Hindawi
Mathematical Problems in Engineering
Volume 2019, Article ID 7010389, 6 pages
https://doi.org/10.1155/2019/7010389

Research Article

Direction Finding of Coherent Sources Using a MIMO Array of Triaxial Velocity Sensors

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Received 21 August 2019; Accepted 11 October 2019; Published 11 November 2019

Academic Editor: Raffaele Solimene

In the received side, triaxial velocity sensors of MIMO array are used to solve the problem of coherent source direction-finding in this paper. A new velocity field smoothing algorithm is presented to decorrelate coherent sources. The identically oriented velocity sensors of whole array are divided into three subarrays. Then, the covariance matrices of the three subarrays are smoothed to restore the rank of source covariance matrix (SCM). Lastly, the cross-correlation coefficients of the SCM after smoothing processing are calculated to analyze the performance of decorrelation. The proposed decorrelation algorithm (1) does not need the information of locations of velocity vector sensors; (2) is suitable for arbitrary configuration array; and (3) has no loss of array effective aperture. Simulation results prove the effectiveness of the proposed algorithm.

1. Introduction

Traditional parameter estimation techniques with array signal are based on a scalar sensor array. In order to improve parameter estimation accuracy, the acoustic vector sensor is proposed [1, 2] (it can be also the electromagnetic vector sensor). An acoustic vector sensor comprises two or three orthogonally collocated velocity sensors and a pressure sensor. Compared with the conventional scalar sensor, the vector sensor has more output information and provides more manners of signal processing.

An estimation of signal parameters via rotational invariance techniques- (ESPRIT-) based algorithm is investigated for arbitrarily spaced three-dimensional arrays of vector hydrophones to estimate the two-dimensional direction of arrival (DOA) [3]. But the array aperture is not fully utilized. Aperture extension is achieved for a uniform rectangular array of vector hydrophones spaced much farther apart than a half-wavelength by using a two-step ESPRIT-based method [4]. The same idea of [4] can be applied to the identical subarrays on a sparse uniform rectangular array, but not limited to the acoustic vector sensor [5]. Multiple signal classification (MUSIC) is another popular super-resolution algorithm. A self-initiating MUSIC-based algorithm for the acoustic vector sensor is proposed, which does not require the initial value of the MUSIC algorithm by using beamspace information [6]. Then, the Root-MUSIC-based algorithm without the angle search is studied to reduce the complexity [7]. The above results [3–7] assume that all signals impinging from far-field. Therefore, an ESPRIT-based algorithm for the near-field is addressed by using a single vector hydrophone [8]. A direction-finding and blind interference rejection algorithm for acoustic vector sensor array is presented, which can solve up to three fast frequency-hop wideband signals [9]. Although these papers have made outstanding contributions, they are all based on the acoustic vector sensors with spatially collocated in a point-like geometry. Therefore, the spatially spread vector-sensor is proposed [10, 11], which not only retains the advantages of the collocated acoustic vector sensors, but also will significantly extend the spatial aperture to improve the direction-finding accuracy by orders of magnitude without additional sensors.

On the other hand, the Multiple-Input Multiple-Output (MIMO) concept was first introduced for radar in 2004, [12, 13]. MIMO radars transmit multiple waveforms and receive signals at multiple antennas. Waveform diversity in colocated MIMO radar enables significant superiority over its phased-array counterpart, including much improved parameter identifiability and estimation accuracy [14]. MIMO radar with widely separated sensors is also a high performance radar [15]. Virtual sensors which can extend array aperture can be obtained by using spatially orthogonal signal transmission. These virtual sensors provide higher performance in target detection, angle estimation accuracy, and angular resolution [16]. In this paper, we study the DOA estimation in colocated MIMO array radar or sonar.

The ESPRIT algorithm is used to estimate the direction of departure (DOD) and DOA by using the invariance property of both the transmit and receive array in a bistatic MIMO radar [17], but it needs an extra pairing processing. The unitary ESPRIT algorithm with real-valued processing can provide better estimation performance than that of ESPRIT with reduced computational complexity and automatic pairing [18]. In order to achieve the signal-to-noise ratio (SNR) gain, transmit beamspace processing is proposed in DOA estimation of MIMO [19].

Combined with the velocity vector sensor and waveform diversity offered by MMO radar, the signal model of the MIMO velocity vector array is given [20–22], and then the MUSIC [20] and ESPRIT [21, 22] angle estimation method for the signal model are developed. The results prove that these additional degrees of freedom can enhance spatial resolution, strengthen parameter identifiability, and improve target detection performance.

Although they made outstanding contributions, they are based on the assumption that signals are uncorrected. In practice, coherent signals are often encountered in multipath or some other scenarios, since multipath signals arise from different propagation paths of the same target. For coherent sources direction-finding, the spatial smoothing (SS) algorithm can be used to restore the rank of the covariance matrix. The disadvantage of the SS algorithm is that it will decrease the array aperture. Therefore, the "velocity-field smoothing" (VFS) algorithm restores the rank of the covariance matrix proposed in [23–25], whose algorithm will not decrease the array aperture.

In this paper, we extend the VFS algorithm into the MIMO array to solve the coherent sources estimation problem and analyze the performance of decorrelation. The detailed operations of decorrelation method are dividing the identically oriented sensors within all velocity vectors into three identical subarrays and then smoothing the covariance matrices of the three subarrays to restore the rank of source covariance matrix (SCM). The main idea of this paper is to find the identical subarrays in the vector sensor MIMO array. The remainder of the paper is organized as follows. Signal model is addressed in Section 2. In Section 3, we describe the proposed VFS algorithm in MIMO array. In Section 4, performance of the proposed method is evaluated. Section 5 gives the extensive simulations, and Section 6 concludes the paper.

Notation: Superscript $(\cdot)*$, $(\cdot)^T$, and $(\cdot)^H$ denote complex conjugation, transpose, and conjugate transpose, respectively. $\otimes$ denotes Kronecker product. $\mathbf{I}_{\mathrm{MN}}$ denotes $MN \times MN$ identity matrix. diag$[\cdot]$ denotes the diagonalization of the entity inside. rank$[\mathbf{A}]$ denotes the rank of matrix $\mathbf{A}$.

2. Signal Model

We consider a monostatic MIMO sonar with M transmitted scalar sensors and N triaxial received velocitysensors. Assume that the sensors of transmitted and received array are located in arbitrarily three-dimensional positions. Each of the N received sensors consists of three identical but orthogonally oriented velocity sensors, aligning along with the x-axis, y-axis, and z-axis, respectively. The location of the mth sensor in the transmitted array is defined as $\mathbf{l}_{tm} \triangleq [x_{tm}, y_{tm}, z_{tm}]$, and the location of the nth sensor in received array is defined as $\mathbf{l}_{rn} \triangleq [x_{rn}, y_{rn}, z_{rn}]$. The transmitted sensors transmit M orthogonal waveform signals. In each received sensor, the echoes are matched by M transmitted waveforms. Assume that K coherent signals are located at the far field. After matched filtering processing, the whole output in received side can be written as [22]

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{x}_x(t) \\ \mathbf{x}_y(t) \\ \mathbf{x}_z(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}\mathbf{\Phi}_x \\ \mathbf{A}\mathbf{\Phi}_y \\ \mathbf{A}\mathbf{\Phi}_z \end{bmatrix} s(t) + n(t) \in \mathbb{C}^{3MN\times 1}, \tag{1}$$

where

$$\mathbf{A} = [\mathbf{a}_t(\theta_1,\varphi_1) \otimes \mathbf{a}_r(\theta_1,\varphi_1), \ldots, \mathbf{a}_t(\theta_K,\varphi_K) \otimes \mathbf{a}_K(\theta_K,\varphi_K)] \in \mathbb{C}^{MN\times K}, \tag{2}$$

in which

$$\begin{aligned} \mathbf{a}_t(\theta,\varphi) &= [\exp(-j\kappa(\mathbf{l}_{t1}\mathbf{p}(\theta,\varphi))), \ldots, \exp(-j\kappa(\mathbf{l}_{tM}\mathbf{p}(\theta,\varphi)))]^T \in \mathbb{C}^{M\times 1}, \\ \mathbf{a}_r(\theta,\varphi) &= [\exp(-j\kappa(\mathbf{l}_{r1}\mathbf{p}(\theta,\varphi))), \ldots, \exp(-j\kappa(\mathbf{l}_{rN}\mathbf{p}(\theta,\varphi)))]^T \in \mathbb{C}^{N\times 1}, \end{aligned} \tag{3}$$

denote the transmit steering vector and the receive steering vector, respectively. $\mathbf{p}(\theta,\varphi) = [\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta]^T$ is the propagation vector. θ and φ are the two-dimensional DOA, namely, azimuth and elevation, respectively. $\kappa = 2\pi/\lambda$ is defined as wave number. $s(t) = [s_1(t), s_2(t), \ldots, s_K(t)]^T$ represents the transmitted signals, where $s_k(t) = \beta_k e^{j2\pi f_k t}$, and β_k and f_k represent amplitude and Doppler frequency of the k signal, respectively. The K sources are set as coherent. The noise $n(t)$ is assumed to be white noise with zero mean and $\sigma^2 \mathbf{I}_{3MN}$ is covariance. The matrices $\mathbf{\Phi}_x, \mathbf{\Phi}_y$, and $\mathbf{\Phi}_z$ are equal to

$$\begin{cases} \mathbf{\Phi}_x = \operatorname{diag}(\mathbf{p}_x) = \operatorname{diag}[\sin\theta_1\cos\varphi_1, \ldots, \sin\theta_K\cos\varphi_K]^T, \\ \mathbf{\Phi}_y = \operatorname{diag}(\mathbf{p}_y) = \operatorname{diag}[\sin\theta_1\sin\varphi_1, \ldots, \sin\theta_K\sin\varphi_K]^T, \\ \mathbf{\Phi}_z = \operatorname{diag}(\mathbf{p}_z) = \operatorname{diag}[\cos\theta_1, \ldots, \cos\theta_K]^T. \end{cases} \tag{4}$$

About equation (1), we give more explanations so that readers can understand it better. The data $\mathbf{x}_x(t), \mathbf{x}_y(t)$, and $\mathbf{x}_z(t)$ are the output of matched filtering using the received data in x, y, z axes, respectively. Our system is active sonar, but our channel model is very simplistic with line of sight, no diffraction, and only additive spatio-temporally white Gaussian noise. This paper is our preliminary results.

The source covariance matrix will be rank-deficiency because of the coherent of sources. Therefore, the traditional super-resolution algorithms, such as MUSIC and ESPRIT, will be failed. In the following, we proposed a new VFS algorithm to restore the rank of source covariance matrix (SCM).

3. "Velocity-Field Smoothing" Algorithm for MIMO Array

It can be seen from equation (1) that the received data $\mathbf{x}_x(t), \mathbf{x}_y(t)$, and $\mathbf{x}_z(t)$ in x, y, z axes have identical form except for the items $\mathbf{\Phi}_x$, $\mathbf{\Phi}_y$, and $\mathbf{\Phi}_z$. Therefore, we can use weighted average of the different items to complete decorrelation for coherent sources. This method is called as VFS algorithm because the velocity field is used in the smoothing processing. In fact, the smoothing processing can be seen as a directly transplantation of spatial smoothing algorithm from spatial domain to velocity-field domain. To make more clear to readers, the following describes VFS smoothing processing with mathematical formulas.

We firstly compute the covariance matrix of the received data of x, y, z axes separately, and then perform smoothing processing as follows:

$$\begin{aligned}\mathbf{R}_{\text{smoothing}} &= \frac{1}{3}\sum_{i=1}^{3} E\left\{\mathbf{x}_i(t)\mathbf{x}_i(t)^H\right\} \\ &= \mathbf{A}\underbrace{\left(\frac{1}{3}\sum_{i=1}^{3}\mathbf{\Phi}_i\mathbf{R}_s\mathbf{\Phi}_i^H\right)}_{\triangleq\mathbf{R}_{\text{s\_smoothing}}}\mathbf{A}^H + \sigma_n^2\mathbf{I}_{MN},\end{aligned} \tag{5}$$

where $i = x, y, z$. It is well known that the rank of steering matrix $\mathbf{A}$ equals the number of sources K. The K sources are coherent, so we can set $s_k(t) = g_k s_0(t)$, where g_k denotes nonzero complex constant. Then, the SCM is equal to $\mathbf{R}_s = E\left\{g_k s_0(t)[g_k s_0(t)]^H\right\} = \sigma_s^2\mathbf{g}\mathbf{g}^H$, where σ_s^2 denotes the power of $s_0(t)$ and the vector of $\mathbf{g}$ equals $\mathbf{g} = [g_1, g_2, \ldots, g_K]^T$. Therefore, the rank of $\mathbf{R}_s$ equals 1. Thus, the traditional super-resolution algorithms are failed. But after the smoothing processing, the rank of $\mathbf{R}_{\text{s\_smoothing}}$ will restore to the number of sources K. In the following, the derivation of the conclusion will be demonstrated. Firstly, $\mathbf{R}_{\text{s\_smoothing}}$ can be rewritten as

$$\begin{aligned}\mathbf{R}_{\text{s\_smoothing}} &= \frac{1}{3}\sum_{i=1}^{3}\mathbf{\Phi}_i\mathbf{R}_s\mathbf{\Phi}_i^H \\ &= \frac{1}{3}\sigma_s^2\sum_{i=1}^{3}\mathbf{\Phi}_i\mathbf{g}\mathbf{g}^H\mathbf{\Phi}_i^H \\ &= \frac{1}{3}\sigma_s^2\mathbf{G}\left\{\sum_{i=1}^{3}\mathbf{p}_i\mathbf{p}_i^H\right\}\mathbf{G}^H,\end{aligned} \tag{6}$$

where $\mathbf{G} = \text{diag}(\mathbf{g})$. Note that matrix $\mathbf{G}$ is a diagonal matrix; therefore, rank$(\mathbf{G}) = K$. Recall equation (4), we know that $\mathbf{p}_i$ and $\mathbf{p}_j$ are linearly independent for arbitrary $\{(\theta_k, \varphi)_k, k = 1, \ldots, K\}$ according to the results of [23–25]. Therefore, rank$(\sum_{i=1}^{3}\mathbf{p}_i\mathbf{p}_i^H) = 3$, and then rank$(\mathbf{G}\{\sum_{i=1}^{3}\mathbf{p}_i\mathbf{p}_i^H\}\mathbf{G}^H) = \min(K, 3)$. From the above analysis, one has rank$(R_{\text{s_smoothing}}) = \min(K, 3)$. Now, when the number of sources $K \leq 3$, the rank of SCM will be restored. Therefore, the number of coherent signals can be resolved up to three by using the proposed algorithm.

Remark 1. This algorithm can only resolve up to three coherent sources. The resolvable of coherent sources will be increased to six when forward and backward smoothing technology is used.

Remark 2. It is worth noting that the array structure information is not used, so this proposed method is applicable to arbitrary configuration array.

4. Performance Analysis

It can be seen clearly that the performance of decorrelation depends on the value of $\mathbf{R} \overset{\text{def}}{=} \sum_{i=1}^{3}\mathbf{p}_i\mathbf{p}_i^H$, we call it as after-smoothing source covariance matrix (AS-SCM). It is easy to calculate the diagonal elements of $\mathbf{R}$, which are equal to $\mathbf{R}_{i,i} = \sin^2\theta_i\cos^2\varphi_i + \sin^2\theta_i\sin^2\varphi_i + \cos^2\theta_i = 1,\ i = 1, \ldots, K$. The nondiagonal element values of AS-SCM are equal to

$$\begin{aligned}\mathbf{R}_{i,j} &= \sin\theta_i\cos\varphi_i\sin\theta_j\cos\varphi_j + \sin\theta_i\sin\varphi_i\sin\theta_j\sin\varphi_j \\ &\quad + \cos\theta_i\cos\theta_j, \quad i \neq j.\end{aligned} \tag{7}$$

It can be seen from equation (7) that $\mathbf{R}_{i,j}$, $i \neq j$, is a function of two-dimensional DOA. It shows that the performance of decorrelation of the proposed VFS method depends on the parameters of two-dimensional DOA. The smaller the AS-SCM $\mathbf{R}_{i,j}$, $i \neq j$, the better the performance of decorrelation will be. If $\mathbf{R}_{i,j} = 0$, $i \neq j$, the coherent signals will be decorrelated completely.

In order to make the reader more clearly, we give two examples to observe the elements of matrix $\mathbf{R}$. Firstly, assume that there are two coherent sources, which are located in $(\theta_1, \varphi_1) = (12°, 28°)$ and $(\theta_2, \varphi_2) = (42°, 57°)$, respectively. Then, we have

$$\mathbf{R} = \begin{bmatrix} 1 & 0.8486 \\ 0.8486 & 1 \end{bmatrix}. \tag{8}$$

Then, the rank of $\mathbf{R}$ equals rank$(\mathbf{R}) = 2$. When an extra source with $(\theta_3, \varphi_3) = (61°, 67°)$ is added, we have

$$\mathbf{R} = \begin{bmatrix} 1 & 0.8486 & 0.6155 \\ 0.8486 & 1 & 0.8151 \\ 0.6155 & 0.8151 & 1 \end{bmatrix}. \tag{9}$$

Then, the rank of $\mathbf{R}$ equals rank$(\mathbf{R}) = 3$.

5. Computer Simulations

In this section, simulations are used to demonstrate the effectiveness of the proposed algorithm. In the following simulations, we set that the transmitted sensors are equal to $M = 6$ and received velocity vector sensors are equal to $N = 6$.

We firstly verify the effectiveness of decorrelation by using eigenvalues of eigendecomposition for AS-SCM. Assume that there are two spatially closed coherent sources with the same power, and sources' angles are $(\theta_1, \varphi_1) = (10^\circ, 15^\circ)$ and $(\theta_2, \varphi_2) = (20^\circ, 25^\circ)$, respectively. We set that snapshots $L = 200$ and SNR = 20 dB. In order to verify that the proposed algorithm is suitable for arbitrary array configuration, the received velocity vector sensors are assumed in the x-axis with $[x_{r1}, \ldots, x_{r6}] = [2, 3, 7, 10, 11, 12] (\lambda/2)$, $y_{rm} = 0$, $z_{rm} = 0$, $m = 1, \ldots, 6$, and the transmitted scalar sensors located in the y-axis with $[x_{t1}, \ldots, x_{t6}] = [0, 1, 3, 6, 9, 13] (\lambda/2)$, $y_{tm} = 0$, $z_{tm} = 0$, $m = 1, \ldots, 6$. It is clear that the array configuration has no rotational invariance. Therefore, the spatial smoothing algorithm is failed for the above array. For convenient comparison, Figure 1 gives the largest MN eigenvalues of SCM and AS-SCM. We can clearly see from the figure that there is only one large eigenvalue without smoothing processing. There are two large eigenvalues using VFS smoothing processing, which has the same number to the number of coherent sources. The result demonstrates that the VFS smoothing processing can recover the loss rank of SCM and the proposed algorithm is suitable for arbitrary array configuration. Overall, the results verify the effectiveness of the proposed VFS smoothing algorithm.

In the second simulation, we prove that the performance of our proposed smoothing method depends on the angle of DOA. Without loss of generality, we set $\theta = 90^\circ$. Then, the nondiagonal elements values of AS-SCM are simplified to $\mathbf{R}_{i,j} = \cos\varphi_i \cos\varphi_j + \sin\varphi_i \sin\varphi_j$. Figure 2 shows the nondiagonal elements of AS-SCM versus angle of DOA. It can be seen from the figure that the values of nondiagonal elements are changed with the angle of DOA. The different values of nondiagonal elements values of AS-SCM demonstrate the different performances of the proposed algorithm. A graceful mathematical relationship between decorrelated performance and angle of DOA will be focus of our future work.

In the last simulation, we use the eigenvalue distribution and RMSE of DOA estimation to verify the effectiveness of the algorithm further. Root mean square error (RMSE) of azimuth estimation is defined as RMSE $= \sqrt{(1/2Q)\sum_{q=1}^{Q}\sum_{k=1}^{K}[(\widehat{\varphi}_{k,q} - \varphi_{k,q})^2]}$, where Q is the number of Monte Carlo experiments, $\widehat{\varphi}_{k,q}$ and $\varphi_{k,q}$ denote azimuth estimation and real azimuth value of q th Monte Carlo experiment with k th source, respectively. RMSE of elevation estimation is defined the same as that of azimuth. Here we have two groups' signals: $(\theta_1, \varphi_1, \theta_2, \varphi_2) = (10^\circ, 15^\circ, 20^\circ, 25^\circ)$ and $(\theta_1, \varphi_1, \theta_2, \varphi_2) = (80^\circ, 90^\circ, 60^\circ, 0^\circ)$. Two groups' signals mean that the same MATLAB program needs to run two times, and then the two results are compared to prove the performance of decorrelation. According to equation (7), the

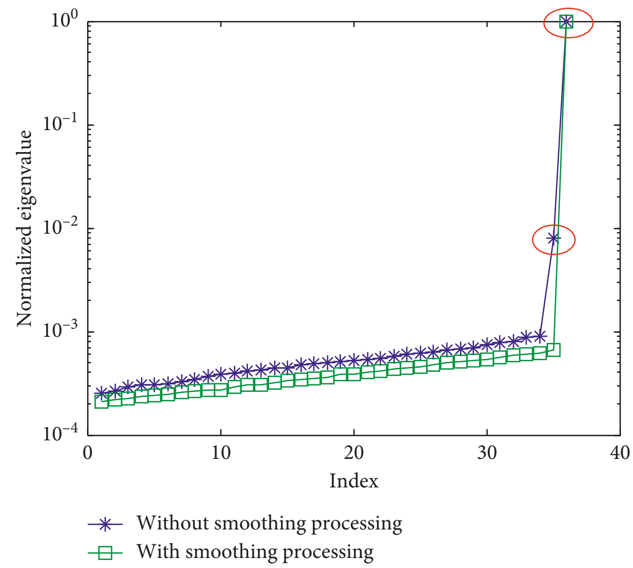

FIGURE 1: Eigenvalues of covariance matrix with and without VFS processing.

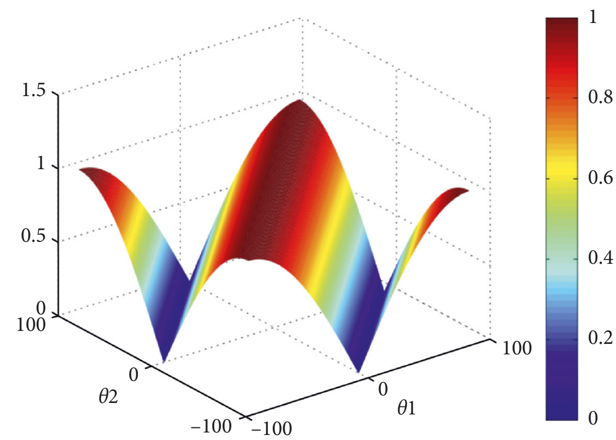

FIGURE 2: The nondiagonal elements of AS-SCM versus DOA.

nondiagonal elements of $\mathbf{R}$ of the first group signal equals $\mathbf{R}_{12} = 0.9839$ and the second group equals $\mathbf{R}_{12} = 0.0868$. It implies that the performance of decorrelation of the second group is much better than that of the first group. The same result can be expected in the following figures. Snapshots are equal to $L = 200$. The number of Monte Carlo experiment equals 1000. Assume that the received velocity vector sensors are located in the x-axis with half-wavelength uniform linear array and the transmitted scalar sensors are located in the y-axis also with half-wavelength uniform linear array. The ESPRIT algorithm is used to estimate sources' azimuth and elevation after velocity-field smoothing. Figure 3 shows the eigenvalue distribution of AS-SCM of two group signals. It is indicated that the two eigenvalues of the second group are closer to each other than those of the second group, so the performance of decorrelation of the second group is better. The above analysis results are successfully verified. Figure 4 shows the RMSE of two-dimensional DOA estimation along

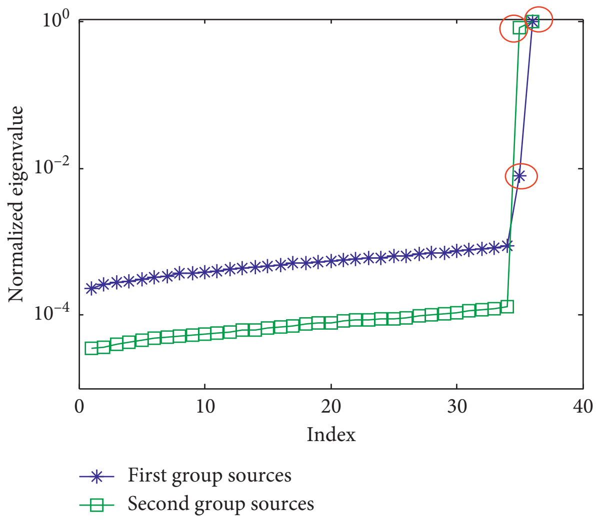

Figure 3: The eigenvalue distribution of AS-SCM with SNR = 20 dB.

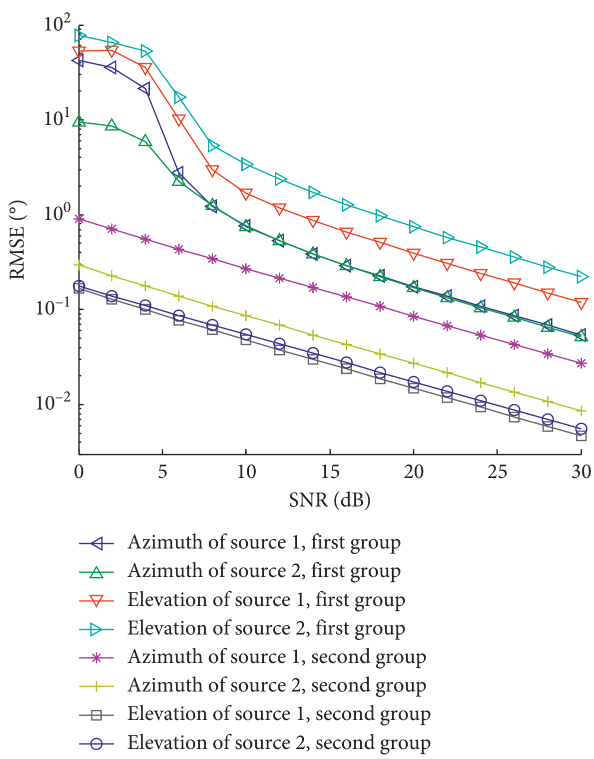

Figure 4: RMSE of two-dimensional DOA estimation versus SNR.

with SNR. We can see from Figure 4 that the DOA estimation performance of the second group is better than those of the first group. It is indicated that the greater the performance of decorrelation, the better estimation accuracy will be.

6. Conclusions

This paper has studied the coherent source DOA estimation for MIMO array with received velocity vector sensors. The proposed VFS preprocessing can effectively restore the rank of AS-SCM. The performance of the algorithm is analyzed with computing the nondiagonal elements of AS-SCM. We find that (1) it is a function of DOA and (2) the better smaller the correlation, the better the estimation accuracy will be. In addition, the algorithm is suitable for arbitrary array configuration.

Data Availability

The MATLAB code data used to support the findings of this study are included within the supplementary information file.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The manuscript is supported by the National Natural Science Fund with grant no. 61971438.

Supplementary Materials

The code of velocity-field smoothing algorithm for MIMO array. (*Supplementary Materials*)

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