ω	0.0092	0.0089	0.0102	0.0097	0.0103	0.0114	0.0121	0.0089	0.0086
C_2/KF	0.8031	0.8347	0.8890	0.8912	0.9121	0.9330	0.8062	0.9241	0.9364
α	0.7034	0.7002	0.6997	0.0731	0.0726	0.0703	0.0691	0.0701	0.7102

However, it is still difficult to apply the least squares method to the parameter estimation of the fractional order system [15]. Based on the fractional equivalent circuit model, the parameter identification process of the least squares method in the fractional order system is derived, and the parameter identification of the fractional equivalent circuit is completed.

The fractional order equivalent circuit is analyzed based on Kirchhoff's law, available

$$\begin{gathered} V_o = E_o + IR_1 + V_2 \\ D^\alpha V_2 = -\frac{V_2}{R_2 C_2} + \frac{I}{C_2} \end{gathered} \tag{4}$$

Performing a Laplace transform on (4) yields an S-domain expression

$$\begin{aligned} V_o(s) &= E_o(s) + I(s) \cdot R_1 + V_2(s) \\ V_2(s) &= \frac{R_2}{R_2 C_2 s^\alpha + 1} I(s) \end{aligned} \tag{5}$$

Defining the input of the fractional order system as I and the output as V_2, it means $Y(s) = V_2(s)$ and $U(s) = I(s)$. The transfer function of the S-domain is

$$\frac{Y(s)}{U(s)} = \frac{R_2}{R_2 C_2 s^\alpha + 1} \tag{6}$$

The bilinear transformation relationship of fractional order systems is

$$s^\alpha = \left(\frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}} \right)^\alpha, \quad |z| > 1 \tag{7}$$

The bilinear transformation relationship of the fractional order system described by (7) corresponds to the convolution of two binomial sequences corresponding to the numerator and the denominator, as

$$s^\alpha = \left(\frac{2}{T} \right)^\alpha \sum_{k=0}^{n} \frac{(-a)_k}{k!} \frac{(-1)^{n-k} (a)_{n-k}}{(n-k)!} \tag{8}$$

where $n! = (-1)^k (-n)_k (n-k)!$ $k \le n$, $(\alpha)_n = (-1)^k (-\alpha - n + 1)_k (\alpha)_{n-k}$. So (7) can be rewritten as

$$\begin{aligned} s^\alpha &= \left(\frac{2}{T} \right)^\alpha \frac{(-1)^{n-k} (\alpha)_n}{n!} \sum_{k=0}^{n} \frac{(-\alpha)_k (-n)_k (-1)^k}{k! (-\alpha - n + 1)_k} \\ &= \left(\frac{2}{T} \right)^\alpha \frac{(-1)^{n-k} (\alpha)_n}{n!} F(-\alpha, -n; -\alpha - n + 1, -1) \end{aligned} \tag{9}$$

where $F(-\alpha, -n; -\alpha - n + 1, -1)$ is a Gaussian hypergeometric equation obeying the second-order recursive formula.

Using (9), the difference equation of the Z-domain of (5) can be approximated as

$$\begin{aligned} y(k) = &-\frac{T^\alpha - 2R_2 C_2}{T^\alpha + 2R_2 C_2} y(k-1) + \frac{2R_2 T^\alpha}{T^\alpha + 2R_2 C_2} u(k) \\ &+ \frac{R_2 T^\alpha}{T^\alpha + 2R_2 C_2} u(k-1) \end{aligned} \tag{10}$$

Define $\theta = [-(T^\alpha - 2R_2C_2)/(T^\alpha + 2R_2C_2), 2R_2T^\alpha/(T^\alpha + 2R_2C_2), R_2T^\alpha/(T^\alpha + 2R_2C_2)]$ and $\Phi = [y(k-1), u(k), u(k-1)]$, the least squares vector expression is

$$y = \Phi\theta + \varepsilon \tag{11}$$

where y is the output vector at time k, Φ is a known input and output vector, θ is a parameter estimation matrix vector, and ε is a residual vector.

By using the least squares method to minimize the sum of the squares of the residuals, the optimal estimate of the parameter matrix can be obtained, as

$$\theta = \left[\Phi^T \Phi \right]^{-1} \Phi^T y \tag{12}$$

Since the lithium battery changes at the moment of current change, the voltage change at the battery terminal is equivalent to pure resistance, and the influence of polarization resistance is small enough and can be approximately zero. Therefore, the estimation of the internal resistance of a lithium battery can be equivalent to

$$R_1 = \frac{\Delta V_o}{\Delta t} \tag{13}$$

The fractional equivalent circuit model parameters of the lithium battery are identified by the least squares method under different SOC from HPPC test, and the parameter estimation of the whole discharge interval can be obtained. The parameter identification results are shown in Table 1.

4. Adaptive Online Parameter Identification Method

Although the fractional order equivalent circuits have a more accurate battery performance description, the battery has obvious nonlinear, time-varying characteristics in the actual working condition. In other words, the accurate battery mode is suitable for static performance description, but not for dynamic performance description.

Therefore, in order to improve the estimation accuracy of the state of charge and enhance the adaptability of the system, the battery model parameters need to be estimated online and the battery model should be updated.

According to the fractional order equivalent circuit model, the fractional order is set as $\alpha = 0.7$ from the contents of the previous section. The model parameters that the internal resistance unit and the polarization impedance unit need to recognize include R_1, R_2, and C_2. Under normal operating conditions, the battery charge and discharge rate is not high, so the charge state changes slowly. Open circuit voltage (E_o) is a function of state of charge (SOC), so the battery open circuit voltage change is relatively slow, in order to obtain a more accurate parameter model, the E_o also needs to be identified online.

The system of fractional equivalent circuits can be described as a fractional differential equation of (3) and (4). And the derivative of (4) can be rewritten

$$\begin{aligned} D^\alpha V_o &= D^\alpha E_o + D^\alpha I R_1 + D^\alpha V_2 \\ &= R_1 D^\alpha I + \frac{R_1 + R_2}{R_2 C_2} I - \frac{V_o}{R_2 C_2} + \frac{E_o}{R_2 C_2} \\ &= \begin{bmatrix} R_1 & \frac{R_1 + R_2}{R_2 C_2} & \frac{1}{R_2 C_2} & \frac{E_o}{R_2 C_2} \end{bmatrix} \begin{bmatrix} D^\alpha I & I & -V_o & 1 \end{bmatrix} \\ &= \theta \mu^{\mathrm{T}} \end{aligned} \tag{14}$$

where

$$\begin{aligned} \theta &= \begin{bmatrix} R_1 & \frac{R_1 + R_2}{R_2 C_2} & \frac{1}{R_2 C_2} & \frac{E_o}{R_2 C_2} \end{bmatrix} \in \mathbb{R}, \\ \mu &= \begin{bmatrix} D^\alpha I & I & -V_o & 1 \end{bmatrix}^{\mathrm{T}} \in \mathbb{R} \end{aligned} \tag{15}$$

Since the system parameters are unknown to scene θ, using its estimation $\widehat{\theta}$ instead, the corresponding estimated output is

$$D^\alpha \widehat{V}_o = \widehat{\theta} \mu^{\mathrm{T}} \tag{16}$$

Set the output voltage prediction error and the parameter vector estimation error respectively as

$$e(t) = V_o - \widehat{V}_o \tag{17}$$

$$\widetilde{\theta}(t) = \theta - \widehat{\theta} \tag{18}$$

Thus, a model for estimation of parameters is constructed

$$\begin{bmatrix} D^\beta \widehat{\theta}_1 \\ D^\beta \widehat{\theta}_2 \\ D^\beta \widehat{\theta}_3 \\ D^\beta \widehat{\theta}_4 \end{bmatrix} = \begin{bmatrix} \rho_1 D^\alpha I \\ \rho_2 I \\ -\rho_3 V_o \\ \rho_4 \end{bmatrix} e(t) \tag{19}$$

where β is the parameter update order limited by inequality $0 < \beta < 1$, and ρ_1, ρ_2, ρ_3, ρ_4 are positive constants.

In order to reduce the $\widetilde{\theta}(t)$, the parameter update law is designed and the normalized quadratic cost function is defined as

$$J(\widehat{\theta}) = \frac{e^2(t)}{2\varpi^2(t)} \tag{20}$$

where $\varpi^2(t) = 1 + \mu^T \Lambda \mu$, and Λ is blamed for the positive definite matrices and it is called the normalizing weighting coefficient matrix.

Design system parameters update law as

$$D^\beta \widehat{\theta} = -\Gamma \nabla J \tag{21}$$

where Γ is the positive definite matrix of the appropriate dimension.

Take (17) and (20) into (21), we can obtain

$$D^\beta \widetilde{\theta}(t) = -\Gamma \frac{\mu}{1 + \mu^T \Lambda \mu} e(t) \tag{22}$$

Define $\Theta(t) = \mu\mu^T / \varpi^2(t) \geq 0$, so (22) is rewritten as

$$D^\beta \widetilde{\theta}(t) = -\Gamma \Theta(t) \widetilde{\theta}(t) \tag{23}$$

According to Lemma 2.1 in [22], the corresponding frequency distributed model of the system can be described by

$$\begin{aligned} \frac{\partial z(\omega, t)}{\partial t} &= -\omega z(\omega, t) - \Gamma \Theta(t) \widetilde{\theta}(t) \\ \widetilde{\theta}(t) &= \int_0^\infty \phi_\beta(\omega) z(\omega, t) \, d\omega \end{aligned} \tag{24}$$

where $z(\omega, t) \notin \mathbb{R}$, $\phi_\beta(\omega) = \sin \beta\pi / \pi\omega^\beta$.

Define the Lyapunov function as

$$V = \frac{1}{2} \int_0^\infty \phi_\beta(\omega) z^T(\omega, t) \Gamma^{-1} z(\omega, t) \, d\omega \tag{25}$$

The time derivative of (24) can be written as

$$\begin{aligned} \dot{V} &= \int_0^\infty \phi_\beta(\omega) z^T(\omega, t) \Gamma^{-1} \frac{\partial z(\omega, t)}{\partial t} d\omega = \int_0^\infty \phi_\beta(\omega) \\ &\quad \cdot z^T(\omega, t) \Gamma^{-1} \left(-\omega z(\omega, t) - \Gamma \Theta(t) \widetilde{\theta}(t) \right) d\omega \\ &= -\int_0^\infty \omega \phi_\beta(\omega) z^T(\omega, t) \Gamma^{-1} z(\omega, t) \, d\omega - \widetilde{\theta}^T(t) \\ &\quad \cdot \Theta(t) \widetilde{\theta}(t) \end{aligned} \tag{26}$$

It is obvious that $\dot{V} < 0$, which means that it satisfies the Lyapunov stability criterion, so the adaptive parameter estimation algorithm is asymptotically stable.

Based on LaSalle Lemma, one can conclude that such a fractional order gradient estimator is asymptotically stable when $0 < \beta < 1$. In other words, $\widetilde{\theta}(t)$ can converge to zero

$$\lim_{t \longrightarrow \infty} \widetilde{\theta} = 0 \tag{27}$$

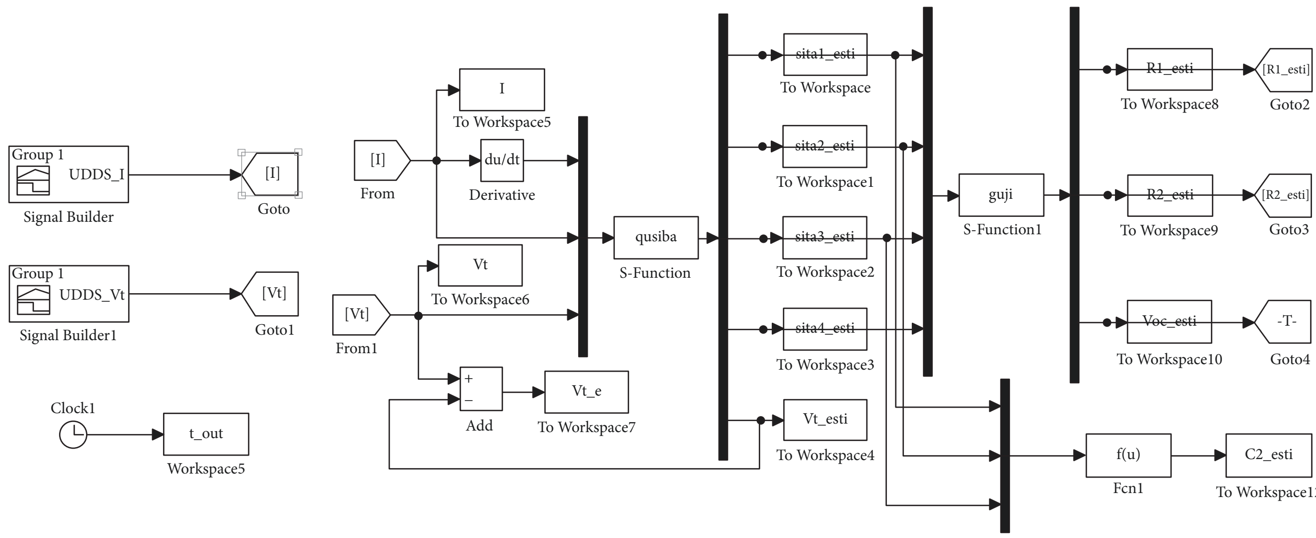

Figure 4: Parameter online simulation mode.

Further, the equivalent circuit parameters can be expressed as

$$\begin{bmatrix} \widehat{R}_1 \\ \widehat{R}_2 \\ \widehat{E}_o \\ \widehat{C}_2 \end{bmatrix} = \begin{bmatrix} \widehat{\theta}_1 \\ \dfrac{\widehat{\theta}_2}{\widehat{\theta}_3} - \widehat{\theta}_1 \\ \dfrac{\widehat{\theta}_4}{\widehat{\theta}_3} \\ \dfrac{\left(\widehat{R}_1 + \widehat{R}_2\right) I - V_o + \widehat{E}_o}{\left(\dot{V}_o - \widehat{R}_1 D^{\alpha} I\right) \widehat{R}_2} \end{bmatrix} \tag{28}$$

5. Simulation and Experimental Verification

In order to verify the accuracy of the adaptive online parameter identification, a Simulink model is designed in MATLAB/Simulink, as shown in Figure 4. The battery current and voltage data is collected in real time on the battery test platform, according to the discharge requirements of the UDDS driving cycle. The simulation model is used to continuously evaluate the ohmic resistance, polarization resistance, and polarization capacitance of the battery under this condition.

The initial parameters and feedback coefficients of adaptive online parameter identification algorithm are set as follows: $\widehat{\theta}_{1(0)} = 0.2$, $\widehat{\theta}_{2(0)} = 0.01$, $\widehat{\theta}_{3(0)} = 0.2$, $\widehat{\theta}_{4(0)} = 1$, $\rho_1 = 0.01$, $\rho_2 = 0.002$, $\rho_3 = 0.02$, and $\rho_4 = 0.005$. The results of the adaptive online estimation of fractional equivalent circuit parameters are shown in Figure 5. The blue curves represent the results of the adaptive online identification of the equivalent circuit parameters, and the red curves represent the results of the parameter identification shown in Table 1 under offline conditions.

It can be seen that, under the condition of randomly given initial values, the estimated curve of each parameter quickly converges to the reference curve, and then the fluctuation tends to be stable. This shows that the proposed algorithm can quickly correct the initial capacity deviation under dynamic conditions, accurately estimate the internal parameters of the battery, and have good rapidity, robustness, and stability.

6. Conclusions

In this paper, we analyzed the EIS and the HPPC data of lithium-ion batteries and proposed a simplified fractional order RC equivalent circuit impedance model. By offline identification of fractional order circuit model parameters based on HPPC data, it can be seen that the fractional order equivalent circuit model is more accurate than the integer-order equivalent circuit model. However, there still exists the problem that static battery models cannot match dynamic conditions. In order to solve this problem, an online parameter estimation method for the simplified fractional equivalent circuit model is proposed. Using the Lyapunov stability theorem, the stability and convergence of the method is proved. This method can realize real-time collection of the battery current and voltage data and continuously estimate the ohmic resistance, polarization resistance, and polarization capacitance of the battery under working conditions. Simulation results show that the adaptive online parameter estimation method can be used for parameter identification and parameter updating of the fractional order equivalent circuit model and can well reflect the dynamic characteristics of the battery.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

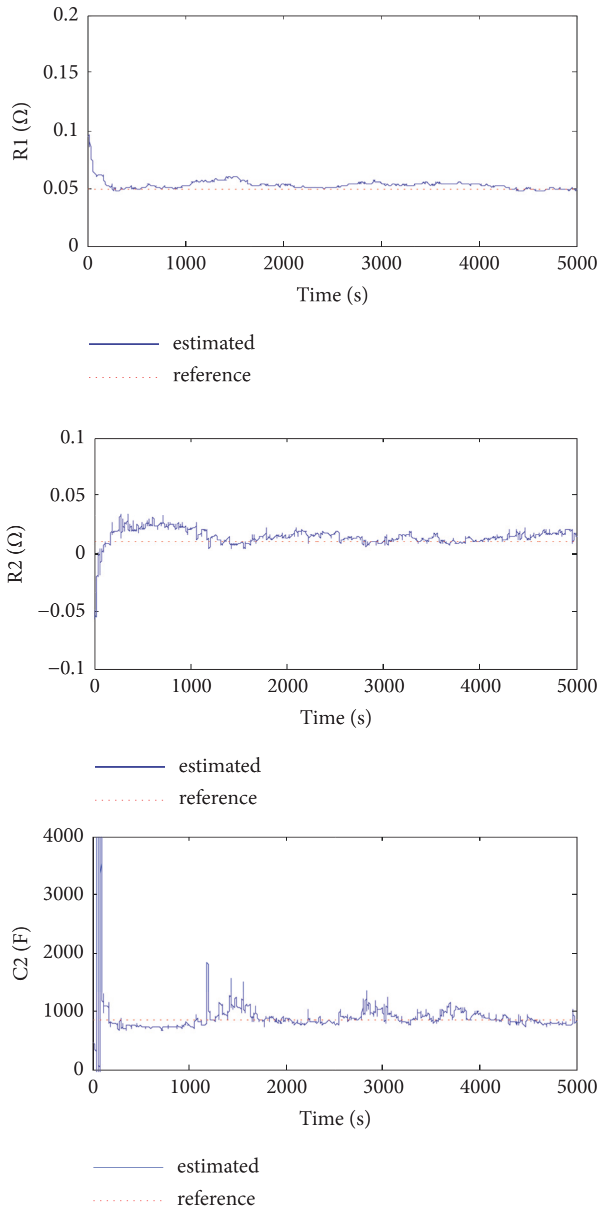

Figure 5: Online parameter estimation curves.

Authors' Contributions

WJL (Jianlin Wang) and XD (Dan Xu) wrote the main manuscript text; ZL (Le Zhang), ZP (Peng Zhang), and ZGR (Gairu Zhang) assisted in electrochemical testing of batteries. All authors reviewed the manuscript.

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