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Research Article

Fatigue Strength Assessment of Trimaran Cross-Deck Structure Based on Spectral and Simplified Fatigue Method

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In order to investigate the fatigue behaviour of trimaran cross-deck structural details, the spectral and simplified fatigue analysis approaches are proposed. In spectral fatigue approach, three-dimensional (3D) linear potential flow theory and global FE analysis are used for wave loads and stress transfer functions calculation; the stochastic spectral fatigue analysis is carried out considering the weighted wave headings factors. In simplified fatigue approach, based on the direct calculation procedure of LR rules, the evaluation of simplified fatigue loads and loading conditions are presented, and the stress ranges are obtained by global finite element (FE) analysis. Then the fatigue lives of a few hot spots are computed to demonstrate the application of the proposed method. The result shows that the method given in this paper has a good applicability. This study offers methodology for the fatigue analysis of trimaran cross-deck structure, which may be regarded as helpful references for structural design of these types of ships.

1. Introduction

Trimarans, being high-performance ships, have attracted more and more people's attention in recent years. Because of the unique configuration, trimarans have a lower resistance and a better transverse stability compared with conventional monohull ships. Further, they have large open decks for diverse design, transport capacity, and arrangement convenience, due to the suitable arrangement of the side hulls. Consequently, trimarans have become a high-performance vehicle type, which have been considered as alternative to monohulls in high speed transportation and in naval applications [1].

Because of the trimaran having so many advantages, many investigations have been performed on trimaran in the past years. Related researches are still going on worldwide. Most of the researches focused on hydrodynamics including the resistance and seakeeping performance. With regard to resistance, reducing the wave-making resistance and the optimization of placement of side hulls are widely investigated [2–4]. In terms of seakeeping performance study of a trimaran, several researches have been conducted to examine the seakeeping performance of the trimaran using numerical and experimental methods [5–7]. For the trimaran wave loads study, Bingham et al. investigated the trimaran's wave induced motion and loads in regular head waves using three-dimensional pulsating source method [8]. In addition, Fang et al. used 3D translating-pulsating source distribution model to study the global loads of a trimaran in oblique waves, and the wave loads based spectral analysis for the optimal selection of trimaran side hulls arrangement are proposed [9, 10].

Fatigue is one of the most significant failure modes for marine structures [11]. Ships are prone to fatigue due to high cyclic loads predominantly caused by waves and varying loading conditions. Due to the unique structure, the stress concentration of trimaran's cross-deck structure is serious and the fatigue problem of cross-deck structure appears particularly serious [12]. Hence, evaluation of fatigue strength is vital for trimaran design.

The fatigue strength assessment of marine structure usually includes the $S-N$ curve approach and fracture mechanics approach [13]. The fracture mechanics approach is based on the existence of an initial crack and subsequent growth under cyclic loading. The $S-N$ curve approach is based on experimental fatigue test data along with Miner

linear cumulative damage criteria, which has been widely used for marine structure fatigue assessment. It can be further subdivided into the "simplified fatigue method" and "spectral fatigue method". This approach is adopted by most of the world major classification societies such as ABS, BV, DNV GL, LR, and CCS. The fatigue analysis procedures of the conventional ship structures have been established by the major classification societies, in order to meet the fatigue strength requirement at the design stage. Lots of investigators have adopted $S-N$ curve approach for the study of marine structure's fatigue strength, using both the "simplified fatigue method" and the "spectral fatigue method" [14–17]. Previous studies show that an extensive and in depth research in recent years on fatigue strength assessment of ship structures mainly focused on conventional vessels like bulk carriers, oil tankers, and container ships. However, few researchers have investigated the trimaran fatigue strength and relatively very little reference material is available in literature. Peng et al. made some important contributions to trimaran fatigue strength assessment [18]. They performed coarse meshed global finite element analysis of trimaran to identify the hot spots and computed fatigue damage by spectral fatigue analysis in conjunction with hot spot stress approach. Zhen et al. calculated the fatigue damage of a trimaran by using spectral analysis and discussed the effect of different sea area and heading angles' time factors [19].

Furthermore, for the novel trimaran, there is only conceptual guideline providing for fatigue analysis, and no clear fatigue assessment procedure is available. At present, the only available rule for trimaran is the Lloyd's Register Rules for Classification of Trimaran (LR Rules) [20]. But in LR Rules, only formulation of design loads and strength analysis are illustrated and the fatigue strength assessment is suggested to refer to the fatigue guidelines of conventional vessels. When the fatigue strength assessment is done according to conventional vessels' fatigue guidelines, especially using the "simplified fatigue method", there are some difficulties for the unique cross-deck structure. Firstly, we cannot use the conventional ship beam-theory to calculate the stress of the cross-deck structure. Secondly, the shape parameter h of the Weibull distribution is usually obtained by empirical formula about ship length for conventional vessel; it is obviously not appropriate for the unique cross-deck structure. Presently, because of the above problems, the literature regarding fatigue strength assessment of trimaran, which uses the "simplified fatigue method", is very rare.

Therefore, there is a great significance of an in depth research on fatigue strength assessment of trimaran especially fatigue problems related to the cross-deck structure. In the present study, in order to investigate the fatigue behaviour of cross-deck structural details, a suitable simplified fatigue approach for trimaran is explored, which refers to the existing fatigue guidelines of conventional ships and the direct calculation procedure of LR rules for trimaran. At the same time, the stochastic spectral fatigue analysis is carried out with considering the weighted wave headings factors. Finally, the fatigue lives of a few hot spots are computed to demonstrate the application of the proposed method. Eventually the fatigue characteristic of trimaran cross-deck structure is summarized, which includes the most dangerous fatigue position and the fatigue damage proportion of various wave heading directions.

2. Basic Theory of Ship Fatigue Analysis

2.1. S-N Curve and Palmgren-Miner Linear Cumulative Damage Theory. The fatigue strength of structural components is described by using the $S-N$ curve, which is obtained from fatigue tests. For ship structural details, the $S-N$ curve is usually represented by the following formula:

$$S^m \cdot N = A \tag{1}$$

where S is the stress range; N is the number of cycles to failure; m and A are constants, which are obtained from the fatigue test data.

Fatigue damage is defined as the ratio between the number of cycles in the design lifetime and the number of cycles to fatigue failure. Based on Palmgren-Miner linear cumulative damage theory, the total fatigue damage can then be calculated as

$$D = \sum_{i=1}^{K} D_i = \sum_{i=1}^{K} \frac{n_i}{N_i} = \frac{1}{A}\sum_{i=1}^{K} n_i S_i^m = \frac{N_L}{A}\overline{S^m} \tag{2}$$

where D_i is the fatigue damage in the stress amplitude S_i; n_i is the number of cycles under the stress range S_i; N_i is the number of cycles corresponding to fatigue failure at the same stress range S_i, based on the $S-N$ curve; N_L represents the total number of cycles in the duration; and $\overline{S^m}$ is the mathematical expectation of S^m.

When the stress range is a continuum function and its probability density function is $f_S(S)$, the total fatigue damage can be denoted as follows:

$$D = \frac{N_L}{A}\int_0^{+\infty} S^m f_S(S)\, dS \tag{3}$$

2.2. Short-Term and Long-Term Stress Range Distribution. Under the assumption of a stationary zero mean Gaussian wave elevation process within each short-term period, the stress response for the linear system is also a stationary zero mean Gaussian process, and the peak values of the stress follow Rayleigh probability density function:

$$f_Y(y) = \frac{y}{\sigma_X^2}\exp\left(-\frac{y^2}{2\sigma_X^2}\right) \quad 0 \le y < +\infty \tag{4}$$

where y is the peak stress and σ_X is the standard deviation of the stress process.

The standard deviation of the stress process σ_X in terms of spectral moment can be described as

$$\sigma_X = \sqrt{\int_0^{+\infty} G_{XX}(\omega)\, d\omega} = \sqrt{m_0} \tag{5}$$

where ω is the frequency of the stress process; $G_{XX}(\omega)$ is the stress response spectrum; m_0 is the zero order spectral moment.

On the other hand, for a narrow band process, the following relationship applies to stress range and peak stress.

$$S = 2y \tag{6}$$

Then the stress range probability density function and distribution function can be expressed as

$$f_S(S) = \frac{S}{4m_0} \exp\left(-\frac{S^2}{8m_0}\right) \quad 0 \le S < +\infty \tag{7a}$$

$$F_S(S) = 1 - \exp\left(-\frac{S^2}{8m_0}\right) \quad 0 \le S < +\infty \tag{7b}$$

The stress range distribution over the entire structure life is referred as the long-term distribution of the stress range. The commonly used theoretical distribution to adequately approximate the long term distribution of wave induced stress range is two-parameter Weibull distribution [21]. The probability density function and distribution function of two-parameter Weibull distribution are expressed as

$$f_S(S) = \frac{h}{q}\left(\frac{S}{q}\right)^{h-1} \exp\left[-\left(\frac{S}{q}\right)^h\right] \quad 0 \le S < +\infty \tag{8a}$$

$$F_S(S) = 1 - \exp\left[-\left(\frac{S}{q}\right)^h\right] \quad 0 \le S < +\infty \tag{8b}$$

where h and q are Weibull shape and scale parameters, respectively.

The distribution function of two-parameter Weibull distribution $F_S(S)$ is usually obtained from weighted combination of short-term distribution functions of stress range:

$$\begin{aligned} F_S(S) &= \frac{\sum_{i=1}^{n_S}\sum_{j=1}^{n_H} \nu_{ij}\cdot p_i \cdot p_j \cdot F_{S\theta ij}(S)}{\sum_{i=1}^{n_S}\sum_{j=1}^{n_H} \nu_{ij}\cdot p_i \cdot p_j} \\ &= \sum_{i=1}^{n_S}\sum_{j=1}^{n_H} r_{ij}\cdot p_i \cdot p_j \cdot F_{S\theta ij}(S) \end{aligned} \tag{9}$$

where n_S is the number of sea states; n_H is the number of wave headings; p_i is the probability of occurrence of the individual sea state; p_j is the probability of occurrence of the wave heading; ν_{ij} is the average zero crossing rate of stress alternating response in the sea state i and wave heading j; r_{ij} is the ratio of average zero-crossing rate to total average response zero-crossing rate in the sea state i and wave heading j.

The shape parameter h of the Weibull distribution depends on the parameters of the ship, locations of the structure details, structure types, response characteristics, and the sailing routes during the design life. The values of shape parameter generally vary from 0.7 to 1.3. In this paper, the least square method is used to fit the shape parameter. In order to fit Weibull shape parameters by least square method, (8b) is rewritten in linear form as follows:

$$1 - F(s) = \exp\left[-\left(\frac{s}{q}\right)^h\right] \tag{10}$$

Taking natural logarithms on (10) and setting a function $Q(s)$ about s, (10) can be rewritten as

$$Q(s) = \ln\{-\ln[1 - F(s)]\} = h\cdot \ln s - h\cdot \ln q \tag{11}$$

From (11) we can find that $Q(s)$ is linear with $\ln s$, and the shape parameter h of the Weibull distribution is the slope of the line.

In practical application, $Q(s)$ and $\ln s$ can be obtained from a series values of stress range S by using the above method; then a set of sample values are obtained. The least square method is used to fit the sample data linearly, and the shape parameter is obtained.

The scale parameter q is described from the shape parameter and a reference stress response S_0. The reference stress response S_0 is exceeded once out of the corresponding reference number of the stress cycles N_0. The scale parameter q can be determined as

$$q = \frac{S_0}{(\ln N_0)^{1/h}} \tag{12}$$

2.3. Spectral Fatigue Damage Calculation. The "spectral fatigue method" usually involves direct wave load analysis in frequency domain and the stress response analysis to establish complex stress transfer functions [21, 22]. On the assumption of narrow band, the Rayleigh probability density function, which is used to describe the short-term stress range distribution, can be obtained by using the various orders of spectral moments of the stress response. The total fatigue damage of a structural element is calculated by adding up the short-term damages over all the applicable sea states in a specific wave scatter diagram. It is considered as the most reliable method for fatigue life estimation of ship structure due to its ability to take into account different sea states as well as their probabilities of occurrence.

The environmental wave spectrum for the different sea states can be defined as the Pierson-Moskowitz wave spectrum and expressed as

$$\begin{aligned} &G_{\eta\eta}(\omega \mid H_s, T_z) \\ &\quad = \frac{H_s^2}{4\pi}\left(\frac{2\pi}{T_z}\right)^4 \omega^{-5} \exp\left(-\frac{1}{\pi}\left(\frac{2\pi}{T_z}\right)^4 \omega^{-4}\right) \end{aligned} \tag{13}$$

where H_s is significant wave height; T_z is zero crossing period and ω is wave frequency.

For vessel with forward speed U and heading angle θ, the relation between wave frequency ω and the encounter frequency ω_e is given by

$$\omega_e = \omega\left(1 + \frac{2\omega U}{g}\cos\theta\right) \tag{14}$$

According to conservation of energy, using the value of ω_e from (14) and after some mathematical manipulation, wave spectrum $G_{\eta\eta}(\omega_e \mid H_s, T_z, \theta)$ for a given heading angle is given by the relation

$$G_{\eta\eta}(\omega_e \mid H_s, T_z, \theta) = \frac{G_{\eta\eta}(\omega \mid H_s, T_z)}{1 + (2\omega U/g)\cos\theta} \tag{15}$$

Assuming that ship structure is a linear system, the stress energy spectrum $G_{XX}(\omega_e|H_s, T_z, \theta)$ can be obtained from

$$G_{XX}(\omega_e \mid H_s, T_z, \theta) = \left|H_\sigma(\omega_e \mid \theta)\right|^2 \cdot G_{\eta\eta}(\omega_e \mid H_s, T_z, \theta) \tag{16}$$

where $H_\sigma(\omega_e \mid \theta)$ is the stress transfer function.

Then the nth order spectral moment of the response process for a given heading may be described as

$$m_n = \int_0^{+\infty} \omega_e^n \cdot G_{XX}(\omega_e \mid H_s, T_z, \theta)\, d\omega_e \tag{17}$$

According to (3) and (7a) and (7b) and after some mathematical manipulations, the short-term fatigue damage D_{ij} during the time of sailing T_{ij} in the specific sea state i and wave heading j can be written as

$$D_{ij} = \frac{T_{ij} f_{0ij}}{A}\left(2\sqrt{2m_{0ij}}\right)^m \Gamma\left(1 + \frac{m}{2}\right) \tag{18}$$

where f_{0ij} is zero-up crossing frequency of the stress response.

The cumulative fatigue damage D for the structural detail in the design life is calculated as

$$\begin{aligned} D &= \sum_{i=1}^{n_S}\sum_{j=1}^{n_H} D_{ij} \\ &= \frac{T}{A}\Gamma\left(1 + \frac{m}{2}\right)\sum_{i=1}^{n_S}\sum_{j=1}^{n_H} p_i p_j f_{0ij}\left(2\sqrt{2m_{0ij}}\right)^m \end{aligned} \tag{19}$$

where T denotes the design life of a ship in seconds.

2.4. Simplified Fatigue Damage Calculation. The "simplified fatigue method" is based on the long-term stress range distribution, which follows the two parameter Weibull probability distribution [21]. The shape and scale parameters are obtained from the previous section. Then choosing basic design $S-N$ curves and according to the linear cumulative damage theory, the fatigue damage can be obtained.

Substituting the value of Weibull scale factor q from (12) into (8a) and then putting the value of long term stress range probability density function $f_S(S)$ from (8a) into (3), after mathematical manipulation, we get the fatigue damage:

$$\begin{aligned} D &= \frac{N_L}{A}\int_0^{+\infty} S^m \frac{h}{q}\left(\frac{S}{q}\right)^{h-1} \exp\left[-\left(\frac{S}{q}\right)^h\right] dS \\ &= \frac{N_L}{A}\frac{{S_0}^m}{(\ln N_0)^{m/h}}\Gamma\left(1 + \frac{m}{h}\right) \end{aligned} \tag{20}$$

where Γ denotes gamma function.

When predicting the long-term response for trimaran per LR rules, the design life is generally to be taken as 20 years, which corresponds to a long-term probability level of 10^{-8} [20]. So the value of N_0 is 10^8 cycles, and the value of N_L is 0.6×10^8 cycles. Substituting these values and $S-N$ curve parameter m = 3, incorporating variables α (proportion of ship's life in different loading condition) and μ (coefficient taking into account the change in slope of $S-N$ curve) in (20), we get

$$D = \alpha\frac{0.6}{A}\frac{{S_0}^3}{18.42^{3/h}}\mu\Gamma\left(\frac{3}{h} + 1\right)\times 10^8 \tag{21}$$

From (21), we can find that only the reference stress response S_0 is the important parameter to be calculated. It can be obtained by complex empirical formulas global FE analysis.

Table 1: Wave load calculation parameters.

Parameter		Range	Increment
Frequency	ω (rad/s)	0.1~2.9	0.1
Heading angle	θ (°)	0~330	30

3. FE Model and Location to Be Checked

Because of the complexity of trimaran's hullform and structure, the local fine meshes at hot spot location in the whole FE model are used for the calculation of hot spot stress. This whole FE model extends over the full length, breadth, and depth of trimaran. All primary structures, such as deck plating, bottom and side shell plating, longitudinal and transverse bulkhead plating, transverse floors, and internal structural walls, are represented by plate elements. Secondary stiffening members are modeled using line elements having axial and bending stiffness (beams and bars). And the net thinkness is used for the fatigue analysis. The whole FE model is shown in Figure 1(a). Structure analysis of trimaran based on direct calculation procedure of Lloyd's Register Rules for the classification of trimarans is performed to identify locations of hot spots [19, 20]. Then the connection of the main hull and cross-deck at the front (Figure 1(b)) and two transverse sections of the main hull at the center of the cross-deck are selected to perform fatigue analysis.

4. Spectral Fatigue Analysis for Trimaran

4.1. Calculation of Wave Load. The calculation of wave load and stress transfer function is the key step in spectral method. The responses of trimaran motion and wave load are calculated using 3D linear potential flow theory. Parameters used in wave load calculation are summarized in Table 1.

After these parameters are determined, the responses of trimaran motion and hydrodynamic pressure on trimaran's surface in unit wave amplitude regular waves are calculated. Under the condition of head sea and the wave frequency ω =0.7rad/s, the hydrodynamic pressure on trimaran's surface is shown in Figure 2.

4.2. Calculation of Stress Transfer Function. A global FE analysis of trimaran structure is performed for 4176 load cases by applying inertial load of trimaran motion and spectral fatigue pressure loads generated during previous step to generate stress transfer functions at hot spots. Stress transfer functions of selected hot spots are shown in Figure 3.

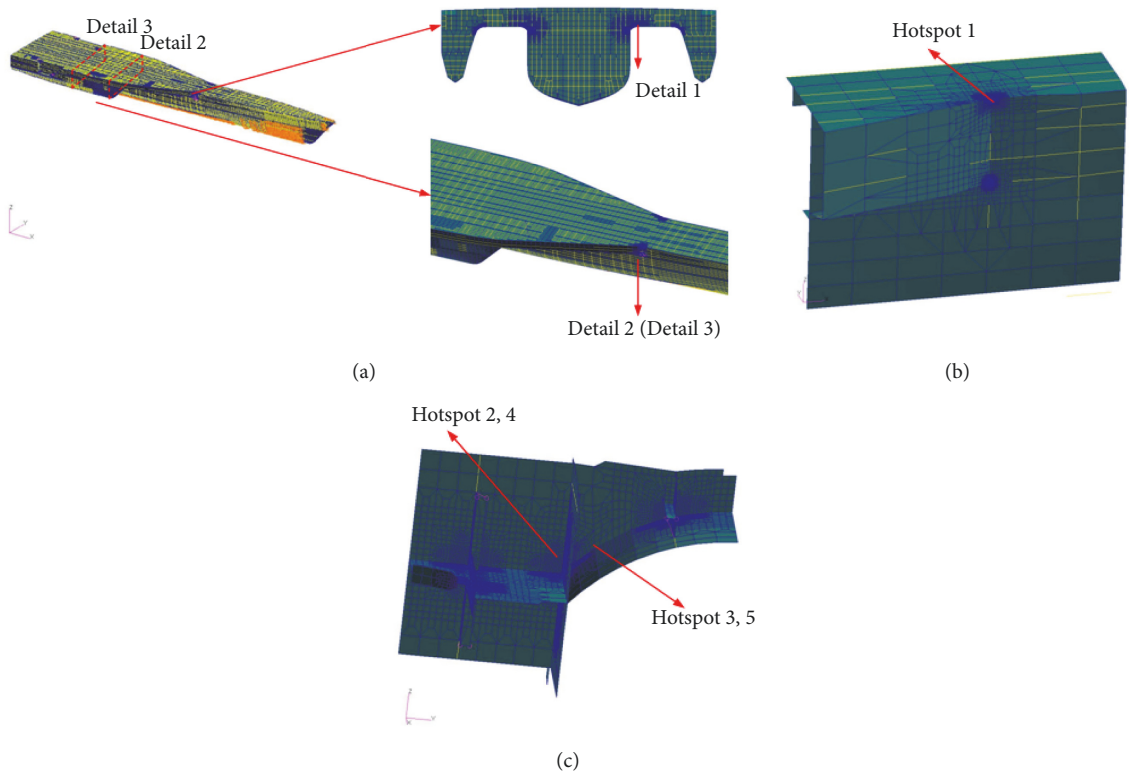


Figure 1: The whole FE model and fine mesh models. (a) The whole FE model and details. (b) Detail 1. (c) Detail 2 (Detail 3).

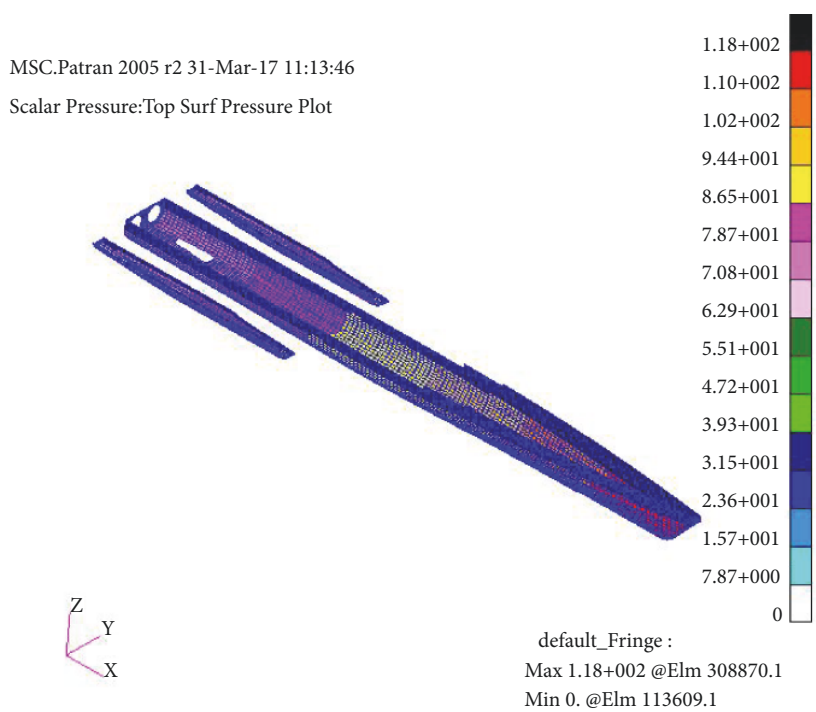


Figure 2: The hydrodynamic pressure on ship's surface.

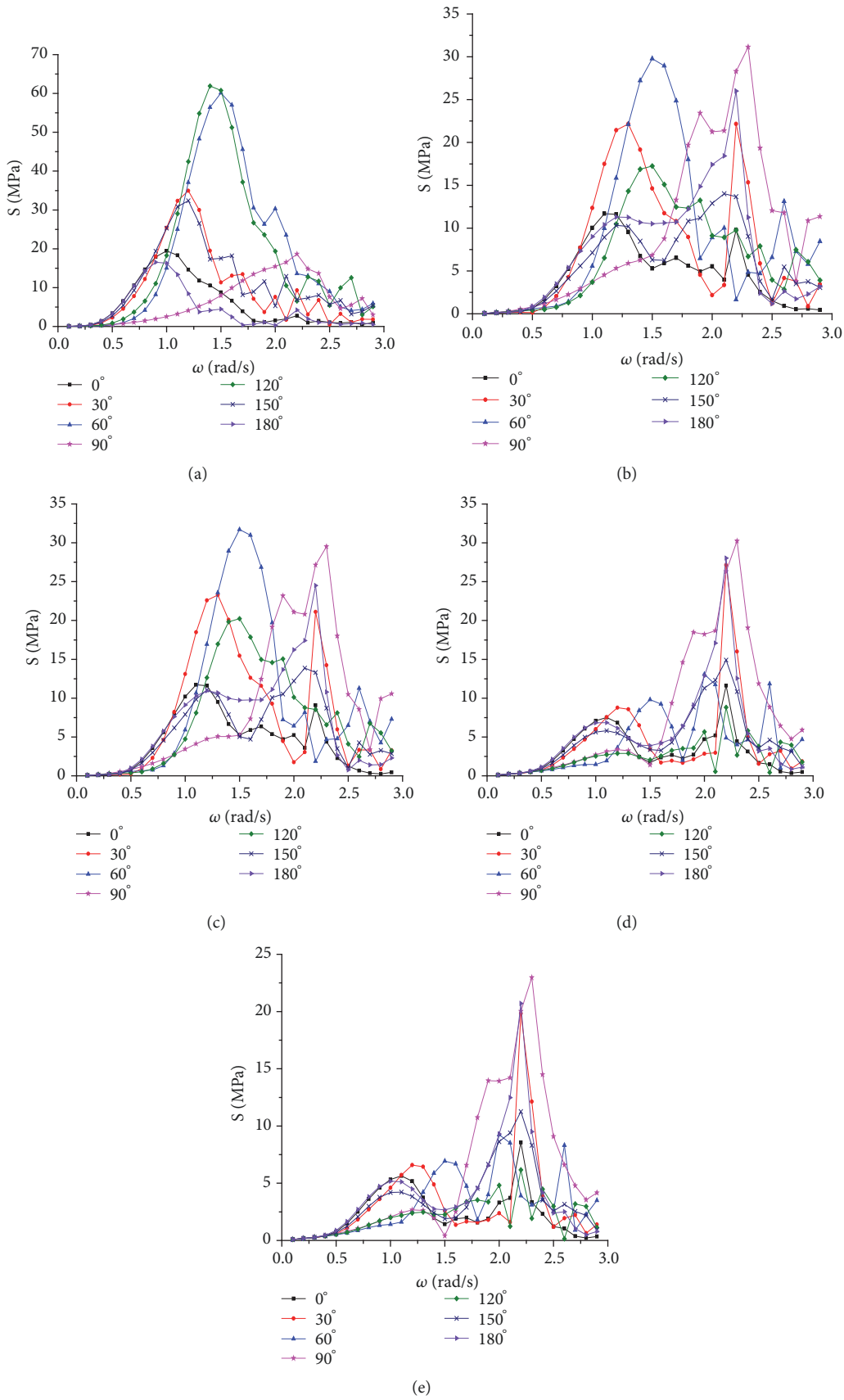


Figure 3: Stress transfer function. (a) Hot spot 1. (b) Hot spot 2. (c) Hot spot 3. (d) Hot spot 4. (e) Hot spot 5.

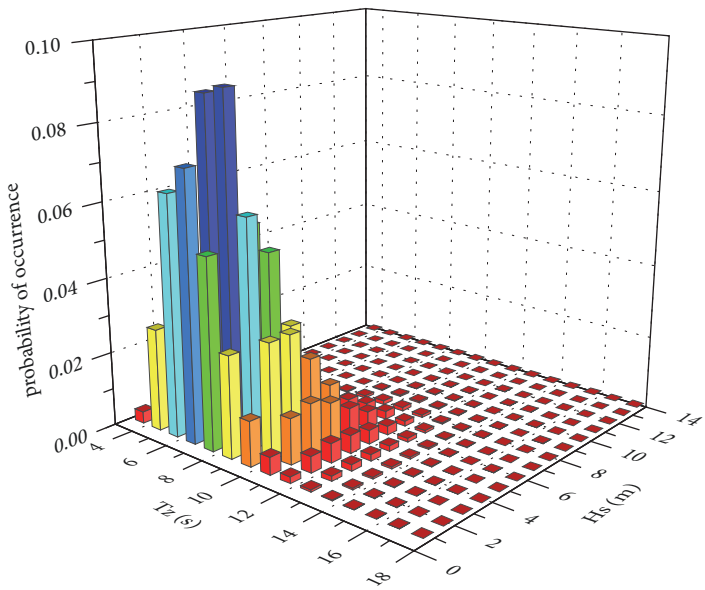


Figure 4: World Wide Trade wave scatter diagrams.

Figure 3 shows that the stress transfer function values of hot spot 1 are larger in the wave heading 60° and 120°. It can be inferred that the longitudinal torsional moment is the main load for structure detail 1. The stress transfer function values of hot spots 2, 3, 4, and 5 in the wave heading 60° and 90° are almost equivalent; the values in the wave heading 180° are also larger. It means that structural response in oblique sea is almost as large as that caused in beam sea, so the splitting moment and transverse torsional moment are the main load components for structure details 2 and 3. At the same time, following sea can also cause larger structural response.

4.3. Spectral Fatigue Analysis Results and Discussion. In this paper, fatigue damage is calculated by using wave scatter data of World Wide Trade (shown in Figure 4). And the Pierson-Moskowitz spectrum is chosen as wave energy spectrum.

Meanwhile, it is usually assumed that the ship has equal probability of encountering waves from all directions in spectral fatigue analysis. Corresponding to simplified fatigue method, the time allocation ratios of 50% head sea and following sea, 25% beam sea and 25% oblique sea are used for calculating the fatigue damage. In this paper, 0° is defined as head sea, 180° is defined following sea, 90° and 270° are defined as beam sea, and others are defined as oblique sea. Fatigue damage calculation results of hot spots in all wave headings are shown as Figure 5. The proportion of spectral fatigue damage in head sea, beam sea, and oblique sea is summarized in Figure 6.

From Figure 5, we can clearly find that the fatigue cumulative damage values of hot spot 1 are larger in all wave headings than that of other hot spots. It indicates that the fatigue problem of the connection of the main hull and cross-deck at the front is serious. Meanwhile, the fatigue cumulative damage values of hot spots 2, 3, 4, and 5 in the oblique wave headings are larger than other wave headings, so the oblique sea contributes the majority of fatigue damage. Figure 6

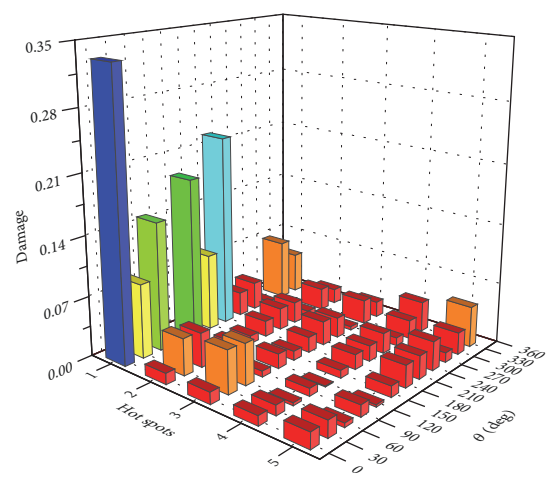


Figure 5: The spectral fatigue damage results.

shows that the fatigue damage caused in oblique seas are larger than beam sea and head sea for hot spots 2, 3, 4, and 5. Meanwhile, the fatigue damage is caused in oblique sea nearly as large as that caused in head sea for hot spot 1. Figure 6 also shows that the fatigue damage caused in beam sea is smaller than other sea conditions, especially for hot spot 1.

5. Simplified Fatigue Analysis for Trimaran

The simplified fatigue method is generally the first level procedure of the classification societies for fatigue assessment of ship structures. The analysis procedures have been established for conventional ships by the major classification societies. However, because the positions with severe fatigue problems are mainly located at trimaran cross-deck structure, application of this method for trimaran is complex since guidelines for fatigue loads, load cases, loading conditions,

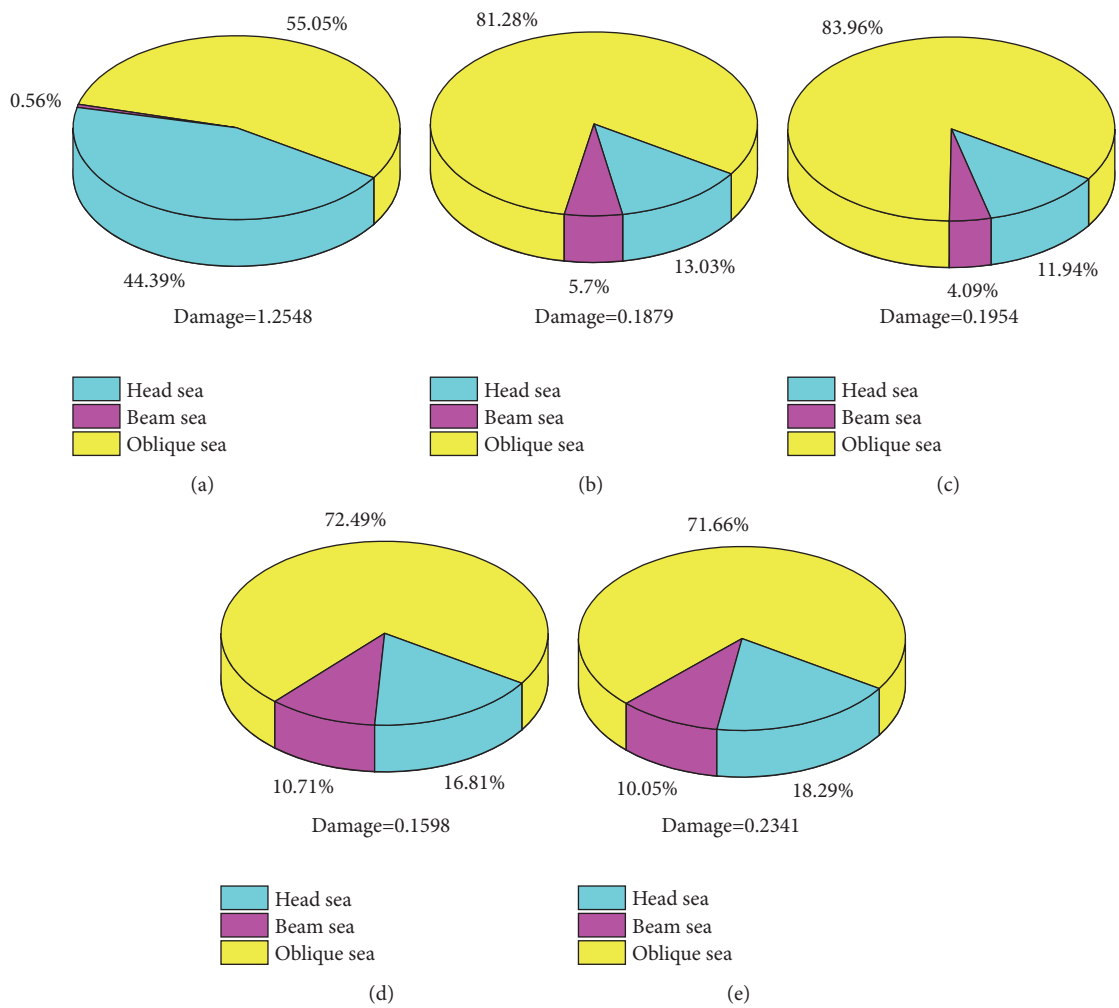


Figure 6: The proportion of spectral fatigue damage. (a) Hot spot 1. (b) Hot spot 2. (c) Hot spot 3. (d) Hot spot 4. (e) Hot spot 5.

and stress range evaluation are not available. The simplified fatigue method for trimaran in this study, the fatigue loads, and loading conditions are based on the manipulation of load combinations of the direct calculation procedure of LR Rules [20]. Stress ranges are computed by global FE analysis.

5.1. Calculation of Wave Load. The vertical wave bending moment M_w, at any position along the length of trimaran, is given by the following:

$$M_w = F_f \cdot D_f \cdot M_0 \tag{22}$$

where F_f is the correction factor of the hogging or sagging; D_f is the vertical bending moment distribution factor; M_0 is given by the following:

$$M_0 = 0.1 L_f f_{serv} L_R{}^2 B_{WL} (C_b + 0.7) \tag{23}$$

where L_f is the factor varying with length of trimaran; f_{serv} is the service group factor; L_R is the Rule length; B_{WL} is the breadth of water line; C_b is the block coefficient.

The horizontal bending moment M_h is calculated as follows:

$$M_h = D_f L_f f_{serv} L_R^2 D (C_b + 0.7) \tag{24}$$

where D is the depth of main hull.

The splitting moments, for hog, M_{sph}, and for sag, M_{sps}, are given by the following:

$$\begin{aligned} M_{sph-\mathrm{I}} &= 9.81 \mathrm{f}_{serv} W_{sh} (1 + \alpha_z) \left(y_{sh} - \frac{B_{mh}}{2} \right) \\ M_{sph-\mathrm{O}} &= 9.81 \mathrm{f}_{serv} W_{sh} (1 + \alpha_z) (y_{sh} - y_{\mathrm{O}}) \\ M_{sps-\mathrm{I}} &= 9.81 \mathrm{f}_{serv} \frac{(\Delta - 2\Delta_{sh})}{2} \alpha_z \left(y_{sh} - \frac{B_{mh}}{2} \right) \\ M_{sps-\mathrm{O}} &= 9.81 \mathrm{f}_{serv} \frac{(\Delta - 2\Delta_{sh})}{2} \alpha_z (y_{sh} - y_{\mathrm{O}}) \end{aligned} \tag{25}$$

where W_{sh} is the total weight of one side hull; α_Z is the vertical acceleration; B_{mh} is the main hull breadth; Δ is the total displacement of the side hulls and the main hull; Δ_{sh}

Table 2: Load combinations.

Wave direction	Load cases	Load components						
		M_{wh}	M_{ws}	M_h	M_{sph}	M_{sps}	M_{lt}	M_{tt}
Head seas	1	1.0	—	—	0.3	—	—	0.2
	2	—	1.0	—	—	0.3	—	0.2
Beam seas	3	0.1	—	—	1.0	—	0.2	—
	4	—	0.1	—	—	1.0	0.2	—
Oblique seas	5	—	—	0.3	0.4	—	1.0	0.3
	6	—	—	1.0	0.4	—	—	0.2
	7	—	0.2	0.2	0.6	—	—	1.0

is the total displacement of one side hull. Points I and O are referred to as LR Rules [21].

The longitudinal torsional moment M_{lt} is calculated as follows:

$$M_{lt} = 7.5\left(T_f f_{serv}\rho V_{sh} + V_{cd} + \frac{V_{msh}}{2} y_{cs} a_{heave}\right) \tag{26}$$

where T_f is the distribution coefficient along trimaran length; ρ is the sea water density; V_{sh} is the volume of one side hull; V_{cd} is the volume of the cross-deck structure; V_{mhs} is the volume of the main hull; y_{cs} is the transverse distance from centerline to the centre of area of a cross-section taken at mid length of the side hull; a_{heave} is the heave acceleration.

The transverse torsional moment M_{tt}, is uniform along the breadth of the cross-deck structure and is given by the following:

$$M_{tt} = 3.75 f_{serv}\rho\left(V_{sh} + V_{cd}\right) L_{sh} a_{heave} \tag{27}$$

where L_{sh} is the length of side hull.

5.2. Load Case. Fatigue analyses are carried out for the representative loading conditions according to the intended operation of trimaran. Fatigue loading conditions corresponding to LR Rules are summarized in Table 2 and categorized as three wave heading directions including head sea, beam sea, and oblique sea.

5.3. Evaluation of Stress Range. In simplified fatigue analysis of conventional ship, the stress range *Sri* required for the calculation of accumulated damage is usually calculated based on simple beam theory assumptions. This method is obviously not appropriate for trimaran cross-deck structure due to its unique configuration. Alternatively, for trimaran, the global FE analysis is employed to directly calculate stress values, and the stress range evaluation is based on the results of that. According to various deflection pattern of trimaran for different loading conditions, load cases 1 and 2 of head seas cause the longitudinal hogging and sagging, whereas load cases 3 and 4 of beam seas result in transverse hogging and sagging. Therefore the stress ranges for head seas condition and beam seas condition can be expressed as

$$S_{head} = \sigma_{LC1} - \sigma_{LC2} \tag{28a}$$

$$S_{beam} = \sigma_{LC3} - \sigma_{LC4} \tag{28b}$$

For oblique seas condition, the relationship $S = 2\sigma$ is assumed between stress range S and stress amplitude σ. Thus, the stress range for oblique seas condition can be calculated as follows:

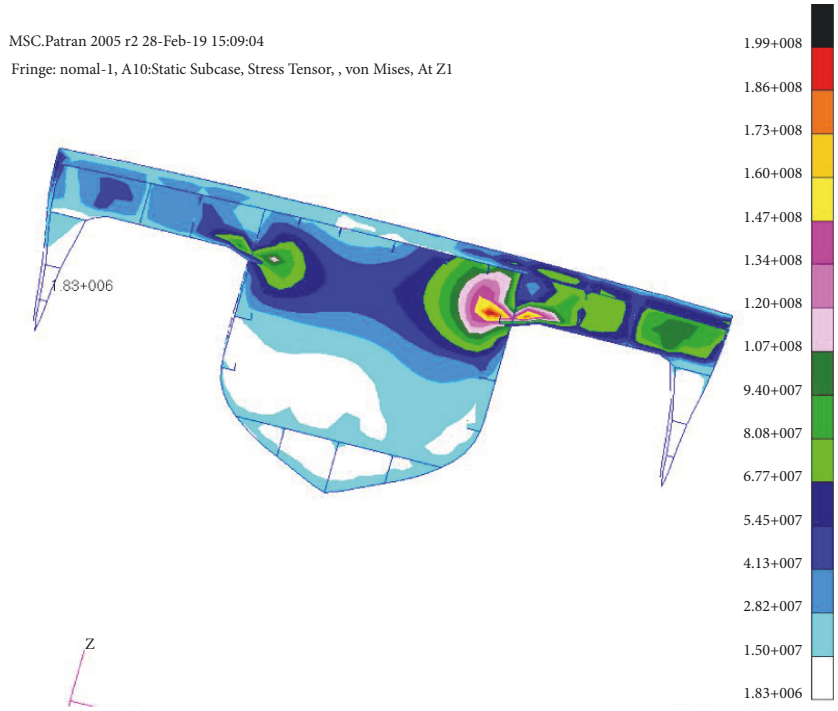
$$\begin{aligned} S_{oblique} &= \frac{1}{3}\left(S_{oblique-1} + S_{oblique-2} + S_{oblique-3}\right) \\ &= \frac{1}{3}\left(2\sigma_{LC5} + 2\sigma_{LC6} + 2\sigma_{LC7}\right) \end{aligned} \tag{29}$$

5.4. Simplified Fatigue Analysis Results and Discussion. A global FE analysis of trimaran in the seven load cases above is done. To limit the length of the paper, only the results of structural details 2 and 3 in load case 4 are given in Figure 7. The shape parameter h of the Weibull distribution is calculated by using the method given in Section 2.2. The short-term distribution functions of stress range are obtained by using the spectral method. And the parameters used in wave load calculation are the same as those for spectral fatigue analysis, which has given in Section 4.1. In this paper, 45 sample data (3-25MPa with 0.5MPa increments) are selected for fitting. The fitting results are shown in Figure 8.

Figure 7 shows that the connections of the main hull and wet-deck in the cross-deck structure have serious stress concentration. From Figure 8, it can be seen that the data points are distributed near the fitting lines for all the five hot spots. The correlation coefficients are all close to 1; there is a strong linear correlation. The values of the Weibull distribution's shape parameter h are all near to 1 for these hot spots. But for hot spots 2 and 4, the values are larger.

The fatigue strength of trimaran cross-deck structure's hot spots, which are illustrated at Section 3, is evaluated by simplified fatigue method to demonstrate its application. The fatigue damage calculation results of hot spots in all wave headings are shown as Figure 9. The proportion of fatigue damage in head seas, beam seas, and oblique seas is summarized in Figure 10.

From Figures 9 and 10, we can see that the fatigue damage value of hot spot 1 is larger than other hot spots, which indicates that the connection of the main hull and cross-deck at the front has serious fatigue problem. The same conclusion can be obtained from spectral fatigue analysis result. Figure 9 shows that the fatigue damage values of hot spot 1 in the head seas and load case 6 of oblique seas are larger than other sea conditions. It can be inferred that the vertical and horizontal wave bending moment are the main loads for hot



(a)

(b)

Figure 7: The max principal stress contour in load case 4. (a) Detail2. (b) Detail3.

spot 1, which cause the majority of the fatigue damage. From Figure 10, we can also find that beam seas nearly do not cause fatigue damage; head seas cause the majority of fatigue damage for hot spot 1. But, for hot spots 2, 3, 4, and 5, the fatigue damage caused in oblique seas is larger than head seas and beam seas. Meanwhile, the fatigue damage values caused in beam seas are also larger, which is mainly caused by the splitting moments.

5.5. Further Discussion. From the spectral and simplified analysis calculation results, it is found that the connection of the main hull and cross-deck at the front suffers maximum

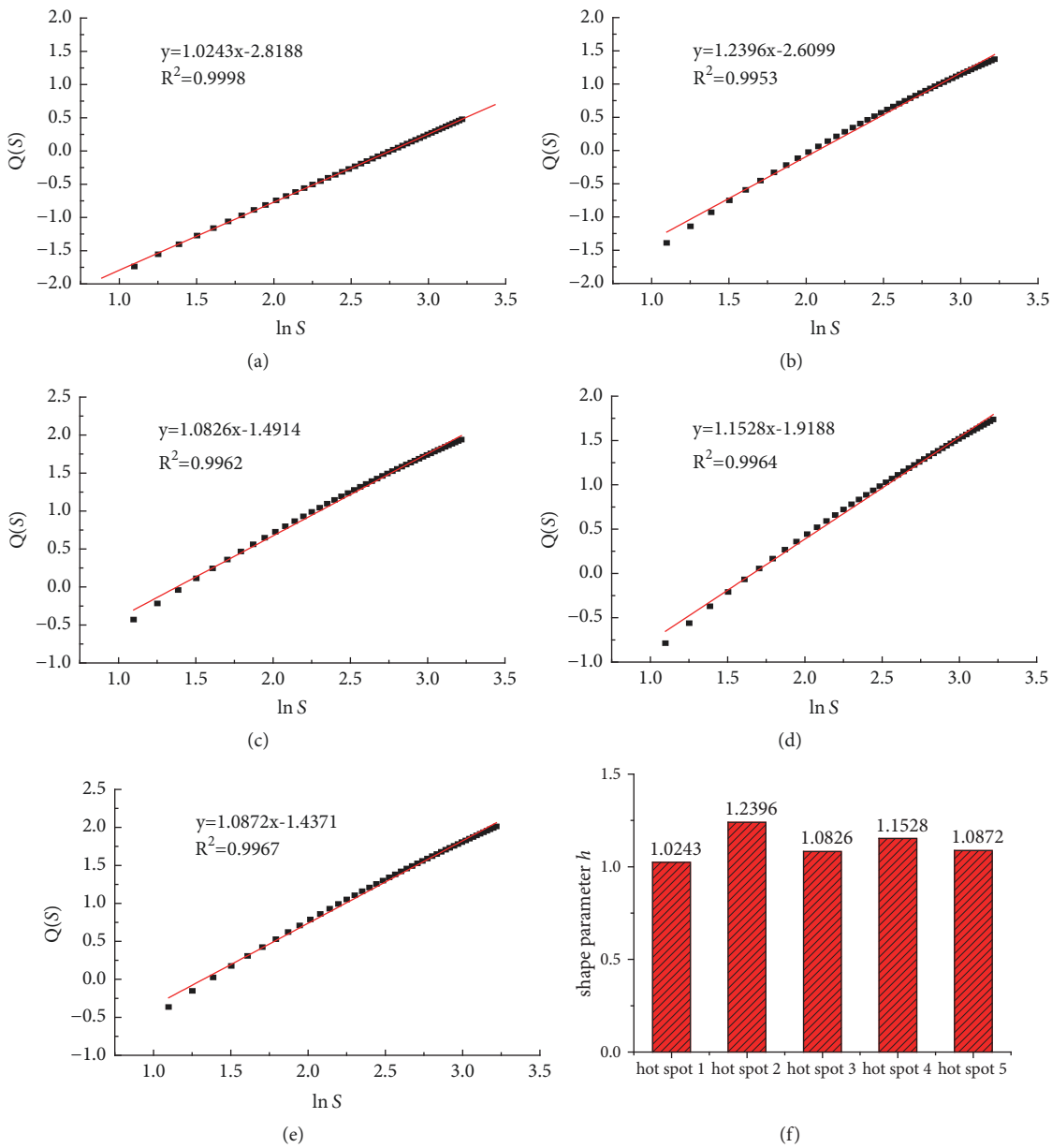


Figure 8: The fitting results of shape parameter. (a) Hot spot 1. (b) Hot spot 2. (c) Hot spot 3. (d) Hot spot 4. (e) Hot spot 5. (f) Values of hot spots' shape parameter.

fatigue damage for trimaran. The fatigue damage results in three wave headings obtained from both spectral and simplified fatigue method show that the cumulative damage from the beam sea can be neglected for hot spot 1. The cumulative damage values from the oblique sea are so large in all hot spots. It can be inferred that severe loads from head sea and oblique sea are the major factor of trimaran cross-deck structure's fatigue. Moreover, the predicted fatigue damage value of hot spot 1 exceeded the allowance, which indicates the requirement of structure strengthening and optimization at this location.

6. Conclusion

In this paper, the methodology of spectral and simplified fatigue analysis is adopted to investigate the fatigue characteristic of trimaran cross-deck structure. The following conclusions can be drawn based on the above study.

A methodology to perform simplified fatigue analysis of trimaran cross-deck structure is presented. In this simplified fatigue method, the total stress ranges are obtained by combining the stress ranges under different sea conditions. Due to the unique structure, they are achieved by global FE analysis based on direct calculation procedure.

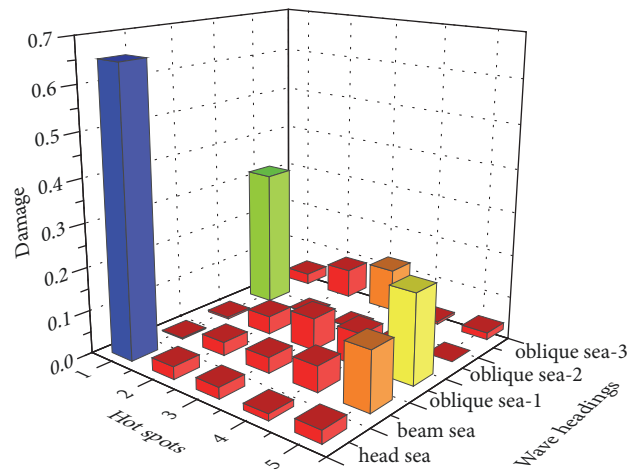


Figure 9: The simplified fatigue damage results.

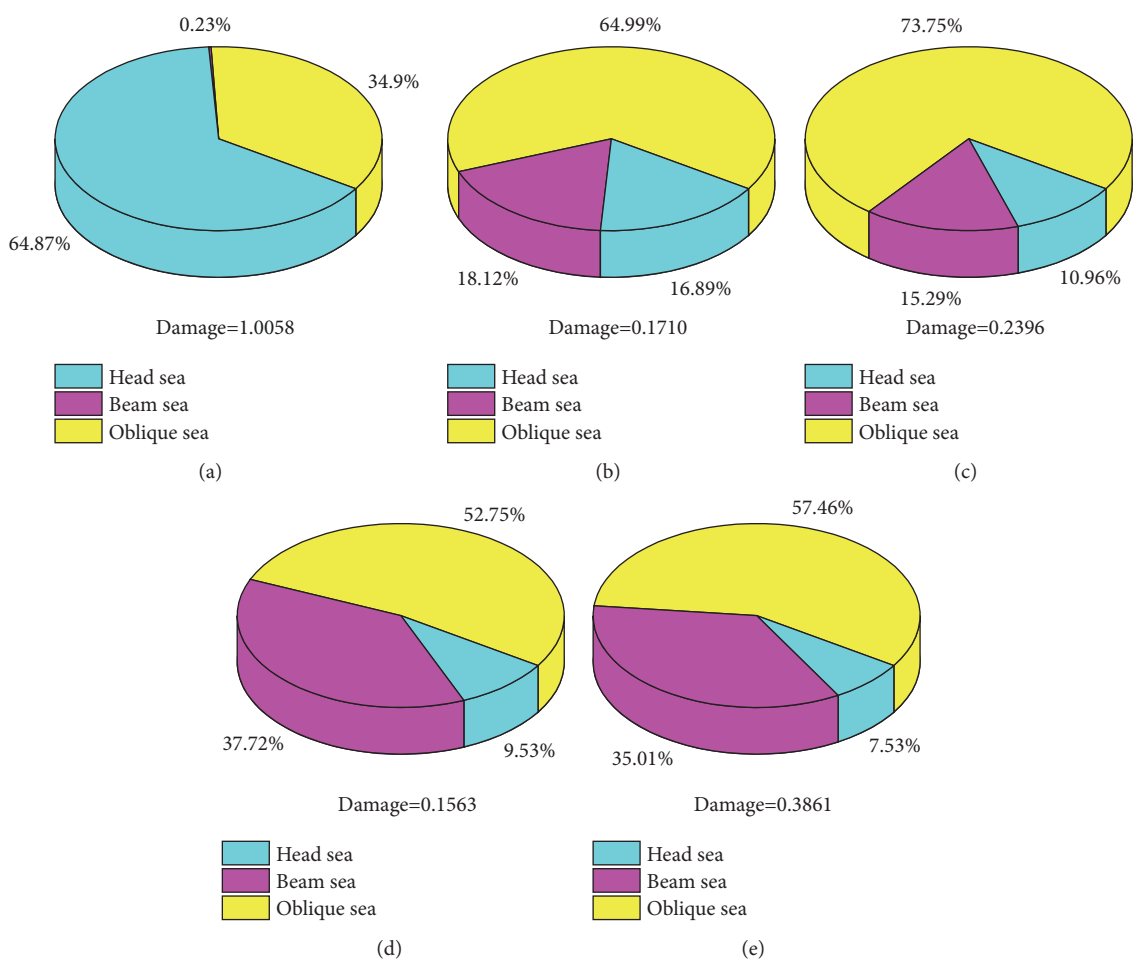


Figure 10: The proportion of simplified fatigue damage. (a) Hot spot 1. (b) Hot spot 2. (c) Hot spot 3. (d) Hot spot 4. (e) Hot spot 5.

The spectral and simplified fatigue analysis results give the basically same fatigue characteristics of trimaran cross-deck structure details. The fatigue analysis methodology of trimaran cross-deck structure presented in this paper is validated. Furthermore, the connection of the main hull and cross-deck at the front is the most dangerous fatigue position; it must attract sufficient attention. Meanwhile, the cross-deck structure's fatigue damage is mainly caused from head sea and oblique sea.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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