

## Research Article

# Robust Adaptive Sliding Mode Fault Tolerant Control for Nonlinear System with Actuator Fault and External Disturbance

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In this paper, an adaptive sliding mode fault tolerant control (ASMFTC) approach is proposed for a class of nonlinear systems with actuator fault, uncertainty, and external disturbance. Specifically, first, a novel observer is proposed to estimate the state, actuator fault, and external disturbance. Then, by utilising the observed information, a novel output sliding mode observer is constructed. In the control method, an adaptive law and two compensators are designed to attenuate the unknown estimation errors, actuator fault, and disturbance. Furthermore, the reaching ability of the sliding motion is analysed and the H-infinite performance is introduced to ensure the robustness of the system. Finally, a flexible single joint manipulator system and a two-cart system are used to verify the effectiveness of the proposed method.

## 1. Introduction

Industrial control system is inevitable to suffer from faults caused by the actuators, sensors, and other components of the system because of long time operation or unexpected operation. When fault occurs, the system performance will be certainly reduced and even the stability of the system cannot be guaranteed [1, 2]. With the growing requirement of the reliability and maintainability of the system, fault tolerant control (FTC) becomes an import and indispensable way when designing the controller [3, 4]. Generally, fault tolerant control (FTC) can be divided into two parts, i.e., passive FTC and active FTC. For passive FTC, the controller is designed based on the prior knowledge. On the contrary, the active FTC is constructed and integrated with the fault diagnosis, and the fault is detected online. Many literatures have been achieved about the FTC for the manipulator system; see [5–7] and the references therein.

It is worth noting that disturbance exists widely in many industry processes [8], which will directly lead to the instability of the control system. There are many reasons for disturbance for mechanical systems, such as load fluctuation, noise in the working environment, and parameters vibration.

Disturbance rejection control is also of great importance when designing the controller. Many control methods have been proposed in the last decades to attenuate, such as  $H_\infty$  control [8], robust control [9], and sliding mode control [4]. In [8], the problem of reliable  $H_\infty$  control is investigated for discrete-time T-S fuzzy systems with infinite distributed delay disturbance and actuator faults; the stability of the system and the  $H_\infty$  performance index were guaranteed. In [9], the disturbance rejection control strategy for attitude tracking of an aircraft with both internal uncertainties and external disturbances was investigated; a robust feedback controller was designed to achieve the desired tracking performance. In [4], an adaptive sliding controller was proposed for a class of single-link flexible-joint robot with mismatched disturbance; a back stepping method was used to deal with the disturbance such that the tracking performance can be guaranteed.

In most practical industry control systems, fault and external disturbance are unknown or are too expensive to measure, which drives people regard the fault or disturbance signal as an unknown input of the system. Aiming at this problem, the observer based method is of great importance in order for the accommodate fault and disturbance. Actually, the observer based methods have achieved abundant research

results since the last 1970s. For example, in [10], a class of linear time-invariant systems with unknown inputs was discussed, and the design procedure of the observer to estimate the maximum subspace of the state was presented. In [11], a class of Lipschitz system was considered; an asymptotic observer with respect to estimate both the state and input with the features of considering additive disturbance and uncertainty was designed. In [12], an adaptive unknown input observer was constructed and the robustness performance was analysed. Reference [13] addressed the problem of the disturbances observer constructing; the disturbance rejection was considered simultaneously; and the method was applied to a robot manipulator regulation.

Sliding mode control (SMC), which has potential advantages of robustness to uncertainties and disturbances and low sensitivity to the system parameter variations, has been widely investigated in nonlinear system [14]. However, the inherent robustness properties of conventional sliding mode control approaches are guaranteed only after the occurrence of a sliding mode [15]. In order to circumvent this problem, the idea of integral sliding mode control (ISM) control was introduced. In the ISMC scheme, the reaching phase is eliminated; i.e., the sliding motion occurs from the initial time. In [16], an adaptive robust controller for robot manipulators system with parameter variations and disturbances was developed. By using adaptive integral sliding mode control and time-delay estimation, the robustness and stability of the system were achieved. In addition, the observer based sliding mode control has an advantage of the disturbance attenuation control, because the estimated information is concluded in the controller scheme. In [17], the disturbance observer based SMC was considered for a special second order system with mismatched uncertainties. In [18], a disturbance observer based integral sliding mode control approach for continuous system was investigated; by incorporating the estimated information in the controller to counteract the disturbance, the chattering problem was eased and the stability was guaranteed.

Motivated by [16, 17], this paper focuses on the robust fault tolerant control for a class of nonlinear system with actuator fault, uncertainty, and external disturbance. The main works and contributions are as follows: (1) a novel observer is presented to estimate the states, fault signal, and disturbance simultaneously; (2) a new sliding mode robust control scheme is constructed, in which two compensators are included to accommodate the fault signal and to attenuate the disturbance; (3) the reaching ability of the sliding motion is proved. Finally, the proposed method is applied to the flexible single joint manipulator system; the simulation results illustrate the effectiveness of the proposed method.

The remaining parts of this paper are organized as follows. In Section 2, the system description and some assumptions are given. In Section 3, the construction of observer and the design of the controller are presented. In Section 4, a flexible single joint manipulator system and a two-cart system are presented and some conclusions are given in Section 5.

## 2. System Descriptions

The following nonlinear system with actuator fault and mismatched disturbance is

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) + Ew(t) + B_f f(t) + \Delta\xi(x, t) \\ &\quad + Mg(x, t) \\ y(t) &= Cx(t)\end{aligned}\quad (1)$$

where  $x(t) \in R^n$  is the state vector,  $u(t) \in R^m$  is the control input of the system, and  $y(t) \in R^p$  is the output.  $w(t) \in R^k$  is the external disturbance of the system and satisfies  $\|w(t)\| \leq \bar{\omega}$ ,  $\|\dot{w}(t)\| \leq \lambda$ , where  $\bar{\omega}$  and  $\lambda$  are known constants.  $f(t) \in R^p$  is the actuator fault signal, which satisfies  $\|f(t)\| \leq f^*$ , where  $f^*$  is known constant.  $\Delta\xi(x, t)$  represents the system uncertainty with  $\|\Delta\xi(x, t)\| \leq \bar{\omega}_\kappa$ .  $g(x, t) \in R^n$  is a nonlinear Lipschitz function representing the unmodeled dynamics, i.e.,  $g(x(t), t) - g(\hat{x}(t), t) \leq \eta\|x(t) - \hat{x}(t)\|$ , where  $\eta$  is the Lipschitz constant.  $A, B, C, E, B_f$ , and  $M$  are known as system parameter matrices with proper dimensions.

## 3. Main Results

*3.1. Observer Design.* Motivated by [12, 13], in this part, a novel composite observer is designed as follows:

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + B_f v + L(y(t) - \hat{y}(t)) \\ &\quad + Mg(\hat{x}, t) \\ \dot{p} &= -HCEp - HCEHy(t) - HCA\hat{x}(t) \\ &\quad - HCB(u(t) + Mg(\hat{x}, t)) - u_s \\ \hat{y}(t) &= C\hat{x}(t) \\ \hat{w} &= p + Hy(t)\end{aligned}\quad (2)$$

where  $\hat{x}$ ,  $\hat{w}$ ,  $\hat{y}$ ,  $\Delta\xi(\hat{x}, t)$ , and  $g(\hat{x}, t)$  are estimates of  $x$ ,  $w$ ,  $y$ ,  $\Delta\xi(x, t)$ , and  $g(x, t)$ ;  $L$  and  $H$  are the observer gains.  $p$  is an intermediate variable of the observer,  $u_s$  is a compensator, and  $v$  is the estimation of  $f(t)$ .  $u_s$  and  $v$  are designed as follows:

$$u_s = \frac{P_1 e_w}{\|P_1 e_w\|^2} \alpha(t) \quad (3)$$

$$v = \frac{B_f^T P_2 e_x}{\|e_x^T P_2 B_f\|} (f^* + \delta) \quad (4)$$

where  $e_w = w - \hat{w}$ ,  $e_x = x - \hat{x}$ ,  $\alpha(t) = -(2\|e_w^T P_1 HC\| \bar{\omega}_\kappa + 2\|e_w^T P_1\| \lambda + \|2e_x^T P_2\| \bar{\omega}_\kappa)$ , and  $\delta$  is a positive scalar;  $P_1$  and  $P_2$  are two positive definite symmetric matrices. From (2), one can deduce that

$$\begin{aligned}\dot{\hat{w}} &= -HCE\hat{w} - HCA\hat{x}(t) - HCBu(t) + HC(Ax(t) \\ &\quad + Bu(t) + Ew(t) + B_f f(t) + Mg(x, t) + \Delta\xi(x, t)) \\ &\quad - u_s - HCMg(\hat{x}, t)\end{aligned}\quad (5)$$

Through further simplification, one can obtain that

$$\begin{aligned} \dot{\hat{w}} = & HCEe_w + HCA\Delta\xi(x, t) + HCB_f f(t) \\ & + HCM\tilde{g}(x, t) + HCAe_x - u_s \end{aligned} \quad (6)$$

where  $\tilde{g}(x, t) = g(x, t) - g(\hat{x}, t)$ .

Then, the error dynamic systems can be obtained as follows:

$$\begin{aligned} \dot{e}_w = & -HCEe_w - HCM\tilde{g}(x, t) - HCB_f f(t) - HCAe_x \\ & - HCA\Delta\xi(x, t) + u_s + \dot{w} \end{aligned} \quad (7)$$

$$\begin{aligned} \dot{e}_x = & Ax(t) + Bu(t) + Ew(t) + B_f f(t) + \Delta\xi(x, t) \\ & + Mg(x, t) - A\hat{x}(t) - Bu(t) - Gv \\ & - L(y(t) - \hat{y}(t)) - Mg(\hat{x}, t) \\ = & (A - LC)e_x + \Delta\xi(x, t) + Ew(t) + M\tilde{g}(x, t) \\ & + B_f(f(t) - v) \end{aligned} \quad (8)$$

Define  $e = [e_w \ e_x]^T$ , then the augmented system can be written as

$$\begin{aligned} \dot{e} = & \bar{A}e + \bar{M}\tilde{g}(x, t) + \bar{G}\zeta_w + \bar{w} + G^*(f(t) - v) \\ & + H^*\Delta\xi(x, t) + \bar{u}_s \end{aligned} \quad (9)$$

where

$$\begin{aligned} \bar{A} = & \begin{bmatrix} -HCE & -HCA \\ 0 & A - LC \end{bmatrix}, \\ \bar{M} = & \begin{bmatrix} -HCM \\ M \end{bmatrix}, \\ \bar{w} = & \begin{bmatrix} \dot{w} \\ 0 \end{bmatrix}, \\ \bar{M} = & \begin{bmatrix} -HCM \\ M \end{bmatrix}, \\ \bar{u}_s = & \begin{bmatrix} u_s \\ 0 \end{bmatrix}, \\ \bar{G} = & \begin{bmatrix} -HCG & 0 \\ 0 & E \end{bmatrix}, \\ H^* = & \begin{bmatrix} -HC \\ I \end{bmatrix}, \\ G^* = & \begin{bmatrix} 0 \\ B_f \end{bmatrix}, \\ \zeta_w = & [f(t) \ w(t)]^T. \end{aligned} \quad (10)$$

$\zeta_w$  is the equivalent disturbance, and the  $H_\infty$  performance index is defined as

$$J = \int_0^\infty e(s)^T e(s) ds - \int_0^\infty \zeta_w(s)^T \zeta_w(s) ds \quad (11)$$

**Theorem 1.** *The dynamic error systems (7) and (8) are stable and satisfy the  $H_\infty$  performance index, if there exist two positive definite symmetry matrices  $P_1$  and  $P_2$ , such that the following (12) holds:*

$$\begin{bmatrix} \psi_m & P_1 HCA & -P_1 \bar{H} & P_1 HCM & 0 \\ * & \psi_n & P_2 \bar{E} & 0 & P_2 M \\ * & * & -\gamma^2 & 0 & 0 \\ * & * & * & -\sigma_1^{-1} & 0 \\ * & * & * & * & -\sigma^{-1} \end{bmatrix} < 0 \quad (12)$$

where  $\psi_m = -P_1 HCE - (P_1 HCE)^T + I$ ,  $\bar{H} = [HCB_f \ 0]$ ,  $\bar{E} = [E \ 0]$ , and  $\psi_n = P_2(A - LC) + (A - LC)^T P_2 + I$ .

*Proof.* The Lyapunov candidate function is chosen as

$$V = e^T P e \quad (13)$$

where  $P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$ .

From (13), the following (14) can be obtained

$$\dot{V} = 2e_w^T P_1 \dot{e}_w + 2e_x^T P_2 \dot{e}_x \quad (14)$$

According to (7) and (8), it can be deduced that

$$\begin{aligned} \dot{V} = & 2e_w^T P_1 \dot{e}_w + 2e_x^T P_2 \dot{e}_x = 2e_w^T P_1 (-HCEe_w \\ & - HCM\tilde{g}(x, t) + u_s - HCB_f f(t) - HCAe_x \\ & - HCA\Delta\xi(x, t) + \dot{w}) + 2e_x^T P_2 ((A - LC)e_x \\ & + \Delta\xi(x, t) + Ew(t) + M\tilde{g}(x, t) + B_f(f(t) - v)) \end{aligned} \quad (15)$$

From (4), one can check that

$$\begin{aligned} e_x^T P_2 B_f (f(t) - v) = & e_x^T P_2 B_f f(t) - e_x^T P_2 B_f v \\ \leq & \|e_x^T P_2 B_f\| \|f^*\| \\ & - e_x^T P_2 B_f \frac{B_f^T P_2 e_x}{\|e_x^T P_2 G\|} (f^* + \delta) \\ \leq & -\delta \|e_x^T P_2 B_f\| < 0 \end{aligned} \quad (16)$$

Then (15) can be written as

$$\begin{aligned} \dot{V} = & 2e_w^T P_1 \dot{e}_w + 2e_x^T P_2 \dot{e}_x \leq 2e_w^T P_1 (-HCEe_w \\ & - HCM\tilde{g}(x, t) + u_s - \bar{H}\zeta_w - HCAe_x \\ & - HCA\Delta\xi(x, t) + \dot{w}) + 2e_x^T P_2 ((A - LC)e_x \\ & + \Delta\xi(x, t) + \bar{E}\zeta_w + M\tilde{g}(x, t)) \end{aligned} \quad (17)$$

where  $\bar{E} = [0 \ E]$ .

For any two positive scalars  $\sigma$  and  $\sigma_1$ , the following inequalities are satisfied:

$$\begin{aligned} & 2e_x^T P_2 M \bar{g}(x, t) \\ & \leq \sigma e_x^T P_2 M M^T P_2 e_x + \sigma^{-1} \bar{g}^T(x, t) \bar{g}(x, t) \end{aligned} \quad (18)$$

$$\begin{aligned} & \leq e_x^T (\sigma P_2 M M^T P_2 + \sigma^{-1} \eta^2) e_x \\ - 2e_w^T P_1 H C M \bar{g}(x, t) \\ & \leq \sigma_1 e_w^T P_1 H C M M^T C^T H^T P_1 e_w + e_x^T \sigma_1^{-1} \eta^2 e_x \end{aligned} \quad (19)$$

From (3), one can check that

$$\begin{aligned} & 2e_w^T P_1 u_s + 2e_w^T P_1 (-H C \Delta \xi(x, t) + \dot{w}) \\ & + 2e_x^T P_2 \Delta \xi(x, t) \leq 2e_w^T P_1 \frac{P_1 e_w}{\|P_1 e_w\|^2} \alpha(t) \\ & + 2 \|e_w^T P_1 H C\| \|\Delta \xi(x, t)\| + 2 \|e_w^T P_1\| \|\dot{w}\| \\ & + \|2e_x^T P_2\| \|\Delta \xi(x, t)\| \leq 0 \end{aligned} \quad (20)$$

Combining (18) and (19), (17) can be written as

$$\dot{V}(t) \leq \xi(t)^T \Xi \xi(t) \quad (21)$$

where  $\xi(t) = [e_w(t)^T \ e_x(t)^T \ \zeta_w^T]^T$ ,  $\Xi = \begin{bmatrix} \psi_1 & P_1 H C A & -P_1 \bar{H} \\ * & \psi_2 & P_2 \bar{E} \\ * & * & 0 \end{bmatrix}$ ,

$$\begin{aligned} \psi_1 & = -P_1 H C E - (P_1 H C E)^T \\ & + \sigma_1 P_1 H C M M^T C^T H^T P_1, \\ \psi_2 & = P_2 (A - LC) + (A - LC)^T P_2 + \frac{\eta^2}{\sigma} \\ & + \sigma P_2 M M^T P_2 + \frac{\eta^2}{\sigma_1}. \end{aligned} \quad (22)$$

Then the  $H_\infty$  performance index (11) is changed to the following (21):

$$\begin{aligned} J & = \int_0^\infty e(s)^T e(s) ds - \int_0^\infty \gamma^2 \zeta_w(s)^T \zeta_w(s) ds \\ & \leq \int_0^\infty e(s)^T e(s) ds - \int_0^\infty \gamma^2 \zeta_w(s)^T \zeta_w(s) ds \\ & + \int_0^\infty \dot{V} \leq \xi(t)^T \Xi_1 \xi(t) \end{aligned} \quad (23)$$

where  $\Xi_1 = \begin{bmatrix} \psi_a & P_1 H C A & -P_1 \bar{H} \\ * & \psi_b & P_2 \bar{E} \\ * & * & -\gamma^2 \end{bmatrix}$ ,  $\psi_a = \psi_1 + I$ ,  $\psi_b = \psi_2 + I$ .

Thus, according to (12) and the Schur complement, the dynamic error system is stable and the  $H_\infty$  performance index is satisfied. This completes the proof.  $\square$

From Theorem 1, the estimation errors  $e_x$ ,  $e_w$ , and  $e_f = f(t) - v(t)$  will converge to zero asymptotically; i.e., there exist

two positive scalars  $\bar{\kappa}_1, \bar{\kappa}_2$ , such that  $\|e_f\| \leq \bar{\kappa}_1$ ,  $\|e_w\| \leq \bar{\kappa}_2$ . In the following part, we will reconstruct the fault signal.

Define the following sliding mode surface

$$S = \{S(t) \mid S = e_x\} \quad (24)$$

**Theorem 2.** *With the observer (2) and the condition (24), if (12) has a feasible solution, then the state of the system will be driven to the sliding surface  $S$  in finite time, if the following condition is satisfied*

$$\begin{aligned} & \delta \\ & > \|B_f\|^{-1} \{ \|\Delta \xi(x, t)\| + \|E\| \|w(t)\| + \|M\| \|\bar{g}(x, t)\| \} \end{aligned} \quad (25)$$

and the fault signal can be approximated by

$$f(t) \approx \frac{B_f^T P_2 e_x}{\|e_x^T P_2 B_f\|} (f^* + \delta) \quad (26)$$

*Proof.* The Lyapunov candidate function is defined as follows:

$$V_1 = e_x^T P_2 e_x \quad (27)$$

The derivation of (27) can be obtained as

$$\begin{aligned} \dot{V}_1 & = 2e_x^T P_2 \dot{e}_x = 2e_x^T P_2 ((A - LC) e_x + \Delta \xi(x, t) \\ & + Ew(t) + M\bar{g}(x, t) + B_f(f(t) - v)) \end{aligned} \quad (28)$$

From Theorem 1, if (12) holds, then  $\psi_n = P_2(A - LC) + (A - LC)^T P_2 + I < 0$ ; one can check that  $2e_x^T P_2(A - LC)e_x < 0$ ; then (28) can be rewritten as follows (29):

$$\begin{aligned} \dot{V}_1 & \leq 2e_x^T P_2 (\Delta \xi(x, t) + Ew(t) + M\bar{g}(x, t) \\ & + B_f(f(t) - v)) \leq \|2e_x^T P_2 \Delta \xi(x, t)\| \\ & + \|2e_x^T P_2 Ew(t)\| + \|2e_x^T P_2 M\bar{g}(x, t)\| \\ & + \|2e_x^T P_2 B_f\| (f(t) - v) \leq \|2e_x^T P_2\| \|B_f\| \\ & \cdot (\|B_f\|^{-1} \|\Delta \xi(x, t)\| + \|B_f\|^{-1} \|Ew(t)\| \\ & + \|B_f\|^{-1} \|M\bar{g}(x, t)\|) - \|2e_x^T P_2\| \|B_f\| \delta \\ & = -2 \|e_x^T P_2\| \|B_f\| (\delta - \|B_f\|^{-1} \|\Delta \xi(x, t)\| \\ & + \|E\| \|w(t)\| + \|M\| \|\bar{g}(x, t)\|) \leq -2 \|e_x^T P_2 B_f\| \\ & \leq -2 \|B_f\| \sqrt{\lambda_{\min}(P_2)} V_1 \end{aligned} \quad (29)$$

Thus, we can conclude that the state of the system will reach the sliding mode surface in finite time. When the state of the system reaches the sliding surface, one can obtain that

$$S = e_x = 0 \quad (30)$$

From (30), it can be obtained that

$$0 = \Delta\xi(x, t) + Ew(t) + M\bar{g}(x, t) + B_f(f(t) - v) \quad (31)$$

From Theorem 1, there exist a normal scalar  $\mu_a$  which satisfies  $\|\bar{g}(x, t)\| \leq \mu_a$ ; it can be obtained by (31) that

$$\begin{aligned} f(t) - v &= -B_f^\dagger (\Delta\xi(x, t) + Ew(t) + M\bar{g}(x, t)) \\ &\leq \lambda_{\min}(B_f^\dagger) \|\Delta\xi(x, t)\| \\ &\quad + \lambda_{\min}(B_f^\dagger E) \|w(t)\| \\ &\quad + \lambda_{\min}(B_f^\dagger M) \|\bar{g}(x, t)\| \\ &\leq \lambda_{\min}(B_f^\dagger) \omega_\kappa + \lambda_{\min}(B_f^\dagger E) \omega \\ &\quad + \lambda_{\min}(B_f^\dagger M) \mu_a \leq \sigma_\omega \end{aligned} \quad (32)$$

where  $B_f^\dagger = (B_f^T B_f)^{-1} B_f^T$  and

$$\begin{aligned} \sigma_\omega &\geq \lambda_{\min}(B_f^\dagger) \omega_\kappa + \lambda_{\min}(B_f^\dagger E) \omega \\ &\quad + \lambda_{\min}(B_f^\dagger M) \mu_a \end{aligned} \quad (33)$$

It can be seen from (33) that if  $\sigma_\omega$  is small enough, (26) is established. It can be seen that (26) is a discontinuous functions; (34) formula can be used to replace (26)

$$f(t) \approx \frac{B_f^T P_2 e_x}{\|e_x^T P_2 B_f\| + \alpha_m} (f^* + \delta) \quad (34)$$

where  $\alpha_m$  is a positive constant.  $\square$

**3.2. Controller Design.** The main purpose of this part is to design a controller to guarantee the stability of the system. In this paper, a novel integral sliding mode method based on the observer is presented, and the reaching ability is proved; the controller is designed as follows:

$$u(t) = u_1(t) + u_2(t) \quad (35)$$

The sliding mode surface is designed as

$$\begin{aligned} s(t) &= G(y(t) - y(0) - C \int_0^t (A\hat{x}(\theta) + Bu_2(\theta) \\ &\quad + E\hat{w}(\theta) + B_f v(\theta) + M\bar{g}(\hat{x}, t)) d\theta \end{aligned} \quad (36)$$

where  $G$  is a parameter matrix, which need to be chosen to satisfy that  $GCB$  is inevitable.

From (36), (37) can be obtained

$$\begin{aligned} \dot{s}(t) &= GC(\dot{x}(t) - A\hat{x}(t) - Bu_2(t) - E\hat{w}(t) \\ &\quad - B_f v(t) - M\bar{g}(\hat{x}, t)) = GCBu_1(t) + GC(Ae_x \\ &\quad + Ee_w(t) + B_f e_f(t) + M\bar{g}(x, t) + \Delta\xi(x, t)) \end{aligned} \quad (37)$$

From Theorems 1 and 2, the estimation errors  $e_x$ ,  $e_w$ ,  $e_f$ , and  $\bar{g}(x, t)$  are bounded; in accordance with (1), one can define the following equivalent uncertain term:

$$\begin{aligned} \phi &= \|Ae_x\| + \|Ee_w(t)\| + \|Ge_f(t)\| + \|M\bar{g}(x, t)\| \\ &\quad + \|\Delta\xi(x, t)\| \end{aligned} \quad (38)$$

Then, one can check that  $\phi$  is bounded.

Then the controller  $u_1(t)$  is given by

$$u_1(t) = -\psi(t) \frac{(GCB)^T s(t)}{\|s^T GCB\|} \quad (39)$$

where  $\psi(t) = \hat{\phi} + \delta_k$ ,  $\delta_k$  is a positive scalar and  $\hat{\phi}$  is the estimation of  $\phi$  with the following adaptive rules:

$$\dot{\hat{\phi}} = \beta_m (\|s^T GCB\| + \delta_m) \quad (40)$$

where  $\beta_m$  and  $\delta_m$  are two positive scalars.

**Lemma 3** (see [19]). *With the sliding mode surface (40) and controller (39),  $\hat{\phi}$  is bounded; i.e., there exist a positive constant such that the following inequality is satisfied:*

$$\hat{\phi} \leq \bar{\phi}, \quad \forall t > 0 \quad (41)$$

**Theorem 4.** *With the controller (39), the system trajectory will converge to the sliding mode surface  $s$  in finite time.*

*Proof.* The Lyapunov function is given as

$$V_2(t) = \frac{1}{2} s^T(t) s(t) + \frac{1}{2\beta_m} (\hat{\phi} - \bar{\phi})^2 \quad (42)$$

The derivation of (42) can be obtained as

$$\begin{aligned} \dot{V}_2(t) &= s^T(t) \dot{s}(t) + (\hat{\phi} - \bar{\phi}) \dot{\hat{\phi}} = s^T(t) GCBu_1(t) \\ &\quad + s^T(t) GCB(Ae_x + Ee_w(t) + Ge_f(t) + M\bar{g}(x, t) \\ &\quad + \Delta\xi(x, t)) + (\hat{\phi} - \bar{\phi}) \dot{\hat{\phi}} \leq -\psi(t) s^T(t) GC \\ &\quad \cdot \frac{(GCB)^T s(t)}{\|s^T GCB\|} + \|s^T(t) GCB\| \phi + (\hat{\phi} - \bar{\phi}) \dot{\hat{\phi}} \\ &= -\|s^T GCB\| (\phi - \hat{\phi}) - \delta_k \|s^T GCB\| + (\hat{\phi} - \bar{\phi}) \\ &\quad \cdot (\|s^T GCB\| + \delta_m) \leq -\delta_k \|s^T GCB\| - \delta_m |\hat{\phi} - \bar{\phi}| \\ &< 0 \end{aligned} \quad (43)$$

Then the trajectory of the system will reach the sliding mode surface in finite time. This completes the proof.  $\square$

When the system reaches the sliding surface, i.e.,  $\dot{s}(t) = 0$ , then the equivalent control of the system can be obtained

$$\begin{aligned} u_{eq}(t) &= -(GCB)^{-1} GC(Ae_x + Ee_w(t) + B_f e_f(t) \\ &\quad + M\bar{g}(x, t) + \Delta\xi(x, t)) \end{aligned} \quad (44)$$

In this paper, the idea of equivalent control is applied to design the controller. In the next part, the design of  $u_2(t)$  and the stability analysis of the system are given.  $u_2(t)$  is designed as

$$u_2(t) = -(GCB)^{-1} \cdot GC \left( A\hat{x}(t) + E\hat{x}(t) + B_f v(t) + M\hat{g}(x, t) \right) - K_1 \hat{x}(t) - K_2 \hat{w}(t) - K_3 \hat{f}(t) + u_k(t) \quad (45)$$

where  $u_k(t)$  is given as follows:

$$u_k(t) = -\frac{B^T \bar{N}^T Q x(t)}{\|x^T(t) Q \bar{N} B\|^2} \|x^T(t) Q \bar{N}\| \bar{\omega}_\kappa \quad (46)$$

where  $\bar{N} = I - B(GCB)^{-1}GC$  and  $Q$  is a positive symmetric matrix.

Substituting (45) and (46) into (1), the closed loop system can be obtained as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B(u_2(t) + u_{eq}(t)) + Ew(t) \\ &\quad + B_f f(t) + \Delta\xi(x, t) + Mg(x, t) \\ &= (\bar{A} - BK_1)x(t) + (\bar{E} - BK_2)w(t) \\ &\quad + (\bar{B}_f - BK_3)f(t) + \bar{M}(x, t) + \bar{N}\Delta\xi(x, t) \\ &\quad + BK_1 e_x + BK_2 e_w + BK_3 e_f + Bu_k(t) \end{aligned} \quad (47)$$

where  $\bar{A} = A - B(GCB)^{-1}GCA$ ,  $\bar{E} = E - B(GCB)^{-1}GCE$ ,  $\bar{B}_f = B_f - B(GCB)^{-1}GCB_f$ , and  $\bar{M} = M - B(GCB)^{-1}GCM$ .

Take  $\mathcal{H}_e = [\bar{E} - BK_2 \quad \bar{B}_f - BK_3 \quad \bar{M} \quad BK_1 \quad BK_2 \quad BK_3]$ ;  $w_e = [w(t)^T \quad f(t)^T \quad g^T(x, t) \quad e_x^T \quad e_w^T \quad e_f^T]^T$ ; then the upper (48) can be written as (49)

$$\begin{aligned} \dot{x}(t) &= (\bar{A} - BK_1)x(t) + \mathcal{H}_e w_e(t) + \bar{N}\Delta\xi(x, t) \\ &\quad + Bu_k(t) \end{aligned} \quad (48)$$

The following theorem gives the stability analysis of the system.

**Theorem 5.** With (44)-(46), system (48) is asymptotically stable and the  $H_\infty$  performance

$J = \int_0^t y(\tau)^T y(\tau) d\tau \leq \gamma^2 \int_0^t \mathcal{H}_e(\tau)^T \mathcal{H}_e(\tau) d\tau$  is satisfied, if there exist a positive symmetric matrix  $Q$ , such that the following conditions holds:

$$\begin{bmatrix} Q(\bar{A} - BK_1) + (\bar{A} - BK_1)^T Q & Q\mathcal{H}_e & C^T \\ * & -\gamma^2 & 0 \\ * & * & -I \end{bmatrix} < 0 \quad (49)$$

*Proof.* The Lyapunov function is designed as

$$V(t) = x(t) Q x(t) \quad (50)$$

The derivation of (51) can be obtained as

$$\begin{aligned} \dot{V}(t) &= 2x^T(t) Q \dot{x}(t) \\ &= 2x^T(t) Q (\bar{A} - BK_1)x(t) \\ &\quad + 2x^T(t) Q \mathcal{H}_e w_e(t) \\ &\quad + 2x^T(t) Q (\bar{N}\Delta\xi(x, t) + Bu_k(t)) \end{aligned} \quad (51)$$

From (46), one can obtain that

$$\begin{aligned} &x^T(t) Q (\bar{N}\Delta\xi(x, t) + Bu_k(t)) \\ &\leq \|x^T(t) Q \bar{N}\| \|\Delta\xi(x, t)\| \\ &\quad - x^T(t) Q \bar{N} B \frac{B^T \bar{N}^T Q x(t)}{\|x^T(t) Q \bar{N} B\|} \|x^T(t) Q \bar{N}\| \bar{\omega}_\kappa \\ &\leq 0 \end{aligned} \quad (52)$$

Then, (51) can be rewritten as

$$\begin{aligned} \dot{V}(t) &\leq 2x^T(t) Q (\bar{A} - BK_1)x(t) \\ &\quad + 2x^T(t) Q \mathcal{H}_e w_e(t) \end{aligned} \quad (53)$$

Then, on can obtain that

$$\begin{aligned} J &= \int_0^t y(\tau)^T y(\tau) d\tau - \gamma^2 \int_0^t w_e(\tau)^T w_e(\tau) d\tau \\ &\leq \int_0^t y(\tau)^T y(\tau) d\tau - \gamma^2 \int_0^t w_e(\tau)^T w_e(\tau) d\tau + \dot{V}(t) \\ &= 2x^T(t) Q (\bar{A} - BK_1)x(t) + 2x^T(t) Q \mathcal{H}_e w_e(t) \\ &= \zeta^T(t) \Xi_a \zeta(t) \end{aligned} \quad (54)$$

where  $\zeta^T(t) = [x^T(t) \quad \mathcal{H}_e^T]^T$ ,  $\Xi_a = \begin{bmatrix} Q(\bar{A} - BK_1) + (\bar{A} - BK_1)^T Q + C^T C & Q\mathcal{H}_e \\ * & -\gamma^2 \end{bmatrix}$ .

According to (49),  $J < 0$ , which implies that the system is stable and the  $H_\infty$  performance is satisfied. The proof is completed.  $\square$

## 4. Simulation

In this part, two examples are used to verify the effectiveness of the proposed method.

*Example 1.* In this part, the flexible single joint manipulator system borrowed from [7] is used to verify the effectiveness of the proposed method, the diagram for the flexible single joint

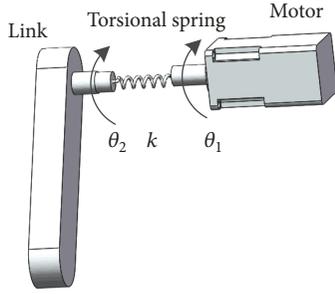


FIGURE 1: Diagram for flexible single joint structure.

is shown as Figure 1 [5], the joint is driven by a DC motor, and its dynamic system can be represented as

$$\begin{aligned}
 \dot{\theta}_m &= w_m \\
 \dot{w}_m &= \frac{k}{J_m} (\theta_1 - \theta_m) - \frac{B}{J_m} w_m + \frac{k_\tau}{J_m} u \\
 \dot{\theta}_1 &= w_1 \\
 \dot{w}_1 &= \frac{k}{J_1} (\theta_1 - \theta_m) - \frac{mgq}{J_1} \sin(\theta_1)
 \end{aligned} \tag{55}$$

where  $J_m$  is the inertia of the DC motor,  $J_1$  is the inertia of the link, and  $\theta_m$  and  $\theta_1$  are the angle positions of the motor and the link, respectively.  $w_m$  and  $w_1$  represent the angular velocities of the motor and link.  $k$  is the torsional spring constant,  $k_\tau$  is the amplifier gain,  $m$  is the mass,  $B$  is the viscosity coefficient,  $g$  is the gravitational constant, and  $q$  is the distance between the rotor center and the center of gravity of the link;  $u$  is the control input delivered by the motor. Taking the system state as  $x(t) = [\theta_m \ w_m \ \theta_1 \ w_1]^T$ , the system can be rewritten in the form of (1), where

$$\begin{aligned}
 A &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 10 \\ 1.95 & 0 & -1.95 & 0 \end{bmatrix}, \\
 B &= \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix}, \\
 M &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.333 \end{bmatrix},
 \end{aligned}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$E = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}.$$

(56)

Assuming  $B_f = B$ , the system nonlinear term  $g(x, t) = \sin(x_3)$ . The actuator fault and disturbance signal are assumed as

$$\begin{aligned}
 f(t) &= \begin{cases} 0 & (0 \leq t < 8) \\ 0.5 \sin(0.5t) & (8 \leq t < 12) \\ 0.2 & (12 \leq t < 20), \end{cases} \\
 w(t) &= \begin{cases} 0 & (0 \leq t < 5) \\ 0.4 \sin(0.4t) & (5 \leq t < 10) \\ 0.5 & (10 \leq t < 20). \end{cases}
 \end{aligned} \tag{57}$$

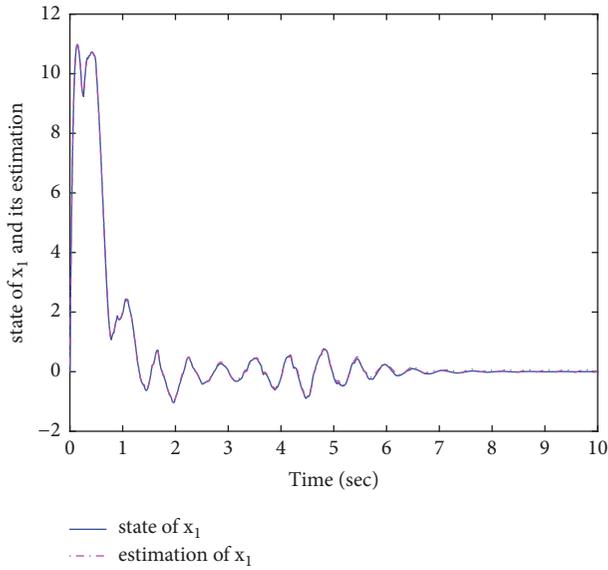
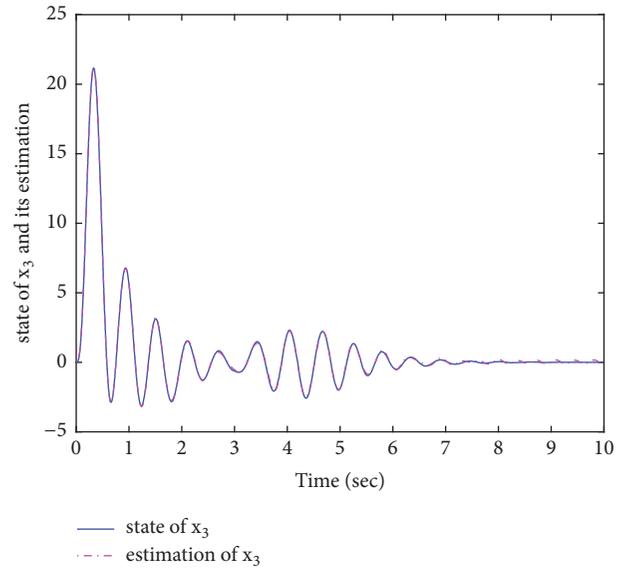
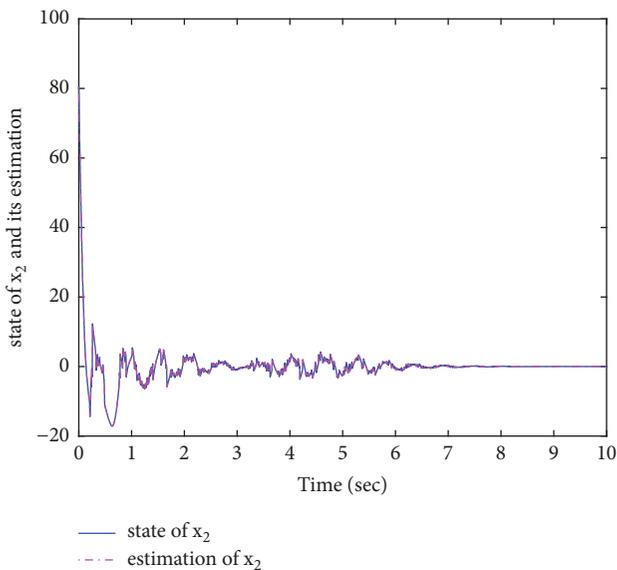
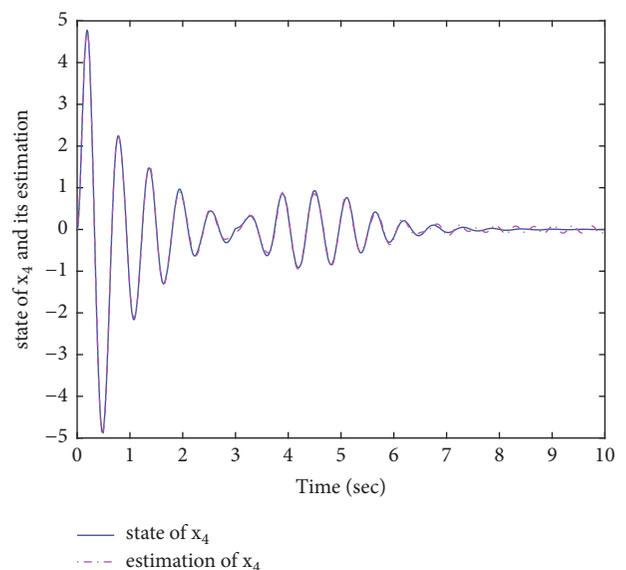
In addition, we assume that system uncertainty has the following forms:  $\Delta\xi(x, t) = \vartheta_m q(x) + \vartheta_n$ , in which  $\|q(x)\| \leq q_m$ ,  $q_m$ ,  $\vartheta_n$  are known parameters.

By solving inequalities (12) and (49), we can obtain that

$$\begin{aligned}
 P_2 &= \begin{bmatrix} 0.6848 & 0.0014 & -0.2221 & -0.1254 \\ 0.0014 & 0.8652 & 0.0015 & 0.0016 \\ -0.2221 & 0.0015 & 0.6223 & -1.1022 \\ -0.1254 & 0.0016 & -1.1022 & 0.4829 \end{bmatrix}, \\
 L &= \begin{bmatrix} 0.3333 & -14.8916 & -0.2074 \\ -5.8147 & 0.1869 & 7.0873 \\ 0.2728 & 13.8298 & 0.6310 \\ -0.6510 & 0.0010 & 2.7158 \end{bmatrix}, \\
 H &= [9.6590 \ 0.0030 \ 0.1060], \\
 P_1 &= 0.1. \\
 Q &= \begin{bmatrix} 2.4916 & 0.7847 & -0.7115 & 0.8784 \\ 0.7847 & 0.7428 & -0.4084 & 0.4250 \\ -0.7115 & -0.4084 & 0.5576 & -0.2803 \\ 0.8784 & 0.4250 & -0.2803 & 2.1097 \end{bmatrix}.
 \end{aligned} \tag{58}$$

The simulation results are as given in Figures 2–8

Figures 2–5 show the states of the system and their estimated response curves, respectively. Figure 6 shows the external disturbance and the estimation curve of the system. Figure 7 expresses the fault and its estimation curves. Figure 8 illustrates the control input signal. From Figures 2–5, it can

FIGURE 2: State of  $x_1$  and its estimation.FIGURE 4: State of  $x_3$  and its estimation.FIGURE 3: State of  $x_2$  and its estimation.FIGURE 5: State of  $x_4$  and its estimation.

be observed that the states of the system are stable in a limited time in the presence of and fault signals and external disturbance. These results also show that the system has better convergence with the controller, which means the proposed control method can effectively deal with the above problems.

Seen from Figure 6, the accuracy of the observer in this paper is good, which can effectively detect the disturbance signals of the system, thus helping the subsequent controller design. From Figure 7, it can be seen that the fault signal of the system can be reconstructed effectively by adjusting the parameters of the sliding mode observer, which indicates that the observed signal can be used in the design of the controller, and the effectiveness of the method is verified. From Figure 8, one can observe that the controller works when fault occurs

and in the presence of disturbance. The simulation results illustrate the effectiveness of the proposed method.

*Example 2.* In this section, the two-cart system which borrowed from [20] is provided to illustrate the effectiveness of the proposed method. The geometric of the system is shown as in Figure 9 [20].

As shown in Figure 9, the first cart is connected to a rigid wall via a damper, and is connected to a second cart by a spring. The external force is applied to a second cart via an actuator. Both carts have a nominal mass of  $a = 1\text{kg}$ , the damper have a constant of  $b_0 = 1\text{N/m}$  and the spring constant  $c_0 = 1\text{N/m}$ . The time constant of the actuator is  $\tau = 0.2$ . The states are the force, velocities and positions of

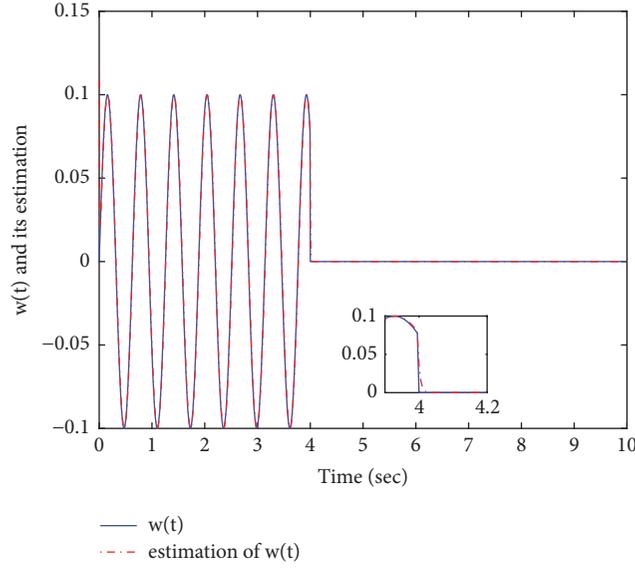


FIGURE 6: Disturbance and its estimation.

the two carts. The actuator fault and unmatched disturbance are considered. The system parameters are given as follows:

$$A = \begin{bmatrix} -\frac{1}{\tau} & 0 & 0 & 0 & 0 \\ 0 & -\frac{b_0}{a} & 0 & -\frac{c_0}{a} & \frac{c_0}{a} \\ \frac{1}{a} & 0 & 0 & \frac{c}{a} & -\frac{c_0}{a} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} \frac{1}{\tau} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$B_F = \begin{bmatrix} \frac{1}{\tau} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$B_d = \begin{bmatrix} 1 & 1 \\ 0 & 0.2 \\ 0 & 0.2 \\ 0 & 0.1 \\ 0 & 0.2 \end{bmatrix}.$$

(59)

The nonlinear unmodeled uncertainty and unmatched nonlinearity are assumed to be  $\Delta f(x, t) = \sin(x_d)$ ,  $\xi(x, t) = \sin(x_1)$ . The actuator fault and external disturbance are supposed to be as follows:

$$f(t) = \begin{cases} 0 & (0 \leq t < 25) \\ 0.5 \sin(0.2t) & (25 \leq t < 40) \\ 1 & (40 \leq t < 80), \end{cases}$$

$$w_1(t) = \begin{cases} 0 & (0 \leq t < 15) \\ 0.5 \sin(t) & (15 \leq t < 50) \\ 1 & (50 \leq t < 80), \end{cases} \quad (60)$$

$$w_2(t) = \begin{cases} 0.5 \sin(3t) & (0 \leq t < 20) \\ 0.5 \sin(3t) & (20 \leq t < 50) \\ 1 & (50 \leq t < 80). \end{cases}$$

Choosing the matrix  $G = [1 \ 0 \ 0 \ 0 \ 0]$ , and one can check that  $GCB$  is invertible. By solving (52), one can obtain that

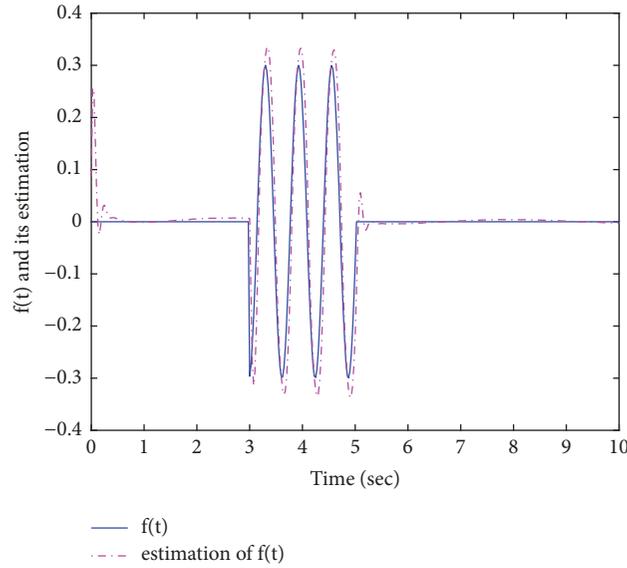
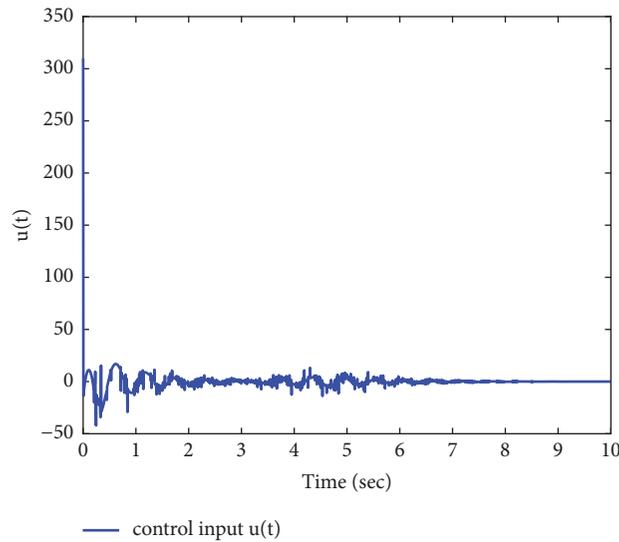


FIGURE 7: Fault and its estimation.

FIGURE 8: Control input  $u(t)$ .

$$P = \begin{bmatrix} 0.0799 & -0.000 & -0.000 & -0.000 & -0.000 \\ -0.000 & 1.1046 & 0.000 & -0.000 & -0.000 \\ -0.000 & 0.000 & 1.1046 & 0.000 & -0.000 \\ -0.000 & -0.000 & 0.000 & 1.1046 & 0.000 \\ -0.000 & -0.000 & -0.000 & 0.000 & 1.1046 \end{bmatrix}. \quad (61)$$

The results of the simulation are as in Figures 10–14.

Figures 10 and 11 characterize the trajectories of the system; from Figures 10–11, it can be seen that the stability of the positions and velocities of the first and second cart can be guaranteed. Figure 12 shows the estimation of the actuator fault; one can clearly check that the proposed method performance better than the intermediate method

proposed in [20], precisely, the method proposed response faster than the method in [20], and the proposed method has less chattering.

To verify the proposed method, the responses of the system are shown in Figures 13–14 without controller. From the two figures, one can conduct that the closed loop system becomes unstable when removing the observer and compensator. The simulation results illustrate the effectiveness of the proposed method.

## 5. Conclusion

In this paper, a class of nonlinear systems with disturbance, fault, and uncertainty is studied. The state and disturbance

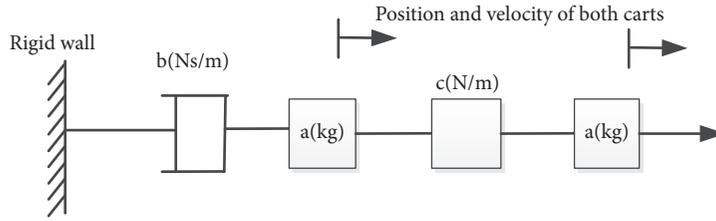


FIGURE 9: Geometric structure of the two-cart system.

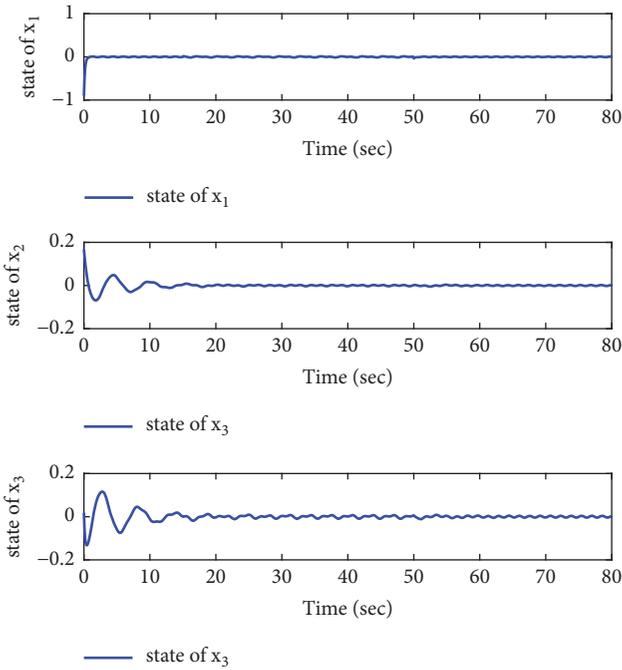


FIGURE 10: Response of state  $x_1(t)$ ,  $x_2(t)$ , and  $x_3(t)$ .

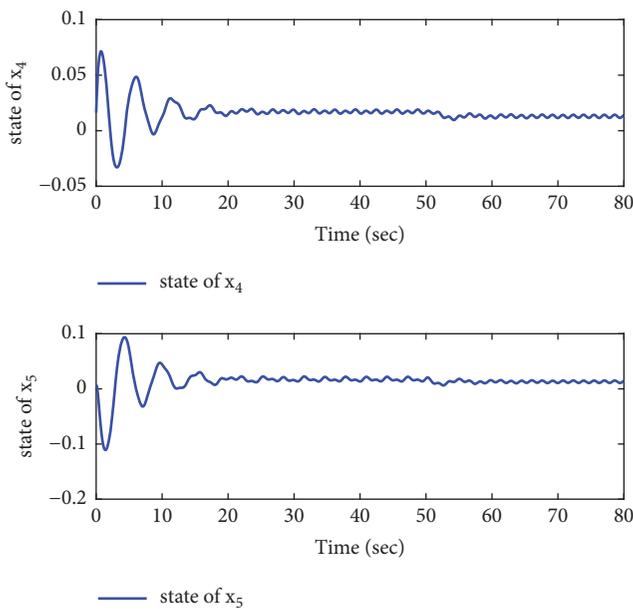


FIGURE 11: Response of state  $x_4(t)$  and  $x_5(t)$ .

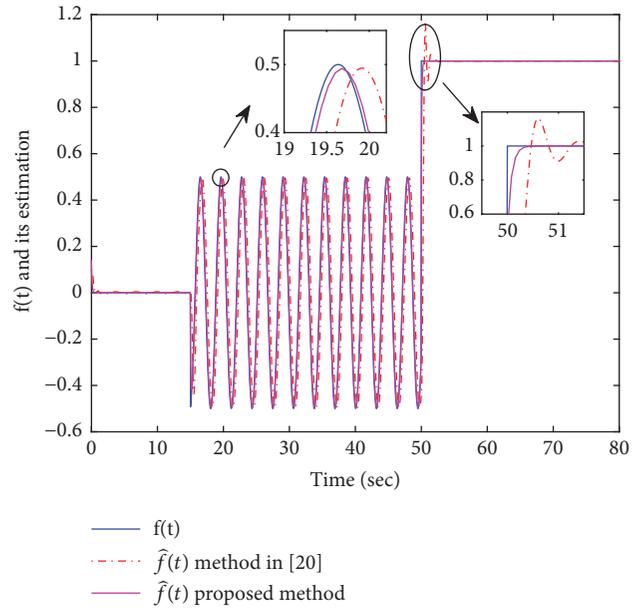


FIGURE 12: Estimation of actuator fault  $f(t)$ .

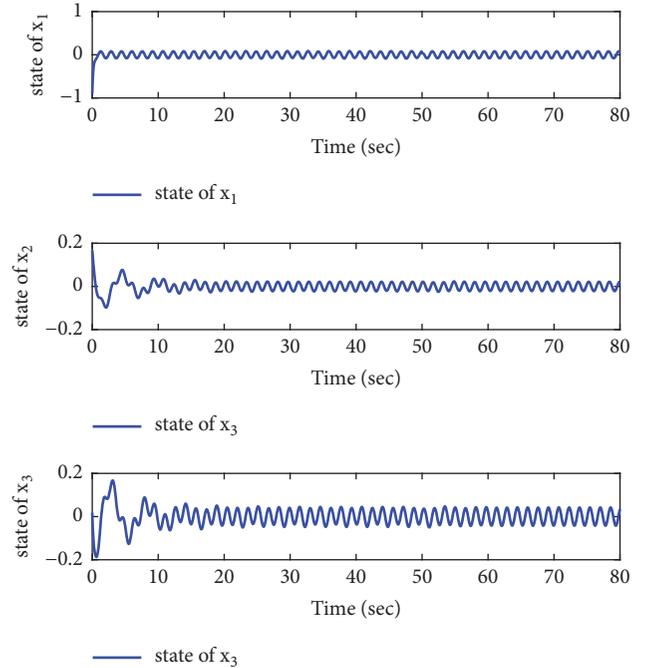


FIGURE 13: Response of state  $x_1(t)$ ,  $x_2(t)$ , and  $x_3(t)$  without controller.

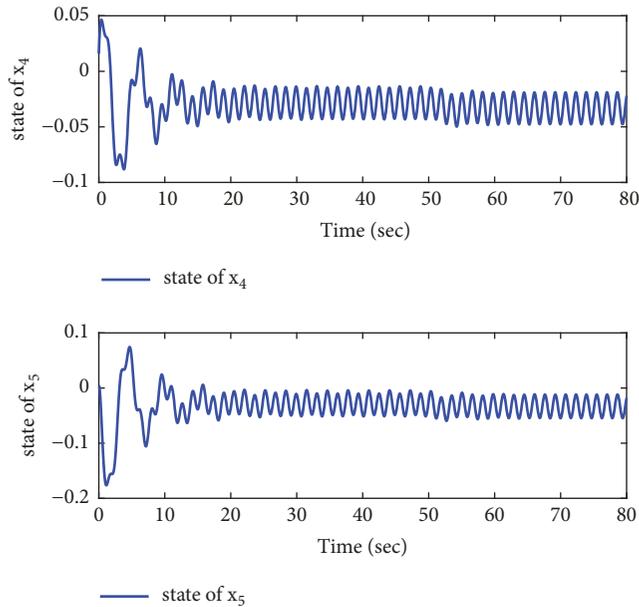


FIGURE 14: Response of state  $x_4(t)$  and  $x_5(t)$  without controller.

estimation of the system are realized by designing an observer first, while the sliding mode observer technology is introduced to reconstruct the fault signal. Then, by using the observed information, a novel robust integral sliding mode controller is presented. Furthermore, the reaching ability is proved and the stability of the closed loop system is analysed. Finally, the method is applied to a class of single joint manipulator system and a two-cart system. The simulation results show that the observer has good response characteristics, and the controller has good fault tolerant ability.

## Data Availability

The data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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