

# Research Article **Updating Road Information in Open-Pit Mines Using Truck Trajectories**

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Accurate road information is crucial for the effective planning and management of open-pit mines. However, the parameters related to the roads in open-pit mines change constantly, and their shapes are often complicated. These factors indicate the difficulties in updating road information in open-pit mines. This study reports a method for updating the road information in open-pit mines using the trajectories of trucks. The method employs an algorithm for compressing trajectory data, an algorithm for extracting new roads, and an algorithm for old-road classification. Data from the Fushun open-pit mine is used to validate this method, and our experiments show that it can identify more than 90% of new roads and the majority of disappearing roads. In addition, we analyze several factors affecting the results obtained using this approach and provide a detailed description of the limitations of this method. Overall, our findings indicate that this is a promising approach to road-information retrieval.

# 1. Introduction

Trucks are used in the majority of open-pit operations because of their flexibility compared to other transportation systems [1, 2]. However, their use leads to higher associated costs and lower efficiency than other transportation methods. For example, a mining operation in northern China has a transportation cost that accounts for more than 60% of the total budget, and the effective working time accounts for less than 70% of the potential working time. A series of scheduling algorithms have been suggested to solve this problem, though few of these are viable [3]. After studying these algorithms, we found that all these scheduling algorithms are based on real-time road information. The roads in open-pit mines, however, change frequently and are difficult to update [4].

In general, the parameters related to roads in open-pit mines are measured manually, which always takes a long time due to the wide area contained in open-pit mines. As a result, the road information is often not timely. In recent years, researchers have tried to extract road information from satellite images [5–7]. Shi extracted road maps from high-resolution satellite images [8]. This technique was based on the mathematical morphology of the search area and employed a method that matches line segments. The results indicated that this method can provide highly accurate road information. J. Senthilnath presented a method for extracting city road information using the structural, spectral, and geometric features of the roads [9]. The results showed that this method is efficient in extracting city road information from high-resolution satellite images. Although the road information extracted by these methods is highly accurate, these methods are sensitive to weather [10], as shown in Figure 1.

With the increased use of GPS devices in vehicles, researchers studied methods for using the vehicles' trajectory to extract road information [11–13]. Ekpenyong et al. analyzed GPS trajectory datasets and organized these into categories using a snap-drift neural network (SDNN) [14]. The result shows that the SDNN is able to extract the travelled roads. Sevgen et al. segmented each trajectory into a group and then clustered each group into a similar line, based on a clustering method [15]. The roads were then fitted using a regression algorithm. The experiment indicated that this method can provide promising results in the case of city roads. Cao et al.



FIGURE 1: Certain deep open-pit mines are usually covered by smoke and fog, rendering road identification difficult.

combined the color of the roads with their trajectories in order to detect the road centerline and developed an algorithm to extract each individual road from the data [16]. Extensive experiments show that this method can be used to rapidly and accurately extract the centerlines of roads. Qiu et al. proposed a point segmentation and grouping method based on the Hidden Markov Model in order to generate a map from GPS traces [17]. This algorithm was shown to be robust in terms of noise, generating a highly accurate map.

In addition, researchers have attempted to update road information using GPS trajectories. Ouyang et al. proposed a new method for road-information extraction based on trajectory clustering [18]. Their results show that this method can not only extract the centerline of roads but also be used to update the road network.

These previous studies have inspired the present work. However, the roads in open-pit mines change more frequently than city roads, and the shape of the roads is more complex. We propose a method to extract and update road information using truck trajectories obtained by the GPS module, which is based on the particular features of roads in open-pit mines. Our experiments show that this is a promising approach to road-information retrieval. Further, we analyze several factors affecting the results obtained using this approach and provide a detailed description of the limitations of this method.

#### 2. Methods

A road in an open-pit mine is subject to three stages: generation, development, and disappearance. In general, roads are purposefully built, as opposed to roads that are formed where vehicles are frequently driven. Irrespective of how a road is generated, there will be vehicles passing through it. A road disappears when it is covered or excavated or disconnected from the major roads caused by two formers. The development of a road refers to the road's extension, and this extension can be regarded as the generation of a new road. Therefore, in order to analyze the transformation of roads in an open-pit mine, we may only consider the generation and disappearance of roads. 2.1. Algorithm for Compressing Trajectory Data. Vehicles driving in an open-pit mine send a large amount of position information back every day, which presents challenges in terms of computing efficiency. All the location information collected by the vehicles can be represented as a set

$$\mathbf{L} = \left\{ l_1, l_2, \dots, l_n \right\},\tag{1}$$

where  $l_i$ , i = 1, 2, ..., n, represents the position of a vehicle, including its coordinates and elevation. New data continue to be added to the dataset over time. This implies that **L** is theoretically infinite. If we examine the data in set **L** closely, we find that the truck trajectories along a road where vehicles pass frequently are generally concentrated on the road. This indicates that the location information is spatially bounded. As there is an error in the positioning information sent back by the GPS module, two similar pieces of information can come from the same location. This implies that the set **L** exhibits redundancy. The density of truck trajectories on the same road does not change significantly. Examples of a section of the truck trajectories are shown in Figure 2.

We therefore propose a trajectory-compression algorithm for use with our datasets. The position data obtained using GPS is not absolutely precise, and for a location l = (x, y, z) given by a GPS terminal, the real position might be l'. For simplicity, we assume that this migration obeys a normal distribution.

For a point *P*, with nearby points  $P_1 = (x_1, y_1, z_1)$ ,  $P_2 = (x_2, y_2, z_2)$ , ...,  $P_k = (x_k, y_k, z_k)$ , ...,  $P_t = (x_t, y_t, z_t)$  provided by GPS terminals, the possibilities that every point's real position is *P* are  $p_1, p_2, ..., p_t$ . We then obtain an expression for the possibility that point *P* has at least one vehicle passing through it:

$$p = 1 - (1 - p_1)(1 - p_2)(1 - p_3) \cdots (1 - p_t).$$
(2)

We define a rectangular area G, so that every position is within G. We divide G into  $m \times n$  subregions, denoted by  $G_{ij}$ , where  $1 \le i \le m$  and  $1 \le j \le m$ . We define an  $m \times n$  matrix **R** and an  $m \times n$  matrix **H**, where

$$r_{ij} = p_{ij} \tag{3}$$

$$h_{ij} = \frac{\sum_{k=1}^{t} p_k z_k}{\sum_{k=1}^{t} p_k}.$$
 (4)

Thus, we obtain the average elevation of a region and the probability that this region is part of a road. Regardless of increases in the amount of positioning data, there is an upper limit on the compressed output, which will not exceed m  $\times$  n. We have obtained higher precision elevation data due to averaging of the elevations at a particular position. As the positioning data increases, the elevation is more accurate.

2.2. Algorithm for Extracting New Road Information. The fact that vehicles are driven where previously there was no road is a sufficient condition for road generation. A new road must have at least one vehicle passing through it; otherwise, there would be no purpose to its generation. This implies that we

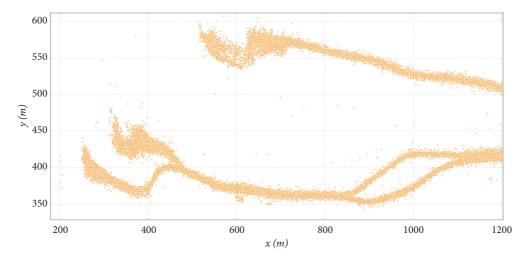


FIGURE 2: x-axis and y-axis represent the length of the north-south and east-west directions in the mining area.

may detect new roads using this characteristic of at least one car passing along the trajectory.

**R** may be regarded as an aerial view that can be used to indicate the presence of roads on the terrain. An imagethinning algorithm is used here to obtain the centerline of the road network. Thinning is one of the most important techniques in the field of image processing. It is applied in order to erode the image of an object layer-by-layer until only the skeleton structure remains.

In order to obtain the centerline of the road, the thinning algorithm must respect the following principles:

- (1) The results of the thinning algorithm must preserve the topology of the roads.
- (2) The results of the thinning algorithm must retain the one-pixel thickness.
- (3) The results of the thinning algorithm must avoid the presence of extraneous pixels.

A popular and well-proven thinning algorithm is the ZS algorithm proposed by Zhang and Suen [19], although results obtained using this method do not exhibit the one-pixel thickness. By changing the neighborhood condition, Lu and Wang proposed an improved ZS algorithm, referred to as the LW algorithm [20]. This algorithm retains the same diagonal lines, but these are two pixels wide. Wu and Tsai proposed a new parallel thinning algorithm for use with binary images, referred to as the WT algorithm [21], which may be considered to be a fully parallel thinning algorithm. However, this algorithm does not preserve the structures of the original objects contained in some patterns, producing biased skeletons and cutting corners. Ahmed and Ward proposed a rotation-invariant rule-based thinning algorithm (referred to as the AW algorithm) for character recognition [22]. Although the AW algorithm produces rotation-invariant skeletons that map central lines, preserving the shape of the original objects, as in the case of the ZS algorithm, this algorithm fails to produce a one-pixel-width skeleton. Yang and Guo proposed the YG algorithm, an index-thinning

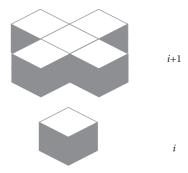


FIGURE 3: If we mine the i<sup>th</sup> level, the blocks on level i+1 must also be mined; otherwise the slope across the point will be unstable.

algorithm [23] that can preserve the topology of objects and provides images containing almost no extraneous pixels after processing. The results of this algorithm also exhibit one-pixel thickness. These advantages led us to use the YG algorithm in order to extract road centerlines.

2.3. Algorithm for Old-Road Classification. In general, there are no overpasses in open-pit mines. Therefore, we assert that every point in the same period on the road corresponds to only one elevation. We assume that the elevation of P(x, y) is  $z_n$  during  $t_n$ , and the elevation of this point is  $z_{n+1}$  during  $t_{n+1}$ . We may then determine the status of this point between  $t_n$  and  $t_{n+1}$  according to the following rule:

if 
$$z_{n+1} > z_n$$
 then point is covered,  
else if  $z_{n+1} < z_n$  then point is excavated, (5)

Else point remains in use.

There are also certain spatial constraints in an open-pit mine. As shown in Figure 3, when a block in the  $i^{\text{th}}$  level is mined, the blocks above *i* must also have been mined.

A block on the  $i^{th}$  level and the blocks that must be mined above the  $i^{th}$  level can be considered to form a cone shape. If

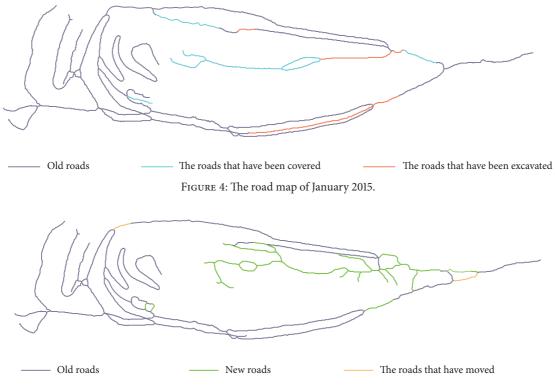


FIGURE 5: The road map of December 2015.

the coordinate of the block on level *i* is  $(x_0, y_0, z_0)$ , then the cone can be expressed as follows:

$$z = z_0 + \tan \alpha \times \sqrt{(x - x_0)^2 + (y - y_0)^2}$$
(6)

where  $\alpha$  is the angle between the cone and the horizontal plane. For a point  $P_n(x_n, y_n, z_n)$ , we may determine the status according to the following rule:

i

f 
$$z_n \ge z$$
 then point is excavated,  
Else point remains in use. (7)

Similarly, when a block is added to the i+1 level, several blocks must be added to level i. We may obtain another expression to determine if the road is covered:

$$z = z_0 - \tan \beta \times \sqrt{(x - x_0)^2 + (y - y_0)^2}$$
(8)

where  $\beta$  is the angle between the cone and the horizontal plane. We may determine the status of the point according to the following rule:

if 
$$z_n \le z$$
 then point is covered,  
else point remains in use. (9)

Therefore, the key to the classification of roads is to accurately determine  $\alpha$  and  $\beta$ , as the results of this method depend on the accuracy of the elevation calculation. In order to reduce the influence of this error, we input the trajectories within a certain range into the classification model, and the most common status is adopted as the final result.

#### 3. Results

We implemented the above approach using the Python code. The code is available on GitHub. The data used for the validation of this method was collected in 2015 from the Fushun West open-pit mine, located in Fushun City, Liaoning Province. There are 38,795,543 data sets available from that year. We compressed the data for each quarter of the year into two matrices, in which every element represents the average elevation of a 2 m × 2 m subregion, or the probability that this subregion is a part of a road. We let  $\alpha = 30^{\circ}$  and  $\beta = 40^{\circ}$  and obtained the changes of the road network for 2015, as shown in Figures 4 and 5.

In order to better understand the way in which the size of the subregion, angle  $\alpha$ , and angle  $\beta$  affect the road recognition results, we conducted various experiments.

3.1. The Effect of Subregion Size on Accuracy. We calculated the offset of each road intersection and used the maximum of these offsets to characterize the accuracy of the results. In order to determine the way in which subregion size affects accuracy, we calculated the accuracy of the results for various subregion sizes. The obtained relationship between accuracy and the size of the subregion is shown in Table 1. These results indicate that the accuracy of the results reduces with increasing subregion size.

3.2. The Effect of Subregion Size on Completeness. In order to analyze the effect of subregion size on the completeness of the results, we define coverage as the ratio of the number of roads extracted using our method to the number of real roads.

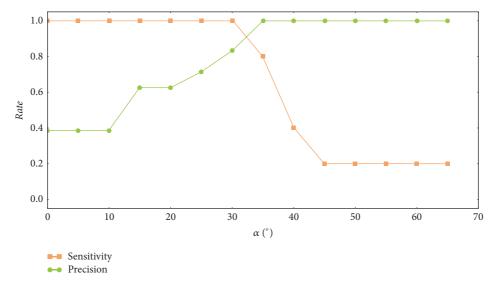


FIGURE 6: This figure shows the relationship between the classification results and  $\alpha$ . For  $\alpha$  values greater than 65°, the present method is not able to find the excavated road.

TABLE 1: The relationship between accuracy and the size of the subregion.

Size (m)	1	2	3	4	5	6
Accuracy (m)	0.73	1.51	1.97	3.03	5.32	7.80

TABLE 2: The relationship between completeness and the size of the subregion.

Size (m)	1	2	3	4	5	6
Completeness (%)	64.63	92.68	93.90	96.34	100	100

Calculations of the coverage of the results were performed for various subregion sizes, as shown in Table 2. These calculations indicate that the completeness of our results improves with increasing subregion size. A completeness value of 100% was reached for subregion sizes above 5 m, after which the completeness was stable.

3.3. The Effect of  $\alpha$  on Road-Mapping Results. In order to determine the way in which  $\alpha$  affects classification results for roads, we conducted the following experiment. In terms of information retrieval, precision may be defined as the fraction of relevant instances among the retrieved instances, while sensitivity is the fraction of relevant instances that have been retrieved with respect to the total amount of relevant instances [24, 25].

A perfect precision score of 1.0 implies that every result that was retrieved by a search was relevant (though this does not indicate whether all relevant documents were retrieved). A perfect sensitivity score of 1.0 implies that all relevant documents were retrieved by the search (though this does not state the number of irrelevant documents that were also retrieved) [26]. Here, we define sensitivity as the fraction of relevant roads (excavated roads) among the retrieved roads (those considered to be excavated). The precision is defined as the fraction of relevant and retrieved roads (excavated roads that are also considered to be excavated) among the excavated roads.

A method may be thought to provide good performance only if both the precision and sensitivity approach a value of 1.0. We calculated the sensitivity and precision of road recognition at various angles  $\alpha$ , with the results shown in Figure 6.

As  $\alpha$  increases, the sensitivity gradually decreases from a value of 1.0, and the precision gradually increases to 1.0. When the angle  $\alpha$  is equal to 30°, sensitivity reaches 1.0, and the precision can be greater than 0.8. This implies that the present method is able to accurately identify most of the excavated roads in the open-pit mine.

3.4. The Effect of  $\beta$  on Road-Mapping Results. Another similar experiment was conducted in order to analyze the effect of the angle  $\beta$  on the classification results, with the results shown in Figure 7.

The two curves in Figure 7 exhibit similar trends to those that describe the effect of  $\alpha$  on the road-mapping results. Unlike in the previous findings, the sensitivity does not reach a value of 1.0. This implies that certain roads cannot be recognized. After an investigation, we found that the roads in question had been destroyed by a landslide.

## 4. Discussion

4.1. Accuracy and Completeness of the Road Skeleton. A smaller subregion leads to accurate but less complete road mapping, while a larger subregion leads to more complete but less accurate road mapping. This indicates that there must be a trade-off between the accuracy and completeness of the results. It is also apparent from these results that the subregions represent a unit that cannot be subdivided. The smaller the subregion size is, the more precise the results of

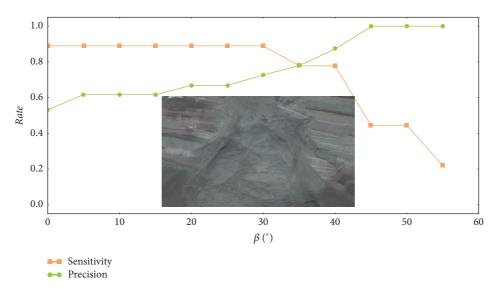


FIGURE 7: This figure shows the relationship between the classification results and  $\beta$ . For  $\beta$  values greater than 55°, the method is no longer able to find the excavated road. Due to phenomena such as landslides, the sensitivity does not reach a value of 1.0.

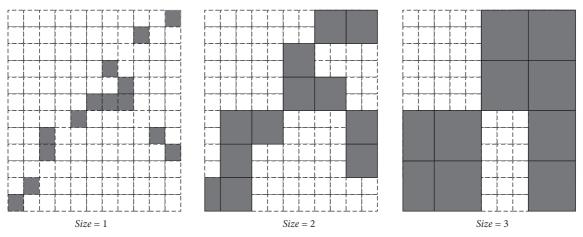


FIGURE 8: The relationship between the accuracy and completeness of the road skeleton for subregions of 1, 2, and 3 m.

road extraction will be. As the subregions become larger, the error also increases.

However, since both the amount of data and the size of the mining area are constant, the smaller the subregion size, the greater the number of subregions that are contained in the mining area. As not every subregion contains data, the connectivity between subregions is degraded. As a result, the completeness of the road extraction results is degraded, as shown in Figure 8.

4.2. The Precision and Sensitivity of Classification Results. For this classification model, the sensitivity of the results with increasing  $\alpha$  remains unchanged at first and then gradually decreases, while the precision gradually increases and finally stabilizes. The reasons for these phenomena are as follows:

- When α (or β) is small, most roads, including all excavated or covered roads, are within the cone.
- (2) As  $\alpha$  (or  $\beta$ ) increases, the number of roads within the cone gradually decreases, but the road being

excavated and covered is still in the cone. As a result, the sensitivity remains unchanged, but the precision is gradually increasing.

- (3) When α (or β) reaches a particular angle, the number of roads that are excavated or covered within the cone gradually decreases. This leads to a decline in the sensitivity.
- (4) When there is only a road that is excavated or covered in the cone, the precision rises to 1.0.

At the point at which the sensitivity begins to decline, the precision has not yet reached a value of 1.0. This implies that there are always roads that cannot be classified using this model. We will discuss the reasons for this limitation in the next section.

4.3. Limitations of the Classification Methods. Investigations into the effect of angle  $\beta$  on the classification results indicated that the classification model could not effectively identify

roads that were covered due to a landslide. Clearly, when the two roads meet the spatial relationship mentioned previously, the older road must have disappeared. However, when older roads disappear, they do not necessarily meet this spatial relationship.

As a result of on-site inspections, we found that the reasons for this phenomenon are as follows:

- (1) Landslides cause incorrect road identification.
- Dumping by rail renders certain covered roads unrecognizable.

Therefore, although this classification approach is able to identify changes in the majority of roads, it cannot be used to effectively identify road changes caused by processes other than the truck-shovel process, as well as by landslides.

# 5. Conclusions

In this paper, we propose a mathematical method for updating the information related to roads in open-pit mines using truck trajectories. Our experiments show that this method can identify more than 90% of new roads and the majority of disappearing roads. The maximum offset of the center point of a road intersection is found to be less than 2 m, which implies that this method could be successfully employed in rapidly changing road-mapping applications. However, there are still limitations related to this approach. This method cannot effectively identify road changes caused by processes other than the truck-shovel process, such as landslides. This issue must be addressed in order to further develop this promising mapping approach, and, in the future, we will focus on the identification of such road changes.

#### **Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

# **Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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