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Research Article

A Modelling and Control Approach for a Type of Mixed Logical Dynamical System Using in Chilled Water System of Refrigeration System

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The chilled water system of central air conditioning is a typical hybrid system. The dynamic adjustment of cooling capacity based on hybrid system can achieve accurate temperature control and real-time energy saving. Mixed logical dynamical (MLD) systems have advantage for solving constrained optimization problems of this type of case by numerical methods. This paper proposes a novel modified type of MLD system, which enhances the model applicability and improves the switching flexibility and control effect for the framework. In order to meet the needs of cooling capacity and energy saving, the optimal control problem is transformed as MIQP problem by defined performance index. As a numeral example of application, the model and control method is used in pumps group control for variable water volume in chilled water system of central air conditioning. At last, the dynamic and energy-saving effects of the system are simulated, which shows the ideal control results.

1. Introduction

In recent years, hybrid systems have attracted many researchers from various fields, and more and more investigations focused on the theoretical exploration and engineering applications of hybrid systems in a variety of industries and fields, such as electric systems [1, 2], vehicle dynamic control [3, 4], aircraft design and control [5–7], traffic control [8, 9], robot systems [10, 11], chemical industries [12], and so on. Actually, modern complex industrial processes are typical hybrid systems, in which continuous dynamic processes interact with discrete events. According to the application background, the traditional single continuous or discrete systems hardly illustrate the process accurately, and the effect of corresponding control measure is not ideal. However, hybrid systems show advantages to reflect the issues of modelling and control for modern complex industrial applications. Because hybrid system includes both continuous and discrete components, and the control strategy is based on hybrid model. Hybrid systems adapt the characteristics of complexity in industrial control, therefore requiring further developments in the future.

For solving different fields of modelling and control problems, many researchers have built types of hybrid systems to fit different engineering cases, i.e., the mixed logical dynamical (MLD) systems [13]. MLD system is described by interdependent physical laws, logical rules, and operational constraints. In the process of modelling, qualitative knowledge and expert experience of the system are taken into account. The heuristic knowledge, logical judgment, and constraints that must be observed in operation of the object are transformed into propositional logic. The truth and falsity of propositions are represented by binary variables 1 and 0 introduced and expressed by logic. Connectors $\wedge$ (and), $\vee$ (or), $\sim$ (non), $\longrightarrow$ (contain), $\longleftrightarrow$ (and only), $\oplus$ (exclusive or) transform simple propositions into compound propositions and then express them in the form of integer linear inequalities containing both continuous variables and logical variables. Based on the MLD system, it is more convenient to establish and solve the problems concerned in the field of system, for example, reachability analysis [14], controller synthesis [15], state estimation [16], and system fault detection and so on [17].

Based on the above model rules and characteristics, MLD system can easily establish numerical methods for solving constrained optimization problems. The advantage of MLD model is that the systems can be represented by a hybrid logical dynamic model with the characteristics of linear dynamic systems in form after introducing appropriate auxiliary logical variables and auxiliary continuous variables. In order to achieve optimal control, the control measures [18–20] were transformed to dynamic programming problems, such as mixed-integer linear programming (MILP) form [21] and mixed-integer quadratic programming (MIQP) form [22, 23].

The study of MLD systems has gained growing interest in nearly two decades; some theoretical and applied results already exist in the literature. In [24], the model of a hydroelectric power plant in the framework of MLD systems was derived, and solved in a systematic way using a MILP method. In [25], a semi-active suspension model and constrained optimization problem was studied; the performance of different finite-horizon hybrid MPC controllers is tested in simulation using MIQP measure. In [26], an autonomous Lane-Change Controller via MLD systems was designed. An approach of complex mission optimization for multiple-UAVs using linear temporal logic was found in [27]. In [28], an algorithm named optimal pointer placement (OPP) scheduling algorithm was investigated and applied to the control and scheduling of a car suspension system. In [29], some recast types of biped motion were provided and controlled by on line optimal control approach. In [30], a unified modelling mode for control problems of multi-contact hand manipulation can be seen. In [31], a kind of MLD model to predict the back bead width of pulsed GTAW process with misalignment was addressed. In [32], a control strategy for a continuous stirred tank reactor (CSTR) system was introduced, and a controller of MPC was designed.

Many researchers focused on the existing MLD framework, and few people improve the model and control effect. Since the MLD system is an inequality form, the existing relationships between logical express and inequalities are based on zero, so there is only a couple of 0/1 switching signals. Based on that, there is only a pair of opposite switching directions of the system. The corresponding control strategy is also relatively simple, unable to deal with more complex control problems [33].

This paper proposes a novel type of MLD systems, which shows improvements for the inequalities. In framework of the inequalities, some new contain relations between logical expresses and inequalities are inferred by discussing non-zero switching point. A universal state-dependent switching law is obtained by dividing state region and setting three switching directions, which is not a 0/1 relationship confined to zero. For solving the optimal control problem of the improved MLD system, it is transformed as MIQP problem by defined performance index. The MLD system is firstly used in the refrigeration system for energy-saving control. Under the conditions of limited inputs and finite time, the inside temperature is reduced to setting value for minimizing power consumption. Via the state-dependent switching law and MIQP control strategy, pumps are used in variable water volume method for the chilled water system in central air conditioning, which transfer the cooling capacity to inside room by air system. Moreover, a simulation shows the control dynamical the energy-saving results.

The paper's structure is as follows. Section 2 presents the framework of the type of MLD system with new switching points and law. The optimal control strategy is shown in Section 3. Section 4 uses the system in refrigeration system and gives a numeral inequality. Section 5 uses a simulation example to illustrate the dynamics of refrigeration system under the framework. Finally, some concluding remarks are made in Section 6.

2. Framework of the Improved MLD System

The improved MLD system contains *n+1* subsystems, which is described as an inequality and composed of state variables, control variables, logical variables, auxiliary variables, novel defined switching points, and switching law. The constraints of the variables are gained via the inequality by (1), which is transformed from the constraints (2c) of the MLD standard form.

$$\begin{bmatrix} -\begin{bmatrix} \vdots \\ 0_i \\ \vdots_{M(i)} \\ M_{a-M(i)} \\ \vdots \end{bmatrix} \\ \begin{bmatrix} \vdots \\ M_{b-m(i)} \\ \vdots_{m(i)} \\ 0_i \\ \vdots \end{bmatrix} \\ M_{c-i} \\ -M_{d-i} \\ \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \\ -M_i \\ m_i \\ M_i \\ -m_i \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \delta(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} z(t)$$

$$
\leq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -B_i \\ B_i \\ 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} u(t) + \begin{bmatrix} -\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix} \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ -A_i \\ A_i \\ 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} x(t) + \begin{bmatrix} -\left(M_i{}^* - \varepsilon\right) \\ -\left(m_i{}^* + \varepsilon\right) \\ -\left(M_i{}^* + \varepsilon\right) \\ m_i{}^* - \varepsilon \\ 1 \\ 0 \\ 0 \\ M_i \\ -m_i \\ -m_i \\ M_i \\ -u_{i-\min} \\ u_{i-\max} \end{bmatrix} \tag{1}
$$

where state variable $x(t) \in R^n$, input variable $u(t) \in R^n$, logic variable $\delta_i(t) \in \{0,1\}$, $i = 0, 1, \ldots, n$, and auxiliary variable are defined as $z_i(t) \triangleq [A_i x(t) + B_i u(t)]\delta_i(t)$. t is time scale, A_i and B_i with $i = 0, 1, \ldots, n$ are state and input matrices of subsystems, respectively. ε is a small tolerance, M_i^* and $m_i{}^*$ with $i = 0, 1, \ldots, n$ are the upper and lower switching critical values, of course the switching points, respectively. M_i and m_i are state upper and lower bound values of vectors $M = [M_1, \ldots, M_n]'$ and $m = [m_1, \ldots, m_n]'$, respectively. $M(i)$ and $m(i)$ are the switching laws/switching directions and the condition that it is triggered beyond the upper and lower boundaries, respectively. For convenience, equalities are set as $M_{a-i} = M_i + \varepsilon - M_i^*$, $M_{b-i} = m_i - \varepsilon - m_i^*$, $M_{c-i} = M_i + \varepsilon - m_i^*$, and $M_{d-i} = m_i - \varepsilon - M_i^*$.

The standard discrete-time MLD system is

$$
\begin{aligned} x(t+1) = {} & A_t x(t) + B_{1t} u(t) + B_{2t} \delta(t) \\ & + B_{3t} z(t) \end{aligned} \tag{2a}
$$

$$
\begin{aligned} y(t) = {} & C_t x(t) + D_{1t} u(t) + D_{2t} \delta(t) \\ & + D_{3t} z(t) \end{aligned} \tag{2b}
$$

$$
E_{2t} \delta(t) + E_{3t} z(t) \leq E_{1t} u(t) + E_{4t} x(t) + E_{5t} \tag{2c}
$$

where $x = \left[\begin{smallmatrix} x_c \\ x_d \end{smallmatrix}\right]$, $x_c \in \mathbb{R}^{n_c}$, $x_d \in \{0,1\}^{n_d}$, $n \triangleq n_c + n_d$ is the continuous and discrete state of the system, respectively. $y = \left[\begin{smallmatrix} y_c \\ y_d \end{smallmatrix}\right]$, $y_c \in \mathbb{R}^{p_c}$, $y_d \in \{0,1\}^{p_d}$, $p \triangleq p_c + p_d$ is the output vector. $u = \left[\begin{smallmatrix} u_c \\ u_d \end{smallmatrix}\right]$, $u_c \in \mathbb{R}^{m_c}$, $u_d \in \{0,1\}^{m_d}$, $m \triangleq m_c + m_d$ is the continuous and discrete input variables, respectively. A_t, $B_{1\text{-}3t}$, C_t, $D_{1\text{-}3t}$, $E_{1\text{-}5t}$ are matrices of suitable dimensions.

2.1. Mathematical Model. Consider the following dynamic system:

$$
x(t+1) = \begin{cases} A_0 x(t) + B_0 u(t) & \begin{bmatrix} x & u \end{bmatrix}' \in X_0 \\ \vdots & \\ A_n x(t) + B_n u(t) & \begin{bmatrix} x & u \end{bmatrix}' \in X_n \end{cases} \tag{3}
$$

Each region X_i ($i = 0, 1, \ldots, n$) is a continuous linear time-invariant subsystem, which indicates a mutually disjoint convex polyhedron, subjected to $N_i x + R_i u \leq T_i$. N_i, R_i, and T_i are the matrices of state, input and constraint, respectively. The system switches in such finite *n+1* regions and satisfies the following conditions:

$$
X_i \cap X_j = \varnothing, \quad \forall i \neq j \tag{4a}
$$

$$
\bigcup_{i=0}^{n} X_i = X \tag{4b}
$$

For each region X_i, δ_i subjected to the exclusive-or condition:

$$
[x \in X_i] \longleftrightarrow [\delta_i = 1] \tag{5a}
$$

$$
\bigoplus_{i=0}^{n} [\delta_i = 1] \tag{5b}
$$

Due to constraints of the subsystems and the transformational relation with the logic variables,

$$
[\delta_i = 1] \longleftrightarrow \left[\begin{bmatrix} x \\ u \end{bmatrix} \in X_i\right] \tag{6a}
$$

$$
X_i = \left\{\begin{bmatrix} x \\ u \end{bmatrix} : N_i x + R_i u \leq T_i\right\} \tag{6b}
$$

This infers that

$$
[\delta_i = 0] \longleftrightarrow \bigvee_{j=1}^{n_i} \left[N_i^j x + R_i^j u > T_i{}^j\right] \tag{7}
$$

Clearly, N_i^j denotes the jth row and ith rank. According to the exclusive-or and equivalence relation [16], (7) can be transformed to equivalent mixed integer inequality constraints:

$$
N_i x(t) + R_i u(t) - T_i \leq M^*{}_i \left[1 - \delta_i(t)\right] \tag{8a}
$$

$$N_i x(t) + R_i u(t) - T_i \geq \varepsilon_i + (m_i^* - \varepsilon_i)\,\delta_i(t) \tag{8b}$$

$$\sum_{i=0}^{n} \delta_i = 1 \tag{8c}$$

where $M_i^* = \max_{x \in X_i}\{N_i x(t) + R_i u(t) - T_i\}$, $m_i^* = \min_{x \in X_i}\{N_i x(t) + R_i u(t) - T_i\}$. Due to the logic variable δ_i, the standard form of the system can be rewritten as follows:

$$x(t+1) = \sum_{i=0}^{n} [A_i x(t) + B_i u(t)]\,\delta_i(t) \tag{9}$$

Equation (9) is nonlinear and contains a logic variable, state, and input for the sake of easy control. Thus, equation $y = \delta f(x)$ can be rewritten as mixed integer linear inequalities:

$$y \leq M\delta \tag{10a}$$

$$y \geq m\delta \tag{10b}$$

$$y \leq f(x) - m(1-\delta) \tag{10c}$$

$$y \geq f(x) - M(1-\delta) \tag{10d}$$

where $M \triangleq \max_{x \in X} f(x)$ and $m \triangleq \min_{x \in X} f(x)$, which are finite-estimated or taken as the correct values by quadratic linear programming.

Assuming that

$$x(t+1) = \sum_{i=0}^{n} z_i(t) \tag{11a}$$

$$z_i(t) \triangleq [A_i x(t) + B_i u(t)]\,\delta_i(t) \tag{11b}$$

Finally, because of z, the standard form can be updated as follows:

$$z_i(t) \leq M\delta_i(t) \tag{12a}$$

$$z_i(t) \geq m\delta_i(t) \tag{12b}$$

$$z_i(t) \leq Ax_i(t) + Bu_i(t) - m(1-\delta_i(t)) \tag{12c}$$

$$z_i(t) \geq Ax_i(t) + Bu_i(t) - M(1-\delta_i(t)) \tag{12d}$$

2.2. Novel Defined Switching Points. In [14], it just shows some simple inequalities (switched point is 0). For improving the freedom degree of switching point, and letting it not be confined to a specific point, a non-zero point should be presented. If the switched point is not 0 but S, $m \leq S \leq M$ can be set, S is any value of the region, which infers some inequalities. The relationship between logical express and inequalities is shown in Table 1.

2.3. Switching Law. Based on the Novel defined switching points, switching law has new definition. Compared with one switching point (switched point is 0 mentioned in [16]), there are three intervals for switching decision, but not two.

Table 1: Relationship between logical express and inequalities.

Relation	Logic	Inequalities
contain $m \leq S \leq M$	$[f(x) \leq S] \longrightarrow [\delta = 1]$	$f(x) \geq S+\varepsilon+(m-\varepsilon-S)\delta$
	$[\delta = 1] \longrightarrow [f(x) \leq S]$	$f(x) \leq M(1-\delta) + S\delta$
	$[f(x) \geq S] \longrightarrow [\delta = 1]$	$f(x) \leq (M+\varepsilon-S)\delta-\varepsilon+S$
	$[\delta = 1] \longrightarrow [f(x) \geq S]$	$f(x) \geq m(1-\delta) + S\delta$

Thus, the switching law can be defined as follows: the state variable always changes in a region with time (whatever infinite or finite region). Here, two points can be set (the switching upper and lower bounds M_i^* and m_i^*), which divide the region into 3 intervals, $m_i^* < x_i(t) < M_i^*$, $x_i(t) \geq M_i^*$, and $x_i(t) \leq m_i^*$. Once the state variable occurs $x_i(t) \geq M_i^*$ or $x_i(t) \leq m_i^*$, the switching behavior will be triggered. If the state variable is $m_i^* < x_i(t) < M_i^*$, system still stays in current subsystem without triggering switching behavior.

If the system is in the ith subsystem currently with $i = 0, 1, \ldots, n$, on account of the state value, switching points and intervals, the switching law can be set as follows [33]:

$$m_i^* < x_i(t) < M_i^*, \quad \delta_i = 1 \tag{13a}$$

$$x_i(t) \geq M_i^*, \quad \delta_{M(i)} = 1 \tag{13b}$$

$$x_i(t) \leq m_i^*, \quad \delta_{m(i)} = 1 \tag{13c}$$

Remark

(1) By the switching law/control strategy, $M(i) = j$ is a mapping functions, the same as $m(i)$.

(2) $M(i) \neq m(i)$ indicates that $M(i)$ has different switching direction with $m(i)$.

(3) If state variable $x_i(t) \geq M_i^*$, the system transfers from ith subsystem to $M(i)$ th subsystem; if state variable $x_i(t) \leq m_i^*$, the system transfers from ith subsystem to $m(i)$ th subsystem.

(4) i denotes the ith subsystem, the same as j.

(5) i belongs to the integer set which contains $n+1$ numbers, i just is one number of the set, the same as j.

(6) Generally, $M(i) = j$, $i \neq j$. Just under some special conditions, $i = j$ occurs. Otherwise, the law loses switching meaning.

(7) Because the law is state-dependent, switching behavior is triggered by state variable, which is not decided by switching sequence and switching time.

(8) Switching sequence and switching time are produced autonomously by state variable. Thus, both of them are random and synchronous.

The basic switching condition is known: only one subsystem can be selected each time, which abides by $\sum_0^n \delta_i = 1$. $\delta_i = 1$ or $\delta_i = 0$ indicates the ith subsystem is selected or not, respectively.

Using the MLD logic expression, the law can be modified as follows:

$$[x_i(t) < M_i^*] \wedge [x_i(t) > m_i^*] \longrightarrow [\delta_i(t) = 1] \tag{14a}$$

$$[x_i(t) \geq M_i^*] \longrightarrow [\delta_{M(i)}(t) = 1] \tag{14b}$$

$$[x_i(t) \leq m_i^*] \longrightarrow [\delta_{m(i)}(t) = 1] \tag{14c}$$

with the constraint $\sum_0^n \delta_i(t) = 1$.

Because the system only selects one subsystem,, after switching behavior happens, the logical relations can be found:

$$[\delta_{M(i)}(t) = 1] \longrightarrow [\delta_i(t) = 0] \tag{15a}$$

$$[\delta_{m(i)}(t) = 1] \longrightarrow [\delta_i(t) = 0] \tag{15b}$$

From (14a), (14b), (14c), (15a), and (15b), we obtain the mixed integer inequality constraints:

$$\begin{aligned} m_i^* - \varepsilon + (M + \varepsilon - m_i^*)\,\delta_i &\leq x_i(t) \\ &\leq M_i^* + \varepsilon + (m - \varepsilon - M_i^*)\,\delta_i \end{aligned} \tag{16a}$$

$$x_i(t) \leq M_i^* - \varepsilon + (M + \varepsilon - M_i^*)\,\delta_{M(i)} \tag{16b}$$

$$x_i(t) \geq m_i^* + \varepsilon + (m - \varepsilon - m_i^*)\,\delta_{m(i)} \tag{16c}$$

$$\delta_i + \delta_{M(i)} + \delta_{m(i)} \leq 1 \tag{16d}$$

This paper uses the continuous auxiliary variable z and (12a), (12b), (12c), and (12d) to derive MLD inequality (1).

3. Optimal Control Strategy

Many literatures [13, 14, 18–23] show that mixed-integer quadratic programming (MIQP) method is a more effective way to deal with the problem of MLD system. This method can not only easily deal with the optimization control problem of quadratic performance objective, but, more importantly, it can conveniently handle the state of the system and the constraints of the control input and establish a unified method for the constraint optimization of the MLD system. Especially in continuous-time hybrid systems, it inevitably meets the problem of dimensionality in solving the Hamilton-Jacobi-Bellman and Euler-Lagrange differential equations by the principle of maximum value or the method of dynamic programming.

For a giving positive MLD system, the state transfers from initial state x_0 to final state x_f in a finite time T. We search (if it exists) the control sequence $u_0^{T-1} \triangleq \{u(0), u(1), \ldots u(T-1),\}$ to ensure the state transferring and minimize the performance index:

$$\begin{aligned} J\left(u_0^{T-0}, x_0\right) \triangleq \sum_{t=0}^{T-1} & \left\| u(t) + u_f \right\|_{Q_1}^2 \\ & + \left\| \delta\left(t, x_0, u_0^t\right) + \delta_f \right\|_{Q_2}^2 \\ & + \left\| z\left(t, x_0, u_0^t\right) - z_f \right\|_{Q_3}^2 \\ & + \left\| x\left(t, x_0, u_0^{t-1}\right) - x_f \right\|_{Q_4}^2 \\ & + \left\| y\left(t, x_0, u_0^t\right) - y_f \right\|_{Q_5}^2 \end{aligned} \tag{17}$$

subject to $x(T, x_0, u_0^{T-1}) = x_f$ and (2a), (2b), and (2c), where $\|x\|_Q^2 \triangleq x'Qx$, $Q_i = Q_i' \geq 0$, $i = 1, \ldots, 5$, are given weight matrices, and $u_f, x_f, y_f, \delta_f, z_f$ are given offset vectors satisfying (2b) and (2c).

From (2a), (2b), and (2c), the solution formula is

$$\begin{aligned} x(t) = A^t x_0 + \sum_{i=0}^{t-1} A^i \,[&B_1 u(t-1-i) + B_2 \delta(t-1-i) \\ &+ B_3 z(t-1-i)] \end{aligned} \tag{18}$$

with these following vectors:

$$\begin{aligned} \Omega &\triangleq \begin{bmatrix} u(0) \\ \vdots \\ u(T-1) \end{bmatrix}, \\ \Delta &\triangleq \begin{bmatrix} \delta(0) \\ \vdots \\ \delta(T-1) \end{bmatrix}, \\ \Xi &\triangleq \begin{bmatrix} z(0) \\ \vdots \\ z(T-1) \end{bmatrix}, \\ \gamma &\triangleq \begin{bmatrix} \Omega \\ \Delta \\ \Xi \end{bmatrix}. \end{aligned} \tag{19}$$

Then, the object of constrained optimization control is transformed into a typical MIQP problem:

$$\min \gamma \quad \gamma' S_1 \gamma + 2\left(S_2 + x_0' S_3\right)\gamma \tag{20a}$$

$$\text{s.t.} \quad F_1 \gamma \leq F_2 + F_3 x_0 \tag{20b}$$

where matrices S_1, F_i, $i = 1, 2, 3$ are suitably defined.

4. Using in Refrigeration System

In the air conditioning system, the cooling capacity transfers from chiller unit to chilled water system by evaporator. The pumps drive water as cycle in the chilled water system, and send cooling capacity to air condition unit for exchanging from chilled water system to air system. After mixed with fresh air, the cold air is sent to air conditioning room to drop the temperature. In the process of temperature change in air conditioning room, the heat balance is influenced by several aspects: cold air, random heat of occupants and equipment, latent heat inside, and heat transfer from building structure. The processes of the cooling capacity transfer and temperature change are shown in Figure 1.

This paper uses constant air volume and variable water volume/constant temperature difference measures to transfer cooling capacity. In chilled water system, switching and variable frequency behaviors of pumps adjust the flow for saving energy, which denote the discrete and continuous time process, respectively. So we give an equation to show the heat balance as follows [34]:

$$\begin{aligned} C_a m_a \frac{d\theta}{dt} &= -\gamma\eta C_w \Delta\theta \left(q_b + q_G + k q_{sw}\right) \\ &\quad - \left(1 - R_r\right) q_{sa} C_a \theta + \left(1 - R_r\right) q_{sa} C_a \theta_{out} \\ &\quad + Q_{rd} + Q_{qr} - \sum K_j A_j \theta + \sum K_j A_j \theta_j \end{aligned} \tag{21}$$

On the left side of the equation, $C_a m_a (d\theta/dt)$ means the time differential of the heat capacity of a room. On the left side of the equation, $-\gamma\eta C_w \Delta\theta(q_b + q_G + k q_{sw})$ reflects the cooling capacity for chilled water system; $-(1 - R_r) q_{sa} C_a \theta$ and $(1 - R_r) q_{sa} C_a \theta_{out}$ denote the cooling capacity from return and fresh air systems, respectively; Q_{rd} is random heat of occupants and equipment; Q_{qr} shows latent heat inside; $-\sum K_j A_j \theta + \sum K_j A_j \theta_j$ represents heat transfer from building structure. The description of the symbols and values can be seen in Table 2.

In the MLD system, the inequality is shown in (22), in which the values and calculations are from Table 2. $x(t)$ is the inside temperature with variable boundaries, $u(t)$ is the flow volume of the variable pump with limited range of 50-100%, $\delta(t)$ denotes different amount of switchable pumps (fixed and variable frequency pumps are always work), and $z(t)$ is auxiliary variable without physical meaning.

$$\begin{bmatrix} 0 \\ -49.52 \\ -49.52 \\ -49.52 \\ 10.48 \\ 10.48 \\ 10.48 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 50.52 \\ 50.52 \\ 50.52 \\ 50.52 \\ -9.48 \\ -9.48 \\ -9.48 \\ -9.48 \\ -50 \\ -50 \\ -50 \\ -50 \\ 10 \\ 10 \\ 10 \\ 10 \\ 50 \\ 50 \\ 50 \\ 50 \\ -10 \\ -10 \\ -10 \\ -10 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \delta(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} z(t)$$

$$\leq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.035 \\ -0.035 \\ -0.035 \\ -0.035 \\ 0.035 \\ 0.035 \\ 0.035 \\ 0.035 \\ 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} u(t) + \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.018 \\ -0.018 \\ -0.018 \\ -0.018 \\ 0.018 \\ 0.018 \\ 0.018 \\ 0.018 \\ 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} x(t) + \begin{bmatrix} -0.48 \\ -0.48 \\ -0.48 \\ -0.48 \\ 0.48 \\ 0.48 \\ 0.48 \\ 0.48 \\ 1 \\ 1 \\ 1 \\ 1 \\ -0.52 \\ -0.52 \\ -0.52 \\ -0.52 \\ -0.52 \\ -0.52 \\ -0.52 \\ -0.52 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 50 \\ 50 \\ 50 \\ 50 \\ -10 \\ -10 \\ -10 \\ -10 \\ -10 \\ 50 \\ -0.01 \\ 0.02 \end{bmatrix} \quad (22)$$

4.1. Model of Cooling Capacity Transferring. The principle of refrigeration is the progress of coldness transferring in essence; the cooling capacity is produced by chiller units, as a form of temperature difference between inlet and outlet water, which transfers from water system to air system. The cold air is sent into room to realize temperature shifting. Earlier central air conditioning usually uses a constant water volume and constant air volume to transfer coldness. Obviously, a constant water volume or air volume leads to a constant refrigerating or cooling capacity, even constant energy consumption. However, because of real-time variations in the cooling load, especially at low load, central air conditioning must reduce the refrigerating or cooling capacity to save energy.

In many modelling cases of central air conditioning, ether toward water system, or air system, few researchers combine both of them to model the system completely. Here, we describe the whole transfer process of coldness from water system to air system and model it overall. For air system, variable air volume technique usually uses several constant gears to adjust air speed; thus, it can be seen as constant air volume in some stage actually. For water system, variable water volume is an effective method too. Common techniques often use variable frequency pumps or switchable pumps (constant volume); few researchers combine variable frequency and switching measure together. Because central air conditioning with variable frequency pumps and switchable pumps has continuous and discrete features, a hybrid system can reflect the properties exactly. Here, we use this type of hybrid system and switching law to control it and save energy. We build the model between coldness transferring and inside temperature variation, by the mode of variable water volume and constant air volume. On account of same energy-saving principle, we just discuss chilled water system but cooling water system. And we focus on energy saving, for satisfying the requirement of temperature control, without regard to comfort, such as humidity.

The equation of heat balance in the test room is as follows:

$$C_a m_a \frac{d\theta}{dt} = q_{\mathrm{sa}} C_a \left(\theta_{sa} - \theta\right) + \sum K_j A_j \left(\theta_j - \theta\right) + Q_{rd} \quad (23)$$

In (23), it clearly shows that the cooling capacity comes from air being sent, and (24) indicates the transfer of the cooling capacity from the chilled water system to the air system. The model contains the chilled water system and the air system and uses a constant air volume and variable water volume as the transferring mode, to express the refrigeration progress as exactly as possible.

The returned air rate is set as R_r; hence, the new air rate is $1-R_r$. The returned air volume can then be set as q_r; hence, the fresh air volume is q_n. η is the heat transfer efficiency from the water system to the air system $(0 \leq \eta \leq 1)$, Q_{xr} is the sensible heat, and Q_{qr} is the latent heat. Thus, the relationship for transferring coldness from the water system to the air system is as follows:

$$\eta Q_w = Q_a = Q_{qr} + Q_{xr} \quad (24)$$

Table 2: Thermal parameters and control coefficients.

Parameter	Value	Description
m_a (kg)	301.83	indoor air mass
q_{sw} (kg/s)	0.03	constant of switchable pump
q_b (kg/s)	0.02	rated volume of variable frequency pump
q_G (kg/s)	0.1	constant volume of whole fixed pumps
q_{sa} (kg/s)	0.635	constant volume of sending air
A_1 (m^2)	68	area of walls
A_2 (m^2)	22	area of windows
A_3 (m^2)	6	area of roof
C_a (J/kg∗k)	1010	specific heat of air
C_w (J/kg∗k)	4180	specific heat of water
k	3	amount of switchable pumps
K_1 ($W/m^2{*}k$)	0.049	heat transfer coefficient of walls
K_2 ($W/m^2{*}k$)	0.051	heat transfer coefficient of windows
K_3 ($W/m^2{*}k$)	0.05	heat transfer coefficient of roof
Q_{qr} (J)	70	latent heat load
R_r	0.13	return air rate
$\Delta\theta$ (°C)	5	temperature difference
η	0.88	transfer efficiency from water system to air system
γ	0.082	coefficient of cooling capacity allocation
f_{fb}	0.00032	coefficient of feedback control
θ_{ini} (°C)	35, 35, 36	initial temperature
θ_j (°C)	35, 35, 36	temperature of walls, windows, and roof, respectively
θ_{set} (°C)	19.5	setting temperature
P_G (kW)	35	power of whole fixed pumps
P_{sw} (kW)	37	power of switchable pump
P_b (kW)	75	rated power of variable frequency pump

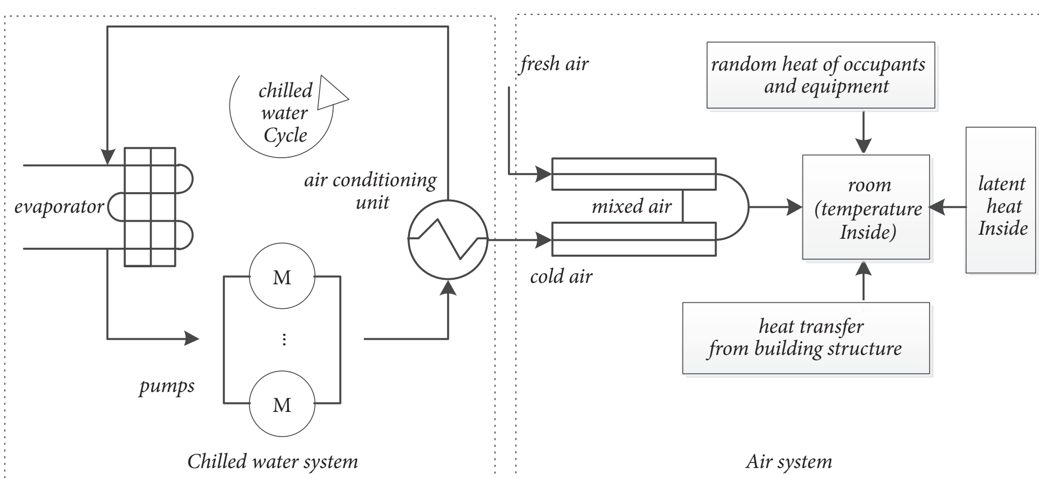

Figure 1: Cooling capacity transfer and temperature change processes.

And because

$$\gamma\eta q_w C_w \Delta T = q_{sa} C_a (T - T_{sa}) + Q_{qr} \tag{25}$$

$$q_{sa} C_a (\theta - \theta_{sa}) = (1 - R_r) q_{sa} C_a (\theta_{out} - \theta_{sa}) + R_r q_{sa} C_a (\theta - \theta_{sa}) \tag{26}$$

we can substitute (24), (25), and (26) into (23) and obtain

$$C_a m_a \frac{d\theta}{dt} = -\gamma\eta q_w C_w \Delta\theta + \left[(R_r - 1) q_{sa} C_a - \sum K_j A_j\right]\theta$$

$$+ (1 - R_r) q_{sa} C_a \theta_{out} + \sum K_j A_j T_j + Q_{rd} + Q_{qr} \quad (27)$$

The pumps are divided between fixed frequency pumps, variable frequency pump, and switchable pumps; thus, $q_w = q_G + kq_s + q_b$, where q_G is the water volume of all fixed frequency pumps, q_s is the water volume of one switchable pump, k is the amount of working switchable pumps, and q_b is the water volume of the variable frequency pump. Equation (27) can then be modified as (21).

4.2. Switching Law. In this example, there are three switchable pumps. Thus, the switching law can be defined as

$$M(i) = i + 1 \quad (\text{if } i = 3,\ M(i) = 3,\ 3 \text{ is maximum}) \quad (28a)$$

$$m(i) = i - 1 \quad (\text{if } i = 0,\ m(i) = 0,\ 0 \text{ is minimum}) \quad (28b)$$

In industrial system, state monitoring is periodical, assuming that the sampling period is T_L. After each T_L, the state value can be obtained. According to the switching law, system compares the value of $x_i(t)$ with the switching critical value of each T_L and decides which subsystem should be chosen next. Thus, switching behavior is decided autonomously.

If the system instantly switches to the ith subsystem, the state variable $x_i(t)$ can be divided into 3 intervals. After T_L seconds, the system checks the value of $x_i(t)$. If $x_i(t) \geq M_i^*$, the system switches from the ith subsystem to the i+1th subsystem; if $x_i(t) \leq m_i^*$, the system switches from the ith subsystem to the i-1th subsystem; if $m_i^* < x_i(t) < M_i^*$, the system remains in the ith subsystem. Note that if the system selects the 0^{th} subsystem and $x_i(t) \leq m_i^*$, the system still stay in the 0^{th} subsystem. Because the switchable pumps are out of work, the system is in the most energy-saving level. This is the same in the 2^{th} subsystem; i.e., if $x_i(t) \geq M_i^*$, the system will remain in 2^{th} subsystem, because the inside temperature must be dropped primarily. In a similar way, all the switchable pumps are full-load working for refrigerating. This phenomenon is the same for the next T_L seconds.

The switching law can be expressed by the MLD logic expressions:

$$[x_i(t) \geq 0.5] \longrightarrow [\delta_{i+1}(t) = 1] \quad (29a)$$

$$[\delta_{i+1}(t) = 1] \longrightarrow [\delta_i(t) = 0] \quad (29b)$$

$$[x_i(t) \leq -0.5] \longrightarrow [\delta_{i-1}(t) = 1] \quad (29c)$$

$$[\delta_{i-1}(t) = 1] \longrightarrow [\delta_i(t) = 0] \quad (29d)$$

$$[x_i(t) < 0.5] \wedge [x_i(t) > -0.5] \longrightarrow [\delta_i(t) = 1] \quad (29e)$$

$$[x_6(t) \geq 0.5] \longrightarrow [\delta_6(t) = 1] \quad (29f)$$

$$[x_0(t) \leq -0.5] \longrightarrow [\delta_0(t) = 1] \quad (29g)$$

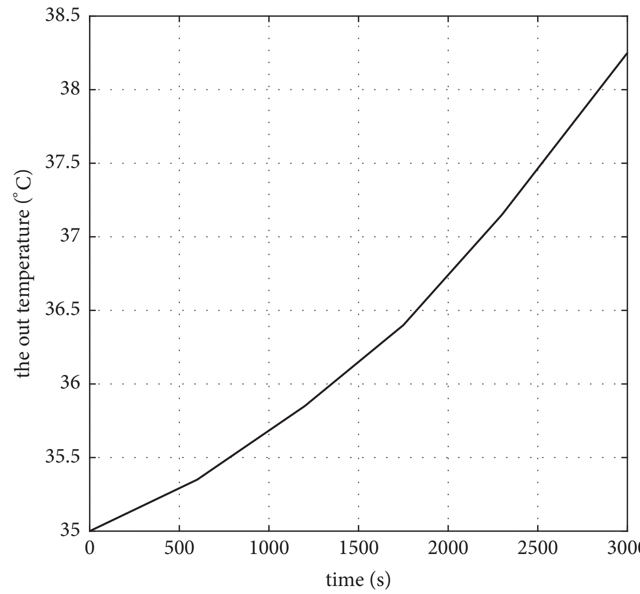

Figure 2: The outside temperature.

The MLD equation transforms the mixed integer inequality constraints:

$$x_i(t) \leq 0.5 - 0.02 + (50 + 0.02 - 0.5)\,\delta_{i+1} \quad (30a)$$

$$\delta_i + \delta_{i+1} \leq 1 \quad (30b)$$

$$x_i(t) \geq -0.5 + 0.02 + (10 - 0.02 + 0.5)\,\delta_{i-1} \quad (30c)$$

$$\delta_i + \delta_{i-1} \leq 1 \quad (30d)$$

$$-0.5 - 0.02 + (50 + 0.02 + 0.5)\,\delta_i \leq x_i(t) \leq 0.5 + 0.02 + (10 - 0.02 - 0.5)\,\delta_i \quad (30e)$$

$$x_6(t) \leq 0.5 - 0.02 + (50 + 0.02 - 0.5)\,\delta_6 \quad (30f)$$

$$x_0(t) \geq -0.5 + 0.02 + (10 - 0.02 + 0.5)\,\delta_0 \quad (30g)$$

Finally, the MLD inequalities and parameters are deduced in (22).

5. Numeral Example

A special items warehouse (length: 12 m, width: 6.5 m and height: 3 m) is the simulation object, which needs a constant temperature environment for storage. Simulink 2007 is used to simulate the modelling and controlling process, where the simulation time is 3000 seconds. The inside temperature starts off at 19.5, the setpoint, which has a small amplitude fluctuation near the set value in the simulation times due to the factors of outdoor temperature, indoor lighting, radiating equipment, and number of occupants. Assuming in a hot summer, the outdoor temperature rises from 35°C to 38.2°C in 3000 seconds (see Figure 2). Because of the two handling operations by the workers entering the warehouse, the indoor random heat capacity has two obvious growths (see Figure 3).

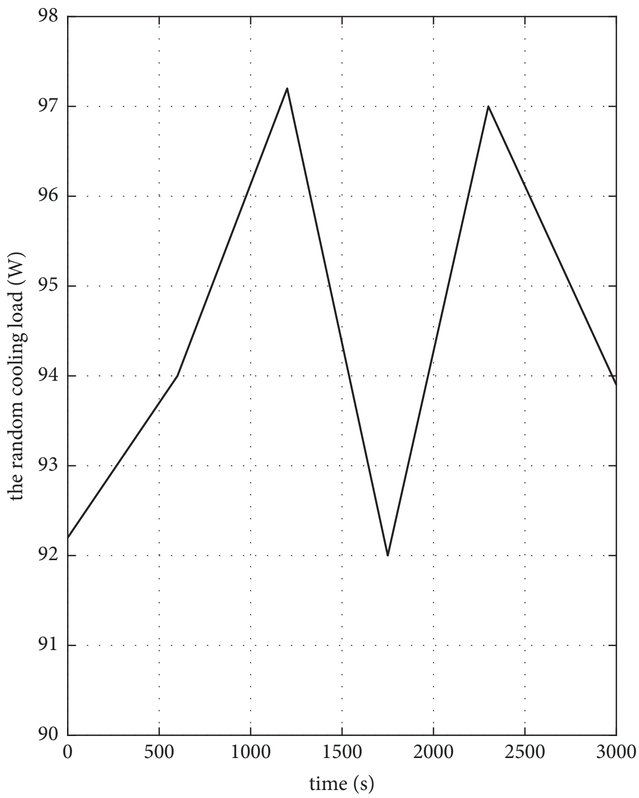

FIGURE 3: The random cooling load.

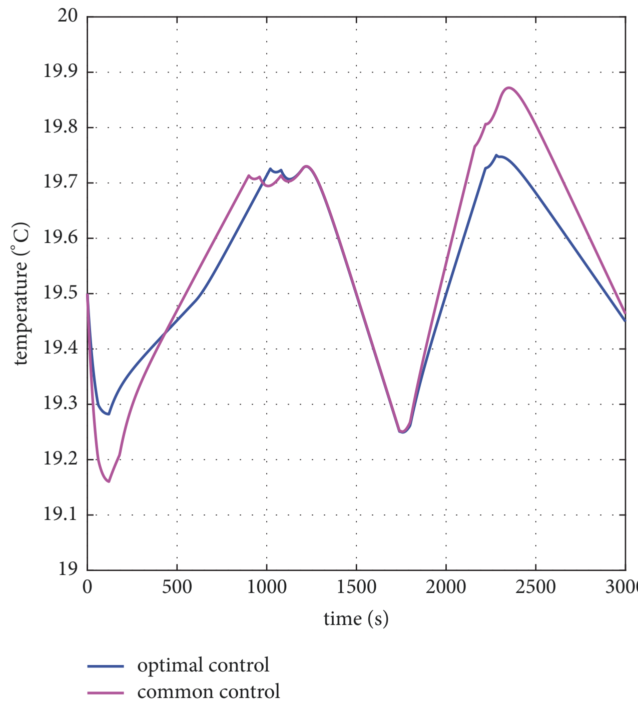

FIGURE 4: The indoor temperature.

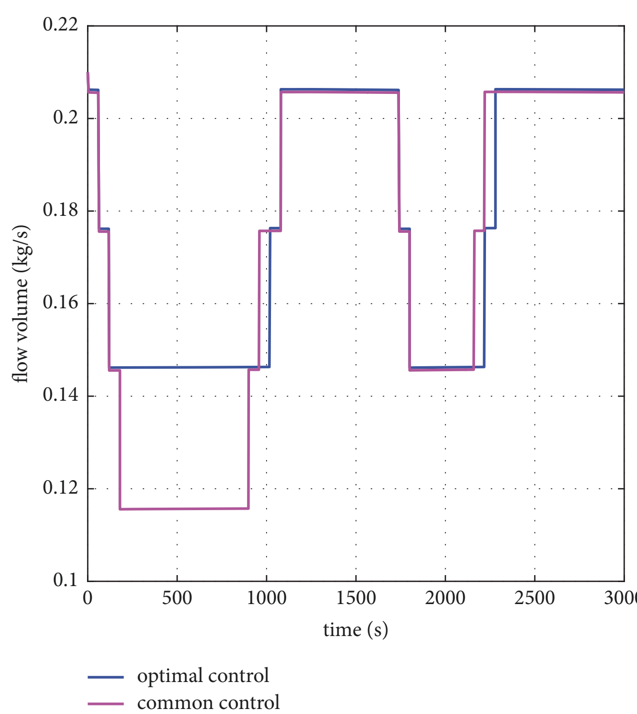

FIGURE 5: The volume of chilled water system.

The switching law is state-dependent mode; the upper and low switching boundaries are defined as 0.2 and -0.2, respectively. Thus, the system checks the indoor temperature frequently and set the switch time T_L as 60 seconds; once the boundaries are crossed, switching actions are triggered. The system begins at the 3th subsystem, in other works, rated power and flow volume. Under the optimal control strategy given in Section 3, we set Q_1=1, Q_2=0.01, Q_3=0.001, Q_4=10, Q_5=0 and use HYBRID TOOLBOX to simulate the dynamics. A common feedback control method is used to compare with the optimal control strategy, which shows the different effects in changes of flow volume, indoor temperature, and energy consumptions. The thermal parameters and control coefficients are given in Table 2.

Figure 4 shows the changes of indoor temperature near the set point under the two different control strategies above. It is obvious that the temperature has less amplitude of fluctuation by the optimal control than common feedback control method, which reflects more stable dynamical trajectory. Actually, the amplitude is approximate ±0.25°C during the whole process, which is unlike approximate ±0.35°C under common control method.

Figures 5 and 6 show the volume of chilled water system and the dynamic of switchable pumps under the two control strategies, respectively. These two figures look similar, because the total flow volume is composed of switchable, fixed, and variable frequency pumps; the change of whole volume is based on the dynamic of the switchable pumps. Under the optimal control strategy, it shows less switching behaviors and smaller flow volume than common control method. In engineering applications, the advantage presents less change of water pressure, which produces less impact and better hydraulic characteristics for pipe network.

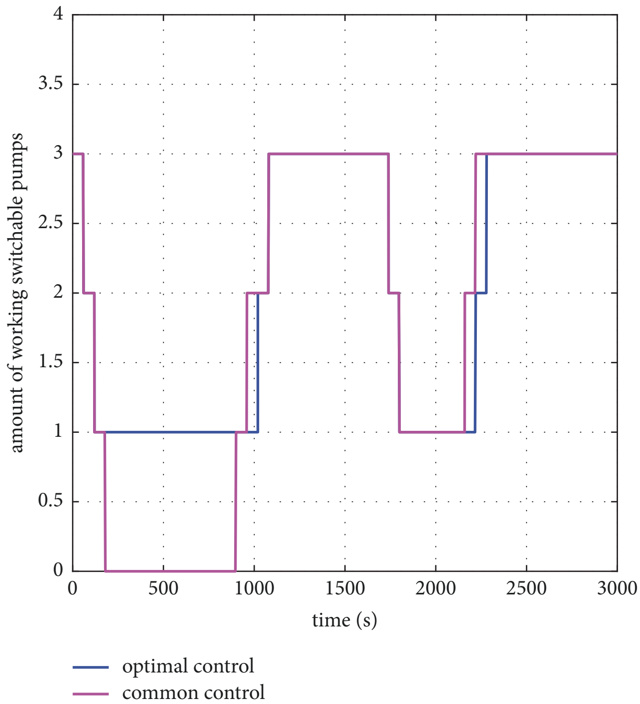

Figure 6: The switching dynamic of pumps.

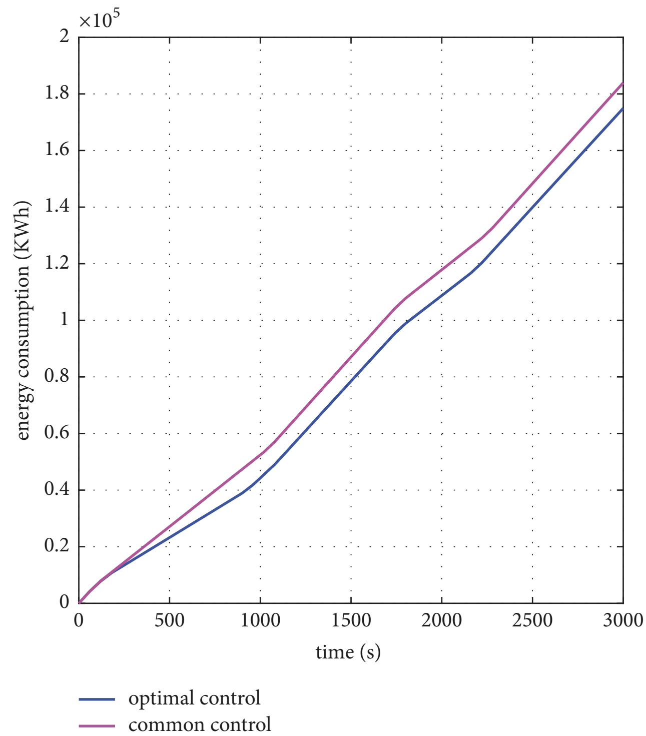

Figure 7: The energy consumptions.

Figure 7 shows the energy consumptions under the two control strategies in the 3000 seconds, which presents approximate 1.75×10^5 and 1.84×10^5 kWh, respectively. According to the cubic relationship between power and speed and the proportional relationship between speed and flow rate of pumps, the energy consumptions are calculated. Compared with the common control method, it provides approximate 5% energy saving. The power of various pumps is given in Table 2.

6. Conclusions

This paper reported a type of mixed logical dynamical system with novel structure, which shows an inequality contained state variables, control variables, logical variables, auxiliary variables, switching points, and switching law. The inequality is the constraints of system based on the state-dependent switching law, in where the switching points present more directions by the new relations between logical expresses and inequalities. A MIQP optimal control strategy is introduced for solving the problem. A case of refrigeration system is illustrated for the model and control dynamics, which use a thermal balance process to describe. From the simulation results, the changes of indoor temperature, flow volume, switching behaviors, and energy consumption show appeal effects for control and optimization of complex systems.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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