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Research Article

H_∞ Guaranteed Cost Fault-Tolerant Control of Double-Fault Networked Control Systems: Piecewise Delay Method

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The term double-fault networked control system means that sensor faults and actuator faults may occur simultaneously in networked control systems. The issues of modelling and an H_∞ guaranteed cost fault-tolerant control in a piecewise delay method for double-fault networked control systems are investigated. The time-varying properties of sensor faults and actuator faults are modelled as two time-varying and bounded parameters. Based on the linear matrix inequality (LMI) approach, an H_∞ guaranteed cost fault-tolerant controller in a piecewise delay method is proposed to guarantee the reliability and stability for the double-fault networked control systems. Simulations are included to demonstrate the theoretical results of the proposed method.

1. Introduction

Networked control systems (NCS) are frequently encountered in many fields of applications due to their suitable and flexible structure [1–9]. However, in practical NCS, there inevitably exists time delay and data packet dropouts because of the introduction of the communication network [10–13]. Sensor faults and actuator faults also occur easily at the device level because of its large scale and complicated structure [14], which can have a negative impact on the system, such as performance decline and instability. Thus, guaranteed cost and fault-tolerant control of networked control systems has become a new popular issue in the network control field.

To achieve stability requirements concerning sensor faults or actuator faults, fault-tolerant control has been investigated in many works [15–26]. Based on the Lyapunov stability theorem, a methodology for the design of fault-tolerant control systems for chemical plants with distributed interconnected processing units was presented by N. H. El-Farra and A. P. D. Gani [15]. Z. Qu and C. M. Ihlefeld devised a fault-tolerant robust controller for a class of nonlinear uncertain systems considering possible sensor faults and developed a robust measure to identify the stability- and performance-vulnerable failures [16]. Based on the integrity control theory, a robust fault-tolerant controller for NCS with actuator faults was discussed by Y. N. Guo [17]. The diagnosis of actuator component faults and fault-tolerant control for a class of networked control systems using adaptive observer techniques was investigated in [18]. A switched model based on probability for NCS was proposed in [19] to research issues of fault-tolerant control when actuators age or become partially disabled.

Recently, guaranteed cost control that can guarantee the stability of a system and make it meet a certain performance indicator has become popular in NCS [4–7, 27–29]. Stability guaranteed active fault-tolerant control against actuators failures in NCS was addressed by S. Li [27]. X. Y. Luo and M. J. Shang proposed the so-called guaranteed cost active fault-tolerant controller (AFTC) strategy in [28]. The issue of guaranteed cost reliable control with regional pole constraint against actuator failures was investigated by H. M. Soliman in [6]. In [7], X. Li investigated the issue of integrity against actuator faults for NCS under variable-period sampling, in which the existence conditions of a guaranteed cost fault-tolerant control law was tested in terms of the Lyapunov stability theory. The resilient reliable dissipativity performance index for systems including actuator faults and probabilistic time delay signals is investigated by authors in [30, 31]. Unfortunately, all the previously mentioned literature investigated the guaranteed cost fault-tolerant control for NCS of single-faults (just considering the condition that only actuator faults occur or sensor faults occur). Few articles examine the double-fault issue. In practical application, it is easy for the sensors and actuators to become faulty simultaneously when the NCS works in a poor environment and is affected by external disturbance. Improving the control performance of NCS when double-faults occur is important. This motivates us to investigate H_∞ guaranteed cost fault-tolerant control of double-fault NCS, which is a necessary but challenge task.

This paper develops a H_∞ guaranteed cost fault-tolerant control for a double-fault NCS to guarantee its stability. The time-varying properties of sensor faults and actuator faults are modelled as two time-varying and bounded parameter matrices, and the networked control system is built as a linear closed-loop system with transmission delays and data packet dropouts. Different from [30, 31], here it is necessary to deal with two dynamic matrices. One of them is located at the left hand side of the gain matrix, and the other one is located at the right hand side of the gain matrix. This brings challenges to searching a feasible control gain. To solve such problem, a piecewise delay method is proposed to analyse the delay-dependent faulty system for reducing the conservatism. The delay falling in each subinterval is treated as a case. For different cases, different weighted technology is used to derive the H_∞ guaranteed cost fault-tolerant condition, and the controller parameter for this NCS is obtained by solving several sets of LMIs. Compared with [26, 32–34], the delay considered in this paper can be continuously changed with time.

Notation. R^n denotes the n-dimensional Euclidean space. The superscript "T" stands for matrix transposition. The notation $X > 0$ means that the matrix X is a real positive definite matrix. I is the identity matrix of appropriate dimensions. $\left[\begin{smallmatrix} X & Z \\ * & Y \end{smallmatrix}\right]$ denotes a symmetric matrix, where$*$denotes the entries implied by symmetry.

2. Modelling of Double-Fault NCS

The linear control plant of NCS with uncertain parameters and external disturbance can be expressed as

$$\begin{aligned} x'(t) &= (A+\Delta A)\,x(t) + (B+\Delta B)\,u(t) + H\omega(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^r$, and $\omega(t) \in L_2[0,\infty) \in R^n$ represent state value, input, output, and external disturbance, respectively. Separately, A, B, H, and C are matrices with appropriate dimensions; ΔA and ΔB are matrices with uncertain time-varying parameters, satisfying $[\Delta A, \Delta B] = DF(t)[E_1, E_2]$; $F(t)$ is an unknown matrix function with Legesgue measurable properties, satisfying $F^T(t)F(t) \le I$; and D, E_1, and E_2 are constant matrices with appropriate dimensions.

For the convenience of the later formulation, two concepts can be initially introduced.

Definition 1. The sensor faults and actuator faults could occur simultaneously, that is, there may be two types of faults at one time; this kind of fault is called a *double-fault*.

Definition 2. The sensor faults and actuator faults do not occur simultaneously, that is, there is only one type of fault at one time; this kind of fault is called a *single-fault*.

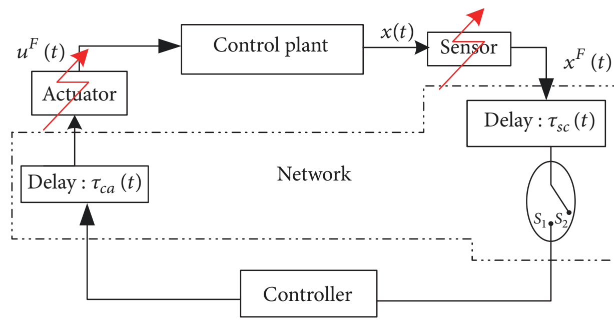


FIGURE 1: The structure of double-fault NCS.

The structure of a double-fault NCS is shown in Figure 1. Transmission delays induced by the network are the sensor-to-controller delay τ_{sc} and the controller-to-actuator delay τ_{ca}. These two delays can be combined when the feedback controller is static. The state of the system is assumed to be completely measurable. A piecewise continuous feedback controller, which is realized by a zero-order hold (ZOH), is employed:

$$u(t) = K\tilde{x}(t-\tau_{ca}), \quad t \in [t_k, t_{k+1}), \; k = 1, 2, \cdots \tag{2}$$

where K is the state feedback gain matrix to be designed and t_k is the sampling instant.

Considering that sensor faults may occur, $x_i^F(t)$ is used to represent the data from the *i-th* sensor. In this paper, we consider faults that include outage and loss of effectiveness. If the *i-th* sensor is an outage, the corresponding sampling work is interrupted, and the sampling data keeps the default value $x_i^F(t) = 0$. If the *i-th* sensor loses effectiveness, the sampling data is inaccurate and nonzero. We denote the sensor fault model as

$$x_i^F(t) = g_i(t)\,x_i(t), \quad i = 1, 2, \cdots, n \tag{3}$$

where $g_i(t)$ is the time-varying sensor efficiency factor, $g_i = 1$ represents that the *i-th* sensor is normal, $g_i = 0$ represents that its fault is outage, and $0 < g_i < 1$ or $g_i > 1$ represents that its fault is loss of effectiveness. The upper bound of sensor efficiency factor $g_i(t)$ is denoted by a constant g_{ui} satisfying

$g_{ui} > 1$, while its lower bound is denoted by a constant g_{li} satisfying $0 \leq g_{li} < 1$.

Denoting $x^F(t) = \left[x_1^F(t) \;\; x_2^F(t) \;\; \cdots \;\; x_n^F(t)\right]^T$, we have

$$x^F(t) = G(t)\,x(t) \tag{4}$$

where $G(t) = \operatorname{diag}(g_1(t), g_2(t), \cdots, g_n(t))$ is the sensor fault indicator matrix. Correspondingly, its upper bound matrix is $G_u = \operatorname{diag}(g_{u1}, g_{u2}, \cdots, g_{un})$, and its lower bound matrix is $G_l = \operatorname{diag}(g_{l1}, g_{l2}, \cdots, g_{ln})$.

Data packet dropouts in NCS are also unavoidable because of limited bandwidth. Considering that data packet dropouts may occur, the network is modelled as a switch. When the switch is located in S_1 position, the data packet containing $x(t_k)$ is transmitted, and the controller utilizes the updated data. When it is located in the S_2 position, the data packet dropouts occur, and the controller uses the old data. For a fixed sampling period h, the dynamics of the switch can be expressed as follows:

The NCS with no packet dropout at time t_k:

$$\tilde{x}(t) = G(t_k - \tau_{sc})\,x(t_k - \tau_{sc}) \tag{5}$$

The NCS with one packet dropout at time t_k:

$$\tilde{x}(t) = G(t_k - \tau_{sc} - h)\,x(t_k - \tau_{sc} - h) \tag{6}$$

The NCS with $d_k \in Z^+$ packet dropout at time t_k:

$$\tilde{x}(t) = G(t_k - \tau_{sc} - d_k h)\,x(t_k - \tau_{sc} - d_k h). \tag{7}$$

Because the feedback controller is static, (2) can be expressed as

$$\begin{aligned} &u(t) \\ &= KG(t_k - \tau_{ca} - \tau_{sc} - d_k h)\,x(t_k - \tau_{ca} - \tau_{sc} - d_k h) \end{aligned} \tag{8}$$

Considering that actuator faults may also occur, $u_j^F(t)$ is used to represent the signal from the *j-th* actuator. Similarly, we denote the actuator fault model as

$$u_i^F(t) = \delta_j(t)\,u_j(t), \quad j = 1, 2, \cdots, m \tag{9}$$

where $\delta_j(t)$ is the time-varying actuator efficiency factor, $\delta_j = 1$ represents that the *j-th* actuator is normal, $\delta_j = 0$ represents that its fault is an outage, and $0 < \delta_j < 1$ or $\delta_j > 1$ denotes that its fault is a loss of effectiveness. The upper bound of sensor efficiency factor $\delta_j(t)$ is denoted by a constant δ_{uj} satisfying $\delta_{uj} > 1$, while its lower bound is denoted by a constant δ_{lj} satisfying $0 \leq \delta_{lj} < 1$.

Denoting the faulty control signal $u^F(t) = [u_1^F(t) \;\; u_2^F(t) \;\; \cdots \;\; u_m^F(t)\,]^T$, we can obtain the fault-tolerant control law as

$$\begin{aligned} u^F(t) &= \Theta(t)\,u(t) = \Theta(t)\,KG(t_k - \tau_{ca} - \tau_{sc} - d_k h) \\ &\quad \cdot x(t_k - \tau_{ca} - \tau_{sc} - d_k h) \end{aligned} \tag{10}$$

where $\Theta(t) = \operatorname{diag}(\delta_1(t), \delta_2(t), \cdots, \delta_m(t))$ is the actuator fault indicator matrix. Correspondingly, its upper bound matrix is $\Theta_u = \operatorname{diag}(\delta_{u1}, \delta_{u2}, \cdots, \delta_{um})$, and its lower bound matrix is $\Theta_l = \operatorname{diag}(\delta_{l1}, \delta_{l2}, \cdots, \delta_{lm})$.

Let $\eta(t) = t - t_k + \tau_{ca} + \tau_{sc} + d_k h$; $t \in [t_k, t_{k+1})$; (10) can now be expressed as follows:

$$u^F(t) = \Theta(t)\,u(t) = \Theta(t)\,KG(t - \eta(t))\,x(t_k - \eta(t)) \tag{11}$$

Obviously, the delay part $\eta(t)$ may vary with time t, and it satisfies

$$\begin{aligned} &\eta_m \leq \eta(t) = t - t_k + \tau_{ca} + \tau_{sc} + d_k h \leq \eta_M \\ &\qquad t \in [t_k, t_{k+1}) \end{aligned} \tag{12}$$

From the upper bounds G_u, Θ_u of fault indicator matrices and lower bounds G_l, Θ_l of fault indicator matrices, the nonsingular mean-value matrices are separately obtained as

$$\begin{aligned} G_0 &= \operatorname{diag}(g_{01}, g_{02}, \cdots, g_{0n}), \quad g_{0i} = \frac{g_{ui} + g_{li}}{2} \\ \Theta_0 &= \operatorname{diag}(\delta_{01}, \delta_{02}, \cdots, \delta_{0m}), \quad \delta_{0j} = \frac{\delta_{uj} + \delta_{lj}}{2} \end{aligned} \tag{13}$$

Moreover, the following two time-varying matrices are introduced:

$$\begin{aligned} &L(t - \eta(t)) \\ &\quad = \operatorname{diag}(l_1(t - \eta(t)), l_2(t - \eta(t)), \cdots, l_n(t - \eta(t))), \\ &l_i(\eta(t)) = \frac{g_i(\eta(t)) - g_{0i}}{g_{0i}} \\ &\Gamma(t) = \operatorname{diag}(\lambda_1(t), \lambda_2(t), \cdots, \lambda_m(t)), \\ &\lambda_i(t) = \frac{\delta_j(t) - \delta_{0j}}{\delta_{0j}} \end{aligned} \tag{14}$$

Obviously, we have

$$\begin{aligned} -1 &\leq \frac{g_{li} - g_{0i}}{g_{0i}} \leq l_i(t - \eta(t)) = \frac{g_i - g_{0i}}{g_{0i}} \leq \frac{g_{ui} - g_{0i}}{g_{0i}} \\ &= \frac{g_{ui} - g_{li}}{g_{ui} + g_{li}} \leq 1 \end{aligned} \tag{15}$$

Similarly, we have

$$-1 \leq \lambda_j(t) \leq 1 \tag{16}$$

In expressions from (13) to (16), i and j meet $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$. Based on (15) and (16), we have

$$\begin{aligned} -I_{n\times n} &\leq L(t - \eta(t)) \leq I_{n\times n} \\ -I_{m\times m} &\leq \Gamma(t) \leq I_{m\times m} \end{aligned} \tag{17}$$

From on (14), the following can be obtained:

$$\begin{aligned} g_i(t - \eta(t)) &= g_{0i}(1 + l_i(t - \eta(t))), \quad i = 1, 2 \cdots, n; \\ \delta_i(t) &= \delta_{0i}(1 + \lambda_i(t)), \quad j = 1, 2 \cdots, m. \end{aligned} \tag{18}$$

Naturally, the time-varying fault indicator matrices can be rewritten as

$$G(t-\eta(t)) = G_0(I + L(t-\eta(t))) \qquad \Theta(t) = \Theta_0(I+\Gamma(t)) \tag{19}$$

Inserting (19) into (11), we have

$$u(t) = \Theta_0(I+\Gamma(t))KG_0(I+L(t-\eta(t)))x(t-\eta(t)) \tag{20}$$

Then, the new model of double-fault NCS can be obtained as follows:

$$\begin{aligned} x'(t) &= (A+\Delta A)x(t) + (B+\Delta B)\Theta_0(I+\Gamma(t)) \\ &\quad \cdot KG_0(I+L(t-\eta(t)))x(t-\eta(t)) + H\omega(t) \\ y(t) &= Cx(t) \end{aligned} \tag{21}$$

Remark 3. The networked control systems in faulty case can be modelled as system (21) with the effects of time-varying delay $\eta(t)$ whose upper bound and lower bound are described in (12). Unlike the previous models, [17, 21, 27, 35], this model is related to both sensor faults and actuator faults, the faults are time-varying, which are reflected by the time-varying parameters $L(t-\eta(t))$ and $\Gamma(t)$. From (17), we undoubtedly know the time-varying parameter $L(t-\eta(t))$ satisfies $L^TL = L^2 \le I$, while the parameter $\Gamma(t)$ satisfies $\Gamma^T\Gamma = \Gamma^2 \le I$.

Remark 4. From the descriptions of faults matrices, we undoubtedly know if $l_i = 1/g_{0i} - 1$ $(i = 1, 2\cdots, n)$, we can obtain $g_i = 1$ and $G_0(I + L(t-\eta(t))) = I$, and then model (21) is an actuator fault model. Similarly, if $\lambda_i = (\delta_j - \delta_{0j})/\delta_{0j}$ $(j = 1, 2\cdots, m)$, we can obtain $\delta_j = 1$ and $\Theta_0(I+\Gamma(t)) = I$, and then model (21) represents a sensor fault model. Therefore, model (21) of a double-fault NCS contains cases of single-faults, and the single-faults are a special form of double-faults.

In the following section, a fundamental preliminary result is presented to guarantee the performance of a double-fault NCS based on the delay information.

3. Performance Analysis of Double-Fault NCS

For the system model (21) established in Section 2, the cost function is given as follows:

$$\begin{aligned} J &= \int_0^\infty \left[x^T(t)S_1x(t) + u^T(t)S_2u(t)\right]d_t \\ &= \int_0^\infty \{x^T(t)S_1x(t) \\ &\quad + \left[\Theta_0(I+\Gamma)KG_0(I+L)x(t-\eta(t))\right]^T \\ &\quad \cdot S_2\Theta_0(I+\Gamma)KG_0(I+L)x(t-\eta(t))\} d_t \end{aligned} \tag{22}$$

where S_1 and S_2 are symmetric positive definite matrices.

Definition 5. For model (21) and its cost function (22), if there exists a control gain matrix K satisfying the conditions

(1) the closed-loop system is asymptotically stable when $\omega(t) = 0$;

(2) for any zero initial condition and any nonzero vector $\omega(t) \in L_2[0,\infty)$, given $\gamma > 0$, the output $y(t)$ satisfies $\|y(t)\|_2 \le \gamma\|\omega(t)\|_2$;

(3) there exists a constant J_0, and the cost function defined as (22) satisfies $J_\infty \le J_0$,

then matrix K is the H_∞ guaranteed cost control gain of double-faults NCS.

To analyse the stability of the system expediently, the following lemmas are introduced.

Lemma 6 (see [36, 37]). *For any matrices W, M, N, $F(t)$ with $F^TF \le I$, and any scalar $\varepsilon > 0$, the inequality holds as*

$$\begin{aligned} &W + MF(t)N + N^TF^T(t)M^T \\ &\quad \le W + \varepsilon MM^T + \varepsilon^{-1}N^TN \end{aligned} \tag{23}$$

Lemma 7 (see [38]). *If $\eta_1 \le \eta(t) \le \eta_2$, for any matrices Π_1, Π_2, and Φ, the following inequalities are equivalent:*

(1) $[\eta(t)-\eta_1]\Pi_1 + [\eta_2 - \eta(t)]\Pi_2 + \Phi < 0$;

(2) $[\eta_2 - \eta_1]\Pi_1 + \Phi < 0$, $[\eta_2 - \eta_1]\Pi_2 + \Phi < 0$.

The fundamental preliminary result is presented in the following theorem.

Theorem 8. *Given symmetric positive definite matrices S_1 and S_2, a set of constant η_m, η_M, $\rho_1 > 0$, $\rho_2 > 0$, and $\alpha = (\eta_M - \eta_m)/2$. If there exists a set of symmetric positive definite matrices R_i $(i = 1, 2, 3)$ and matrix $P > 0$, as well as matrices $M_{l\beta}$, $M_{2\beta}$, $N_{l\beta}$, $N_{2\beta}$, U_β $(\beta = 1, 2, 3, 4, 5, 6, 7, 8)$, K, and a set of constants, $\varepsilon > 0$ and $\gamma > 0$, satisfying the LMIs*

$$\begin{bmatrix} -\varepsilon I & 0 & 0 & 0 & \varepsilon\widetilde{D}^T & 0 & 0 & 0 & 0 & 0 \\ * & -\left(S_1 + C^TC\right)^{-1} & 0 & 0 & \widetilde{I} & 0 & 0 & 0 & 0 & 0 \\ * & * & -S_2^{-1} & 0 & \widetilde{\Sigma} & 0 & \Theta_0KG & 0 & \Theta_0 & 0 \\ * & * & * & -\Gamma_l & \Omega^{lk} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \widetilde{\Phi}_l & \widetilde{E}^T & \widetilde{\Theta} & \widetilde{I}_0 & \Delta & \widetilde{K}^T \\ * & * & * & * & * & -\varepsilon I & E_2\Theta_0KG_0 & 0 & E_2\Theta_0 & 0 \\ * & * & * & * & * & * & -\rho_1U_5 & 0 & 0 & (KG_0)^T \\ * & * & * & * & * & * & * & -\rho_1^{-1}U_5^{-1} & 0 & 0 \\ * & * & * & * & * & * & * & * & -\rho_2 I & 0 \\ * & * & * & * & * & * & * & * & * & -\rho_2^{-1}I \end{bmatrix} < 0 \tag{24}$$

$$l = 1, 2; \; k = 1, 2$$

where

$$\widetilde{\Phi}_1 = \begin{bmatrix} -R_1 + U_1A + (U_1A)^T & R_1 + (U_2A)^T + M_{11} & (U_3A)^T - N_{11} & (U_4A)^T \\ * & -R_1 + M_{12} + M_{12}{}^T & M_{13}{}^T - N_{12} & M_{14}{}^T \\ * & * & -\dfrac{R_3}{\alpha} - N_{12} - N_{12}{}^T & \dfrac{R_3}{\alpha} - N_{14}{}^T \\ * & * & * & -\dfrac{R_3}{\alpha} \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \longrightarrow$$

$$\begin{bmatrix} P - U_1 + (U_5A)^T & \widetilde{\Psi}_1 + (U_6A)^T & (U_7A)^T + (U_8A)^T + U_1H & 0 \\ -U_2 + M_{15}{}^T & \widetilde{\Psi}_2 + M_{16}{}^T & U_2H + M_{17}{}^T + M_{18}{}^T & 0 \\ -U_3 - N_{15}{}^T & \widetilde{\Psi}_3 - N_{16}{}^T & U_3H - N_{17}{}^T - N_{18}{}^T & 0 \\ -U_4 & \widetilde{\Psi}_4 & U_4H & 0 \\ \eta_m{}^2R_1 + \alpha(R_2 + R_3) - U_5 - U_5{}^T & \widetilde{\Psi}_5 - U_6{}^T & U_5H - U_7{}^T - U_8{}^T & 0 \\ * & \widetilde{\Psi}_6 & \Xi & 0 \\ * & * & U_7H + U_8H + (U_7H)^T + (U_8H)^T & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\widetilde{\Phi}_2 = \begin{bmatrix} -R_1 + U_1A + (U_1A)^T & R_1 + (U_2A)^T & (U_3A)^T + M_{21} & (U_4A)^T - N_{21} \\ * & -R_1 - \dfrac{R_2}{\alpha} & \dfrac{R_2}{\alpha} + M_{22} & -N_{22} \\ * & * & -\dfrac{R_2}{\alpha} + M_{23} + M_{23}{}^T & M_{24}{}^T - N_{23} \\ * & * & * & -N_{24} - N_{24}{}^T \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \longrightarrow$$

$$\begin{bmatrix} P - U_1 + (U_5A)^T & \widetilde{\Psi}_1{}' + (U_6A)^T & (U_7A)^T + (U_8A)^T + U_1H & 0 \\ -U_2 & \widetilde{\Psi}_2{}' & U_2H & 0 \\ -U_3 + M_{25}{}^T & \widetilde{\Psi}_3{}' + M_{26}{}^T & U_3H + M_{27}{}^T + M_{28}{}^T & 0 \\ -U_4 - N_{25}{}^T & \widetilde{\Psi}_4{}' - N_{26}{}^T & U_4H - N_{27}{}^T - N_{28}{}^T & 0 \\ \eta_m{}^2R_1 + \alpha(R_2 + R_3) - U_5 - U_5{}^T & \widetilde{\Psi}_5{}' - U_6{}^T & U_5H - U_7{}^T - U_8{}^T & 0 \\ * & \widetilde{\Psi}_6{}' & \Xi' & 0 \\ * & * & U_7H + U_8H + (U_7H)^T + (U_8H)^T & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\Gamma_1 = \alpha R_2,$$

$$\Gamma_2 = \alpha R_3,$$

$$\Omega^{11} = \alpha M_1{}^T,$$

$$\Omega^{12} = \alpha N_1{}^T,$$
$$\Omega^{21} = \alpha M_2{}^T,$$
$$\Omega^{22} = \alpha N_2{}^T,$$
$$M_1{}^T = \left[M_{11}{}^T \;\; M_{12}{}^T \;\; M_{13}{}^T \;\; M_{14}{}^T \;\; M_{15}{}^T \;\; M_{16}{}^T \;\; M_{17}{}^T \;\; M_{18}{}^T\right];$$
$$M_2{}^T = \left[M_{21}{}^T \;\; M_{22}{}^T \;\; M_{23}{}^T \;\; M_{24}{}^T \;\; M_{25}{}^T \;\; M_{26}{}^T \;\; M_{27}{}^T \;\; M_{28}{}^T\right];$$
$$N_1{}^T = \left[N_{11}{}^T \;\; N_{12}{}^T \;\; N_{13}{}^T \;\; N_{14}{}^T \;\; N_{15}{}^T \;\; N_{16}{}^T \;\; N_{17}{}^T \;\; N_{18}{}^T\right];$$
$$N_2{}^T = \left[N_{21}{}^T \;\; N_{22}{}^T \;\; N_{23}{}^T \;\; N_{24}{}^T \;\; N_{25}{}^T \;\; N_{26}{}^T \;\; N_{27}{}^T \;\; N_{28}{}^T\right];$$
$$\Xi = (U_7 B\Theta_0 KG_0)^T + (U_8 B\Theta_0 KG_0)^T + U_6 H - M_{17}{}^T - M_{18}{}^T + N_{17}{}^T + N_{18}{}^T;$$
$$\widetilde{\Psi}_i = U_i B\Theta_0 KG_0 + N_{1i} - M_{1i}, \quad (i = 1, 2, 3, 4, 5);$$
$$\widetilde{\Psi}_6 = U_6 B\Theta_0 KG_0 + (U_6 B\Theta_0 KG_0)^T - M_{16} - M_{16}{}^T + N_{16} + N_{16}{}^T;$$
$$\Xi' = (U_7 B\Theta_0 KG_0)^T + (U_8 B\Theta_0 KG_0)^T + U_6 H - M_{27}{}^T - M_{28}{}^T + N_{27}{}^T + N_{28}{}^T;$$
$$\widetilde{\Psi}_i{}' = U_i B\Theta_0 KG_0 + N_{2i} - M_{2i}, \quad (i = 1, 2, 3, 4, 5);$$
$$\widetilde{\Psi}_6' = U_6 B\Theta_0 KG_0 + (U_6 B\Theta_0 KG_0)\, T - M_{26} - M_{26}{}^T + N_{26} + N_{26}{}^T;$$
$$\widetilde{D} = \left[(U_1 D)^T \;\; (U_2 D)^T \;\; (U_3 D)^T \;\; (U_4 D)^T \;\; (U_5 D)^T \;\; (U_6 D)^T \;\; (U_7 D)^T + (U_8 D)^T \;\; 0\right]^T;$$
$$\widetilde{E} = [E_1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; E_2\Theta_0 KG_0 \;\; 0 \;\; 0];$$
$$\widetilde{I}_0 = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; I \;\; 0 \;\; 0]^T;$$
$$\widetilde{I} = [I \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0];$$
$$\widetilde{\Theta} = \left[(U_1 B\Theta_0 KG_0)^T \;\; (U_2 B\Theta_0 KG_0)^T \;\; (U_3 B\Theta_0 KG_0)^T \;\; (U_4 B\Theta_0 KG_0)^T \;\; (U_5 B\Theta_0 KG_0)^T \;\; (U_6 B\Theta_0 KG_0)^T \;\; (U_7 BKG_0)^T + (U_8 BKG_0)^T \;\; 0\right]^T;$$
$$\Delta = \left[(U_1 B\Theta_0)^T \;\; (U_2 B\Theta_0)^T \;\; (U_3 B\Theta_0)^T \;\; (U_4 B\Theta_0)^T \;\; (U_5 B\Theta_0)^T \;\; (U_6 B\Theta_0)^T \;\; (U_7 B\Theta_0)^T + (U_8 B\Theta_0)^T \;\; 0\right]^T;$$
$$\widetilde{K} = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; KG_0 \;\; 0 \;\; 0];$$
$$\widetilde{\Sigma} = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \Theta_0 KG_0 \;\; 0 \;\; 0], \tag{25}$$

then model (21) is asymptotically stable with the H_∞ norm bound γ. In addition, the upper bound J_0 of cost function J is given as

$$\begin{aligned} J_0 = {} & x^T(0t)\, Px(0) + \eta_m \int_{-\eta_m}^{0}\int_{s}^{0} x'^T(\tau)\, R_1 x'(\tau)\, d_s d_\tau \\ & + \int_{-\eta_1}^{-\eta_m}\int_{s}^{0} x'^T(\tau)\, R_2 x'(\tau)\, d_s d_\tau \\ & + \int_{-\eta_M}^{-\eta_1}\int_{s}^{0} x'^T(\tau)\, R_3 x'(\tau)\, d_s d_\tau \end{aligned} \tag{26}$$

Proof. First, with the definition of $\alpha = (\eta_M - \eta_m)/2$ and $\eta_1 = \eta_m + \alpha$, the interval of delay is distributed into two subintervals as follows:

$$\eta(t) \in [\eta_m, \eta_M] = [\eta_m, \eta_1] \cup [\eta_1, \eta_M]. \tag{27}$$

Then, we consider the Lyapunov-Krasovskii functional as follows:

$$\begin{aligned} v(t) = {} & x^T(t)\, Px(t) \\ & + \eta_m \int_{t-\eta_m}^{t}\int_{s}^{t} x'^T(\tau)\, R_1 x'(\tau)\, d_s d_\tau \\ & + \int_{t-\eta_1}^{t-\eta_m}\int_{s}^{t} x'^T(\tau)\, R_2 x'(\tau)\, d_s d_\tau \\ & + \int_{t-\eta_M}^{t-\eta_1}\int_{s}^{t} x'^T(\tau)\, R_3 x'(\tau)\, d_s d_\tau \end{aligned} \tag{28}$$

where matrix P satisfies $P > 0$ and R_i $(i = 1, 2, 3)$ are symmetric positive definite matrices with appropriate dimensions. For the convenience of writing, we denote $L = L(t - \eta(t))$ and $\Gamma = \Gamma(t)$ in the following expressions.

Calculating the derivative of Lyapunov-Krasovskii function and based on (21), we have

$$\begin{aligned} v'(t) &= 2x^T(t)Px'(t) + \eta^2_m x'^T(t)R_1x'(t) \\ &\quad - \eta_m \int_{t-\eta_m}^{t} x'^T(s)R_1x'(s)\,d_s + (\eta_1-\eta_m)x'^T(t) \\ &\quad \cdot R_2x'(t) - \int_{t-\eta_1}^{t-\eta_m} x'^T(s)R_2x'(s)\,d_s + (\eta_M-\eta_1) \\ &\quad \cdot x'^T(t)R_3x'(t) - \int_{t-\eta_M}^{t-\eta_1} x'^T(s)R_3x'(s)\,d_s \\ &\quad + 2\left[\varepsilon^T(t)\ \ \omega^T(t)\right]U\left[(A+\Delta A)x(t)\right. \\ &\quad + (B+\Delta B)\Theta_0(I+\Gamma)KG_0(I+L)x(t-\eta(t)) \\ &\quad \left. + H\omega(t) - x'(t)\right] \end{aligned} \tag{29}$$

where $v'(t) = \lim\sup_{\delta\longrightarrow 0^+}(1/\delta)[v(t+\delta)-v(t)]$ [39].

Based on Jessen's inequality, we have

$$\begin{aligned} -\eta_m \int_{t-\eta_m}^{t} x'^T(s)R_1x'(s)\,d_s &\le \left[x^T(t), x^T(t-\eta_m)\right] \\ &\quad \cdot \begin{bmatrix} -R_1 & R_1 \\ R_1 & -R_1 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\eta_m) \end{bmatrix} \end{aligned} \tag{30}$$

$$\begin{aligned} -\int_{t-\eta_1}^{t-\eta_m} x'^T(s)R_2x'(s)\,d_s &\le \frac{1}{\alpha}\left[x^T(t-\eta_m), x^T(t-\eta_1)\right] \\ &\quad \cdot \begin{bmatrix} -R_2 & R_2 \\ R_2 & -R_2 \end{bmatrix} \begin{bmatrix} x(t-\eta_m) \\ x(t-\eta_1) \end{bmatrix} \end{aligned} \tag{31}$$

$$\begin{aligned} -\int_{t-\eta_M}^{t-\eta_1} x'^T(s)R_3x'(s)\,d_s &\le \frac{1}{\alpha}\left[x^T(t-\eta_1), x^T(t-\eta_M)\right] \\ &\quad \cdot \begin{bmatrix} -R_3 & R_3 \\ R_3 & -R_3 \end{bmatrix} \begin{bmatrix} x(t-\eta_1) \\ x(t-\eta_M) \end{bmatrix} \end{aligned} \tag{32}$$

where $\alpha = (\eta_M - \eta_m)/2$. For the convenience of the following discussion, we define

$$\begin{aligned} \varepsilon^T(t) &= \left[x^T(t), x^T(t-\eta_m), x^T(t-\eta_1), x^T(t-\eta_M)\right. \\ &\quad \left. \cdot x'^T(t), x^T(t-\eta(t)), \omega^T(t)\right]; \end{aligned} \tag{33}$$

Case 1. If $\eta(t) \in [\eta_m, \eta_1]$, weighted technology based on the principle of Newton-Leibniz is introduced as follows:

$$\begin{aligned} &2\left[\varepsilon^T(t)\ \ \omega^T(t)\right]M_1\left[x(t-\eta_m) - x(t-\eta(t))\right. \\ &\quad \left. - \int_{t-\eta(t)}^{t-\eta_m} x'^T(s)\,d_s\right] = 0 \end{aligned} \tag{34}$$

and

$$\begin{aligned} &2\left[\varepsilon^T(t)\ \ \omega^T(t)\right]N_1\left[x(t-\eta(t)) - x(t-\eta_1)\right. \\ &\quad \left. - \int_{t-\eta_1}^{t-\eta(t)} x'^T(s)\,d_s\right] = 0 \end{aligned} \tag{35}$$

Because $R_2 > 0$, we have

$$\begin{aligned} &-2\left[\varepsilon^T(t)\ \ \omega^T(t)\right]M_1\int_{t-\eta(t)}^{t-\eta_m} x'(s)\,d_s \le (\eta(t)-\eta_m) \\ &\quad \cdot \left[\varepsilon^T(t)\ \ \omega^T(t)\right]M_1R_2^{-1}M_1{}^T\left[\varepsilon^T(t)\ \ \omega^T(t)\right]^T \\ &\quad + \int_{t-\eta(t)}^{t-\eta_m} x'^T(s)R_2x'(s)\,d_s \end{aligned} \tag{36}$$

and

$$\begin{aligned} &-2\left[\varepsilon^T(t)\ \ \omega^T(t)\right]N_1\int_{t-\eta_1}^{t-\eta(t)} x'(s)\,d_s \le (\eta_1-\eta(t)) \\ &\quad \cdot \left[\varepsilon^T(t)\ \ \omega^T(t)\right]N_1R_2^{-1}N_1{}^T\left[\varepsilon^T(t)\ \ \omega^T(t)\right]^T \\ &\quad + \int_{t-\eta_1}^{t-\eta(t)} x'^T(s)R_2x'(s)\,d_s \end{aligned} \tag{37}$$

Therefore,

$$\begin{aligned} &-2\left[\varepsilon^T(t)\ \ \omega^T(t)\right]M_1\int_{t-\eta(t)}^{t-\eta_m} x'(s)\,d_s \\ &\quad -2\left[\varepsilon^T(t)\ \ \omega^T(t)\right]N_1\int_{t-\eta_1}^{t-\eta(t)} x'(s)\,d_s \\ &\quad \le (\eta(t)-\eta_m) \\ &\quad \cdot \left[\varepsilon^T(t)\ \ \omega^T(t)\right]M_1R_2^{-1}M_1{}^T\left[\varepsilon^T(t)\ \ \omega^T(t)\right]^T \\ &\quad + (\eta_1-\eta(t)) \\ &\quad \cdot \left[\varepsilon^T(t)\ \ \omega^T(t)\right]N_1R_2^{-1}N_1{}^T\left[\varepsilon^T(t)\ \ \omega^T(t)\right]^T \\ &\quad + \int_{t-\eta_1}^{t-\eta_m} x'^T(s)R_2x'(s)\,d_s \end{aligned} \tag{38}$$

In addition, $y^T(t)y(t) - \gamma^2\omega^T(t)\omega(t)$ both on the left and right sides of equality (29) and $x^T(t)S_1x(t) + [\Theta_0(I+\Gamma)KG_0(I+L)x(t-\eta(t))]^TS_2\Theta_0(I+\Gamma)KG_0(I+L)x(t-\eta(t))$ on the right of the equal sign "=", and then inserting (30), (32), (34), (35), (38) to the obtained inequality, we have

$$\begin{aligned} &v'(t) + y^T(t)y(t) - \gamma^2\omega^T(t)\omega(t) \\ &\quad \le \left[\varepsilon^T(t)\ \ \omega^T(t)\right]\Phi_1\left[\varepsilon^T(t)\ \ \omega^T(t)\right]^T \\ &\quad + (\eta(t)-\eta_m) \\ &\quad \cdot \left[\varepsilon^T(t)\ \ \omega^T(t)\right]M_1R_2^{-1}M_1{}^T\left[\varepsilon^T(t)\ \ \omega^T(t)\right]^T \\ &\quad + (\eta_1-\eta(t)) \\ &\quad \cdot \left[\varepsilon^T(t)\ \ \omega^T(t)\right]N_1R_2^{-1}N_1{}^T\left[\varepsilon^T(t)\ \ \omega^T(t)\right]^T \end{aligned} \tag{39}$$

where

$$\Phi_1 = \begin{bmatrix} \Xi_1 & R_1 + [U_2(A+\Delta A)]^T + M_{11} & [U_3(A+\Delta A)]^T - N_{11} & [U_4(A+\Delta A)]^T & P - U_1 + [U_5(A+\Delta A)]^T & \Psi_1 + [U_6(A+\Delta A)]^T & [U_7(A+\Delta A)]^T + [U_8(A+\Delta A)]^T + U_1 H & 0 \\ * & -R_1 + M_{12} + M_{12}{}^T & M_{13}{}^T - N_{12} & M_{14}{}^T & -U_2 + M_{15}{}^T & \Psi_2 + M_{16}{}^T & U_2 H + M_{17}{}^T + M_{18}{}^T & 0 \\ * & * & -\frac{R_3}{\alpha} - N_{12} - N_{12}{}^T & \frac{R_3}{\alpha} - N_{14}{}^T & -U_3 - N_{15}{}^T & \Psi_3 - N_{16}{}^T & U_3 H - N_{17}{}^T - N_{18}{}^T & 0 \\ * & * & * & -\frac{R_3}{\alpha} & -U_4 & \Psi_4 & U_4 H & 0 \\ * & * & * & * & \eta_m{}^2 R_1 + \alpha(R_2 + R_3) - U_5 - U_5{}^T & \Psi_5 - U_6{}^T & U_5 H - U_7{}^T - U_8{}^T & 0 \\ * & * & * & * & * & \Psi_6 & \Xi_2 & 0 \\ * & * & * & * & * & * & U_7 H + U_8 H + (U_7 H)^T + (U_8 H)^T & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} \quad (40)$$

$$\Xi_1 = -R_1 + U_1(A+\Delta A) + [U_1(A+\Delta A)]^T + C^T C + S_1,$$

$$\Xi_2 = [U_7(B+\Delta B)\Theta_0(I+\Gamma)KG_0(I+L)]^T + [U_8(B+\Delta B)\Theta_0(I+\Gamma)KG_0(I+L)]^T + U_6 H - M_{17}{}^T - M_{18}{}^T + N_{17}{}^T + N_{18}{}^T,$$

$$\Psi_i = U_i(B+\Delta B)\Theta_0(I+\Gamma)KG_0(I+L) + N_{1i} - M_{1i}, \quad (i = 1,2,3,4,5);$$

$$\Psi_6 = U_6(B+\Delta B)\Theta_0(I+\Gamma)KG_0(I+L) + [U_6(B+\Delta B)\Theta_0(I+\Gamma)KG_0(I+L)]^T [\Theta_0(I+\Gamma)KG_0(I+L)]^T S_2 \Theta_0(I+\Gamma)KG_0(I+L) - M_{16} - M_{16}{}^T + N_{16} + N_{16}{}^T$$

if

$$\Phi_1 + (\eta(t) - \eta_m) M_1 R_2^{-1} M_1{}^T + (\eta_1 - \eta(t)) N_1 R_2^{-1} N_1{}^T < 0 \quad (41)$$

Next, we need to acquire the inequality (24) through a transformation based on inequality (41), which is equivalent to the following inequalities by applying the theory given in Lemma 7 and the Schur complement, also used by D. Yue [38] in the previous study:

$$\begin{bmatrix} -(\eta_1 - \eta_m)^{-1} R_2 & M_1{}^T \\ * & \Phi_1 \end{bmatrix} < 0 \quad (42)$$

$$\begin{bmatrix} -(\eta_1 - \eta_m)^{-1} R_2 & N_1{}^T \\ * & \Phi_1 \end{bmatrix} < 0 \quad (43)$$

Premultiplying and postmultiplying the inequalities above by $\mathrm{diag}((\eta_1 - \eta_m)I, I)$, we have

$$\begin{bmatrix} -\alpha R_2 & \Omega^{1k} \\ * & \Phi_1 \end{bmatrix} < 0 \quad (44)$$

Applying the theory of the Schur complement to inequality (44), we have

$$\begin{bmatrix} -(S_1 + C^T C)^{-1} & 0 & 0 & \widetilde{I} \\ * & -S_2{}^{-1} & 0 & \Sigma \\ * & * & -\alpha R_2 & \Omega^{1k} \\ * & * & * & \Phi_1' \end{bmatrix} < 0 \quad (45)$$

where

$$\Phi_1{}' = \begin{bmatrix} \Xi_1{}' & R_1 + [U_2(A+\Delta A)]^T + M_{11} & [U_3(A+\Delta A)]^T - N_{11} & [U_4(A+\Delta A)]^T \\ * & -R_1 + M_{12} + M_{12}{}^T & M_{13}{}^T - N_{12} & M_{14}{}^T \\ * & * & -\frac{R_3}{\alpha} - N_{12} - N_{12}{}^T & \frac{R_3}{\alpha} - N_{14}{}^T \\ * & * & * & -\frac{R_3}{\alpha} \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \longrightarrow$$

$$\begin{bmatrix} P - U_1 + [U_5(A+\Delta A)]^T & \Psi_1 + [U_6(A+\Delta A)]^T & [U_7(A+\Delta A)]^T + [U_8(A+\Delta A)]^T + U_1 H & 0 \\ -U_2 + M_{15}{}^T & \Psi_2 + M_{16}{}^T & U_2 H + M_{17}{}^T + M_{18}{}^T & 0 \\ -U_3 - N_{15}{}^T & \Psi_3 - N_{16}{}^T & U_3 H - N_{17}{}^T - N_{18}{}^T & 0 \\ -U_4 & \Psi_4 & U_4 H & 0 \\ \eta_m{}^2 R_1 + \alpha(R_2 + R_3) - U_5 - U_5{}^T & \Psi_5 - U_6{}^T & U_5 H - U_7{}^T - U_8{}^T & 0 \\ * & \Psi_6{}' & \Xi_2 & 0 \\ * & * & U_7 H + U_8 H + (U_7 H)^T + (U_8 H)^T & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} \quad (46)$$

$$\Xi_1{}' = -R_1 + U_1(A+\Delta A) + [U_1(A+\Delta A)]^T$$

$$\Psi_6{}' = U_6(B+\Delta B)\Theta_0(I+\Gamma)KG_0(I+L) + [U_6(B+\Delta B)\Theta_0(I+\Gamma)KG_0(I+L)]^T - M_{16} - M_{16}{}^T + N_{16} + N_{16}{}^T,$$

$$\Sigma = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \Theta_0(I+\Gamma)KG_0(I+L)\ \ 0\ \ 0].$$

Inequality (44) can be written as

$$\begin{bmatrix} -(S_1 + C^T C)^{-1} & 0 & 0 & \tilde{I} \\ * & -S_2{}^{-1} & 0 & \Sigma \\ * & * & -\alpha R_2 & \Omega^{1k} \\ * & * & * & \Phi_1'' \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \tilde{D} \end{bmatrix} F \begin{bmatrix} 0 \\ 0 \\ 0 \\ \hat{E}^T \end{bmatrix}^T + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \hat{E}^T \end{bmatrix} F^T \begin{bmatrix} 0 \\ 0 \\ 0 \\ \tilde{D} \end{bmatrix}^T < 0 \quad (47)$$

where

$$\Phi_1{}'' = \begin{bmatrix} -R_1 + U_1 A + (U_1 A)^T & R_1 + (U_2 A)^T + M_{11} & (U_3 A)^T - N_{11} & (U_4 A)^T \\ * & -R_1 + M_{12} + M_{12}{}^T & M_{13}{}^T - N_{12} & M_{14}{}^T \\ * & * & -\frac{R_3}{\alpha} - N_{12} - N_{12}{}^T & \frac{R_3}{\alpha} - N_{14}{}^T \\ * & * & * & -\frac{R_3}{\alpha} \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \longrightarrow$$

$$\begin{bmatrix} P-U_1+(U_5A)^T & \Psi_1''+(U_6A)^T & (U_7A)^T+(U_8A)^T+U_1H & 0 \\ -U_2+M_{15}^T & \Psi_2''+M_{16}^T & U_2H+M_{17}^T+M_{18}^T & 0 \\ -U_3-N_{15}^T & \Psi_3''-N_{16}^T & U_3H-N_{17}^T-N_{18}^T & 0 \\ -U_4 & \Psi_4' & U_4H & 0 \\ \eta_m^2R_1+\alpha(R_2+R_3)-U_5-U_5^T & \Psi_5''-U_6^T & U_5H-U_7^T-U_8^T & 0 \\ * & \Psi_6'' & \Xi_2'' & 0 \\ * & * & U_7H+U_8H+(U_7H)^T+(U_8H)^T & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\Xi_2''=[U_7B\Theta_0(I+\Gamma)KG_0(I+L)]^T+[U_8B\Theta_0(I+\Gamma)KG_0(I+L)]^T+U_6H-M_{17}^T-M_{18}^T+N_{17}^T+N_{18}^T,$$

$$\Psi_i''=U_iB\Theta_0(I+\Gamma)KG_0(I+L)+N_{1i}-M_{1i},\quad (i=1,2,3,4,5);$$

$$\Psi_6'=U_6B\Theta_0(I+\Gamma)KG_0(I+L)+[U_6B\Theta_0(I+\Gamma)KG_0(I+L)]^T-M_{16}-M_{16}^T+N_{16}+N_{16}^T$$

$$\widehat{E}=[E_1\ \ 0\ \ 0\ \ 0\ \ 0\ \ E_2\Theta_0(I+\Gamma)KG_0(I+L)\ \ 0\ \ 0]. \tag{48}$$

With the definition of scalar $\varepsilon > 0$, we apply the theory given in Lemma 6 to (47), also used by Y. Wang and L. Xie [37], in which the uncertain matrix F can be eliminated, and a sufficient condition of (47) is obtained.

$$\begin{bmatrix} -(S_1+C^TC)^{-1} & 0 & 0 & \widetilde{I} \\ * & -S_2^{-1} & 0 & \Sigma \\ * & * & -\alpha R_2 & \Omega^{1k} \\ * & * & * & \Phi_1'' \end{bmatrix} + \varepsilon\begin{bmatrix}0\\D\\0\\\widetilde{D}\end{bmatrix}\begin{bmatrix}0\\D\\0\\\widetilde{D}\end{bmatrix}^T+\varepsilon^{-1}\begin{bmatrix}0\\0\\0\\\widehat{E}^T\end{bmatrix}\begin{bmatrix}0\\0\\0\\\widehat{E}^T\end{bmatrix}^T<0 \tag{49}$$

Applying the Schur complement to inequality (49), we have

$$\begin{bmatrix} -\varepsilon I & 0 & 0 & 0 & \varepsilon\widetilde{D}^T & 0 \\ * & -(S_1+C^TC)^{-1} & 0 & 0 & \widetilde{I} & 0 \\ * & * & -S_2^{-1} & 0 & \Sigma & 0 \\ * & * & * & -\alpha R_2 & \Omega^{1k} & 0 \\ * & * & * & * & \Phi_1'' & \widehat{E}^T \\ * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{50}$$

There exists $\rho_1 > 0$, $\rho_2 > 0$. Based on (17) and Remark 3 in Section 2, we know that $L^TL \le I$ and $\Gamma^T\Gamma \le I$. According to expressions (47), (49), and (50), we obviously know that $U_5 > 0$. Using Lemma 6 again, we have

$$\begin{aligned}&\Theta_0(I+\Gamma)KG_0L+[\Theta_0(I+\Gamma)KG_0L]^T\\&\quad\le\rho_1^{-1}\Theta_0(I+\Gamma)KG_0U_5^{-1}[\Theta_0(I+\Gamma)KG_0]^T\\&\qquad+\rho_1U_5\end{aligned} \tag{51}$$

and

$$\begin{aligned}&\Theta_0\Gamma KG_0+(\Theta_0\Gamma KG_0)^T\\&\quad\le\rho_2^{-1}\Theta_0\Theta_0^T+\rho_2(KG_0)^TKG_0\end{aligned} \tag{52}$$

Based on inequalities (51), (52) and the Schur complement, we know inequality (24) is a sufficient condition of inequality (50), while inequality (50) is equivalent to inequality (41). Therefore, we can undoubtedly obtain inequality (24) as a sufficient condition of inequality (41). Thus, based on inequality (39) and (41), we know

(1) if $\omega(t)\equiv 0$, obviously, we have $v'(t) < 0$, so system (21) is asymptotically stable;

(2) if $x(0)\equiv 0$, we know $v(0) = 0$. In addition, it can be obtained that $v(\infty)\ge 0$.

Therefore,

$$\begin{aligned}&\int_0^\infty v'(t)+y^T(t)y(t)-\gamma^2\omega^T(t)\omega(t)d_t\\&\quad=v(\infty)+\int_0^\infty y^T(t)y(t)-\gamma^2\omega^T(t)\omega(t)d_t<0\end{aligned} \tag{53}$$

Thus,

$$\int_0^\infty y^T(t)y(t)<\int_0^\infty\gamma^2\omega^T(t)\omega(t)d_t \tag{54}$$

Because $\omega(t) \in L_2[0,\infty)$, we have

$$\|y(t)\| \le \gamma \|\omega(t)\|^T. \quad (55)$$

It is known that model (21) is asymptotically stable with the H_∞ norm bound γ.

Moreover, according to (39) and (41), we have

$$\begin{aligned} v'(t) \le & -x^T(t) S_1 x(t) \\ & - \left[\Theta_0 (I+\Gamma) K G_0 (I+L) x(t-\eta(t))\right]^T \\ & \cdot S_2 \Theta_0 (I+\Gamma) K G_0 (I+L) x(t-\eta(t)) \end{aligned} \quad (56)$$

Through the integral operation, it can be determined that $J \le v(0)$. In addition, by inserting $t = 0$ into the Lyapunov-Krasovskii function shown as expression (28), the upper bound of cost function J can be obtained and shown as expression (26). Therefore, the theorem is verified if $l = 1$.

Case 2. If $\eta(t) \in [\eta_1, \eta_M]$, weighted technology based on the principle of Newton-Leibniz is introduced as follows:

$$2\left[\varepsilon^T(t)\ \ \omega^T(t)\right] M_2 \left[x(t-\eta_1) - x(t-\eta(t)) - \int_{t-\eta(t)}^{t-\eta_1} x'^T(s)\, d_s\right] = 0 \quad (57)$$

$$2\left[\varepsilon^T(t)\ \ \omega^T(t)\right] N_2 \left[x(t-\eta(t)) - x(t-\eta_M) - \int_{t-\eta_M}^{t-\eta(t)} x'^T(s)\, d_s\right] = 0 \quad (58)$$

Because $R_3 > 0$, we have

$$\begin{aligned} -2\left[\varepsilon^T(t)\ \ \omega^T(t)\right] M_2 \int_{t-\eta(t)}^{t-\eta_1} x'(s)\, d_s \le & (\eta(t)-\eta_1) \\ & \cdot \left[\varepsilon^T(t)\ \ \omega^T(t)\right] M_2 R_3^{-1} M_2{}^T \left[\varepsilon^T(t)\ \ \omega^T(t)\right]^T \\ & + \int_{t-\eta(t)}^{t-\eta_1} x'^T(s) R_3 x'(s)\, d_s \end{aligned} \quad (59)$$

$$\begin{aligned} -2\left[\varepsilon^T(t)\ \ \omega^T(t)\right] N_2 \int_{t-\eta_M}^{t-\eta(t)} x'(s)\, d_s \le & (\eta_M-\eta(t)) \\ & \cdot \left[\varepsilon^T(t)\ \ \omega^T(t)\right] N_2 R_3^{-1} N_1{}^T \left[\varepsilon^T(t)\ \ \omega^T(t)\right]^T \\ & + \int_{t-\eta_M}^{t-\eta(t)} x'^T(s) R_3 x'(s)\, d_s \end{aligned} \quad (60)$$

Therefore,

$$\begin{aligned} & -2\left[\varepsilon^T(t)\ \ \omega^T(t)\right] M_2 \int_{t-\eta(t)}^{t-\eta_1} x'(s)\, d_s \\ & -2\left[\varepsilon^T(t)\ \ \omega^T(t)\right] N_2 \int_{t-\eta_M}^{t-\eta(t)} x'(s)\, d_s \\ & \le (\eta(t)-\eta_1) \\ & \cdot \left[\varepsilon^T(t)\ \ \omega^T(t)\right] M_2 R_3^{-1} M_2{}^T \left[\varepsilon^T(t)\ \ \omega^T(t)\right]^T \\ & + (\eta_M - \eta(t)) \\ & \cdot \left[\varepsilon^T(t)\ \ \omega^T(t)\right] N_2 R_3^{-1} N_2{}^T \left[\varepsilon^T(t)\ \ \omega^T(t)\right]^T \\ & + \int_{t-\eta_M}^{t-\eta_1} x'^T(s) R_3 x'(s)\, d_s \end{aligned} \quad (61)$$

In addition, $y^T(t)y(t) - \gamma^2\omega^T(t)\omega(t)$ both on the left and right sides of equality (29) and $x^T(t)S_1x(t) + [\Theta_0(I+\Gamma)KG_0(I+L)x(t-\eta(t))]^T S_2\Theta_0(I+\Gamma)KG_0(I+L)x(t-\eta(t))$ on the right side of equal sign "=". By inserting (30), (31), (57), (58), and (61) into the obtained inequality, the item $-\int_{t-\eta_M}^{t-\eta_1} x'^T(s)R_3x'(s)d_s$ can be offset, while the item $-\int_{t-\eta_1}^{t-\eta_m} x'^T(s)R_2x'(s)d_s$ is offset in Case 1. Then, in the same methods of transformation as Case 1, the inequality (24) when $l = 2$ can be obtained. Therefore, the proof is complete. □

Remark 9. In the two different cases, a variational weighting matrix and Jessen's inequalities are used to derive the H_∞ guaranteed cost fault-tolerant condition of the system, in which more delay information is employed to reduce the conservatism.

The next section will provide sufficient conditions for designing the guaranteed cost fault-tolerant control for a double-fault NCS.

4. Guaranteed Cost Fault-Tolerant Control of Double-Fault NCS

Inequality (24) is not linear with respect to the gain matrices of the controller, so it is needs to be reformulated into LMIs via a change of variables.

Theorem 10. *Given symmetric positive definite matrices S_1 and S_2, a set of constant η_m, η_M, $\rho_1 > 0$, $\rho_2 > 0$, λ_i (i = from 1 to 7), and $\alpha = (\eta_M - \eta_m)/2$. If there exists a set of symmetric positive definite matrices $\widetilde{R}_j$ $(j = 1,2,3)$, X, and matrix $\widetilde{P} > 0$, as well as matrices $\widetilde{M}_{1\beta}$, $\widetilde{M}_{2\beta}$, $\widetilde{N}_{1\beta}$, $\widetilde{N}_{2\beta}$, (β =from 1 to 8),* *Y, and a set of constants,* $\varepsilon > 0$ *and* $\mu > 0$*, satisfying the LMIs*

$$
\begin{bmatrix}
-\varepsilon I & 0 & 0 & 0 & \varepsilon \overline{D}^T & 0 & 0 & 0 & 0 & 0 \\
* & -\left(S_1 + C^T C\right)^{-1} & 0 & 0 & \Pi & 0 & 0 & 0 & 0 & 0 \\
* & * & -S_2^{-1} & 0 & \overline{\Sigma} & 0 & \Theta_0 Y & 0 & \Theta_0 & 0 \\
* & * & * & -\overline{\Gamma}_l & \overline{\Omega}^{lk} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \overline{\Phi}_l & \overline{E}^T & \overline{\Theta} & \overline{X} & \overline{\Delta} & \overline{Y}^T \\
* & * & * & * & * & -\varepsilon I & E_2 \Theta_0 Y & 0 & E_2 \Theta_0 & 0 \\
* & * & * & * & * & * & -\rho_1 X^T & 0 & 0 & Y^T \\
* & * & * & * & * & * & * & -\rho_1^{-1} X & 0 & 0 \\
* & * & * & * & * & * & * & * & -\rho_2 I & 0 \\
* & * & * & * & * & * & * & * & * & -\rho_2^{-1} I
\end{bmatrix} < 0
$$

$$
l = 1, 2; \; k = 1, 2 \qquad (62)
$$

where

$$
\breve{\Phi}_1 = \begin{bmatrix}
-\widetilde{R}_1 + \lambda_1 A X^T + \lambda_1 X A^T & \widetilde{R}_1 + \lambda_2 X A^T + \widetilde{M}_{11} & \lambda_3 X A^T - \widetilde{N}_{11} & \lambda_4 X A^T \\
* & -\widetilde{R}_1 + \widetilde{M}_{12} + \widetilde{M}_{12}^T & \widetilde{M}_{13}^T - \widetilde{N}_{12} & \widetilde{M}_{14}^T \\
* & * & -\dfrac{\widetilde{R}_3}{\alpha} - \widetilde{N}_{12} - \widetilde{N}_{12}^T & \dfrac{\widetilde{R}_3}{\alpha} - \widetilde{N}_{14}^T \\
* & * & * & -\dfrac{\widetilde{R}_3}{\alpha} \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & *
\end{matrix} \longrightarrow
$$

$$
\begin{matrix}
\widetilde{P} - \lambda_1 X^T + X A^T & \overline{\Psi}_1 + \lambda_5 X A^T & \lambda_6 X A^T + \lambda_7 X A^T + \lambda_1 H X^T & 0 \\
-\lambda_2 X^T + \widetilde{M}_{15}^T & \overline{\Psi}_2 + \widetilde{M}_{16}^T & \lambda_2 H X^T + \widetilde{M}_{17}^T + \widetilde{M}_{18}^T & 0 \\
-\lambda_3 X^T - \widetilde{N}_{15}^T & \overline{\Psi}_3 - \widetilde{N}_{16}^T & \lambda_3 H X^T - \widetilde{N}_{17}^T - \widetilde{N}_{18}^T & 0 \\
-\lambda_4 X^T & \overline{\Psi}_4 & \lambda_4 H X^T & 0 \\
\eta_m^2 \widetilde{R}_1 + \alpha\left(\widetilde{R}_2 + \widetilde{R}_3\right) - X^T - X & \overline{\Psi}_5 - \lambda_5 X & H X^T - \lambda_6 X - \lambda_7 X & 0 \\
* & \overline{\Psi}_6 & \overline{\Xi} & 0 \\
* & * & \lambda_6 H X^T + \lambda_7 H X^T + \lambda_6 X H^T + \lambda_7 X H^T & 0 \\
* & * & * & -\mu I
\end{bmatrix}
$$

$$
\widetilde{\Phi}_2 = \begin{bmatrix}
-\widetilde{R}_1 + \lambda_1 A X^T + \lambda_1 X A^T & \widetilde{R}_1 + \lambda_2 X A^T & \lambda_3 X A^T + \widetilde{M}_{21} & \lambda_4 X A^T - \widetilde{N}_{21} \\
* & -\widetilde{R}_1 - \dfrac{\widetilde{R}_2}{\alpha} & \dfrac{\widetilde{R}_2}{\alpha} + \widetilde{M}_{22} & -\widetilde{N}_{22} \\
* & * & -\dfrac{\widetilde{R}_2}{\alpha} + \widetilde{M}_{23} + \widetilde{M}_{23}^T & \widetilde{M}_{24}^T - \widetilde{N}_{23} \\
* & * & * & -\widetilde{N}_{24} - \widetilde{N}_{24}^T \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & *
\end{matrix} \longrightarrow
$$

$$
\left.\begin{array}{cccc}
\widetilde{P}-\lambda_1X^T+XA^T & \overline{\Psi}_1{}'+\lambda_5XA^T & \lambda_6XA^T+\lambda_7XA^T+\lambda_1HX^T & 0\\
-\lambda_2X^T & \overline{\Psi}_2{}' & \lambda_2HX^T & 0\\
-\lambda_3X^T+\widetilde{M}_{25}{}^T & \overline{\Psi}_3{}'+\widetilde{M}_{26}{}^T & \lambda_3HX^T+\widetilde{M}_{27}{}^T+\widetilde{M}_{28}{}^T & 0\\
-\lambda_4X^T-\widetilde{N}_{25}{}^T & \overline{\Psi}_4{}'-\widetilde{N}_{26}{}^T & \lambda_4HX^T-\widetilde{N}_{27}{}^T-\widetilde{N}_{28}{}^T & 0\\
\eta_m{}^2\widetilde{R}_1+\alpha\left(\widetilde{R}_2+\widetilde{R}_3\right)-X-X^T & \overline{\Psi}_5{}'-\lambda_5X & HX^T-\lambda_6X-\lambda_7X & 0\\
* & \breve{\Psi}_6{}' & \overline{\Xi'} & 0\\
* & * & \lambda_6HX^T+\lambda_7HX^T+\lambda_6XH^T+\lambda_7XH^T & 0\\
* & * & * & -\mu I
\end{array}\right]
$$

$$
\begin{gathered}
\Gamma_1=\alpha R_2\\
\overline{\Gamma}_1=\alpha\widetilde{R}_2,\\
\Gamma_2=\alpha\breve{R}_3,\\
\overline{\Omega}^{11}=\alpha\widetilde{M}_1{}^T,\\
\overline{\Omega}^{12}=\alpha\widetilde{N}_1{}^T,\\
\overline{\Omega}^{21}=\alpha\widetilde{M}_2{}^T,\\
\breve{\Omega}^{22}=\alpha\widetilde{N}_2{}^T,\\
\widetilde{M}_1{}^T=\left[\widetilde{M}_{11}{}^T\ \ \widetilde{M}_{12}{}^T\ \ \widetilde{M}_{13}{}^T\ \ \widetilde{M}_{14}{}^T\ \ \widetilde{M}_{15}{}^T\ \ \widetilde{M}_{16}{}^T\ \ \widetilde{M}_{17}{}^T\ \ \widetilde{M}_{18}{}^T\right];\\
\widetilde{M}_2{}^T=\left[\widetilde{M}_{21}{}^T\ \ \widetilde{M}_{22}{}^T\ \ \widetilde{M}_{23}{}^T\ \ \widetilde{M}_{24}{}^T\ \ \widetilde{M}_{25}{}^T\ \ \widetilde{M}_{26}{}^T\ \ \widetilde{M}_{27}{}^T\ \ \widetilde{M}_{28}{}^T\right];\\
\widetilde{N}_1{}^T=\left[\widetilde{N}_{11}{}^T\ \ \widetilde{N}_{12}{}^T\ \ \widetilde{N}_{13}{}^T\ \ \widetilde{N}_{14}{}^T\ \ \widetilde{N}_{15}{}^T\ \ \widetilde{N}_{16}{}^T\ \ \widetilde{N}_{17}{}^T\ \ \widetilde{N}_{18}{}^T\right];\\
N_2{}^T=\left[\widetilde{N}_{21}{}^T\ \ \widetilde{N}_{22}{}^T\ \ \widetilde{N}_{23}{}^T\ \ \widetilde{N}_{24}{}^T\ \ \widetilde{N}_{25}{}^T\ \ \widetilde{N}_{26}{}^T\ \ \widetilde{N}_{27}{}^T\ \ \widetilde{N}_{28}{}^T\right];\\
\overline{\Xi}=\lambda_6\left(B\Theta_0Y\right)^T+\lambda_7\left(B\Theta_0Y\right)^T+\lambda_5HX^T-\widetilde{M}_{17}{}^T-\widetilde{M}_{18}{}^T+\widetilde{N}_{17}{}^T+\widetilde{N}_{18}{}^T;\\
\overline{\Psi}_i=\lambda_iB\Theta_0Y+\widetilde{N}_{1i}-\widetilde{M}_{1i},\quad(i=1,2,3,4);\\
\overline{\Psi}_5=B\Theta_0Y+\widetilde{N}_{1i}-\widetilde{M}_{1i},\\
\overline{\Psi}_6=\lambda_5B\Theta_0Y+\lambda_5\left(B\Theta_0Y\right)^T-\overline{M}_{16}-\overline{M}_{16}{}^T+\overline{N}_{16}+\overline{N}_{16}{}^T;\\
\overline{\Xi'}=\lambda_6\left(B\Theta_0Y\right)^T+\lambda_7\left(B\Theta_0Y\right)^T+\lambda_5HX^T-\widetilde{M}_{27}{}^T-\widetilde{M}_{28}{}^T+\widetilde{N}_{27}{}^T+\widetilde{N}_{28}{}^T;\\
\overline{\Psi}_i{}'=\lambda_iB\Theta_0Y+\widetilde{N}_{2i}-\widetilde{M}_{2i},\quad(i=1,2,3,4);\\
\overline{\Psi}_5{}'=B\Theta_0Y+\widetilde{N}_{2i}-\widetilde{M}_{2i};\\
\overline{\Psi}_6{}'=\lambda_5B\Theta_0Y+\lambda_5\left(B\Theta_0Y\right)^T-\widetilde{M}_{26}-\widetilde{M}_{26}{}^T+\widetilde{N}_{26}+\widetilde{N}_{26}{}^T;\\
\overline{D}=\left[\lambda_1D^T\ \ \lambda_2D^T\ \ \lambda_3D^T\ \ \lambda_4D^T\ \ D^T\ \ \lambda_5D^T\ \ \lambda_6D^T+\lambda_7D^T\ \ 0\right]^T;\\
\overline{E}=\left[E_1X^T\ \ 0\ \ 0\ \ 0\ \ 0\ \ E_2\Theta_0Y\ \ 0\ \ 0\right];\\
\overline{X}=\left[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ X^T\ \ 0\ \ 0\right]^T;\\
\Pi=\left[X^T\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\right];
\end{gathered}
$$

$$\overline{\Theta} = \left[(\lambda_1 B\Theta_0 Y)^T \ (\lambda_2 B\Theta_0 Y)^T \ (\lambda_3 B\Theta_0 Y)^T \ (\lambda_4 B\Theta_0 Y)^T \ (B\Theta_0 Y)^T \ (\lambda_5 B\Theta_0 Y)^T \ (\lambda_6 BY)^T + (\lambda_7 BY)^T \ 0\right]^T;$$
$$\overline{\Delta} = \left[(\lambda_1 B\Theta_0)^T \ (\lambda_2 B\Theta_0)^T \ (\lambda_3 B\Theta_0)^T \ (\lambda_4 B\Theta_0)^T \ (B\Theta_0)^T \ (\lambda_5 B\Theta_0)^T \ (\lambda_6 B\Theta_0)^T + (\lambda_7 B\Theta_0)^T \ 0\right]^T;$$
$$\overline{Y} = [0 \ 0 \ 0 \ 0 \ 0 \ Y \ 0 \ 0];$$
$$\overline{\Sigma} = [0 \ 0 \ 0 \ 0 \ 0 \ \Theta_0 Y \ 0 \ 0], \tag{63}$$

then the H_∞ guaranteed cost control gain $K = YX^{-T}G_0^{-1}$ can render model (21) to be asymptotically stable with the H_∞ norm bound $\gamma = \sqrt{\mu}$. The upper bound J_0 of cost function J is given as

$$\begin{aligned} J_0 &= x^T(0) X^{-1}\widetilde{P}X^{-T}x(0) \\ &+ \eta_m \int_{-\eta_m}^{0}\int_{s}^{0} x'^T(\tau) X^{-1}\widetilde{R}_1 X^{-T} x'(\tau)\, d_s d_\tau \\ &+ \int_{-\eta_1}^{-\eta_m}\int_{s}^{0} x'^T(\tau) X^{-1}\widetilde{R}_2 X^{-T} x'(\tau)\, d_s d_\tau \\ &+ \int_{-\eta_M}^{-\eta_1}\int_{s}^{0} x'^T(\tau) X^{-1}\widetilde{R}_3 X^{-T} x'(\tau)\, d_s d_\tau. \end{aligned} \tag{64}$$

Proof. The proof is based on a suitable transformation and a change of variables allowing us to obtain inequality (24) in Theorem 8. First, we define $U_5 = U_0$, $U_i = \lambda_i U_0$ (i = from 1 to 4), $U_6 = \lambda_5 U_0$ in (24). Because we consider the dimension of state$x$is equal to that of outside disturbance ω in this paper, we can also define $U_7 = \lambda_6 U_0$, $U_8 = \lambda_7 U_0$. Obviously, (24) implies $U_5 > 0$, so U_0 is nonsingular. Then, using the analysis method of D. Yue [40] and Z. Wang [41], pre- and postmultiplying both sides of inequality (24) with $\mathrm{diag}(I, I, I, X, \widetilde{X}, I, X, I, I, I)$ and its transpose, where $\widetilde{X} = \mathrm{diag}(X, X, X, X, X, X, X, X)$ and $X = U_0^{-1}$, introducing new variables $XPX^T = \widetilde{P}$; $XR_jX^T = \widetilde{R}_j$ ($j = 1, 2, 3$); $XM_{l\beta}X^T = \widetilde{M}_{l\beta}$, $XN_{l\beta}X^T = \widetilde{N}_{l\beta}$ ($l = 1, 2; \beta = 1, 2, \cdots, 8$); $KG_0X^T = Y$ and $\mu = \gamma^2$. From the definition of G_0, we know G_0 is invertible, so K can be obtained by calculating $K = YX^{-T}G_0^{-1}$. It is easy to see that 10 and (64) respectively imply (24) and (26). Therefore, from Theorem 8, we can complete the proof. □

To obtain the optimal bound J^* shown in (26), the commonly used method is to consider it as an optimization problem like [42], in which the expression of initial state $x(t)$ ($t \in [-\eta_M, 0]$) needs to be known. The expression of initial state is not given in this paper. Therefore, the optimization method used in [42] cannot be used here. A practical method to obtain J^*, also used by D. Yue [43], is employed as follows.

Suppose $x'(t)$ is bounded if $t \in [-\eta_M, 0]$ and satisfying $x'^T(\tau)x'(\tau) \le \ell$. In addition, suppose that there exists $\beta_i > 0$ ($i = 1, 2, 3$), satisfying

$$\begin{aligned} X^{-1}\widetilde{R}_i X^{-T} &\le \beta_i I \quad i = 1, 2, 3 \\ X^{-1}\widetilde{P}X^{-T} &\le \beta_4 I \end{aligned} \tag{65}$$

Inserting this into (64), we have

$$\begin{aligned} J_0 &\le \beta_4 x^T(0)x(0) + 0.5\ell\beta_1\eta_m^3 + 0.5\ell\beta_2\left(\eta_1^2 - \eta_m^2\right) \\ &+ 0.5\ell\beta_3\left(\eta_M^2 - \eta_1^2\right) = J^* \end{aligned} \tag{66}$$

Applying the Schur complement to the inequalities above, we have

$$\begin{aligned} &\begin{bmatrix} -\beta_i I & X^{-1} \\ * & -R_i^{-1} \end{bmatrix} < 0 \quad i = 1, 2, 3 \\ &\begin{bmatrix} -\beta_4 I & X^{-1} \\ * & -P^{-1} \end{bmatrix} < 0 \end{aligned} \tag{67}$$

Then, combining 10 and (67), $\widetilde{R}_i^{-1}, \widetilde{R}_i$ ($i = 1, 2, 3$), $\widetilde{P}^{-1}, \widetilde{P}, X^{-1}, X$ exist simultaneously. We cannot directly use the LMI tools to solve the problem. Defining $\widetilde{R}_i^{-1} = \widehat{R}_i$ ($i = 1, 2, 3$), $\widetilde{P}^{-1} = \widehat{P}$, $X^{-1} = \widehat{X}$ and using the idea of the cone complementary linearization algorithm, the guaranteed cost fault-tolerant controller of system (21) and the value of optimal performance indicator J^* can be obtained in the following method:

$$\begin{aligned} \text{Minimize:} \quad & \mathit{trace}\left(\widetilde{R}_1\widehat{R}_1 + \widetilde{R}_2\widehat{R}_2 + \widetilde{R}_3\widehat{R}_3 + \widetilde{P}\widehat{P} + \widetilde{X}\widehat{X}\right) \\ & + \beta_4 + 0.5\beta_1\eta_m^3 + 0.5\beta_2\left(\eta_1^2 - \eta_m^2\right) \\ & + 0.5\beta_3\left(\eta_M^2 - \eta_1^2\right) \\ \text{Subject to:} \quad & \mathit{Inequalities}\ (62), \\ & \begin{bmatrix} -\beta_i I & \widehat{X} \\ * & -\widehat{R}_i \end{bmatrix} < 0 \quad (i = 1, 2, 3), \\ & \begin{bmatrix} -\beta_4 I & \widehat{X} \\ * & -\widehat{P} \end{bmatrix} < 0, \\ & \begin{bmatrix} X & I \\ * & \widehat{X} \end{bmatrix} \ge 0, \\ & \begin{bmatrix} \widehat{R}_1 & I \\ * & \widetilde{R}_1 \end{bmatrix} \ge 0, \\ & \begin{bmatrix} \widehat{R}_2 & I \\ * & \widetilde{R}_2 \end{bmatrix} \ge 0, \end{aligned}$$

$$\begin{bmatrix} \widehat{R}_3 & I \\ * & \widetilde{R}_3 \end{bmatrix} \geq 0,$$
$$\begin{bmatrix} \widehat{P} & I \\ * & \widetilde{P} \end{bmatrix} \geq 0,$$
$$\widetilde{R}_i > 0 \quad (i = 1, 2, 3). \tag{68}$$

5. Simulations

Example 11. Consider inverted pendulum model that can usually be modelled as (1), and the system parameters are given as follows:

$$\begin{aligned} A &= \begin{bmatrix} -2 & 1.11 \\ 0 & 4 \end{bmatrix}, \\ B &= \begin{bmatrix} 0.1 \\ -4 \end{bmatrix}, \\ C &= \begin{bmatrix} -0.3151 & 0.1 \\ 0.11 & 0.3 \end{bmatrix}, \\ H &= \begin{bmatrix} 0.15 & -0.56 \\ -0.1 & 0.32 \end{bmatrix}, \\ \Delta A &= \begin{bmatrix} 0.04 \sin t & 0.15 \sin t \\ -0.2 \sin t & 0.75 \sin t \end{bmatrix}, \\ \Delta B &= \begin{bmatrix} 0.02 \sin t \\ -0.1 \sin t \end{bmatrix}, \\ D &= \begin{bmatrix} 0.1 \\ -0.5 \end{bmatrix}, \\ F(t) &= \sin t. \end{aligned} \tag{69}$$

Therefore, we have

$$\begin{aligned} E_1 &= [0.4 \quad 1.5], \\ E_2 &= 0.2. \end{aligned} \tag{70}$$

For this simulation, the initial state of system is assumed $x(0) = [2 \quad -1]^T$, and the external disturbance is considered as $\omega(t) = \begin{cases} [0.15 \quad -0.5]^T & 3s \leq t \leq 4s \\ 0 & others \end{cases}$. Here, we take the upper bound of time-varying delay as $\eta_M = 0.3s$ and its lower bound as $\eta_m = 0s$, namely, $0 \leq \eta(t) \leq 0.3$. In addition, the fault bounds of the system are given in Table 1.

Table 1: The bounds of faults.

Symbol	Upper bound	Lower bound
Actuator faults Θ	1.36	0.09
Sensor Faults G	$\begin{bmatrix} 1.65 & 0 \\ 0 & 1.75 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0.15 \end{bmatrix}$

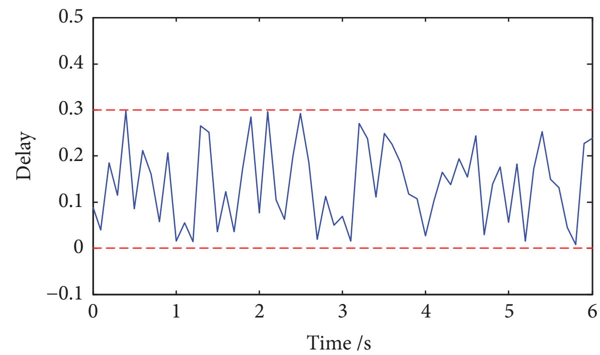


Figure 2: The time-varying delay in double-fault NCS.

We choose the parameters as follows:

$$\begin{aligned} \lambda_1 &= 0.1, \\ \lambda_2 &= 0.15, \\ \lambda_3 &= -0.64, \\ \lambda_4 &= 0.3, \\ \lambda_5 &= \lambda_6 = 0.35, \\ \lambda_7 &= 1.8, \\ S_2 &= 0.31, \\ \rho_1 &= 3000, \\ \rho_2 &= 220.32, \\ \ell &= 1, \\ S_1 &= \begin{bmatrix} 100.13 & 0 \\ 0 & 92.07 \end{bmatrix}. \end{aligned} \tag{71}$$

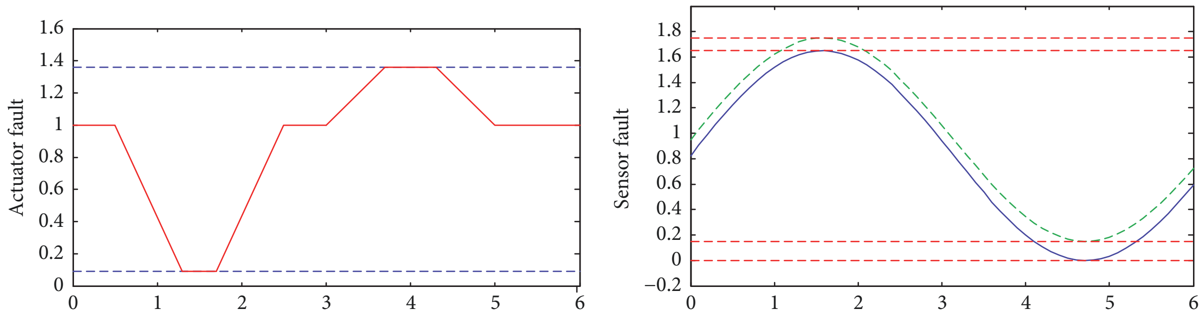
By taking advantage of the LMI tool box and inserting the above parameters into inequalities 10 and (68), we can obtain the H_∞ guaranteed cost control gain

$$K = YX^{-T}G_0^{-1} = [8.5163 \quad 15.1398] \tag{72}$$

with $\gamma = \sqrt{\mu} = 195.2516$.

The corresponding optimal performance indicator (the upper bound value of guaranteed cost function) is $J^* = 8316.2563$.

The time-varying delay is shown in Figure 2. In Figure 3, (a) is the actuator fault, which is a piecewise-linear function.



(a) Time-varying actuator fault (b) Time-varying sensor fault

Figure 3: The time-varying faults.

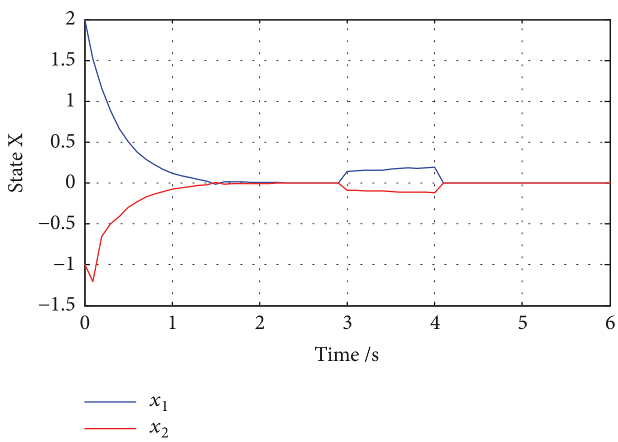


Figure 4: The state response curve of double-fault NCS.

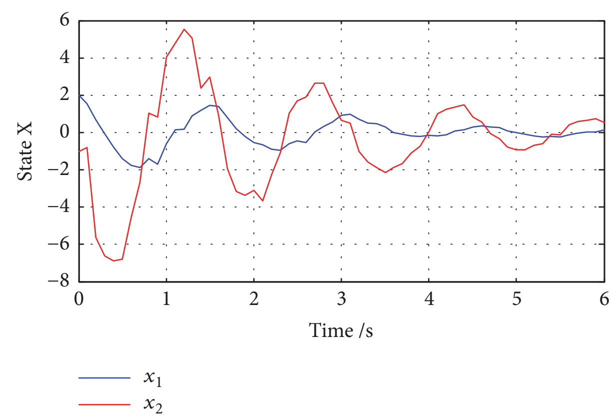


Figure 6: The state response curve of double-fault NCS.

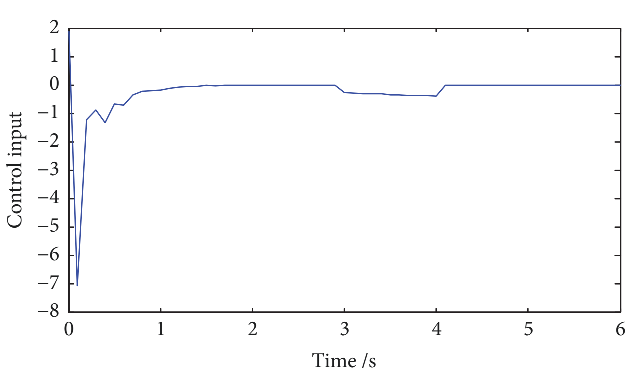


Figure 5: The control input of double-fault NCS.

It keeps the minimum value from 1.3s to 1.7s, while it keeps the maximum value from 3.7s to 4.3s. The sensor faults are shown as (b), which is sinusoidal. It should be noted that the green dotted line represents the fault of sensor 1, while the blue solid line represents the fault of sensor 2. Through the state response of the double-fault NCS shown in Figure 4 and corresponding control signal shown in Figure 5, we know the H_∞ guaranteed cost controller designed in this paper is able to make the double-fault NCS asymptotically stable. The system gets preliminarily steady at 2s, and its state can return to the equilibrium position in a certain period of time when the NCS is affected by external disturbance. Compared with the state response of worse stability shown in Figure 6 when the method proposed in [28] is used for this double-fault problem, it sufficiently proves the effectiveness and feasibility of the method proposed in this paper.

To better illustrate the effectiveness of the method proposed in this paper, the following example is presented.

Example 12. Consider the parameters of system (1) as follows:

$$A = \begin{bmatrix} 0.21 & 0 & 0.35 & 1 \\ 0 & -5.3 & -5.86 & 3.23 \\ 3.65 & -1.1 & -1.56 & -0.89 \\ 0 & 0 & -1.58 & -2.85 \end{bmatrix},$$

$$B = \begin{bmatrix} 5.59 & 1.2 & -0.89 & 1.3 \end{bmatrix}^T,$$

$$C = \begin{bmatrix} 1.7 & 0.2 & 0.15 & -0.18 \end{bmatrix},$$

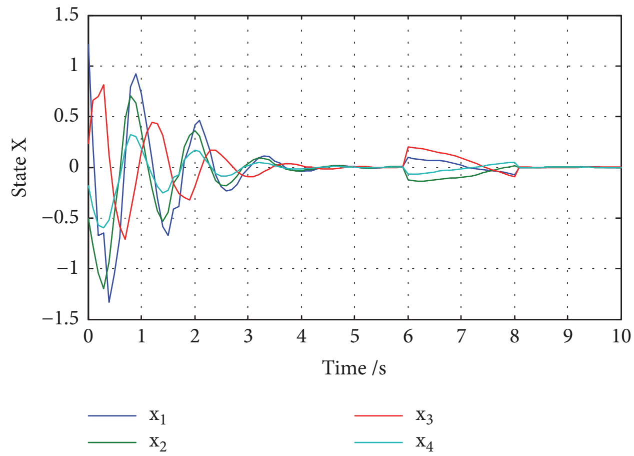


Figure 7: The state response curve of double-fault NCS.

$$\Delta A = \begin{bmatrix} 0.82\sin t & 0 & 0 & 0 \\ 0 & 0.12\sin t & 0 & 0 \\ 0 & 0 & -0.58\sin t & 0 \\ 0 & 0 & 0 & 1.01\sin t \end{bmatrix},$$
$$\Delta B = \begin{bmatrix} -0.15\sin t \\ 0.08\sin t \\ 0 \\ -0.28\sin t \end{bmatrix},$$
$$H = \begin{bmatrix} -0.41 & 0.96 & 0.52 & 0 \\ 0.06 & -0.57 & 0 & 0.21 \\ 1.21 & 0 & 0.59 & 0 \\ -0.32 & -0.11 & -0.35 & -0.03 \end{bmatrix},$$
$$D = I_{4\times 4}, \quad F(t) = \operatorname{diag}(\sin t, \sin t, \sin t, \sin t). \tag{73}$$

Therefore, we have

$$E_1 = \operatorname{diag}(0.82, 0.12, -0.58, 1.01), \quad E_2 = \begin{bmatrix} -0.15 & 0.08 & 0 & -0.28 \end{bmatrix}^T. \tag{74}$$

For this simulation, the initial state of system is assumed as $x(0) = \begin{bmatrix} 1.21 & -0.51 & 0.23 & -0.18 \end{bmatrix}^T$, and the external disturbance is considered as

$$\omega(t) = \begin{cases} \begin{bmatrix} 0.15\cos t & 0.15\cos t & -0.12\sin t & -0.25\cos t \end{bmatrix}^T & 6s \le t \le 8s \\ 0 & others. \end{cases} \tag{75}$$

The uncertain time-varying delay satisfies $0 \le \eta(t) \le 0.23s$. In addition, the fault bounds of the system are given in Table 2. Other parameters are selected as follows:

$$\lambda_1 = 1.2, \quad \lambda_2 = -2.03, \quad \lambda_3 = 1.68, \quad \lambda_4 = -2.3, \quad \lambda_5 = -1.05, \quad \lambda_6 = 6.05, \quad \lambda_7 = 5.12,$$
$$S_1 = \operatorname{diag}(198.88, 156.34, 87.09, 128.76), \quad S_2 = 5.86,$$
$$\rho_1 = 57.98, \quad \rho_2 = 18.79, \quad \ell = 1. \tag{76}$$

By taking advantage of the LMI tool box and submitting these parameters above into inequalities 10 and (68), we can obtain the H_∞ guaranteed cost control gain

$$K = YX^{-T}G_0^{-1} = \begin{bmatrix} -1.7256 & 1.5512 & -0.8571 & 0.7451 \end{bmatrix} \tag{77}$$

with $\gamma = \sqrt{\mu} = 581.9806$.

The corresponding optimal performance indicator is $J^* = 6502.1047$. From the state response shown in Figure 7 and control signal shown in Figure 8, we undoubtedly know

Table 2: The bounds of faults.

Symbol	Upper bound	Lower bound
Actuator faults Θ	1.50	0.12
Sensor Faults G	$\begin{bmatrix} 1.85 & 0 & 0 & 0 \\ 0 & 1.60 & 0 & 0 \\ 0 & 0 & 1.25 & 0 \\ 0 & 0 & 0 & 1.65 \end{bmatrix}$	$\begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.15 \end{bmatrix}$

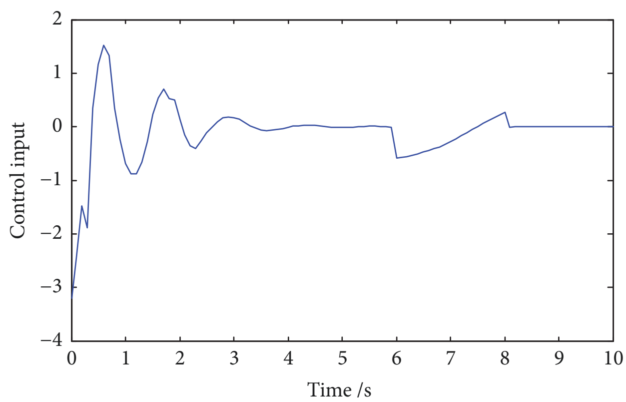


Figure 8: The control input of double-fault NCS.

the double-fault NCS is asymptotically stable when the H_∞ guaranteed cost controller is used. This further demonstrates the feasibility and effectiveness of the method proposed in this paper.

6. Conclusions

The issues of modelling and H_∞ guaranteed cost fault-tolerant control of double-fault networked control systems have been addressed. The closed-loop model of a double-fault NCS is set up with regard to the influences of transmission delay, packet dropout, uncertain parameters, and external disturbance. In addition, the piecewise delay method is proposed to reduce the conservatism when analysing the delay-dependent faulty system. With the help of Lee Y S's lemma, the sufficient condition of guaranteed cost fault-tolerant for time-varying double-fault NCS is introduced using the Lyapunov-Krasovskii theory and weighted technology. The method of designing a guaranteed cost fault-tolerant controller for this NCS is given based on LMI. Our next research task will be choosing more reasonable values of parameters λ_i $(i = 1, 2, 3, 4, 5, 6, 7)$ to reduce the conservatism further. Of course, the study on scheduling policy of double-fault NCS is also a challenge but indispensable work.

Data Availability

The data used in this paper can be got by simulations.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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