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Research Article

Adaptive Aperiodically Intermittent Synchronization for Complex Dynamical Network with Unknown Time-Varying Outer Coupling Strengths

Lihong Yan [1,2] and Junmin Li [1]

[1] *School of Mathematics and Statistics, Xidian University, Xi'an 710071, China*
[2] *College of Mathematics and Information Science, Xianyang Normal University, Xianyang 712000, China*

Correspondence should be addressed to Lihong Yan; 092212@163.com

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In this paper, exponential synchronization problem of complex dynamical networks with unknown periodically coupling strengths was investigated. An aperiodically intermittent control synchronization strategy is proposed. Based on Lyapunov exponential stability theory, inequality techniques, and adaptive learning laws design, some sufficient exponential synchronization criteria for complex dynamical network with unknown periodical coupling weights are obtained. The numerical simulation example is presented to illustrate the feasibility of theoretical results.

1. Introduction

Complex dynamical networks, such as biological network, urban transportation network and human relationship network, are ubiquitous in natural world and social society generally. Researchers were mainly concerned with the issues of modelling, properties analysis, dynamical evolution, and synchronization control of complex dynamical networks. Among these issues involved, synchronization is one of the most interesting topics and has been extensively investigated [1–12]. There are mainly two kinds of strategies for synchronization of complex dynamical networks: one is to improve the network synchronization capability by changing the properties of the network itself, such as topology structure and coupling strengths. The other is to act on the network with external control injection, which is a representative of control theory, mainly including variable feedback control method, pinning control method [2, 4], adaptive control method [5], event-triggered control [8], impulse control method, intermittent pinning control [9], and slide mode control method [10, 11].

We can see that most of the above control methods are continuous, which requires continuous information exchange and increases the cost of control. However, the discontinuous control method such as intermittent control, exerting intermittent control over the controlled objects, can reduce the amount of the transmitted information and be much more economic. Zochowski firstly introduced intermittent control into dynamic control systems [12]. After that, many researchers pay attention to this control strategy and many large-scale dynamical systems are controlled successfully with the help of intermittent control. So, it can also be applied to realize synchronization of complex dynamical networks [13–23].

Over the first few years, the intermittent controller which can help realizing synchronization for complex dynamical networks applied is usually periodical. In papers [13–16], the authors investigated finite-time synchronization of complex dynamical networks with multilink, time delay, and multiswitch periods using periodically intermittent control scheme. Zhao and Cai [17] investigated the exponential synchronization of complex delayed dynamical networks with uncertain parameters adopting the intermittent control scheme, basing on the Lyapunov stability theory combined with the method of the adaptive control. Then semiperiodically intermittent control method was proposed, by applying multiple Lyapunov function method and the mode-dependent average dwell time approach. Qiu [18] derived less conservative synchronization criteria and the synchronization problem for switched complex networks with delayed coupling via semiperiodically intermittent control technique and a mode-dependent average dwell time method was studied. Liu [19] addressed synchronization problems of time-delay coupled network by aperiodically intermittent control. In 2017, with the help of aperiodically intermittent control method, Liu [20] discussed finite-time synchronization of delayed dynamical networks and showed the convergence time does not depend on control widths or rest widths. Combining intermittent control with pinning control scheme, which can greatly reduce control cost, researchers presented some conclusions about intermittent pinning control. Wang [21] proposed a new differential inequality, dealing with the synchronization problem of complex dynamical network with constant coupling, discrete-delay coupling, and distributed-delay coupling. The synchronization problem of switched complex networks with unstable modes and stochastic complex-valued dynamical networks by aperiodically intermittent adaptive control are discussed in [22, 23], respectively.

It is noteworthy that the above aperiodically intermittent adaptive control results can ensure all nodes of complex dynamical network realizing exponential synchronization, which can make the system tend to equilibrium state much more quickly compared with the asymptotic synchronization of networks [24]. In [25], the exponential synchronization problem of a class of hybrid coupled complex dynamical network with time-varying delay is studied. By designing appropriate intermittent feedback controller, a new synchronization criterion is presented. In paper [26], a kind of complex network model with directional topology and time-delay is studied, and sufficient conditions for global exponential synchronization of the system are obtained. Zhang et al. [27] investigated exponential stability of stochastic differential equations with impulse effects. As seen from the above literatures, exponential synchronization of complex dynamical network via aperiodically intermittent control could save the control cost, which is significant in practical applications.

What is more, with the research of complex dynamic network control, researchers found that the differences of nodes and coupling modes play an important role for the evolution of complex dynamical networks. In addition, many practical complex dynamical networks are time-varying generally, in which, the changes of link strengths usually lead to variations of the network topology and coupling configuration [16, 17]. Therefore, it is meaningful to investigate the synchronization problem of complex dynamical networks with unknown time-varying coupling weights. In order to find the time-varying law of the complex dynamical networks, researchers usually use adaptive methods to estimate the coupling strengths of the network [21]. In paper [5], the author analyzed mean square synchronization of time-delay coupled network, and the unknown periodical coupling weight was estimated successfully with the adaptive leaning update laws and proper controller designed. In 2015, Hao and Li [28] combined adaptive control with learning control to estimate the network unknown periodic coupling structure under the stochastic disturbance successfully. However, the controllers designed in [5, 28] are all continuous, the discontinuous method such as intermittent control was not taken into account. Not only that, but also the intrinsic time delay is ubiquitous in neural networks [29], chaotic attractors [30], and even complex dynamical networks [31], which will affect the synchronization performance. So the synchronization of complex dynamical network with time delay is needed to be investigated.

Motivated by the above discussions, we will consider the exponential synchronization problem for time-delay complex dynamical networks with unknown time-varying coupling weights by using an adaptive aperiodically intermittent controller in this paper. The main contribution of our paper is that, combining the advantages of aperiodic intermittent control, adaptive control, and learning control, a new type of aperiodically intermittent synchronization of complex dynamical network is accomplished when the time varying coupling weights among nodes are unknown primitively. The rest of this paper is organized as follows. A new complex dynamical network model is presented, and the problem formulation and preliminaries are presented in Section 2. Section 3 proposes the aperiodically intermittent synchronization approach for complex dynamical networks. In Section 4, a numerical example is given to illustrate the effectiveness of the designed method. Finally, conclusions are presented in Section 5.

2. Problem Statement and Preliminaries

In the paper, the time-delay complex dynamical network with unknown periodical outer coupling strengths is given as follows:

$$\begin{aligned}\dot{x}_i(t) &= f\left(t, x_i(t), x_i\left(t-\tau_0(t)\right)\right) \\ &\quad + \sum_{j=1}^{N} c_{ij}(t)\, a_{ij} \Gamma x_j(t) + u_i, \quad i = 1, 2, \ldots, N.\end{aligned} \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in R^n$ is the state variable of node i and $f : R^+ \times R^n \times R^n \longrightarrow R^n$, $i = 1, 2, \ldots, N$ is a smooth nonlinear function, describing the local dynamics of each node for network (1). $\tau_0(t)$ is the unknown bounded time-varying delay, and there exists a positive constant τ_0 such that $0 \le \tau_0(t) \le \tau_0$. The positive definite $\Gamma \in R^{n\times n}$ is the inner coupling matrix. $u_i(t)$ is the outer controller to be designed later. The matrix $A = (a_{ij})_{N\times N} \in R^{N\times N}$ is the outer coupling matrix, which describes the topology structure of the whole network. And it is defined as follows: if there is a directional connection from node j to node $i (i \ne j)$, then $a_{ij} \ne 0$; otherwise $a_{ij} = 0$. $c_{ij}(t)$ is the unknown periodical coupling strengths between nodes i and j. These elements satisfy the following condition:

$$c_{ii}(t)\, a_{ii} = -\sum_{\substack{j=1 \\ j\ne i}}^{N} c_{ij}(t)\, a_{ij}, \quad i = 1, 2, \ldots, N. \tag{2}$$

The initial conditions of network (1) are assumed to be $x_i(t) = \psi_i(t) \in C([-\tau_0, 0], R^n)$, where $C([-\tau_0, 0], R^n)$ represents the set of all n-dimensional continuous functions defined on interval $[-\tau_0, 0]$.

Without loss of generality, let the solution $s(t) \in R^n$ of (3) be the global exponential synchronization goal orbit:

$$\dot{s}(t) = f(t, s(t), s(t - \tau_0(t))). \tag{3}$$

Let $e_i(t) = x_i(t) - s(t)$; then the error system is

$$\dot{e}_i(t) = \dot{x}_i(t) - \dot{s}(t), \quad i = 1, 2, \ldots, N. \tag{4}$$

In the paper, we will add proper designed controllers $u_i(t)$, $i = 1, 2, \ldots, N$ intermittently to the dynamical network (1) to realize globally exponential synchronization.

Definition 1. The complex dynamical network (1) is said to be globally exponentially synchronized, if there exist two positive constants p and q such that, for any initial state $x_i(t) = \psi_i(t) \in C([-\tau_0, 0], R^n)$, the synchronous error satisfies

$$\|e(t)\| \le pe^{-qt}, \quad \forall t \in [-\tau_0, +\infty). \tag{5}$$

where $e(t) = (e_1(t), e_2(t), \ldots, e_N(t))^T$.

For more discussion, the following assumptions and lemmas are needed to be introduced firstly.

Assumption 2 (see [21]). For the vector-valued function $f(t, x(t), x(t - \tau_0(t)))$, suppose the uniform semi-Lipschitz condition with respect to the time t holds; i.e., for any $x \in R^n$, $y \in R^n$, there exists two positive constants $\bar{l}_1$ and $\bar{l}_2$, such that

$$\begin{aligned}
&(x(t) - y(t))^T (f(t, x(t), x(t - \tau_0(t))) \\
&\quad - f(t, y(t), y(t - \tau_0(t)))) \le \bar{l}_1 (x(t) - y(t))^T \\
&\quad \cdot (x(t) - y(t)) + \bar{l}_2 (x(t - \tau_0(t)) - y(t - \tau_0(t)))^T \\
&\quad \cdot (x(t - \tau_0(t)) - y(t - \tau_0(t))).
\end{aligned} \tag{6}$$

Assumption 3. In network (1), the unknown time-varying coupling strengths $c_{ij}(t)$ are periodical parameters; that is, $c_{ij}(t + T) = c_{ij}(t)$ for $t \in [0, +\infty)$, in which T is the known common period of $c_{ij}(t)$.

For further discussion, the aperiodically intermittent control strategy can be expressed as follows. Each controlling cycle $[w_i, w_{i+1})$ usually contains two types of time zones, one is working time $[w_i, s_i)$ and the other is rest time $[s_i, w_{i+1})$. The controller is activated at each working time and closed at rest time.

As shown in Figure 1, the solid line represents the working time, while the dotted line represents the rest time.

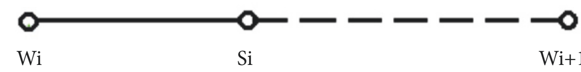

Figure 1: Aperiodical intermittent control sketch.

Assumption 4 (see [32]). For the aperiodically intermittent control strategy, there exist two positive scalars $0 < \theta < \omega < +\infty$, such that, for $i = 0, 1, 2, \cdots$,

$$\begin{aligned}
\inf_i (s_i - w_i) &= \theta, \\
\sup_i (w_{i+1} - w_i) &= \omega.
\end{aligned} \tag{7}$$

Define the maximum proportion of rest width $w_{i+1} - s_i$ in the time span $w_{i+1} - w_i$ as

$$\phi = \limsup_{i \longrightarrow +\infty} \frac{w_{i+1} - s_i}{w_{i+1} - w_i}. \tag{8}$$

Lemma 5 (see [32]). *For any $i = 0, 1, 2, \ldots$, if we denote*

$$\phi(t) = \frac{t - s_i}{t - w_i}, \quad t \in [s_i, w_{i+1}]. \tag{9}$$

Then $\phi(t)$ is an strictly increasing function, so $\phi(t) \le \phi(w_{i+1}) = (w_{i+1} - s_i)/(w_{i+1} - w_i)$.

Lemma 6 (see [32, 33]). *Suppose that function $y(t)$ is continuous and nonnegative for $t \longrightarrow [-\tau, +\infty)$ and satisfies the following condition:*

$$\begin{aligned}
\dot{y}(t) &\le -\gamma_1 y(t) + \gamma_2 \left(\sup_{t-\tau \le s \le t} y(s) \right), \quad w_i \le t \le s_i, \\
\dot{y}(t) &\le \gamma_3 y(t) + \gamma_4 \left(\sup_{t-\tau \le s \le t} y(s) \right), \quad s_i \le t \le w_{i+1}.
\end{aligned} \tag{10}$$

where $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ are positive constants and $i = 0, 1, 2, \cdots$. Suppose that, for the aperiodically intermittent control, there exists a constant ϕ defined in (8). If

$$\begin{aligned}
\gamma_1 &> \gamma^* = \max\{\gamma_2, \gamma_4\} > 0, \\
\rho &= \gamma_1 + \gamma_3 > 0, \\
\omega &= \lambda - \rho\phi > 0,
\end{aligned} \tag{11}$$

then

$$y(t) \le \left(\sup_{-\tau \le s \le 0} y(s) \right) \exp\{-\omega t\}, \quad t \ge 0, \tag{12}$$

where $\lambda > 0$ is the unique positive solution of the equation $\lambda - \gamma_1 + \gamma^* \exp\{\lambda\tau\} = 0$.

3. Aperiodically Intermittent Synchronization for Complex Dynamical Networks

In this section, we consider the synchronization problem of coupled complex dynamical network (1) via aperiodically intermittent control. In order to achieve the synchronization objective (3), we choose the adaptive controller $u_i(t)$ for the i−th node as follows:

$$u_i(t) = \begin{cases} -k_i(t)\Gamma e_i(t) - \sum_{j=1}^{N} \widehat{c}_{ij}(t) a_{ij}\Gamma e_j(t), & t \in [w_i, s_i), \\ 0, & t \in [s_i, w_{i+1}), \end{cases} \quad 1 \le i \le N. \tag{13}$$

The update law of the adaptive parameter $k_i(t)$, $i = 1, 2, \ldots, N$ is

$$\dot{k}_i(t) = \begin{cases} \alpha_i \exp(\beta_1 t) e_i^T(t)\Gamma e_i(t), & t \in [w_i, s_i), \\ 0, & t \in [s_i, w_{i+1}). \end{cases} \tag{14}$$

where α_i and β_1 are positive constants.

For the unknown time-varying periodical coupling strengths $c_{ij}(t)$, $i, j = 1, 2, \ldots, N$, the parameters estimation are designed by

$$\widehat{c}_{ij}(t) = \begin{cases} \widehat{c}_{ij}(t-T) + \varrho_{ij}^* a_{ij} e_i^T(t)\Gamma e_j(t), & t \in [kT, (k+1)T), \ k = 1, 2, \cdots \\ \varrho_{ij}(t) a_{ij} e_i^T(t)\Gamma e_j(t), & t \in [0, T) \\ 0, & t \in [-T, 0). \end{cases} \tag{15}$$

where $\widehat{c}_{ij}(t)$ is the estimation of coupling strength $c_{ij}(t)$. Denote $\widetilde{c}_{ij}(t) = c_{ij}(t) - \widehat{c}_{ij}(t)$ is the estimation error. ϱ_{ij}^* are positive constants, $\varrho_{ij}(t)$ is a continuous and strictly increasing function for $t \in [0, T]$ and satisfies $\varrho_{ij}(0) = 0$, $\varrho_{ij}(T) = \varrho_{ij}^*$.

Basing on the designed controllers, the following synchronization error system can be obtained:

$$\dot{e}_i(t) = \begin{cases} \overline{f}(t, e_i(t), e_i(t-\tau_0(t))) + \sum_{j=1}^{N} \widetilde{c}_{ij}(t) a_{ij}\Gamma e_j(t) - k_i(t)\Gamma e_i(t), & t \in [w_i, s_i), \\ \overline{f}(t, e_i(t), e_i(t-\tau_0(t))) + \sum_{j=1}^{N} c_{ij}(t) a_{ij}\Gamma e_j(t), & t \in [s_i, w_{i+1}), \end{cases} \quad 1 \le i \le N. \tag{16}$$

where $\overline{f}(t, e_i(t), e_i(t-\tau_0(t))) = f(t, x_i(t), x_i(t-\tau_0(t))) - f(t, s(t), s(t-\tau_0(t)))$.

Obviously, we can see that exponential synchronization of complex dynamical network (1) with controller u_i equals the exponential stability of error system (16). So, next, a sufficient condition for the controlled complex network realizing global exponential synchronization will be presented as follows.

Theorem 7. *Suppose that Assumptions 2–4 hold. If there exist positive constants β_1 and β_2 ($\beta_2 > \beta_1$), such that the following conditions hold,*

$$\eta_2 + \lambda_{\max}\left(\widehat{A}_C^s\right) < 0, \tag{17}$$

$$\beta_1 - 2l_2 > 0, \tag{18}$$

$$\xi - \beta_2\phi > 0. \tag{19}$$

where $\xi > 0$ is the unique positive solution of equation $\xi - \beta_1 + l\exp\{\xi\tau_0\} = 0$, $l = \max\{l_2, l_3\}$, then the controlled network (1) with adaptive periodical outer coupling update laws (15) is globally exponentially synchronized under the adaptive aperiodically intermittent controllers (13) and (14).

Proof. Construct the Lyapunov-Krasovskii-like function candidate as follows:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t) e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varrho_{ij}^*}\int_{t-T}^{t} \widetilde{c}_{ij}^2(\tau)\, d\tau + \frac{1}{2}\sum_{i=1}^{N}\exp(-\beta_1 t)\frac{(k_i(t)-k_i^*)^2}{\alpha_i}, \tag{20}$$

where $t \ge T$, k_i^* are some sufficiently large positive constants which will be determined later.

When $t \in [w_i, s_i)$, taking the derivative along the trajectories of the error system, we get

$$\dot{V}(t) = \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varrho_{ij}^*}\left(\widetilde{c}_{ij}^2(t) - \widetilde{c}_{ij}^2(t-T)\right) - \frac{\beta_1}{2}\sum_{i=1}^{N}\exp(-\beta_1 t)\frac{(k_i(t)-k_i^*)^2}{\alpha_i} + \sum_{i=1}^{N}\exp(-\beta_1 t)\frac{(k_i(t)-k_i^*)}{\alpha_i}\cdot\dot{k}_i(t) = \sum_{i=1}^{N} e_i^T(t)$$

$$\cdot\left(\overline{f}\left(t, e_i(t), e_i\left(t-\tau_0(t)\right)\right)+\sum_{j=1}^{N}\tilde{c}_{ij}(t)\, a_{ij}\Gamma e_j(t)\right)$$
$$-\frac{\beta_1}{2}\sum_{i=1}^{N}\exp\left(-\beta_1 t\right)\frac{\left(k_i(t)-k_i^*\right)^2}{\alpha_i}$$
$$+\sum_{i=1}^{N}\exp\left(-\beta_1 t\right)\frac{\left(k_i(t)-k_i^*\right)}{\alpha_i}\cdot\dot{k}_i(t)+\frac{1}{2}$$
$$\cdot\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varrho_{ij}^*}\left(\tilde{c}_{ij}^2(t)-\tilde{c}_{ij}^2(t-T)\right)-\sum_{i=1}^{N}e_i^T(t)\, k_i(t)$$
$$\cdot\,\Gamma e_i(t). \tag{21}$$

Next we will calculate the items separately. Combining with Assumption 2, we can have

$$\begin{aligned} &e_i^T(t)\,\overline{f}\left(t, e_i(t), e_i\left(t-\tau_0(t)\right)\right)\\ &\quad\le \bar{l}_1 e^T(t)\, e(t)+\bar{l}_2 e^T\left(t-\tau_0(t)\right) e\left(t-\tau_0(t)\right). \end{aligned} \tag{22}$$

Considering the adaptive law (14), one can have

$$\begin{aligned} &\sum_{i=1}^{N}\exp\left(-\beta_1 t\right)\frac{\left(k_i(t)-k_i^*\right)}{\alpha_i}\cdot\dot{k}_i\\ &\quad=\sum_{i=1}^{N}\left(k_i(t)-k_i^*\right) e_i^T(t)\,\Gamma e_i(t). \end{aligned} \tag{23}$$

Using the matrix equality $(a-b)^T H(a-b)-(a-c)^T H(a-c)=(c-b)^T H[2(a-b)+(b-c)]$ and the estimation of unknown coupling item (15), we can obtain

$$\begin{aligned} &\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varrho_{ij}^*}\left(\tilde{c}_{ij}^2(t)-\tilde{c}_{ij}^2(t-T)\right)\\ &\quad=-\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varrho_{ij}^*}\left(\hat{c}_{ij}(t)-\hat{c}_{ij}(t-T)\right)^2\\ &\qquad-2\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varrho_{ij}^*}\left(c_{ij}(t)-\hat{c}_{ij}(t-T)\right)\\ &\qquad\cdot\left(\hat{c}_{ij}(t)-\hat{c}_{ij}(t-T)\right)\\ &\quad=-2\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}(t)\, a_{ij}e_i^T(t)\,\Gamma e_j(t)\\ &\qquad-\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varrho_{ij}^*}\left(\hat{c}_{ij}(t)-\hat{c}_{ij}(t-T)\right)^2\\ &\quad\le-2\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}(t)\, a_{ij}e_i^T(t)\,\Gamma e_j(t). \end{aligned} \tag{24}$$

With the above results and properties of Kronecker production, substitute (22), (23), and (24) into (21), one gets

$$\begin{aligned} \dot{V}(t) &\le \bar{l}_1 e^T(t)\, e(t)+\bar{l}_2 e^T\left(t-\tau_0(t)\right) e\left(t-\tau_0(t)\right)\\ &-\sum_{i=1}^{N}e_i^T(t)\, k_i^*\Gamma e_i(t)-\frac{\beta_1}{2}\\ &\cdot\sum_{i=1}^{N}\exp\left(-\beta_1 t\right)\frac{\left(k_i(t)-k_i^*\right)^2}{\alpha_i}\le\eta_1 e^T(t)\left(I_N\otimes\Gamma\right)\\ &\cdot e(t)+\bar{l}_2 e^T\left(t-\tau_0(t)\right) e\left(t-\tau_0(t)\right)-e^T(t)\left(K^*\right.\\ &\left.\otimes\,\Gamma\right) e(t)-\frac{\beta_1}{2}\left(\sum_{i=1}^{N}e_i^T(t)\, e_i(t)\right.\\ &\left.+\sum_{i=1}^{N}\exp\left(-\beta_1 t\right)\frac{\left(k_i(t)-k_i^*\right)^2}{\alpha_i}\right)\le e^T(t)\\ &\cdot\left(\left(R-K^*\right)\otimes\Gamma\right) e(t)+\bar{l}_2 e^T\left(t-\tau_0(t)\right) e\left(t\right.\\ &\left.-\tau_0(t)\right)+\frac{\beta_1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varrho_{ij}^*}\int_{t-T}^{t}\tilde{c}_{ij}^2(\tau)\, d\tau\\ &-\frac{\beta_1}{2}\left(\sum_{i=1}^{N}e_i^T(t)\, e_i(t)\right.\\ &+\sum_{i=1}^{N}\exp\left(-\beta_1 t\right)\frac{\left(k_i(t)-k_i^*\right)^2}{\alpha_i}\\ &\left.+\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varrho_{ij}^*}\int_{t-T}^{t}\tilde{c}_{ij}^2(\tau)\, d\tau\right), \end{aligned} \tag{25}$$

where the vectors $e(t)=(e_1(t), e_2(t), \ldots, e_N(t))^T$, $e(t-\tau_0(t))=(e_1(t-\tau_0(t)), e_2(t-\tau_0(t)), \ldots, e_N(t-\tau_0(t)))^T$, $\eta_1=\bar{l}_1/\lambda_{min}(\Gamma)+\beta_1/2$, $R=\eta_1 I_N$, $K^*=\operatorname{diag}\{k_1^*, \cdots, k_N^*\}$.

So we can see that if we choose $k_i^*>\eta_1$, the matrix $R-K^*<0$. Let $l_2=\max\{2\bar{l}_2, \beta_1\}$; then we can acquire

$$\dot{V}(t)\le-\beta_1 V(t)+l_2\left(\sup_{t-\tau_0\le\varsigma\le t}V(\varsigma)\right). \tag{26}$$

When $t\in[s_i, w_{i+1})$, taking the derivative along the trajectories of the error system (16), we have

$$\begin{aligned} \dot{V}(t) &\le \bar{l}_1 e^T(t)\, e(t)+\bar{l}_2 e^T\left(t-\tau_0(t)\right) e\left(t-\tau_0(t)\right)\\ &+\sum_{i=1}^{N}\sum_{j=1}^{N}\hat{c}_{ij}(t)\, e_i^T(t)\, a_{ij}\Gamma e_j(t)-\frac{\beta_1}{2} \end{aligned}$$

$$\cdot \sum_{i=1}^{N} \exp(-\beta_1 t) \frac{(k_i(t)-k_i^*)^2}{\alpha_i} \le \eta_2 e^T(t)(I_N \otimes \Gamma) \cdot e(t) + \bar{l}_2 e^T(t-\tau_0(t)) e(t-\tau_0(t)) + e^T(t)(\widehat{A}_C^s \otimes \Gamma) e(t) + \frac{\beta_2-\beta_1}{2}\left(\sum_{i=1}^{N} e_i^T(t) e_i(t) + \sum_{i=1}^{N} \exp(-\beta_1 t) \frac{(k_i(t)-k_i^*)^2}{\alpha_i}\right) \le e^T(t)(\widetilde{R} \otimes \Gamma) \cdot e(t) + \bar{l}_2 e^T(t-\tau_0(t)) e(t-\tau_0(t)) + \frac{\beta_2-\beta_1}{2}\left(\sum_{i=1}^{N} e_i^T(t) e_i(t) + \sum_{i=1}^{l} \exp(-\beta_1 t) \frac{(k_i(t)-k_i^*)^2}{\alpha_i} + \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{1}{\varrho_{ij}^*} \int_{t-T}^{t} \tilde{c}_{ij}^2(\tau)\, d\tau\right), \tag{27}$$

where $\widetilde{R} = \eta_2 I_N + \widehat{A}_C^s$, $\widehat{A}_C^s = (\widehat{A}_C + \widehat{A}_C^T)/2$, $\widehat{A}_C = (c_{ij}(t) \cdot a_{ij})_{N\times N}$, $\eta_2 = \bar{l}_1/\lambda_{min}(\Gamma) - (\beta_2-\beta_1)/2$. According to Lemma 6, it follows that $\lambda_{\max}(\widetilde{R}) \le \eta_2 + \lambda_{\max}(\widehat{A}_C^s)$. If condition (17) is satisfied, it is easy to have $\widetilde{R} < 0$. Let $l_3 = 2\bar{l}_2$. Then we can have

$$\dot{V}(t) \le (\beta_2-\beta_1) V(t) + l_3 \left(\sup_{t-\tau_0 \le \varsigma \le t} V(\varsigma)\right). \tag{28}$$

Thus, we obtain

$$\begin{aligned} &\dot{V}(t) \le -\beta_1 V(t) + l_2 \left(\sup_{t-\tau_0 \le \varsigma \le t} V(\varsigma)\right), \quad t \in [w_i, s_i), \\ &\dot{V}(t) \le (\beta_2-\beta_1) V(t) + l_3 \left(\sup_{t-\tau_0 \le \varsigma \le t} V(\varsigma)\right), \quad t \in [s_i, w_{i+1}). \end{aligned} \tag{29}$$

Combining Lemma 6 with conditions (18) and (19), we can get $V(t) \le (\sup_{t-\tau_0\le\varsigma\le t} V(\varsigma)) \exp(-\eta t)$ for $t \ge 0$ immediately, where $\eta = \xi - (l + \beta_2 - \beta_1)\phi$ and the parameter ξ satisfies the equation $\xi - \beta_1 + l \exp\{\xi\tau_0\} = 0$. So the zero solution of the error system (15) is globally exponentially stable. According to the calculation of $V(t)$, we can obtain $\|e(t)\|^2 \le 2\|V(t)\|$; i.e., $\|e(t)\| \le \sqrt{2}\|V(t)\|^{1/2}$. And hence $\|e(t)\| \le \sqrt{2(\sup_{t-\tau_0\le\varsigma\le t} V(\varsigma))} \exp(-(\eta/2)t)$ holds, which implies the global exponential synchronization is achieved. Thus the proof is completed. □

If the complex dynamical network (1) has no any internal time delay, i.e., $\tau_0(t) = 0$, the system (1) can be rewritten as

$$\dot{x}_i(t) = f(t, x_i(t)) + \sum_{j=1}^{N} c_{ij}(t) a_{ij} \Gamma x_j(t) + u_i, \quad i = 1, 2, \ldots, N. \tag{30}$$

and the synchronization target orbit is described by $\dot{s}(t) = f(t, s(t))$. In this case, we can see that $\bar{l}_2 = 0$ holds. By using a similar line of arguments as that in Theorem 7, the following result is easily achieved.

Corollary 8. *Suppose that Assumptions 2–4 hold. If there exist positive constants β_1 and β_2 ($\beta_2 > \beta_1$), such that the following conditions hold,*

$$\eta_2 + \lambda_{\max}(\widehat{A}_C^s) < 0, \tag{31}$$

$$\xi - \beta_2\phi > 0, \tag{32}$$

then the controlled network (1) with adaptive periodical outer coupling update laws (15) is globally exponential synchronized under the adaptive aperiodically intermittent controllers (13) and (14).

Remark 9. When $\bar{l}_2 = 0$, the equation is $\xi - \beta_1 + l_3 \exp\{\xi\tau_0\} = 0$, so inequality (19) can be expressed as (32).

4. Numerical Simulations

In this section, we will illustrate the effectiveness of the proposed approach to achieve globally exponential synchronization of complex networks (1) with unknown time-varying coupling strengths via aperiodically intermittent control. Without loss of generality, we choose the delayed Chua's chaotic models as the nodes' dynamics. The chaotic Chua's system dynamical function is as follows:

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{pmatrix} = \begin{pmatrix} -\alpha(1+m_2) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & -\omega \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\beta\zeta \sin(\upsilon x_1(t-\tau_1(t))) \end{pmatrix} + \begin{pmatrix} -\frac{1}{2}\alpha(m_1-m_2)(|x_1(t)+1| - |x_1(t)-1|) \\ 0 \\ 0 \end{pmatrix}. \tag{33}$$

where the parameters are $\alpha = 10$, $\beta = 18.53$, $\omega = 0.1636$, $m_1 = -1.4325$, $m_2 = -0.7831$, $\zeta = 0.2$, $\upsilon = 0.5$, respectively, and the time delay $\tau_1(t) = 0.02$. By calculation [9], we have $\bar{l}_1 = 11.6435$, $\bar{l}_2 = 0.3088$.

The outer coupling matrix which depicts the structure of network (1) is a 10×10 matrix, which is chosen as follows:

$$A = \begin{pmatrix} -3 & 0 & 1.5 & 0 & 0.5 & 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & -2 & 0 & 0.3 & 0.5 & 0 & 0 & 0.7 & 0 & 0.5 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & -3 & 0 & 0 & 0.5 & 0.5 & 0 & 1 \\ 0.2 & 0 & 0.5 & 0 & -3 & 0.5 & 0 & 0.8 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -4 & 0 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 & 2 & 0 & -5 & 0 & 0 & 1 \\ 1 & 0 & 0.5 & 1.5 & 0 & 0 & 0.5 & -4.5 & 1 & 0 \\ 0.5 & 0 & 1 & 0 & 1 & 1 & 0 & 0.5 & -4 & 0 \\ 0 & 1 & 0.5 & 0 & 0 & 0 & 1 & 0 & 0 & -2.5 \end{pmatrix}. \quad (34)$$

The coupling weight is

$$c_{11}(t) = \frac{1}{6} + \frac{1}{6}\sin \pi t - \frac{13}{60}\cos\frac{\pi}{2}t,$$
$$c_{12}(t) = 2,$$
$$c_{13}(t) = 0,$$
$$c_{14}(t) = 2.8,$$
$$c_{15}(t) = 1 - 1.3\cos\frac{\pi}{2}t,$$
$$c_{16}(t) = -0.5\cos\frac{\pi}{2}t,$$
$$c_{17}(t) = 0,$$
$$c_{18}(t) = \sin \pi t,$$
$$c_{19}(t) = 0,$$
$$c_{110}(t) = \sin \pi t;$$
$$c_{21}(t) = 1,$$
$$c_{22}(t) = \frac{1}{2},$$
$$c_{23}(t) = \cos \pi t,$$
$$c_{24}(t) = 0,$$
$$c_{25}(t) = 0,$$
$$c_{26}(t) = 0.5\cos\frac{\pi}{3}t,$$
$$c_{27}(t) = \cos \pi t,$$
$$c_{28}(t) = 0,$$
$$c_{29}(t) = \cos\frac{\pi}{3}t,$$
$$c_{210}(t) = 1;$$
$$c_{31}(t) = 1 - \sin\frac{\pi}{3}t,$$
$$c_{32}(t) = 1 + 0.5\cos\frac{\pi}{3}t,$$
$$c_{33}(t) = \frac{1}{2} + \frac{1}{2}\cos\frac{\pi}{3}t + \frac{1}{4}\sin\frac{\pi}{3}t,$$
$$c_{34}(t) = 0,$$
$$c_{35}(t) = 0.7,$$
$$c_{36}(t) = 1 + 1.5\cos\frac{\pi}{3}t,$$
$$c_{37}(t) = \cos\frac{\pi}{3}t,$$
$$c_{38}(t) = \sin\frac{\pi}{3}t,,$$
$$c_{39}(t) = -1,$$
$$c_{310}(t) = 0;$$
$$c_{41}(t) = 1 + \cos\frac{\pi}{3}t,$$
$$c_{42}(t) = \sin \pi t,$$
$$c_{43}(t) = 2\sin \pi t,$$
$$c_{44}(t) = \frac{1}{6} + \frac{2}{3}\cos\frac{\pi}{3}t + \frac{1}{3}\sin \pi t,$$
$$c_{45}(t) = 0,$$
$$c_{46}(t) = \cos\frac{\pi}{3}t,$$
$$c_{47}(t) = \sin \pi t,$$
$$c_{48}(t) = -1 + \sin \pi t,$$
$$c_{49}(t) = -\cos\frac{\pi}{3}t,$$
$$c_{410}(t) = \cos\frac{\pi}{3}t;$$
$$c_{51}(t) = 0,$$
$$c_{52}(t) = 0,$$
$$c_{53}(t) = 1,$$
$$c_{54}(t) = 0,$$
$$c_{55}(t) = \frac{1}{3} - \frac{11}{12}\cos\frac{2\pi}{3}t,$$
$$c_{56}(t) = 1 + 0.5\cos\frac{2\pi}{3}t,$$
$$c_{57}(t) = 0,$$
$$c_{58}(t) = 0,$$

$$
\begin{aligned}
c_{59}(t) &= -3\cos\frac{2\pi}{3}t, \\
c_{510}(t) &= 2.3+\cos\frac{2\pi}{3}t; \\
c_{61}(t) &= -2.5-1.4\sin\frac{\pi}{3}t-1.3\cos\frac{2\pi}{3}t, \\
c_{62}(t) &= 0.1+0.1\sin\frac{\pi}{3}t, \\
c_{63}(t) &= 0, \\
c_{64}(t) &= 0, \\
c_{65}(t) &= 1-\cos\frac{2\pi}{3}t, \\
c_{66}(t) &= \frac{11}{40}+\frac{9}{40}\sin\frac{\pi}{3}t-\frac{1}{4}\cos\frac{2\pi}{3}t, \\
c_{67}(t) &= 0, \\
c_{68}(t) &= 1+0.5\sin\frac{\pi}{3}t, \\
c_{69}(t) &= 0, \\
c_{610}(t) &= 0.5\sin\frac{\pi}{3}t; \\
c_{71}(t) &= 1, \\
c_{72}(t) &= -5-\cos\pi t, \\
c_{73}(t) &= 1+\cos\pi t, \\
c_{74}(t) &= 0, \\
c_{75}(t) &= 0, \\
c_{76}(t) &= 0.5\cos\frac{\pi}{3}t, \\
c_{77}(t) &= -\frac{4}{5}-\frac{1}{10}\cos\frac{\pi}{3}t, \\
c_{78}(t) &= 2\cos\pi t, \\
c_{79}(t) &= 0, \\
c_{710}(t) &= -0.5\cos\frac{\pi}{3}t; \\
c_{81}(t) &= 1-\sin\pi t, \\
c_{82}(t) &= 1+1.5\cos\pi t, \\
c_{83}(t) &= 0, \\
c_{84}(t) &= 0, \\
c_{85}(t) &= 0.7, \\
c_{86}(t) &= 1-0.5\cos\pi t, \\
c_{87}(t) &= 0, \\
c_{88}(t) &= \frac{2}{9}-\frac{2}{9}\sin\pi t, \\
c_{89}(t) &= 0, \\
c_{810}(t) &= -\cos\pi t; \\
c_{91}(t) &= 1+\cos\frac{\pi}{3}t, \\
c_{92}(t) &= \sin\pi t, \\
c_{93}(t) &= -2\sin\pi t, \\
c_{94}(t) &= -1, \\
c_{95}(t) &= 0, \\
c_{96}(t) &= 3-0.5\cos\frac{\pi}{3}t, \\
c_{97}(t) &= \sin\pi t, \\
c_{98}(t) &= 0, \\
c_{99}(t) &= \frac{7}{8}-\frac{1}{2}\sin\pi t, \\
c_{910}(t) &= -\cos\frac{\pi}{3}t; \\
c_{101}(t) &= 0, \\
c_{102}(t) &= \cos\frac{\pi}{2}t, \\
c_{103}(t) &= 1-0.5\cos\frac{\pi}{2}t, \\
c_{104}(t) &= -\cos\frac{\pi}{2}t, \\
c_{105}(t) &= -1-2\cos\frac{2\pi}{3}t, \\
c_{106}(t) &= 1+0.5\cos\frac{\pi}{2}t, \\
c_{107}(t) &= -\sin\pi t, \\
c_{108}(t) &= \sin\pi t, \\
c_{109}(t) &= -\sin\pi t, \\
c_{1010}(t) &= -\frac{1}{5}-\frac{2}{5}\sin\pi t+\frac{3}{10}\cos\frac{\pi}{2}t.
\end{aligned}
\tag{35}
$$

By calculation, we get the common period $T=6$ of $c_{ij}(t)$ and the parameters $\Omega^*=(\varrho_{ij}^*)_{10\times 10}$ are designed as

$$
\Omega^* = \left(\varrho_{ij}^*\right)
= \begin{pmatrix}
1.5 & 0.3 & 0.7 & 2 & 1.1 & 0.1 & 1.7 & 0.2 & 0.1 & 0.4 \\
0.1 & 0.2 & 0.4 & 0.5 & 0.6 & 0.3 & 0.6 & 0.2 & 0.9 & 2.1 \\
0.9 & 0.8 & 0.5 & 0.7 & 1.1 & 0.5 & 0.5 & 0.4 & 0.2 & 1.1 \\
0.5 & 1.5 & 0.9 & 2.1 & 1.8 & 0.6 & 0.8 & 0.6 & 0.8 & 1.6 \\
1.2 & 1.4 & 1.8 & 2 & 2.2 & 0.2 & 0.2 & 1.0 & 0.8 & 0.1 \\
0.5 & 1.3 & 1.7 & 1.2 & 0.1 & 0.4 & 0.1 & 0.3 & 0.3 & 0.9 \\
0.1 & 0.8 & 0.4 & 0.5 & 0.9 & 1.1 & 0.7 & 0.6 & 0.5 & 0.4 \\
0.9 & 0.2 & 1.5 & 0.7 & 1.4 & 0.6 & 0.6 & 0.9 & 0.5 & 0.7 \\
0.8 & 1.1 & 0.1 & 0.1 & 0.8 & 0.3 & 0.5 & 0.7 & 1.1 & 1.3 \\
0.2 & 0.4 & 0.8 & 1.0 & 1.2 & 0.8 & 0.1 & 1.2 & 0.1 & 1.7
\end{pmatrix},
\tag{36}
$$

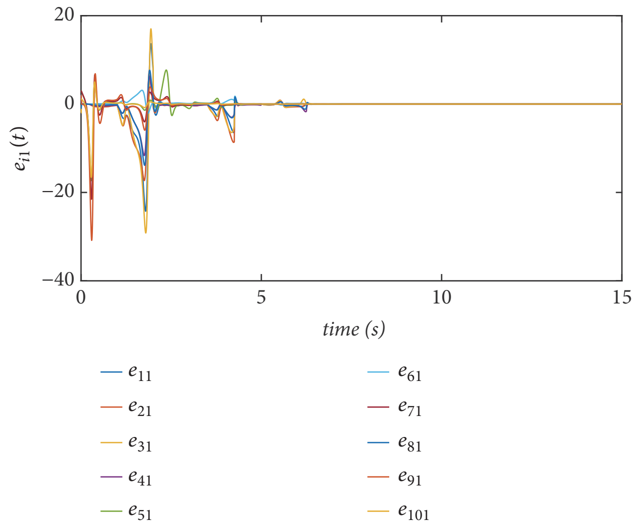

Figure 2: The first component of error evolution for each node.

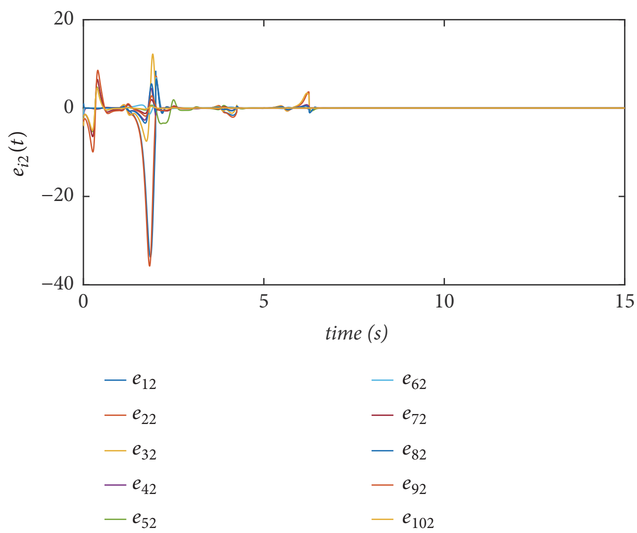

Figure 3: The second component of error evolution for each node.

and $\varrho_{ij}(t) = (t/6)\varrho_{ij}^*$ for $i, j = 1, 2, \ldots, 10$. For simplicity, the inner coupling matrices are designed as follows: $\Gamma = \text{diag}\{0.5, 1, 0.8\}$. And the initial values of target orbit are $s(0) = (-0.2, 0.2, 0.5)$.

The dynamical network with ten nodes can be expressed as follows:

$$\dot{x}_i(t) = f(t, x_i(t), x_i(t - \tau_0(t))) + \sum_{j=1}^{10} c_{ij}(t) a_{ij} \Gamma x_j(t) + u_i, \quad i = 1, 2, \ldots, 10. \tag{37}$$

Choosing initial values of complex dynamical network (37) randomly in $[-5, 5]$, for the time span $[0, 15]$, the work time is $[0, 2) \cup [4, 7) \cup [10, 12.5)$, and the rest time is $[2, 4) \cup [7, 10) \cup [12.5, 15)$. With aperiodically intermittent controller (13), (14), and adaptive update law (15), the synchronization simulations of the complex dynamical network (37) are shown in Figures 2–9.

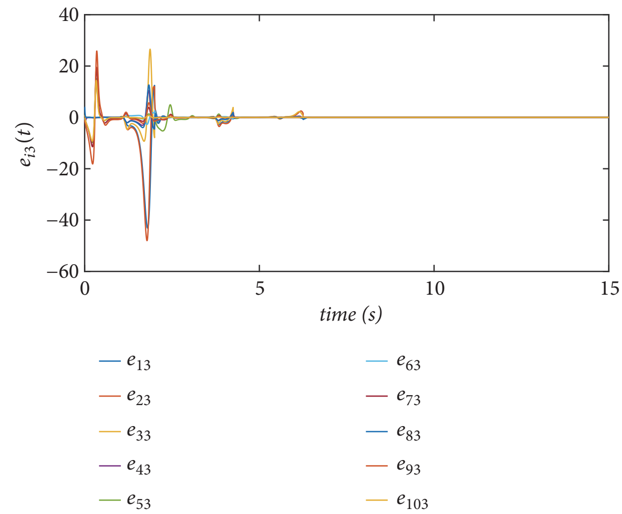

Figure 4: The third component of error evolution for each node.

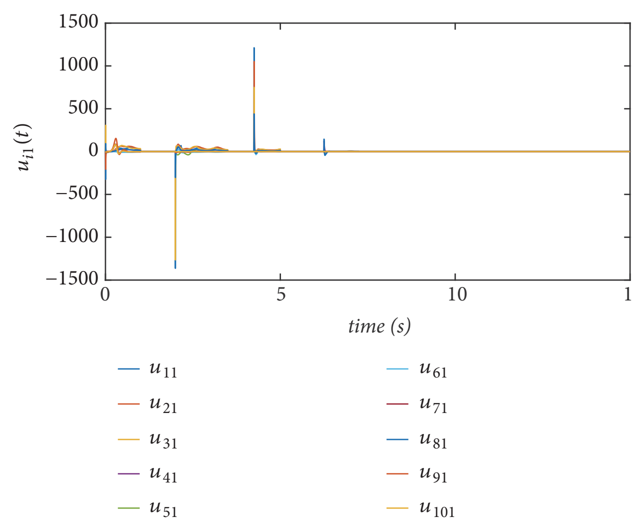

Figure 5: The first component of the controller for each node.

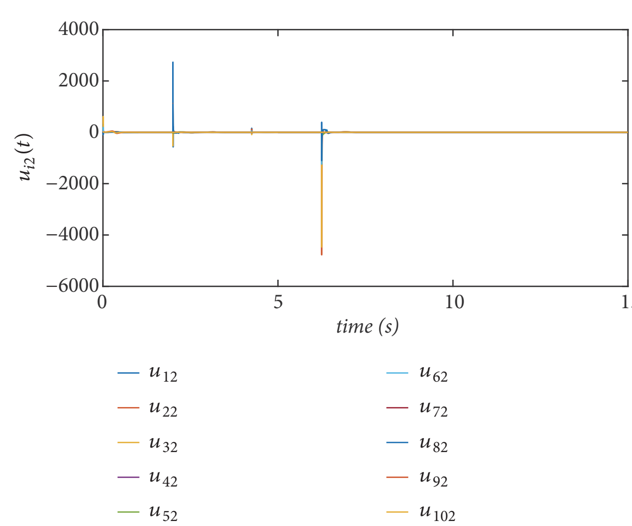

Figure 6: The second component of the controller for each node.

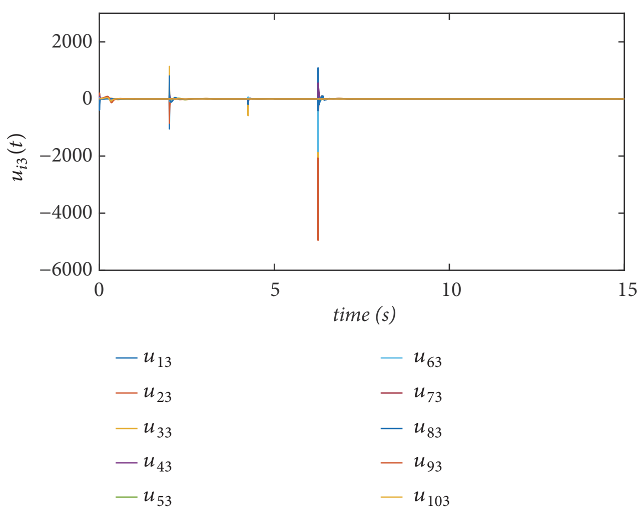

Figure 7: The third component of the controller for each node.

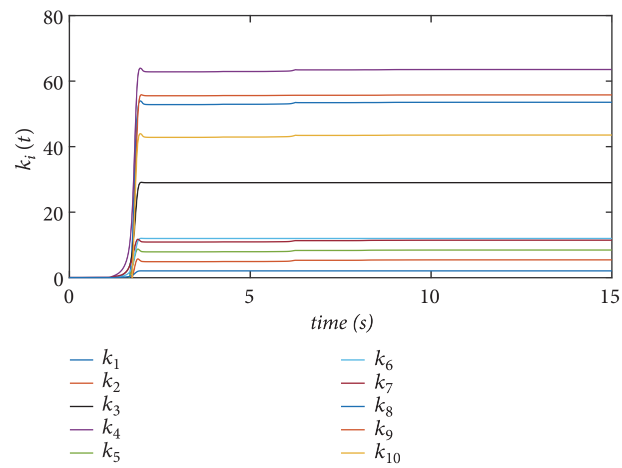

Figure 8: The error evolution of the first node.

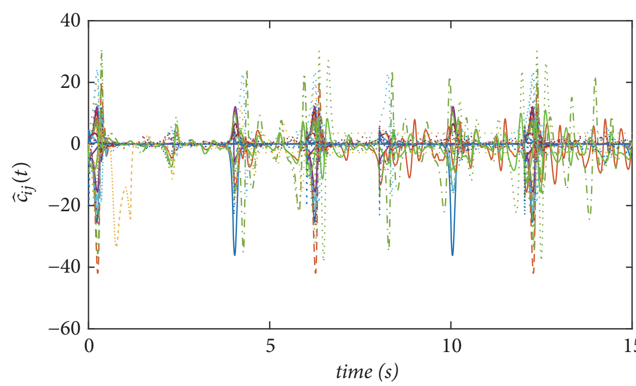

Figure 9: The error evolution of the first node.

Figures 2–4 display the dynamical evolution curves of the synchronization errors $e_i(t)$, $i = 1, 2, \ldots, 10$; as time goes on, the component of errors for each node tends to zero quickly. The aperiodically intermittent controllers are presented in Figures 5–7, from which we can see that all components of the controller are changed violently until they reach zero, which is consistent with the errors evolution results. Moreover, Figure 8 shows the feedback gain convergence to the fixed values: $k_1 = 53.54, k_2 = 5.45, k_3 = 29.01, k_4 = 63.51, k_5 = 8.48, k_6 = 11.97, k_7 = 11.50, k_8 = 2.08, k_9 = 55.79, k_{10} = 43.52$. The estimation of unknown periodical coupling strengths is presented in Figure 9. By simulation, we can see that the controlled complex dynamical network (37) achieved exponential synchronization with the help of controller designed in (13) and (14), and the unknown periodical coupling strengths are all estimated successfully.

5. Conclusions

In this paper, we investigated aperiodically intermittent synchronization problem of complex dynamical network, which contains unknown periodically coupling strengths and bounded time varying delay and is correspondence with the practical complex network system in a great extent. Based on theories of intermittent control, adaptive control, and learning control, some useful aperiodically intermittent synchronization criteria for complex dynamical network with unknown periodical couplings have been obtained.

Also an illustrative example by the numerical simulation is provided to demonstrate the effectiveness and feasibility of the proposed synchronization method. From the simulation results, we can see that the complex dynamical network is exponentially synchronized when the aperiodically intermittent control is injected, and the coupling strengths is estimated successfully. Meanwhile, in the future, the author will take switching and noise disturbance into account, to study the systems' finite-time aperiodically intermittent synchronization.

Data Availability

The Matlab based models used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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