

Research Article

Research on Multiobjective Topology Optimization of Diesel Engine Cylinder Block Based on Analytic Hierarchy Process

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There are alternating impact loads for the diesel engine cylinder block. The topology optimization of the extreme single-working condition cannot guarantee its overall mechanical performance, and the traditional multiworking condition optimization has the problem that the weight coefficients are difficult to determine. Thus, a multiobjective topology optimization method based on analytic hierarchy process is proposed. Firstly, the static, dynamic characteristics and structure efficiency are calculated by the finite element analysis which indicates the direction of topology optimization for the cylinder block. The hierarchical structure model of topology optimization, including 12 weighting coefficients, is constructed considering static multiworking condition stiffness and dynamic multiorder natural frequency. The comprehensive evaluation function for the cylinder block is established by the compromise programming method and the weight coefficients are determined based on analytic hierarchy process. The optimization mathematical model is established and the multiobjective topology optimization of the cylinder block is carried out. The optimization results show that the proposed method can take into account structural multiworking condition performance, which has obvious advantages over the single objective topology optimization. The simulation results show that the static and dynamic characteristics are improved to some extent and the overall mechanical performance of the new model is more uniform with a 5.22% reduction in weight. It shows that the topology structure of the cylinder block is more reasonable.

1. Introduction

With the rapid and sustained development of automobile manufacturing industry all over the world, automobile ownership has increased greatly and the energy and environment issues are becoming more and more prominent. The energy conservation and emission reduction have become an inevitable trend in the development of automobile industry. The diesel engine, as one of the core components in engineering vehicles, is developing towards high-power-density, high-speed, and lightweight [1]. The cylinder block is the main structure and the heaviest part of the diesel engine; it must have sufficient stiffness and strength to support a variety of loads. At present, the design and optimization for the cylinder block mainly adopt traditional method combining finite element analysis (FEA) with engineering experience to check its strength and stiffness [2, 3]. The method is heavy and cumbersome, and it is difficult to effectively play structural bearing capacity [4].

The topology optimization method can provide lightweight and efficient structure form in the conceptual design stage, which has been widely concerned [5–7]. The single objective topology optimization for a V-type twelve-cylinder diesel cylinder block is carried out in [8] and the structural performance is improved. Jia et al. [9] get the optimal topology structure of a single cylinder block in the extreme working condition by using the topology and shape optimization. To achieve a low vibration design for a four-cylinder block, Du et al. [10] obtained the layout of the inner ribs by the topology optimization. Thus, the application of topology optimization for the cylinder block has made some progress and the research mainly focuses on the extreme working condition [11, 12]. However, there are alternating impact loads in the working process



FIGURE 1: The block diagram of multiobjective topology optimization.

of diesel engine. If the explosion of each cylinder for a multicylinder block is regarded as an extreme condition, the topology optimization of the cylinder block belongs to the typical multiworking condition problem. The traditional single objective optimization usually only ensures that the mechanical properties are optimal in a certain working condition while the overall mechanical property may be reduced to a lower value in other working conditions; that is to say, the topology optimization result for the cylinder block will oscillate between different working conditions and the overall mechanical property cannot be guaranteed. In addition, the dynamic characteristics of the cylinder block also need to be considered in the process of optimization.

The multiobjective topology optimization can consider simultaneously several objective functions in the design process [13-15] and the optimal solution can be obtained for each objective function. The intelligent algorithms are used to solve directly to avoid decision of multiobjective weight coefficients [16-18]. However, the calculation for complex structures will cost a lot of time and high economic costs because of numerical instability during the process of topology optimization [19, 20]. Therefore, it is necessary to establish a comprehensive evaluation function to consider several objectives as a whole. But if the weight coefficient of each working condition is decided by the engineering experience, the function will not reflect the overall structural performance in optimization. So the method of determining weight coefficients is the key of the multiobjective topology optimization and whose essence is the multicriterion decision-making problem.

The analytic hierarchy process (AHP) proposed by Saaty [21, 22] is a systematic analysis method for determining

qualitatively and quantitatively the relative importance of a set of activities in a multicriteria decision-making problem. The method can effectively analyze the nonsequential relationship between multiobjective criterion systems by combining mathematical processing with subjective judgment, which has been widely used in the field of resource system analysis, economic management, education management, social science, and so on [23, 24]. The AHP is applied to determinate the weight coefficients of the external economic evaluation model to ensure that the wind power engineering project is constructed and developed in a scientific manner [25]. A multiobjective evolutionary structure optimization method is proposed by combining the AHP and evolutionary structural optimization, which improves the optimization effect [26]. Therefore, it has obvious advantages to bring AHP into the decision of weight coefficients for the multiobjective topology optimization.

Under the above background, this paper presents a multiobjective topology optimization method based on AHP which is applied to a certain four-cylinder diesel engine cylinder block.

2. Multiobjective Topology Optimization Method

The multiobjective topology optimization method of diesel engine cylinder block based on AHP in this paper is mainly divided into four steps, as shown in Figure 1. The first is to introduce the structural geometry characteristic and working condition of the cylinder block in Section 3. Secondly, the topology optimization space is determined on



TABLE 1: The mechanical property of HT300.

FIGURE 2: The cylinder block structure.

the basis of analyzing the static characteristics, vibration mode, and structure efficiency of each working condition in Section 4. Then in Section 5, the hierarchical structure model of topology optimization is constructed considering the static multiworking condition stiffness and dynamic multiorder natural frequency. The comprehensive evaluation function is established by the compromise programming method which can more accurately evaluate the structural overall performance. The weight coefficients are determined by AHP and the mathematical model is established. Finally, multiobjective topology optimization of cylinder block is carried out and the optimization effect is verified in Section 6.

3. Structure Analysis

3.1. Structure Feature. The four-cylinder diesel engine cylinder block, as shown in Figure 2, is a box-type structure obtained by casting and machining and widely used in the heavy engineering vehicle. In order to achieve the lightweight, the topology structure of the cylinder block has been modified many times through finite element analysis and manual experience, but the structure is still too cumbersome and unsatisfactory. Its dimensions are 526.7mm long, 326.1mm wide, and 387.8mm high with a weight of 88.97 kg. The material is gray cast iron HT300 and the mechanical property is shown in Table 1.

As the main structure of the diesel engine, it is covered with various stiffening ribs, convex plates, bearing holes, oil channel holes, water-cooled jacket, and so on. So its mechanical property is directly related to the working efficiency of the diesel engine and it has to possess sufficient strength and stiffness to support a variety of loads.

3.2. Working Condition. For the diesel engine cylinder block, its working condition is a cyclic process including four processes of intake, compression, power, and exhaust. The firing order of cylinder block is 1-3-4-2 and the rotation speed of crank is 3000rpm. Therefore, there are alternating and high-speed impact loads for the cylinder block, and the loads are

very complicated, including the explosion pressure, the wall pressure from crank-link mechanism, the bolt pretightening force between cylinder block and cylinder head, the reaction force of bearing block and thermal load, etc. It is considered that the heat generated at the moment of gas explosion is first transmitted to the cylinder liner, and then to the cylinder wall, the cylinder liner and the water-cooled jacket bear a large amount of heat during the heat transfer process. In order to simplify calculation in this paper, the thermal load on the cylinder wall is ignored. So, the main loads considered are shown in Figure 3.

And the freedom constraints are applied to the six contact faces (a-f) at the bottom of the cylinder block as shown in Table 2, where T_x , T_y , and T_z mean that the displacements of x, y, and z direction are limited, R_x , R_y , and R_z mean that the rotation angles of x, y, and z direction are limited. According to the basic parameters of the cylinder block, the corresponding extreme load values at the moment of each cylinder explosion are calculated, as shown in Table 3. The bolt pretightening force is different in different position of the bolt hole and the number of bolt holes is a great many; only the maximum bolt pretightening force is listed.

4. Optimization Space Analysis

4.1. Static Characteristics Analysis. In order to obtain the topology optimization space, the static and dynamic characteristics of the cylinder block during the working process are obtained by the FEA. The first is to carry out the static characteristics analysis at the moment of each cylinder explosion.

The geometry model is imported into the finite element software, and the bolt hole, chamfer, and oil pipeline are simplified. According to the working condition of the cylinder block in Section 3.2, the finite element model, consisting of the tetrahedral and hexahedral mixing elements, is established as shown in Figure 4. The displacement and stress distribution are calculated and the results are shown in Table 4. It can be seen that the first working condition is

Constraint face	а	Ь	С	d	е	f
Displacement freedoms	$T_{\rm y}$	$T_{\rm z}$	$T_{\rm y}$	$T_{\rm x}$ / $T_{\rm y}$	T_{z}	$T_{\rm x}$ / $T_{\rm y}$
Rotation freedoms		$R_{\rm x}$ / $R_{\rm y}$ / $R_{\rm z}$				

TABLE 2: The constraints of the cylinder block.

TABLE 3: The extreme load value of each cylinder at the time of explosion.

The extreme Loads		No. of e	xplosion	
The extreme Loads	1	2	3	4
Bolt pre-tightening force /N	69007	67586	69474	75693
Reaction force of bearing block/N	63750	63336	47300	63752
The wall pressure /N	17695	17695	17695	17695
The explosion pressure /MPa	17	17	17	17



FIGURE 3: The loads and boundary conditions of cylinder block.



FIGURE 4: The finite element model.

the worst and corresponding displacement and stress distribution cloud charts are shown in Figure 5. The maximum stress is 217.9MPa located at the bolt hole while most of the rest region is about 80 MPa, which is much smaller than the material ultimate strength (300 MPa). It indicates that the cylinder block has optimization space in the worst condition.

4.2. Modal Analysis. The static analysis can only reflect structural stiffness and strength and cannot reflect its vibration performance. Modal analysis is the basis for the dynamic design, analysis, and optimization in modern mechanical products. The structural natural frequencies and vibration modes can be obtained by the modal analysis to evaluate its vibration characteristics.

No. of explosions	Max stress(MPa)	Max displacement (mm)
l	217.9	0.254
2	168.8	0.223
3	168.4	0.214
4	199.2	0.246

TABLE 4: Results of FEA for the cylinder block.

The constrained modal of the cylinder block is analyzed by the FEA and the top 6-order natural frequencies and corresponding vibration modes are shown in Figure 6 and

4

Orders	Frequency	Vibration mode
1	264Hz	First-order torsional vibration around the X axis
2	493 Hz	First-order bending vibration around the Z axis
3	531 Hz	Second-order torsional vibration around the X axis
4	562 Hz	The skirt vibrates with torsion along the X direction
5	778 Hz	Whole bending torsional vibration
6	1038 Hz	Whole torsional vibrating around the X axis

TABLE 5: The top 6-order natural frequencies and vibration modes.



FIGURE 5: Results of FEA under extreme working condition.

Table 5. It can be seen from Figure 6 that the cylinder block firstly appears whole torsional vibration while the whole bending vibration appears in the higher frequency range, which show that the torsional stiffness is less than the bending stiffness for the cylinder block. In addition, the relative displacement near the four corners is large and it is necessary to improve the freedom constraints to lower the extent of the vibration.

To further evaluate its dynamic performance, the working frequency is calculated by (1). The cylinder block studied in this paper is a four-stroke reciprocating piston engine, the crankshaft turns twice, and the cylinder body completes a working cycle, including four times vibration of intake, compression, power, and exhaust. So the corresponding working frequency f is 100Hz calculated, which is much smaller than the first-order natural frequency for the cylinder block. It indicates that the resonance will not occur in working.

$$f = \frac{2 \cdot n}{60} \tag{1}$$

where *n* is the rotation speed of crank, *n*=3000rpm.

4.3. Structure Efficiency Analysis. Structure efficiency [27] refers to the structural comprehensive characterization of the strength and stiffness per unit weight in the case of meeting the load-bearing property. It is commonly used to evaluate the structural overall performance. The greater structural

efficiency, the higher the material utilization, while the smaller the structural efficiency, the larger the optimization space.

In this paper, the structure efficiency of the cylinder block is calculated under four extreme working conditions. The calculation formula of the structure efficiency index η_i is shown in (2). In terms of the multiworking condition topology optimization, its physical meaning is as follows: the value is greater, indicating that the material utilization is higher and the working condition is worse. On the contrary, it shows that the working condition is safer and the optimization space is larger.

$$\eta_i = \frac{\sigma_{i\max} \cdot d_{i\max}}{m} \times 100\% \tag{2}$$

where η_i is the structure efficiency index under the *i*th working condition, $\sigma_{i \max}$ and $d_{i \max}$ are the maximum stress and maximum displacement under the *i*th working condition, and *m* is the structural weight.

Substituting the analysis results of Table 4 into (2), the structure efficiency of the cylinder block is calculated as shown in Figure 7. It can be seen that the cylinder block has the highest structure efficiency at the moment of the first cylinder explosion, and followed by the fourth cylinder, the second cylinder and the third cylinder. It shows that the first cylinder explosion is the worst working condition, and the third cylinder explosion is the safest condition.

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FIGURE 7: The structure efficiency of each cylinder.

Based on the analysis mentioned in Figures 5–7, the cylinder block studied in this paper can meet the stiffness and strength requirements under the worst working condition. The overall stress value (80 MPa) is much lower

than material ultimate strength (300 MPa), which indicates that the cylinder block has surplus material and topology optimization space. Its working frequency (100 Hz) is much lower than the first-order natural frequency (264 Hz), and the resonance does not occur. In addition, the importance for four working conditions is sorted: the first cylinder, the fourth cylinder, the second cylinder, and the third cylinder. Therefore, the first cylinder and the fourth cylinder should be focused when determining the weighting coefficients in multiobjective topology optimization. And the material near the second cylinder and the third cylinder should be considered when improving the topology structure.

5. Topology Optimization Mathematical Model Based on AHP

5.1. The Hierarchical Structure Model. The topology optimization for the diesel engine cylinder block belongs to the typical multiworking condition problem. It is necessary to



FIGURE 8: The hierarchical structure model of topology optimization.

take into account the structural performance requirements, including static and dynamic characteristics. For the static characteristics, structural stiffness has to be considered at the moment of each cylinder explosion. And the top 6-order natural frequencies need to be concerned for dynamic characteristics. Therefore, the hierarchical structure model of topology optimization for the cylinder block is established based on static multiworking condition stiffness and dynamic multiorder natural frequency as shown in Figure 8. It can be seen from the figure that the multiobjective topology optimization of the cylinder block includes 12 weighting coefficients that are static and dynamic topology optimization α_1, α_2 in the criterion layer, static multiworking condition stiffness $w_1 \sim w_4$, and dynamic multiorder natural frequency $w_5 \sim w_{10}$.

5.2. Comprehensive Evaluation of the Cylinder Block. The linear weighting method is usually used to transform the multiobjective problem into a single-objective problem for the traditional multiobjective topology optimization. However, the linear weighting method is to calculate weight average value for all functions and it cannot reflect the prominent influence from some certain functions, which does not guarantee that all functions obtain the relative optimal solution. The compromise programming method [28] can get a group of better relative optimal solutions by calculating the sensitivity of all functions to design variables and adjusting each objective to balance each other. From the hierarchical structure model shown in Figure 8, the topology optimization for the cylinder block includes ten optimization objectives, and the static and dynamic multiobjective optimization problem is converted into the single-objective optimization problem by the compromise programming method.

5.2.1. Static Multiworking Condition Stiffness. The topology optimization oriented by stiffness maximization is to research

material distribution form in the design domain to maximize the structural stiffness. In this paper, the static stiffness of the cylinder block under four extreme conditions is studied, which belongs to the multiworking condition stiffness problem. In this paper, the objective function of static multiworking condition stiffness is obtained by the compromise programming method as shown in (3). $C(\rho)$ is the comprehensive evaluation value of the static stiffness, and the smaller the value, the larger the structural overall stiffness.

$$\min_{\rho} C\left(\rho\right) = \left\{ \sum_{i=1}^{m} w_i^q \left[\frac{C_i\left(\rho\right) - C_i^{\min}}{C_i^{\max} - C_i^{\min}} \right]^q \right\}^{1/q}$$
(3)

where ρ is the relative density in the variable density topology optimization and *m* is the total number of working conditions, *m*=4. w_i is the weight coefficient of the *i*th working condition while *q* is the penalty coefficient ($q \ge 2$). $C_i(\rho)$ is the structural compliance of the *i*th working condition. C_i^{max} and C_i^{min} are the maximum and minimum compliance of the *i*th working condition, respectively.

5.2.2. Dynamic Multiorder Natural Frequency. The topology optimization of dynamic multiorder natural frequency is usually targeted at maximizing the low-order natural frequency, and the material remove ratio is taken as boundary. However, if only one low-order natural frequency is used as the optimization objective, the eigenvalues of other adjacent higher order natural frequency may be reduced because of the gradual material remove in the structure. It will result in the interchange of the low-order natural frequencies and the convergence of topology optimization will be influenced. The average frequency method [29] can consider simultaneously the multiorder natural frequency by defining a smooth objective function and improve the convergence, which is widely used in dynamic topology optimization. In this paper, the objective function of dynamic multiorder natural frequency is defined by the average frequency method as shown in (4). $\Lambda(\rho)$ is the comprehensive evaluation value of the top few order natural frequency and the larger the value, the larger the top few order natural frequency.

$$\max \Lambda(\rho) = \lambda_0 + s \left(\sum_{j=1}^n \frac{w_j}{\lambda_j - \lambda_0}\right)^{-1}$$
(4)

where ρ is the relative density in the variable density topology optimization. λ_j is the *j*th order natural frequency. λ_0 and *s* as given parameters are used to adjust the function value, usually $\lambda_0=0$, *s*=1. w_j is the weight coefficient of the *j*th order natural frequency while *n* is the order of low-order natural frequency that need to be optimized, *n*=6.

In addition, the low-order natural frequency is usually paid to attention during the optimization process and the lower the order, the higher the degree of attention. According to this principle, aiming at reducing the complexity of the weighting coefficients determined by the analytic hierarchy process, the weight coefficients $w_5 \sim w_{10}$ of the top 6 natural frequencies are taken as 0.3, 0.2, 0.2, 0.1, 0.1, and 0.1, respectively. So, the 12 unknown weighting coefficients in the hierarchical structure model are reduced to six.

5.2.3. Comprehensive Evaluation Function. The comprehensive evaluation function of multiobjective topology optimization, considering both the static multiworking condition stiffness and the dynamic multiorder natural frequency, is established by the compromise programming method as shown in (5). By adjusting the position of $C_i(\rho)$ and Λ_{ρ} in the function, the comprehensive evaluation function can uniformly guide the convergence direction of the optimization. And the smaller the value, the better the overall performance of the cylinder block.

$$\min F(\rho) = \left\{ \alpha_1^2 \left[\sum_{i=1}^m w_i \frac{C_i(\rho) - C_i^{\min}}{C_i^{\max} - C_i^{\min}} \right]^2 + \alpha_2^2 \left[\frac{\Lambda^{\max} - \Lambda(\rho)}{\Lambda^{\max} - \Lambda^{\min}} \right]^2 \right\}^{1/2}$$
(5)

where $F(\rho)$ is the objective function value and Λ^{\min} and Λ^{\max} represent minimum and maximum natural frequencies, respectively. Other variables have the same meaning as (3) and (4).

5.3. The Weighting Coefficients. The comprehensive evaluation function of multiobjective topology optimization, shown in (5), has six unknown weighting coefficients including $\alpha_1, \alpha_2, w_1 \sim w_4$. These unknown weighting coefficients are calculated based on the analytic hierarchy process in this paper. The concrete calculating flow chart is shown in Figure 9. The subjective judgment is scaled based on the measure theory and the judgment matrix is established.



FIGURE 9: The calculating flow chart of the weight coefficients.

Then the all weighting coefficients are calculated through the consistency check.

5.3.1. Criteria Layer Decision. There are static stiffness topology optimization and dynamic natural frequency topology optimization in the criterion layer, and the corresponding weighting factors are, respectively, α_1, α_2 . The cylinder block suffers from the alternating impact loads when different cylinder explodes and its stiffness performance directly affects the working reliability. But for the vibration characteristics, it can be seen from Section 4.2 that the maximum working frequency is 100 Hz, which is much smaller than the first-order natural frequency of 264 Hz. Therefore, the static multiworking condition stiffness is more important in the topology optimization for the cylinder block. So the weight coefficients α_1, α_2 are defined as 0.6 and 0.4, respectively.

5.3.2. Index Layer Decision. Firstly, it is necessary to determine the importance of four working conditions. According to the structure efficiency shown in Figure 7, the importance is sorted: the first cylinder, the fourth cylinder, the second cylinder, and the third cylinder. So the weight coefficients are ranked as shown in

$$w_1 > w_4 > w_2 > w_3$$
 (6)

TABLE 6: Meanings of relative scale.

Relative scale	Meanings
1	Two elements have equal importance
3	The former is slightly important than the latter between two elements
5	The former is obviously important than the latter between two elements
7	The former is strongly important than the latter between two elements
9	The former is extremely important than the latter between two elements
2, 4, 6, 8	Indicating the intermediate value above judgment
Reciprocal	If the important ratio between the elements <i>i</i> and <i>j</i> is <i>x</i> , the important ratio between the elements <i>j</i> and i is $1/x$.

Then, according to the standard meaning table of relative scale in the AHP shown in Table 6, the relative importance ratio of four working conditions is determined and the judgment matrix W is constructed as shown in

$$W = \begin{bmatrix} \frac{w_1}{w_1} & \cdots & \frac{w_1}{w_i} & \cdots & \frac{w_1}{w_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{w_j}{w_1} & \cdots & \frac{w_j}{w_i} & \cdots & \frac{w_j}{w_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{w_n}{w_1} & \cdots & \frac{w_n}{w_i} & \cdots & \frac{w_n}{w_n} \end{bmatrix}$$
(7)
$$= \begin{bmatrix} w_{11} & \cdots & w_{1i} & \cdots & w_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{j1} & \cdots & w_{ji} & \cdots & w_{jn} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{n1} & \cdots & w_{ni} & \cdots & w_{nn} \end{bmatrix}$$

where *n* is the number of the weight coefficients, w_i and w_j (*i*, j = 1, 2, ..., 6) represent the weight coefficients and $w_{ii} = w_i/w_i$ denotes the relative importance of w_i to w_i .

According to the results in Section 4, the first cylinder explosion is the worst condition and it is obviously more important than the third working condition and slightly more important than the fourth working condition, so the weight coefficients w_{13} , w_{14} are determined as 5 and 2, respectively. The importance of the second working condition is between the third working condition and the fourth working condition, so the weight coefficient w_{12} is defined as 4. In the same way, the relative importance ratio of the four working conditions is obtained and the judgment matrix is constructed:

$$W = \begin{bmatrix} 1 & 4 & 5 & 2 \\ \frac{1}{4} & 1 & 2 & \frac{1}{3} \\ \frac{1}{5} & \frac{1}{2} & 1 & \frac{1}{4} \\ \frac{1}{2} & 3 & 4 & 1 \end{bmatrix}.$$
 (8)

The judgment matrix *W* is right multiplied by a vector $\boldsymbol{\omega} = (w_1, w_2, w_3, w_4)^{\text{T}}$ consisting of all the weight coefficients, as shown in

$$W \boldsymbol{\omega} = \lambda \boldsymbol{\omega} \Longrightarrow$$

$$(W - \lambda I) \boldsymbol{\omega} = 0$$
(9)

Substituting the judgment matrix *W* into (9), the maximum eigenvalue $\lambda_{max} = 4.0484$ is calculated and the corresponding eigenvector normalized is $\boldsymbol{\omega} = (0.49, 0.12, 0.08, 0.31)^{T}$. So, all weight coefficients are obtained for multiobjective topology optimization of the cylinder block.

In order to ensure the accuracy and reliability of the judgment matrix and avoid influence of individual subjective factor, the consistency test of the judgment matrix is carried out in terms of (10). The consistency ratio *C.R.* of the judgment matrix *W*, calculated by (10), is 0.0179, which is less than 0.1. Therefore, it is considered that the judgment matrix has a satisfactory consistency and the four weight coefficients can well reflect the importance of each working condition.

$$C.R. = \frac{C.I.}{R.I.} \tag{10}$$

where *C.I.* is the consistency index, $C.I. = (\lambda_{\max} - n)/(n-1)$. *R.I.* is the mean random consistency index, whose value can be obtained directly by referring to the standard random consistency index *R.I.-n* table in the analytic hierarchy process, as shown in Table 7. *C.R.* is the random consistency ratio and the inconsistency is acceptable when *C.R.* < 0.1.

In addition, the computing platform of weight coefficient for multiworking condition topology optimization (TOWC) is built in *Matlab* to improve the computational efficiency of the method as shown in Figure 10. According to the number of working conditions and the importance of each working



TABLE 7: The standard random consistency index R.I.-n.



condition, the platform can automatically construct the judgment matrix, output the weight coefficients, and verify its consistency. Taking the cylinder block as an example, the operation steps are as follows.

Step 1. Enter the number of working conditions *n*=4.

Step 2. Rank the importance of each working condition [1, 4, 2, 3].

Step 3. Refer to Table 6, and enter the relative importance between two working conditions expressed in vector form. Before entering the vector, you can click the prompt button to get the number of elements you need to input. The elements in the vector are expressed in sequence as the importance of the first working condition to other working conditions, and the importance of the second working condition to other working conditions and so on. In this paper, six elements need to be input for four working conditions of cylinder block. Based on above analysis, the corresponding vector is [4, 5, 2, 2, 1/3, 1/4].

Step 4. Click the control button "calculating the Judgment Matrix" and the button "calculating the Weight Coefficients" in turn, the judgment matrix and weight coefficients are calculated, and the consistency is checked. If it is satisfied, the weight coefficients are output, or else, the relative importance between two working conditions needs to be modified in Step 3. Finally, for multiobjective topology optimization of the cylinder block, the weight coefficients (0.49, 0.12, 0.08, 0.31) are output.

5.4. Mathematical Model

(1) Objective. The main objective for the cylinder block is to improve the static and dynamic characteristics in the actual



FIGURE 11: Optimized region and non-optimized region.

working process. The comprehensive evaluation function can consider both the static multiworking condition stiffness and the dynamic multiorder natural frequency. Therefore, the comprehensive evaluation function shown in (5) is taken as the optimization objective in this paper.

(2) Design Variable. The classical variable density topology optimization is applied for the cylinder block, and the design variable is set to the relative density of each element in the optimized area. Since the cylinder wall is to cooperate with the cylinder liner and the cylinder head, it is regarded as a nonoptimized area. In addition, the other area connected with the fuel injection pump, supercharger, radiator, bearing block, etc. is also set as nonoptimized area. In Figure 11, the red region represents the nonoptimized region.

(3) Constraint Condition. In the process of topology optimization, it is necessary to ensure that the structure satisfies the equilibrium equation with the continuous material removal in local area. And the relative density of each element is controlled between 0 and 1. In addition, the maximum material remove rate of the cylinder block is set at 10%.

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FIGURE 12: Result of the multiobjective topology optimization.

Thus, the mathematical model of multiobjective topology optimization is established as shown in

Find
$$\rho = (\rho_1, \dots, \rho_n)$$

min $F(\rho) = \left\{ 0.6^2 \cdot \left[\sum_{i=1}^4 w_i \frac{C_i(\rho) - C_i^{\min}}{C_i^{\max} - C_i^{\min}} \right]^2 + 0.4^2 \left(\frac{\Lambda_{\max} - \Lambda(\rho)}{\Lambda_{\max} - \Lambda_{\min}} \right)^2 \right\}^{1/2}$
(11)
Subject to $\mathbf{K}(\rho) \mathbf{u} = \mathbf{P}$
 $V(\rho) \le 0.9 \cdot V_0$
 $0 \le \rho_{\min} \le \rho_i \le 1$

where $F(\rho)$ is the comprehensive evaluation function value. $\mathbf{K}(\rho)$ is the stiffness matrix of finite element model and it is the function of relative density ρ . **u** is the displacement vector and **P** is the force vector. $V(\rho)$ is the objective volume value and V_0 is the initial volume value. ρ_{\min} represents the minimum relative density in all elements and ρ_i is the relative density of *i*th element. Other variables have the same meaning as (3)~ (5).

6. Results and Discussion

6.1. Topology Optimization Result. The finite element model of the cylinder block is imported into the topology optimization software, and the load and boundary condition are the same as those in Section 3.2. The multiobjective topology optimization mathematical model established by (11) is used for the cylinder block and the result is shown in Figure 12, where the areas from blue to red mean that materials become more and more important. According to the result, the areas where materials can be removed are mainly concentrated on stiffening ribs, convex plates, the side edges, and the inner support plates of cylinder block. Refer to the result of stress analysis and modal analysis in Section 4, the new model is obtained as shown in Figure 13, where the partial area is removed, the thickness and height of the ribs are changed in some areas, and the lightening holes are added in the inner support plate. Its weight has been reduced from 88.97 kg to 84.33 kg, accounting for about 5.22%.

6.2. Comparing with Single Objective Topology Optimization. In order to verify the effectiveness of the multiobjective topology optimization proposed in this paper, the single objective topology optimization of four extreme working conditions for the cylinder block is studied, respectively. For the mathematical model, only the objective is replaced with the minimum structural compliance and other variables remain unchanged as shown in

$$\min c\left(\rho\right) = \mathbf{u}^{T}\mathbf{K}\mathbf{u} = \sum_{e=1}^{n} \left(\rho_{e}\right)^{p} \mathbf{u}_{e}^{T}\mathbf{K}_{e}\mathbf{u}_{e}$$
(12)



FIGURE 13: New model of the cylinder block.

where $c(\rho)$ is structural compliance, p is penalty factor, p > 1. \mathbf{u}_e , \mathbf{K}_e are the displacement vector and the element stiffness matrix corresponding to the *e*th element. Other variables have the same meaning as (11).

The results are shown in Figure 14. It shows that the material is removed in the vicinity of the fourth cylinder when the first cylinder explodes as shown in Figure 14(a). Similarly, the material is removed in the vicinity of the first cylinder when the fourth cylinder explodes as shown in Figure 14(b). So, it is very clear that the optimization result is different when different working condition is selected. That is to say, the topology optimization of single working condition usually only ensures that structural mechanical property reaches to optimal in the selected working conditions may be reduced to a lower level.

By comparing with the results, it is necessary to comprehensively consider all working conditions in topology optimization for the cylinder block. If the optimization result of single working condition in a certain working condition is accepted, the structural overall mechanical property may decrease sharply. Therefore, it shows that the method proposed in this paper has obvious advantages comparing with the single objective topology optimization.

6.3. Mechanical Properties Analysis of New Model. The finite element analysis for the new model is used to obtain its static and dynamic characteristics to verify the optimization effect. The calculation process is the same as Section 4 and the displacement and stress distribution are shown in Table 8, and the top 6-order nature frequencies are shown in Table 9.

According to Tables 4 and 8, the comparison of mechanical performance including displacement and stress is, respectively, shown in Figures 15 and 16. From the comparison, the overall stress and displacement of four working conditions keep the same level, and the maximum displacement and maximum stress are slightly reduced in the first and the

No. of explosions	Max stress (MPa)	Max displacement (mm)
1	210.4	0.251
2	177.2	0.202
3	178.3	0.219
4	185.5	0.245

TABLE 8: Results of FEA for the new model.

fourth working condition and others are slightly raised. The first working condition is still the worst and its displacement and stress distribution are shown in Figure 17. The distribution trend of displacement and stress is the same as the original model and the stress is about 80 MPa in most of region, which is much smaller than material ultimate strength (300 MPa). It indicates that the stiffness and strength of the cylinder block can meet working requirements. Comparing Table 5 with Table 9, the 1st natural frequency of new cylinder model is increased by 4 Hz and other order natural frequencies remain basically unchanged, which indicates that the vibration characteristics of the new model meet working requirements.

To reflect intuitively the comprehensive performance of the new model in the explosion of each cylinder, structure efficiency is calculated and the results are shown in Table 10. It can be seen from the table that the structure efficiency is increased in the first, second, and third working condition indicating that the material utilization rate becomes higher. The structure efficiency of the fourth working condition is decreased which indicates that the safety becomes higher. In addition, the variance of the original model and the new model are calculated, which are 0.0108 and 0.0079, respectively. It denotes that the mechanical performance of the new model is more uniform. In general, the topology structure of the cylinder block becomes more reasonable by the multiobjective topology optimization.



TABLE 9: The top 6-order modal analysis results for the new model.





FIGURE 15: Comparison of the displacement.

Structure efficiency	No. of explosions			
	1	2	3	4
Original model	62.21%	42.31%	40.51%	55.08%
New model	62.62%	42.45%	46.30%	53.89%
Variation	+0.41%	+0.14%	+5.79%	-1.19%

TABLE 10: Comparison of structure efficiency (MPa•mm•kg⁻¹).



FIGURE 16: Comparison of the stress.



FIGURE 17: Results of FEA for the new model under extreme working condition.

7. Conclusion

This paper proposes a multiobjective topology optimization method based on AHP. The comprehensive evaluation function for the cylinder block is established by the compromise programming method and the weight coefficients are determined based on AHP. The method is applied to the diesel engine cylinder block and several important conclusions are as follows:

(1) There are alternating impact loads for the diesel engine cylinder block. The traditional single-working condition topology optimization cannot guarantee its overall mechanical performance. The comprehensive evaluation function for the cylinder block is established by the compromise programming method, which can more accurately evaluate the structural performance.

(2) By constructing the hierarchical structure model of topology optimization including 12 weighting coefficients, the establishing process of the comprehensive evaluation function for cylinder block becomes more hierarchical and the determination of the weight coefficients has a theoretical guidance. The method is equally suitable for other multiobjective optimization.

(3) According to the simulation results, the overall structural performance of the cylinder block is improved with a 5.22% reduction in weight. Comparing the structure efficiency variances of the original model and the new model, it can be seen that the mechanical performance becomes more uniform under different conditions, which shows that the topology structure of the cylinder block is more reasonable.

Data Availability

The [DATA TYPE] data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no financial and personal relationships with other people or organizations that can inappropriately influence their work, and there is no professional or other personal interest of any nature or kind in any product, service, and/or company that could be construed as influencing the position presented in this manuscript.

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References

- W. Schoffmann, H. Sorger, F. Zieher et al., "Friction reduction and lightweight design-efficiency improvement at the passenger car base engine," *Internationaler Motorenkongress 2015: Mit Nutzfahrzeugmotoren–Spezial*, pp. 303–330, 2015.
- [2] B. Zheng, Q. L. Yong, R. X. Liu, and J. Meng, "Finite element analysis and structural improvement of diesel engine connecting rod," *Advanced Materials Research*, vol. 291-294, pp. 2413– 2416, 2011.
- [3] A. Ghasemi, "CAE simulations for cylinder block bore distortion," SAE Technical Paper, vol. 2012, no. 1, pp. 1320–1327, 2012.
- [4] X. F. Du, Z. J. Li, F. R. Bi, J. H. Zhang, X. Wang, and K. Shao, "Block design of diesel engine for low vibration level based on topology and shape optimization," *Journal of Mechanical Engineering*, vol. 48, no. 9, pp. 117–122, 2012.
- [5] O. Sigmund and K. Maute, "Topology optimization approaches," *Structural and Multidisciplinary Optimization*, vol. 48, no. 6, pp. 1031–1055, 2013.
- [6] Q. Zhao, X. Chen, Z.-D. Ma, and Y. Lin, "Reliabilitybased topology optimization using stochastic response surface method with sparse grid design," *Mathematical Problems in Engineering*, vol. 2015, Article ID 487686, 13 pages, 2015.
- [7] A. Boonpan and S. Bureerat, "Multi-stage design of an automotive component," *International Journal of Vehicle Design*, vol. 60, no. 1-2, pp. 84–99, 2012.
- [8] H.-S. Lee, Y.-S. Lee, J.-H. Kim, J.-T. Jun, J.-O. Lee, and C.-G. Kim, "A structural analysis and topology optimization on cylinder block of heavy duty diesel engin," *International Journal* of Modern Physics B, vol. 24, no. 15, pp. 2670–2675, 2010.
- [9] W.-X. Jia, Z.-Y. Hao, and H.-M. Xu, "Light-weight design of single cylinder block based on structure optimization," *Journal* of *Zhejiang University: Engineering Science*, vol. 42, no. 2, pp. 224–228, 2008.

- "Low vibration design of diesel engine body based on topology and shape optimization," *Journal of Mechanical Engineering*, vol. 48, no. 9, pp. 117–122, 2012.
- [11] Z.-Y. Hao, W.-X. Jia, and L. Guo, "Application of topology optimization to light-weight design of single cylinder engine," *Journal of Jiangsu University (Natural Science Edition)*, vol. 27, no. 4, pp. 306–309, 2006.
- [12] Y. C. Kang, X. M. Liu, and Y. S. Jiao, "Research on lightweight quantification of six-cylinder engine cylinder based on topology optimization," *Automobile Parts*, vol. 3, no. 6, pp. 80-81, 2011.
- [13] D. Guirguis, K. Hamza, M. Aly, H. Hegazi, and K. Saitou, "Multiobjective topology optimization of multi-component continuum structures via a Kriging-interpolated level set approach," *Structural and Multidisciplinary Optimization*, vol. 51, no. 3, pp. 733–748, 2014.
- [14] H. Li, L. Gao, and P. Li, "Topology optimization of structures under multiple loading cases with a new compliance-volume product," *Engineering Optimization*, vol. 46, no. 6, pp. 725–744, 2014.
- [15] S. Sleesongsom and S. Bureerat, "New conceptual design of aeroelastic wing structures by multi-objective optimization," *Engineering Optimization*, vol. 45, no. 1, pp. 107–122, 2013.
- [16] R. Balamurugan, C. V. Ramakrishnan, and N. Swaminathan, "A two phase approach based on skeleton convergence and geometric variables for topology optimization using genetic algorithm," *Structural and Multidisciplinary Optimization*, vol. 43, no. 3, pp. 381–404, 2011.
- [17] G.-C. Luh, C.-Y. Lin, and Y.-S. Lin, "A binary particle swarm optimization for continuum structural topology optimization," *Applied Soft Computing*, vol. 11, no. 2, pp. 2833–2844, 2011.
- [18] T. Kunakote and S. Bureerat, "Multi-objective topology optimization using evolutionary algorithms," *Engineering Optimization*, vol. 43, no. 5, pp. 541–557, 2011.
- [19] O. Sigmund and J. Petersson, "Numerical instabilities in topology optimization: a survey on procedures dealing with checkerboards, mesh-dependencies and local minima," *Journal* of Structural Optimization, vol. 16, no. 1, pp. 68–75, 1998.
- [20] O. Sigmund, "On the usefulness of non-gradient approaches in topology optimization," *Structural and Multidisciplinary Optimization*, vol. 43, no. 5, pp. 589–596, 2011.
- [21] T. L. Saaty, "Analytic hierarchy process," in *Mathematical Models* for Decision Support, pp. 109–121, 1980.
- [22] A. Darko, A. P. C. Chan, E. E. Ameyaw, E. K. Owusu, E. Pärn, and D. J. Edwards, "Review of application of analytic hierarchy process (AHP) in construction," *International Journal* of Construction Management, pp. 1–17, 2018.
- [23] C.-C. Shih, R. S. Horng, and S.-K. Lee, "Investigation of lab fire prevention management system of combining root cause analysis and analytic hierarchy process with event tree analysis," *Mathematical Problems in Engineering*, vol. 2016, Article ID 3161823, 12 pages, 2016.
- [24] L. Chai and T. Sun, "The design of LQG controller for active suspension based on analytic hierarchy process," *Mathematical Problems in Engineering*, vol. 2010, Article ID 701951, 19 pages, 2010.
- [25] H.-Z. Li and S. Guo, "External economies evaluation of wind power engineering project based on analytic hierarchy process and matter-element extension model," *Mathematical Problems in Engineering*, vol. 2013, Article ID 848901, 11 pages, 2013.

- [26] W. S. Fernandes, M. Greco, and V. S. Almeida, "Application of the smooth evolutionary structural optimization method combined with a multi-criteria decision procedure," *Engineering Structures*, vol. 143, pp. 40–51, 2017.
- [27] H. T. Cen, Structure Bionics Theory, Structure Bionics Design of Lightweight Parts and RP Process Validation, BeiHang University, Beijing, China, 2004.
- [28] E. Ballestero, "Utility functions: a compromise programming approach to specification and optimization," *Journal of Multi-Criteria Decision Analysis*, vol. 6, no. 1, pp. 11–16, 2015.
- [29] Z. D. Ma, N. Kikuchi, and H.-C. Cheng, "Topological design for vibrating structures," *Computer Methods in Applied Mechanics* and Engineering, vol. 121, no. 1–4, pp. 259–280, 1995.





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