Hindawi
Mathematical Problems in Engineering
Volume 2019, Article ID 7453025, 10 pages
https://doi.org/10.1155/2019/7453025

Research Article

Research on Novel Correlation Coefficient of Neutrosophic Cubic Sets and Its Applications

Huiling Xue,[1] Minrong Yu,[2] and Chunfang Chen [1]

[1] *Department of Mathematics, Nanchang University, Nanchang 330031, China*
[2] *School of Software, Nanchang University, Nanchang 330047, China*

Correspondence should be addressed to Chunfang Chen; ccfygd@sina.com

Received 27 August 2018; Accepted 17 September 2018; Published 15 January 2019

Academic Editor: Alberto Campagnolo

Single-valued neutrosophic cubic set is a good tool to solve the vague and uncertain problems because it contains more information. The article first gives the correlation coefficient of single-valued neutrosophic cubic sets. Then, a decision method is proposed, and an application in pattern recognition is considered. Finally, examples are given to explain the feasibility of this method. At the same time, the comparative analysis shows the superiority of this method.

1. Introduction

In 1965, Zadeh [1] defined the fuzzy sets (FSs). Let $X = \{x_1, x_2, \ldots, x_n\}$ be a universe, a fuzzy set A on X is described by membership function $\mu_A(x)$, which satisfies every element in X corresponding to a real number in the interval $[0, 1]$, and the value of $\mu_A(x)$ at x represents grade of membership of $x \in A$. Therefore, the nearer the value of $\mu_A(x)$ to 1, the higher the grade of membership of $x \in A$. The membership is the degree to which every element in X belongs to A. Example, let $X = \{155, 170, 178, 188\}$ denote the height of boys, A is a set of all boys with a high feature, and the membership degree may be $\mu_A(155) = 0$, $\mu_A(170) = 0.5$, $\mu_A(178) = 0.9$, and $\mu_A(188) = 1$. That is, the decision maker thinks 188 is absolutely tall, so the membership value is naturally 1, while the membership value of 170 may be 0.5, indicating that the probability of 170 belonging to set A may be 0.5. Subsequently, some other FSs are proposed [2–7]. In order to contain more information to describe the fuzziness and uncertainty of the decision maker, cubic sets (CSs) were proposed by Jun et al. [8]. After that, some scholars put forward many other related definitions by using the CSs [9–15]. Since FSs and IFSs only consider partial information, therefore, Smarandache [16] introduced neutrosophic sets (NSs), and their values located in the $]^{-}0, 1^{+}[$, the nonstandard interval; these three values are completely independent. NSs can solve the problem of uncertainty quite well. Neutrosophic sets are also widely used in many different fields [17–24]. Wang et al. [25] introduced the interval neutrosophic sets (INSs). It is the generalization of FSs, IVFSs, and NSs. Because the nonstandard interval of NSs cannot be used to solve many specific events, Smarandache [26] proposed the single-valued neutrosophic sets (SVNSs), and it is applied to many fields [27–41]. Later, Ye [42] introduced the single-valued neutrosophic hesitant fuzzy sets (SVNHFSs); SVNHFSs are more widely available than other fuzzy sets.

Alia et al. [43] also proposed the concept of neutrosophic cubic sets (NCSs) and defined some notions. They proposed the concept of NCSs by extending the concept of CSs to NSs, and it has been widely used in many fields [44–49]. By investigating some properties of NCSs, Alia et al. [50] considered distance measures of NCSs. The NCS contains much more information than the general NSs, so it is possible to show more practical value and effectiveness in MADM. At present, there are many researchers on single-valued neutrosophic cubic sets (SVNCSs), as we know, the application of its correlation coefficient has not been studied in SVNCSs environment. However, correlation is an important index and widely used in many places [51–55]. Since the correlation was used in the fuzzy environment [56–58], then the correlation coefficient was proposed and studied in different fuzzy environments [3, 4, 59–65]. Based on its importance, this article considers the correlation coefficient of SVNCSs.

The structure of this article is as follows: Section 2 introduces basic concepts of CSs, NSs, and NCSs. The related concepts of NCSs are proposed in Section 3. In Section 4, a decision method is given, and the feasibility and superiority of the method are illustrated by comparative analysis. In Section 5, an algorithm in pattern recognition is given, and the feasibility and superiority of the modified algorithm are illustrated by comparison and analysis. In Section 6, the paper ends with a conclusion.

2. Preliminaries

Let us start with some concepts, including fuzzy numbers, interval valued fuzzy sets (IVFSs), neutrosophic sets (NSs), interval neutrosophic sets (INSs), single-valued neutrosophic hesitant fuzzy sets (SVNHFSs), and neutrosophic cubic sets (NCSs).

Definition 1 (see [66]). Let $X = \{x_1, x_2, \dots, x_n\}$ be a universe; A is a fuzzy set defined on X. μ_A is a mapping from X to $[0,1]$. The element x_i belongs to the membership function of A, which is expressed as $\{\mu_A(x_1), \mu_A(x_2), \dots, \mu_A(x_n)\}$. And the fuzzy number can be described as

$$\mu_A(X) = \mu_A(x_1) + \mu_A(x_2) + \cdots + \mu_A(x_n), \quad (1)$$

where + denotes the join symbol and does not represent the meaning of addition.

Fuzzy numbers also have some other definitions [67], such as triangular fuzzy numbers, trapezoidal fuzzy numbers, and ordering fuzzy numbers.

Definition 2 (see [2]). Let X be a set, then an IVFS A on X has the form

$$A = \{\langle x, [A^-(x), A^+(x)]\rangle \mid x \in X\}, \quad (2)$$

in which $A^-(x)$ and $A^+(x)$ are the upper and lower limits of the membership degree $x \in X$ where $0 \le A^-(x) + A^+(x) \le 1$, respectively.

Definition 3 (see [68]). Let X be a universe. The form of NSs λ is

$$\lambda = \{\langle x, T(x), I(x), F(x)\rangle \mid x \in X\}, \quad (3)$$

where T, I, F: $X \longrightarrow\ ]^-0, 1^+[$ define the degree of Truth, Indeterminacy, and Falsehood of $x \in X$ to λ and

$$^-0 \le T(x) + I(x) + F(x) \le 3^+. \quad (4)$$

Definition 4 (see [25]). Let X be a set. INSs A in X satisfies $A_T(x)$, $A_I(x)$, $A_F(x) \subseteq [0,1]$.

$$A = \{\langle x, [A_T^-(x), A_T^+(x)], [A_I^-(x), A_I^+(x)], [A_F^-(x), A_F^+(x)]\rangle \mid x \in X\}. \quad (5)$$

Definition 5 (see [26]). Let $X = \{x_1, x_2, \dots, x_n\}$ be a universe; the form of the definition of the SVNSs is

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}, \quad (6)$$

where $T_A, I_A, F_A : X \longrightarrow [0,1]$, with $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 6 (see [39]). Let $X = \{x_1, x_2, \dots, x_n\}$ be a set, then, SVNHFS N on X is defined as

$$N = \{\langle x_i, h_N(x_i), l_N(x_i), g_N(x_i)\rangle \mid x_i \in X\}, \quad (7)$$

in which $h_N(x_i)$, $l_N(x_i)$, and $g_N(x_i)$ are three sets,

$$h_N(x_i) = \{\gamma_{N1}(x_i), \gamma_{N2}(x_i), \dots, \gamma_{Nl_h}(x_i)\},$$
$$l_N(x_i) = \{\delta_{N1}(x_i), \delta_{N2}(x_i), \dots, \delta_{Nl_l}(x_i)\}, \quad (8)$$
$$g_N(x_i) = \{\eta_{N1}(x_i), \eta_{N2}(x_i), \dots, \eta_{Nl_g}(x_i)\},$$

with $0 \le \gamma, \delta, \eta \le 1$ and $0 \le \gamma^+ + \delta^+ + \eta^+ \le 3$, where $\gamma \in h_{N(x_i)}$, $\delta \in l_{N(x_i)}$, $\eta \in g_{N(x_i)}$, and $\gamma^+ \in h_N^+(x_i) = U_{\eta \in g_N(x_i)} \max\{\eta\}$ for $x_i \in X$. Generally, the $n(x_i) = \{h_N(x_i), l_N(x_i), g_N(x_i)\}$ is named a single-valued neutrosophic hesitant fuzzy element (SVNHFE). Denoted by $n = \{h, l, g\}$ such that $h = \{\gamma_{N1}, \gamma_{N2}, \dots, \gamma_{Nl_h}\}$, $l = \{\delta_{N1}, \delta_{N2}, \dots, \delta_{Nl_l}\}$, and $g = \{\eta_{N1}, \eta_{N2}, \dots, \eta_{Nl_g}\}$, where l_h, l_l, and l_g are the number of values in $h_N(x_i)$, $l_N(x_i)$, and $g_N(x_i)$, respectively.

Definition 7 (see [43]). Let X be a universe; the NCS $\mathfrak{I}$ is defined as

$$\mathfrak{I} = \{\langle x, A(x), \lambda(x)\rangle \mid x \in X\}, \quad (9)$$

where A is an INS and λ is a NS. A NCS can be denoted as $\mathfrak{I} = \langle A, \lambda\rangle$.

Because the HFEs are usually disorder, therefore, for convenience of calculation we need to arrange all elements of $h(x)$ in a certain order. And for two HFEs, the numbers of elements are usually different. Chen et al. [60] have defined $l(h(x))$ as the number of elements in $h(x)$. For two HFEs h_A and h_B, let $l = \max\{l_{h(A)}, l_{h(B)}\}$, where $l_{h(A)}$ and $l_{h(B)}$ are the numbers of values in h_A and h_B, respectively. When $l_{h(A)} \ne l_{h(B)}$, for convenience of calculation we need to add some values to make them have common length. And [26, 50] have given the regulation; they have the same length as the longer one.

Based on the correlation coefficient of HFSs, Sahin [41] proposed the correlation coefficient of SVNHFSs.

Definition 8 (see [41]). Let A be a SVNHFS on universe $X = \{x_1, x_2, \dots, x_n\}$; the informational energy of A is defined as

$$E_{SVNHFS}(A) = \sum_{i=1}^{n}\left(\frac{1}{k_i}\sum_{s=1}^{k_i}\gamma^2_{A\sigma(s)}(x_i) + \frac{1}{p_i}\sum_{z=1}^{p_i}\delta^2_{A\sigma(z)}(x_i) + \frac{1}{l_i}\sum_{t=1}^{l_i}\eta^2_{A\sigma(t)}(x_i)\right). \quad (10)$$

Definition 9 (see [41]). Let A, B be two SVNHFSs on $X = \{x_1, x_2, \dots, x_n\}$; the correlation between A and B is defined as

$$C_{SVNHFS}(A, B) = \sum_{i=1}^{n}\left(\frac{1}{k_i}\sum_{s=1}^{k_i}\gamma_{A\sigma(s)}(x_i)\,\gamma_{B\sigma(s)}(x_i)\right.$$

$$+\frac{1}{p_i}\sum_{z=1}^{p_i}\delta_{A\sigma(z)}(x_i)\,\delta_{B\sigma(z)}(x_i) + \frac{1}{l_i}\sum_{t=1}^{l_i}\eta_{A\sigma(t)}(x_i)\,\eta_{B\sigma(t)}(x_i)\Bigg). \quad (11)$$

And we have

(1) $C_{SVNHFS}(A, A) = E_{SVNHFS}(A)$.

(2) $C_{SVNHFS}(A, B) = C_{SVNHFS}(B, A)$.

Definition 10 (see [41]). Let A, B be two SVNHFSs on $X = \{x_1, x_2, \ldots, x_n\}$; the correlation coefficient between A and B is defined as

$$\rho_{SVNHFS}(A,B) = \frac{C_{SVNHFS}(A,B)}{\sqrt{C_{SVNHFS}(A,A)}\sqrt{C_{SVNHFS}(B,B)}} = \frac{\sum_{i=1}^{n}\left((1/k_i)\sum_{s=1}^{k_i}\gamma_{A\sigma(s)}(x_i)\gamma_{B\sigma(s)}(x_i) + (1/p_i)\sum_{z=1}^{p_i}\delta_{A\sigma(z)}(x_i)\delta_{B\sigma(z)}(x_i) + (1/l_i)\sum_{t=1}^{l_i}\eta_{A\sigma(t)}(x_i)\eta_{B\sigma(t)}(x_i)\right)}{\sqrt{\sum_{i=1}^{n}\left((1/k_i)\sum_{s=1}^{k_i}\gamma^2_{A\sigma(s)}(x_i) + (1/p_i)\sum_{z=1}^{p_i}\delta^2_{A\sigma(z)}(x_i) + (1/l_i)\sum_{t=1}^{l_i}\eta^2_{A\sigma(t)}(x_i)\right)}\sqrt{\sum_{i=1}^{n}\left((1/k_i)\sum_{s=1}^{k_i}\gamma^2_{B\sigma(s)}(x_i) + (1/p_i)\sum_{z=1}^{p_i}\delta^2_{B\sigma(z)}(x_i) + (1/l_i)\sum_{t=1}^{l_i}\eta^2_{B\sigma(t)}(x_i)\right)}}. \quad (12)$$

Theorem 11 (see [41]). *For two SVNHFSs A and B, (12) satisfies*

(1) $\rho_{SVNHFS}(A, B) = \rho_{SVNHFS}(B, A)$*;*

(2) $0 \le \rho_{SVNHFS}(A, B) \le 1$*;*

(3) $\rho_{SVNHFS}(A, B) = 1$*, if* $A = B$*.*

Generally, the weights of different elements are different, and also the results will be different. Therefore, it is important to take into account the weights of x_i ($i = 1, 2, \ldots, n$). So the weighted correlation coefficients of SVNHFSs are given.

Definition 12 (see [41]). Let $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be the weighting vector of x_i ($i = 1, 2, \ldots, n$) with $\omega_i \ge 0$ and $\sum_{i=1}^{n}\omega_i = 1$; the weighted correlation coefficient is as follows:

$$\rho_{SVNHFSw}(A,B) = \frac{C_{SVNHFSw}(A,B)}{\sqrt{C_{SVNHFSw}(A,A)}\sqrt{C_{SVNHFSw}(B,B)}} = \frac{\sum_{i=1}^{n}\omega_i\left((1/k_i)\sum_{s=1}^{k_i}\gamma_{A\sigma(s)}(x_i)\gamma_{B\sigma(s)}(x_i) + (1/p_i)\sum_{z=1}^{p_i}\delta_{A\sigma(z)}(x_i)\delta_{B\sigma(z)}(x_i) + (1/l_i)\sum_{t=1}^{l_i}\eta_{A\sigma(t)}(x_i)\eta_{B\sigma(t)}(x_i)\right)}{\sqrt{\sum_{i=1}^{n}\omega_i\left((1/k_i)\sum_{s=1}^{k_i}\gamma^2_{A\sigma(s)}(x_i) + (1/p_i)\sum_{z=1}^{p_i}\delta^2_{A\sigma(z)}(x_i) + (1/l_i)\sum_{t=1}^{l_i}\eta^2_{A\sigma(t)}(x_i)\right)}\cdot\sqrt{\sum_{i=1}^{n}\omega_i\left((1/k_i)\sum_{s=1}^{k_i}\gamma^2_{B\sigma(s)}(x_i) + (1/p_i)\sum_{z=1}^{p_i}\delta^2_{B\sigma(z)}(x_i) + (1/l_i)\sum_{t=1}^{l_i}\eta^2_{B\sigma(t)}(x_i)\right)}}. \quad (13)$$

The upper form satisfies

(1) $\rho_{SVNHFS_w}(A, B) = \rho_{SVNHFS_w}(B, A)$;

(2) $0 \le \rho_{SVNHFS_w}(A, B) \le 1$;

(3) $\rho_{SVNHFS_w}(A, B) = 1$, *if* $A = B$.

3. Correlation Coefficient of Single-Valued Neutrosophic Cubic Sets

Mumtaz Ali [50] introduced the NCSs and also gave some theorems. In this paper, according to the correlation coefficient mentioned above, we give the correlation coefficient of SVNCSs.

Definition 13. Let $\mathfrak{I}$ be SVNCS on $X = \{x_1, x_2, \ldots, x_n\}$, denoted by $\mathfrak{I} = \{\langle x, A(x), \lambda(x)\rangle \mid x \in X\}$, then the informational energy of $\mathfrak{I}$ is defined as

$$E_{SVNCS}(\mathfrak{I}) = \sum_{i=1}^{n}\left[\frac{1}{3}\left((T(x_i))^2 + (I(x_i))^2 + (F(x_i))^2\right) + \frac{1}{6}\left((T^-(x_i))^2 + (I^-(x_i))^2 + (F^-(x_i))^2 + (T^+(x_i))^2 + (I^+(x_i))^2 + (F^+(x_i))^2\right)\right]. \quad (14)$$

Definition 14. Let $\mathfrak{I}_1$ and $\mathfrak{I}_2$ be two SVNCSs on $X = \{x_1, x_2, \ldots, x_n\}$, denoted by $\mathfrak{I}_1 = \{\langle x, A_1(x), \lambda_1(x)\rangle \mid x \in X\}$ and $\mathfrak{I}_2 = \{\langle x, A_2(x), \lambda_2(x)\rangle \mid x \in X\}$. Then, the correlation between $\mathfrak{I}_1$ and $\mathfrak{I}_2$ is defined as

$$C_{SVNCS}(\mathfrak{I}_1, \mathfrak{I}_2) = \sum_{i=1}^{n}\left[\frac{1}{3}\left(T_1(x_i)T_2(x_i) + I_1(x_i)I_2(x_i) + F_1(x_i)F_2(x_i)\right) + \frac{1}{6}\left(T_1^-(x_i)T_2^-(x_i) + I_1^-(x_i)I_2^-(x_i) + F_1^-(x_i)F_2^-(x_i) + T_1^+(x_i)T_2^+(x_i) + I_1^+(x_i)I_2^+(x_i) + F_1^+(x_i)F_2^+(x_i)\right)\right] \quad (15)$$

Definition 15. Let $\mathfrak{I}_1$ and $\mathfrak{I}_2$ be two SVNCSs on $X = \{x_1, x_2, \ldots, x_n\}$, then the correlation coefficient between $\mathfrak{I}_1$ and $\mathfrak{I}_2$ is defined as

$$\rho_{SVNCS}(\mathfrak{I}_1,\mathfrak{I}_2)=\frac{C_{SVNCS}(\mathfrak{I}_1,\mathfrak{I}_2)}{\sqrt{C_{SVNCS}(\mathfrak{I}_1,\mathfrak{I}_1)}\sqrt{C_{SVNCS}(\mathfrak{I}_2,\mathfrak{I}_2)}}=\sum_{i=1}^{n}\Big(\frac{1}{3}\left(T_1(x_i)T_2(x_i)+I_1(x_i)I_2(x_i)+F_1(x_i)F_2(x_i)\right)$$
$$+\frac{1}{6}\left(T_1^-(x_i)T_2^-(x_i)+I_1^-(x_i)I_2^-(x_i)+F_1^-(x_i)F_2^-(x_i)+T_1^+(x_i)T_2^+(x_i)+I_1^+(x_i)I_2^+(x_i)+F_1^+(x_i)F_2^+(x_i)\right)\Big)$$
$$\cdot\Bigg(\sqrt{\sum_{i=1}^{n}\Big(\frac{1}{3}\left((T_1(x_i))^2+(I_1(x_i))^2+(F_1(x_i))^2\right)+\frac{1}{6}\left((T_1^-(x_i))^2+(I_1^-(x_i))^2+(F_1^-(x_i))^2+(T_1^+(x_i))^2+(I_1^+(x_i))^2+(F_1^+(x_i))^2\right)\Big)}$$
$$\cdot\sqrt{\sum_{i=1}^{n}\Big(\frac{1}{3}\left((T_2(x_i))^2+(I_2(x_i))^2+(F_2(x_i))^2\right)+\frac{1}{6}\left((T_2^-(x_i))^2+(I_2^-(x_i))^2+(F_2^-(x_i))^2+(T_2^+(x_i))^2+(I_2^+(x_i))^2+(F_2^+(x_i))^2\right)\Big)}\Bigg)^{-1}. \tag{16}$$

Theorem 16. *Let $\mathfrak{I}_1$ and $\mathfrak{I}_2$ be two SVNCSs; (16) satisfies the following properties:*

(1) $\rho_{SVNCS}(\mathfrak{I}_1,\mathfrak{I}_2)=\rho_{SVNCS}(\mathfrak{I}_2,\mathfrak{I}_1)$;

(2) $0\le\rho_{SVNCS}(\mathfrak{I}_1,\mathfrak{I}_2)\le 1$;

(3) $\rho_{SVNCS}(\mathfrak{I}_1,\mathfrak{I}_2)=1$, if $\mathfrak{I}_1=\mathfrak{I}_2$.

Proof. (1) The proof of (1) and (3) is obvious, so we do not give a detailed proof.

(2) The inequality $0\le\rho_{SVNCS}(\mathfrak{I}_1,\mathfrak{I}_2)$ is evident. Then, we prove $\rho_{SVNCS}(\mathfrak{I}_1,\mathfrak{I}_2)\le 1$.

With the help of Cauchy-Schwarz inequality, $(x_1y_1+x_2y_2+\cdots+x_ny_n)^2\le(x_1^2+x_2^2+\cdots+x_n^2)(y_1^2+y_2^2+\cdots+y_n^2)$, in which $(x_1,x_2,\ldots,x_n)\in R^n$ and $(y_1,y_2,\ldots,y_n)\in R^n$, the following inequality can be obtained:

$$\sum_{i=1}^{n}\Big(\frac{1}{3}\left(T_1(x_i)T_2(x_i)+I_1(x_i)I_2(x_i)+F_1(x_i)F_2(x_i)\right)$$
$$+\frac{1}{6}\left(T_1^-(x_i)T_2^-(x_i)+I_1^-(x_i)I_2^-(x_i)+F_1^-(x_i)F_2^-(x_i)+T_1^+(x_i)T_2^+(x_i)+I_1^+(x_i)I_2^+(x_i)+F_1^+(x_i)F_2^+(x_i)\right)\Big)$$
$$\le\sqrt{\sum_{i=1}^{n}\Big(\frac{1}{3}\left(T_1(x_i)\right)^2+(I_1(x_i))^2+(F_1(x_i))^2\Big)+\frac{1}{6}\left((T_1^-(x_i))^2+(I_1^-(x_i))^2+(F_1^-(x_i))^2+(T_1^+(x_i))^2+(I_1^+(x_i))^2+(F_1^+(x_i))^2\right)}$$
$$\cdot\sqrt{\sum_{i=1}^{n}\frac{1}{3}\left((T_2(x_i))^2+(I_2(x_i))^2+(F_2(x_i))^2\right)+\frac{1}{6}\left((T_2^-(x_i))^2+(I_2^-(x_i))^2+(F_2^-(x_i))^2+(T_2^+(x_i))^2+(I_2^+(x_i))^2+(F_2^+(x_i))^2\right)} \tag{17}$$

It is obvious that we can see $C_{SVNCS}(\mathfrak{I}_1,\mathfrak{I}_2)\le\sqrt{C_{SVNCS}(\mathfrak{I}_1,\mathfrak{I}_1)}\cdot\sqrt{C_{SVNCS}(\mathfrak{I}_2,\mathfrak{I}_2)}$; we can get $0\le\rho_{SVNCS}(\mathfrak{I}_1,\mathfrak{I}_2)\le 1$. So, the theorem has proved completely. □

In a general way, when decision makers give different weight value to each element in the universe, the result of decision may be different. Therefore, it is particularly important to consider the weight of element when making decision. Now, let us give the weighted correlation coefficient of SVNCSs.

Definition 17. Let $\omega=(\omega_1,\omega_2,\ldots,\omega_n)^T$ be the weighting vector of x_i $(i=1,2,\ldots,n)$, where $\omega_i\ge 0$ and $\sum_{i=1}^{n}\omega_i=1$; the weighted correlation coefficient is defined as

$$\rho_{SVNCS_w}(\mathfrak{I}_1,\mathfrak{I}_2)=\frac{C_{SVNCS_w}(\mathfrak{I}_1,\mathfrak{I}_2)}{\sqrt{C_{SVNCS_w}(\mathfrak{I}_1,\mathfrak{I}_1)}\sqrt{C_{SVNCS_w}(\mathfrak{I}_2,\mathfrak{I}_2)}}=\sum_{i=1}^{n}\omega_i\Big(\frac{1}{3}\left(T_1(x_i)T_2(x_i)+I_1(x_i)I_2(x_i)+F_1(x_i)F_2(x_i)\right)$$
$$+\frac{1}{6}\left(T_1^-(x_i)T_2^-(x_i)+I_1^-(x_i)I_2^-(x_i)+F_1^-(x_i)F_2^-(x_i)+T_1^+(x_i)T_2^+(x_i)+I_1^+(x_i)I_2^+(x_i)+F_1^+(x_i)F_2^+(x_i)\right)\Big)$$
$$\cdot\Bigg(\sqrt{\sum_{i=1}^{n}\omega_i\Big(\frac{1}{3}\left(T_1(x_i)\right)^2+(I_1(x_i))^2+(F_1(x_i))^2\Big)+\frac{1}{6}\left((T_1^-(x_i))^2+(I_1^-(x_i))^2+(F_1^-(x_i))^2+(T_1^+(x_i))^2+(I_1^+(x_i))^2+(F_1^+(x_i))^2\right)\Big)}$$
$$\cdot\sqrt{\sum_{i=1}^{n}\omega_i\Big(\frac{1}{3}\left((T_2(x_i))^2+(I_2(x_i))^2+(F_2(x_i))^2\right)+\frac{1}{6}\left((T_2^-(x_i))^2+(I_2^-(x_i))^2+(F_2^-(x_i))^2+(T_2^+(x_i))^2+(I_2^+(x_i))^2+(F_2^+(x_i))^2\right)\Big)}\Bigg)^{-1} \tag{18}$$

Specially, when $\omega_i = 1/n$ $(i = 1, 2, \ldots, n)$, (18) reduced to (16), and (18) satisfies the following properties:

(1) $\rho_{SVNCS_w}(\mathfrak{I}_1, \mathfrak{I}_2) = \rho_{SVNCS_w}(\mathfrak{I}_2, \mathfrak{I}_1)$;

(2) $0 \le \rho_{SVNCS_w}(\mathfrak{I}_1, \mathfrak{I}_2) \le 1$;

(3) $\rho_{SVNCS_w}(\mathfrak{I}_1, \mathfrak{I}_2) = 1, if\ \mathfrak{I}_1 = \mathfrak{I}_2$.

Proof. The proof process of the theorem is identical. □

In the next two parts, we give the decision method and an algorithm via the correlation coefficient of SVNCSs, respectively. The superiority of the proposed methods is illustrated through comparative analysis.

4. Decision-Making Method Based on the Correlation Coefficient of Single-Valued Neutrosophic Cubic Sets

In this section, weighted correlation coefficient is used to select the best alternative in MADM with SVNCS environment.

In MADM problems, let $\mathfrak{I} = \{\mathfrak{I}_1, \mathfrak{I}_2, \ldots, \mathfrak{I}_m\}$ be a set of alternatives; $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ be a set of attributes; $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be the weighting vector of x_i $(i = 1, 2, \ldots, n)$ with $\omega_i \ge 0$ and $\sum_{i=1}^{n} \omega_i = 1$. $\mathfrak{I}_k$ $(k = 1, 2, \ldots, m)$ are evaluation results.

$\mathfrak{I}_{kj} = (\langle [T_{kj}^-, T_{kj}^+], [I_{kj}^-, I_{kj}^+], [F_{kj}^-, F_{kj}^+] \rangle, \langle T_{kj}, I_{kj}, F_{kj} \rangle)(k = 1, 2, \ldots, m; j = 1, 2, \cdots\cdots, n)$, where $T_{kj}^-, I_{kj}^-, F_{kj}^-, T_{kj}^+, I_{kj}^+, F_{kj}^+ \subseteq [0, 1]$ and $T_{kj}, I_{kj}, F_{kj} \in [0, 1]$. Therefore, $A = (\mathfrak{I}_{kj})_{m \times n}$ is the decision matrix.

In MADM problems, we can use the ideal alternative to select the best one in all alternatives. In general, the ideal scheme does not exist, but it can provide theoretical support for decision makers. Jun Ye [46] put forward a method to construct ideal plan, and we can use this idea to construct a relatively ideal plan. The best alternative is selected according to the degree of correlation between known and ideal solution. The selection process is as follows.

Step 1. Construct the neutrosophic cubic decision matrix based on decision information.

Step 2. Set up an ideal scheme $\mathfrak{I}^* = \{\mathfrak{I}_1^*, \mathfrak{I}_2^*, \ldots, \mathfrak{I}_n^*\}$

$$\begin{aligned} \mathfrak{I}_j^* = \Big(\Big\langle & \left[\max\left(T_{kj}^-\right), \max\left(T_{kj}^+\right) \right], \\ & \left[\min\left(I_{kj}^-\right), \min\left(I_{kj}^+\right) \right], \left[\min\left(F_{kj}^-\right), \min\left(F_{kj}^+\right) \right] \Big\rangle, \\ & \left\langle \max\left(T_{kj}\right), \min\left(I_{kj}\right), \min\left(F_{kj}\right) \right\rangle \Big) \end{aligned} \quad (19)$$

It represents the benefit attribute

$$\begin{aligned} \mathfrak{I}_j^* = \Big(\Big\langle & \left[\min\left(T_{kj}^-\right), \min\left(T_{kj}^+\right) \right], \\ & \left[\max\left(I_{kj}^-\right), \max\left(I_{kj}^+\right) \right], \left[\max\left(F_{kj}^-\right), \max\left(F_{kj}^+\right) \right] \Big\rangle, \\ & \left\langle \min\left(T_{kj}\right), \max\left(I_{kj}\right), \max\left(F_{kj}\right) \right\rangle \Big) \end{aligned} \quad (20)$$

It represents the cost attribute.

Step 3. Calculate weight correlation coefficient between $\mathfrak{I}_k$ $(k = 1, 2, \ldots, m)$ and $\mathfrak{I}_j^*$ by using (18).

Step 4. Rank all of the weighted correlation coefficient of SVNCSs and select the best one.

In the following, we give an example from Lu [46] to illustrate the effectiveness of the method.

Example 18. According to [46], we can get the following data:

$$\begin{aligned} \mathfrak{I}_1 &= \begin{bmatrix} \langle [0.5, 0.6], [0.1, 0.3], [0.2, 0.4] \rangle, \langle 0.6, 0.2, 0.3 \rangle \\ \langle [0.5, 0.6], [0.1, 0.3], [0.2, 0.4] \rangle, \langle 0.6, 0.2, 0.3 \rangle \\ \langle [0.6, 0.8], [0.2, 0.3], [0.1, 0.2] \rangle, \langle 0.7, 0.2, 0.1 \rangle \end{bmatrix}, \\ \mathfrak{I}_2 &= \begin{bmatrix} \langle [0.6, 0.8], [0.1, 0.2], [0.2, 0.3] \rangle, \langle 0.7, 0.1, 0.2 \rangle \\ \langle [0.6, 0.7], [0.1, 0.2], [0.2, 0.3] \rangle, \langle 0.6, 0.1, 0.2 \rangle \\ \langle [0.6, 0.7], [0.3, 0.4], [0.1, 0.2] \rangle, \langle 0.7, 0.4, 0.1 \rangle \end{bmatrix}, \\ \mathfrak{I}_3 &= \begin{bmatrix} \langle [0.4, 0.6], [0.2, 0.3], [0.1, 0.3] \rangle, \langle 0.6, 0.2, 0.2 \rangle \\ \langle [0.5, 0.6], [0.2, 0.3], [0.3, 0.4] \rangle, \langle 0.6, 0.3, 0.4 \rangle \\ \langle [0.5, 0.7], [0.2, 0.3], [0.3, 0.4] \rangle, \langle 0.6, 0.2, 0.3 \rangle \end{bmatrix}, \\ \mathfrak{I}_4 &= \begin{bmatrix} \langle [0.7, 0.8], [0.1, 0.2], [0.1, 0.2] \rangle, \langle 0.8, 0.1, 0.2 \rangle \\ \langle [0.6, 0.7], [0.1, 0.2], [0.1, 0.3] \rangle, \langle 0.7, 0.1, 0.2 \rangle \\ \langle [0.6, 0.7], [0.3, 0.4], [0.2, 0.3] \rangle, \langle 0.7, 0.3, 0.2 \rangle \end{bmatrix}, \end{aligned} \quad (21)$$

Step 1. According to the aforementioned four alternatives, construct the neutrosophic cubic decision matrix A:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}. \quad (22)$$

where

$$a_{11} = (\langle [0.5, 0.6], [0.1, 0.3], [0.2, 0.4] \rangle, \langle 0.6, 0.2, 0.3 \rangle);$$

$$a_{12} = (\langle [0.5, 0.6], [0.1, 0.3], [0.2, 0.4] \rangle, \langle 0.6, 0.2, 0.3 \rangle);$$

$$a_{13} = (\langle [0.6, 0.8], [0.2, 0.3], [0.1, 0.2] \rangle, \langle 0.7, 0.2, 0.1 \rangle);$$

$$a_{21} = (\langle [0.6, 0.8], [0.1, 0.2], [0.2, 0.3] \rangle, \langle 0.7, 0.1, 0.2 \rangle);$$

Table 1: The values $\mathfrak{I}^*$ with $\mathfrak{I}_i$ and ranking orders.

	Value	Ranking Order	The Best Alternative
$\rho_{SVNCS_w}(\mathfrak{I}^*, \mathfrak{I}_i)$	0.9542,0.9822,0.6291,0.9944	$\rho_4 > \rho_2 > \rho_1 > \rho_3$	$\mathfrak{I}_4$
$S_{\omega_1}(\mathfrak{I}^*, \mathfrak{I}_i)$	0.9564,0.9855,0.9596,0.9945	$S_4 > S_2 > S_3 > S_1$	$\mathfrak{I}_4$
$S_{\omega_2}(\mathfrak{I}^*, \mathfrak{I}_i)$	0.9769,0.9944,0.9795,0.9972	$S_4 > S_2 > S_3 > S_1$	$\mathfrak{I}_4$
$S_{\omega_3}(\mathfrak{I}^*, \mathfrak{I}_i)$	0.9892,0.9959,0.9897,0.9989	$S_4 > S_2 > S_3 > S_1$	$\mathfrak{I}_4$

$$a_{22} = (\langle [0.6, 0.7], [0.1, 0.2], [0.2, 0.3] \rangle, \langle 0.6, 0.1, 0.2 \rangle);$$
$$a_{23} = (\langle [0.6, 0.7], [0.3, 0.4], [0.1, 0.2] \rangle, \langle 0.7, 0.4, 0.1 \rangle);$$
$$a_{31} = (\langle [0.4, 0.6], [0.2, 0.3], [0.1, 0.3] \rangle, \langle 0.6, 0.2, 0.2 \rangle);$$
$$a_{32} = (\langle [0.5, 0.6], [0.2, 0.3], [0.3, 0.4] \rangle, \langle 0.6, 0.3, 0.4 \rangle);$$
$$a_{33} = (\langle [0.5, 0.7], [0.2, 0.3], [0.3, 0.4] \rangle, \langle 0.6, 0.2, 0.3 \rangle);$$
$$a_{41} = (\langle [0.7, 0.8], [0.1, 0.2], [0.1, 0.2] \rangle, \langle 0.8, 0.1, 0.2 \rangle);$$
$$a_{42} = (\langle [0.6, 0.7], [0.1, 0.2], [0.1, 0.3] \rangle, \langle 0.7, 0.1, 0.2 \rangle);$$
$$a_{43} = (\langle [0.6, 0.7], [0.3, 0.4], [0.2, 0.3] \rangle, \langle 0.7, 0.3, 0.2 \rangle). \tag{23}$$

Step 2. Set up an ideal scheme:

$$\mathfrak{I}^* = \begin{bmatrix} \langle [0.7, 0.8], [0.1, 0.2], [0.1, 0.2] \rangle, \langle 0.8, 0.1, 0.2 \rangle \\ \langle [0.6, 0.7], [0.1, 0.2], [0.1, 0.3] \rangle, \langle 0.7, 0.1, 0.2 \rangle \\ \langle [0.5, 0.7], [0.3, 0.4], [0.3, 0.4] \rangle, \langle 0.6, 0.4, 0.3 \rangle \end{bmatrix}. \tag{24}$$

Step 3. Calculate weight correlation coefficient:

$\rho_{SVNCS_w}(\mathfrak{I}^*, \mathfrak{I}_1) = 0.9542$, $\rho_{SVNCS_w}(\mathfrak{I}^*, \mathfrak{I}_2) = 0.9822$, $\rho_{SVNCS_w}(\mathfrak{I}^*, \mathfrak{I}_3) = 0.6291$, $\rho_{SVNCS_w}(\mathfrak{I}^*, \mathfrak{I}_4) = 0.9944$.

Step 4. Rank all of the weighted correlation coefficient of SVNCSs: $\rho_{SVNCS_w}(\mathfrak{I}, \mathfrak{I}_4) \geq \rho_{SVNCS_w}(\mathfrak{I}, \mathfrak{I}_2) \geq \rho_{SVNCS_w}(\mathfrak{I}, \mathfrak{I}_1) \geq \rho_{SVNCS_w}(\mathfrak{I}, \mathfrak{I}_3)$ and thus, the alternative $\mathfrak{I}_4$ was selected as the best alternative.

Comparative Analysis. From three cosine measures of SVNCSs from Lu [46] and the correlation coefficient of SVNCSs, we have the result in Table 1.

From the results of Table 1, we note that the rank $S_4(\mathfrak{I}, \mathfrak{I}_4) \geq S_2(\mathfrak{I}, \mathfrak{I}_2) \geq S_3(\mathfrak{I}, \mathfrak{I}_3) \geq S_1(\mathfrak{I}, \mathfrak{I}_1)$ is changed to $\rho_{SVNCS_w}(\mathfrak{I}, \mathfrak{I}_4) \geq \rho_{SVNCS_w}(\mathfrak{I}, \mathfrak{I}_2) \geq \rho_{SVNCS_w}(\mathfrak{I}, \mathfrak{I}_1) \geq \rho_{SVNCS_w}(\mathfrak{I}, \mathfrak{I}_3)$. What is more, comparing the results of Lu [46], we can see that all the weighted cosine measure values are quite close; therefore, it is difficult to convince such a ranking result. And the values vary from 0.9564 to 0.9945 of $S_{\omega 1}$, and we note that $S_{\omega 1}(\mathfrak{I}, \mathfrak{I}_1)$ with $S_{\omega 1}(\mathfrak{I}, \mathfrak{I}_3)$ and $S_{\omega 1}(\mathfrak{I}, \mathfrak{I}_2)$ with $S_{\omega 1}(\mathfrak{I}, \mathfrak{I}_4)$ are very close, which makes it difficult to make clear judgments about all alternatives in decision-making. $S_{\omega 2}$ varies from 0.9769 to 0.9972; moreover, the values of $S_{\omega 2}(\mathfrak{I}, \mathfrak{I}_1)$ with $S_{\omega 2}(\mathfrak{I}, \mathfrak{I}_3)$ and $S_{\omega 2}(\mathfrak{I}, \mathfrak{I}_2)$ with $S_{\omega 2}(\mathfrak{I}, \mathfrak{I}_4)$ are basically not any gaps, which can be seen approximately equal, so that the results of the decision making may be suspected. And $S_{\omega 3}$ varies from 0.9892 to 0.9989, the difference between $S_{\omega 3}(\mathfrak{I}, \mathfrak{I}_1)$ with $S_{\omega 3}(\mathfrak{I}, \mathfrak{I}_3)$ is only 0.0005, and the difference between $S_{\omega 3}(\mathfrak{I}, \mathfrak{I}_2)$ with $S_{\omega 3}(\mathfrak{I}, \mathfrak{I}_4)$ is 0.003, so it is difficult to make the most appropriate decision for decision makers, perhaps this is not a desirable decision method.

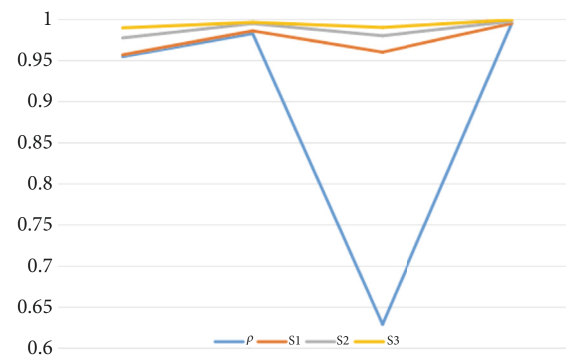

Figure 1: Correlation coefficient and cosine measures of alternatives with ideal alternative.

Obviously, these cosine measure values cannot clearly discriminate the difference of different alternatives. Conversely, the result of this paper lies from 0.6291 to 0.9944; the distinction between different values is relatively large. Moreover, the minimum cell division in the four numbers is also 0.0122, which is more convincing than that article [46]; thus, we can easily select the best alternative. Furthermore, from the table, we can see that several methods are basically consistent with the evaluation results of the fourth schemes. But it is difficult to make a relatively clear and reliable judgment between the alternative 1 and the alternative 3 with the previous cosine measurement method. However, the method of this paper gives a clear comparison between them. As shown in Figure 1, we can clearly see the result $\rho_{SVNCS_w}(\mathfrak{I}, \mathfrak{I}_4) \geq \rho_{SVNCS_w}(\mathfrak{I}, \mathfrak{I}_2) \geq \rho_{SVNCS_w}(\mathfrak{I}, \mathfrak{I}_1) \geq \rho_{SVNCS_w}(\mathfrak{I}, \mathfrak{I}_3)$.

Compared with Lu [46] method, we find the method of this paper more effective and superior.

5. Applications in Pattern Recognition Problems

In this section, we propose a recognition method based on the correlation coefficient of SVNCSs, which is derived from [50]. According to the maximum correlation principle in mathematical statistics, we assume that if the correlation coefficient of ideal pattern with sample pattern is greater than or equal to 0.7, we consider that the sample model belongs to a group of ideal model. The algorithm is as follows.

Algorithm 19.

Step 1. Construct an ideal SVNCS $\mathfrak{I}^*$ on X.

Step 2. Construct SVNCSs $\mathfrak{I}_i$, $i = 1, 2, 3 \cdots, n$ as the sample pattern that is recognized.

Step 3. Calculate the correlation coefficient $\rho_{SVNCSs}(\mathfrak{I}^*, \mathfrak{I}_i)$, $i = 1, 2, \ldots, n$.

Step 4. If $\rho_{SVNCSs}(\mathfrak{I}^*, \mathfrak{I}_i) \geq 0.7$, the $\mathfrak{I}_i$ belongs to the ideal pattern $\mathfrak{I}^*$ and if $\rho_{SVNCSs}(\mathfrak{I}^*, \mathfrak{I}_i) < 0.7$, the $\mathfrak{I}_i$ does not belong to the ideal pattern $\mathfrak{I}^*$.

In the following, we give two examples to illustrate the utility of the correlation coefficient of SVNCSs in pattern recognition.

Example 20. This example is adapted from [50]; three sample patterns $\mathfrak{I}_i$ $(i = 1, 2, 3)$ and the ideal pattern $\mathfrak{I}^*$ are given as

$$\begin{gathered}
\mathfrak{I}_1 = \begin{bmatrix} \langle [0.2, 0.5], [0.4, 0.5], [0.3, 0.5] \rangle, \langle 0.5, 0.6, 0.4 \rangle \\ \langle [0.7, 0.7], [0.1, 0.3], [0.1, 0.3] \rangle, \langle 0.2, 0.2, 0.2 \rangle \\ \langle [0.6, 0.8], [0.5, 0.6], [0.3, 0.4] \rangle, \langle 0.3, 0.1, 0.7 \rangle \end{bmatrix}, \\
\mathfrak{I}_2 = \begin{bmatrix} \langle [0.3, 0.7], [0.3, 0.5], [0.3, 0.9] \rangle, \langle 0.2, 0.5, 0.2 \rangle \\ \langle [0.6, 0.7], [0.1, 0.8], [0.2, 0.3] \rangle, \langle 0.1, 0.5, 0.7 \rangle \\ \langle [0.6, 0.9], [0.9, 1.0], [0.3, 0.4] \rangle, \langle 0.7, 0.1, 0.7 \rangle \end{bmatrix}, \\
\mathfrak{I}_3 = \begin{bmatrix} \langle [0.8, 0.9], [0.1, 0.2], [0.8, 0.9] \rangle, \langle 0.9, 0.8, 0.9 \rangle \\ \langle [0.1, 0.2], [0.8, 0.9], [0.3, 0.9] \rangle, \langle 0.9, 0.8, 0.9 \rangle \\ \langle [0.5, 0.9], [0.1, 1.0], [0.4, 0.7] \rangle, \langle 0.9, 0.9, 0.1 \rangle \end{bmatrix}, \\
\mathfrak{I}^* = \begin{bmatrix} \langle [0.2, 0.4], [0.3, 0.5], [0.3, 0.5] \rangle, \langle 0.1, 0.2, 0.4 \rangle \\ \langle [0.5, 0.7], [0.0, 0.5], [0.2, 0.3] \rangle, \langle 0.1, 0.2, 0.2 \rangle \\ \langle [0.6, 0.8], [0.0, 0.1], [0.3, 0.4] \rangle, \langle 0.3, 0.1, 0.7 \rangle \end{bmatrix}.
\end{gathered} \tag{25}$$

Calculate the correlation coefficient of $\rho_{SVNCS}(\mathfrak{I}^*, \mathfrak{I}_i)$, $i = 1, 2, 3$.

$\rho_{SVNCS}(\mathfrak{I}^*, \mathfrak{I}_1) = 0.93$, $\rho_{SVNCS}(\mathfrak{I}^*, \mathfrak{I}_2) = 0.83$, $\rho_{SVNCS}(\mathfrak{I}^*, \mathfrak{I}_3) = 0.63$.

Since $\rho_{SVNCS}(\mathfrak{I}^*, \mathfrak{I}_1) \geq 0.7$, $\rho_{SVNCS}(\mathfrak{I}^*, \mathfrak{I}_2) \geq 0.7$, and $\rho_{SVNCS}(\mathfrak{I}^*, \mathfrak{I}_3) < 0.7$, we consider the samples $\mathfrak{I}_1$ and $\mathfrak{I}_2$ belong to ideal pattern, and $\mathfrak{I}_3$ does not belong to the ideal pattern.

We notice that the result of Example 20 is consistent with this paper. However, the result of this article is more superior.

Next, we will give another example to illustrate the approach of this article.

Example 21. The sample patterns $\mathfrak{I}_i$ $(i = 1, 2)$ and the ideal pattern $\mathfrak{I}^*$ are given as follows:

$$\begin{gathered}
\mathfrak{I}_1 = \begin{bmatrix} \langle [0.8, 0.9], [0.1, 0.2], [0.8, 0.9] \rangle, \langle 0.9, 0.8, 0.9 \rangle \\ \langle [0.1, 0.2], [0.8, 0.9], [0.3, 0.9] \rangle, \langle 0.9, 0.8, 0.9 \rangle \\ \langle [0.5, 0.9], [0.1, 1.0], [0.4, 0.7] \rangle, \langle 0.9, 0.9, 0.1 \rangle \end{bmatrix}, \\
\mathfrak{I}_2 = \begin{bmatrix} \langle [0.8, 0.9], [0.1, 0.2], [0.8, 0.9] \rangle, \langle 0.8, 0.8, 0.9 \rangle \\ \langle [0.1, 0.2], [0.8, 0.9], [0.7, 0.9] \rangle, \langle 0.3, 0.5, 0.7 \rangle \\ \langle [0.7, 0.9], [0.5, 1.0], [0.5, 0.7] \rangle, \langle 0.7, 0.9, 0.1 \rangle \end{bmatrix}, \\
\mathfrak{I}^* = \begin{bmatrix} \langle [0.2, 0.4], [0.3, 0.5], [0.3, 0.5] \rangle, \langle 0.1, 0.2, 0.4 \rangle \\ \langle [0.5, 0.7], [0.0, 0.5], [0.2, 0.3] \rangle, \langle 0.1, 0.2, 0.2 \rangle \\ \langle [0.6, 0.8], [0.0, 0.1], [0.3, 0.4] \rangle, \langle 0.3, 0.1, 0.7 \rangle \end{bmatrix}.
\end{gathered} \tag{26}$$

Using the method of [50], we can get $d(\mathfrak{I}_1, \mathfrak{I}^*) = 0.51$, $d(\mathfrak{I}_2, \mathfrak{I}^*) = 0.49$, so the sample pattern $\mathfrak{I}_1$ does not belong to the ideal pattern, and $\mathfrak{I}_2$ belongs to the ideal pattern. However, using the method of this paper we will get result $\rho_{SVNCS}(\mathfrak{I}^*, \mathfrak{I}_1) = 0.63$, and $\rho_{SVNCS}(\mathfrak{I}^*, \mathfrak{I}_2) = 0.66$; therefore, according to the principle of maximum correlation, we can judge that $\mathfrak{I}_1$ does not belong to the ideal pattern, and $\mathfrak{I}_2$ also does not belong to the ideal pattern.

Comparative Analysis. In the pattern recognition problems, the results of Example 21 are different. Using the method of [50], the recognition standard is 0.5. According to the minimum distance measures principle (principle of maximum similarity measures), because $d(\mathfrak{I}, \mathfrak{I}_1) = 0.51$, and $d(\mathfrak{I}, \mathfrak{I}_2) = 0.49$ are very close to 0.5, so the judgment may have certain deviation; it is difficult to make accurate identification for some distance measures near 0.5. That is to say, the recognition result will be very fuzzy; therefore, this standard does not have its superiority in this case. However, the method of this paper can clearly identify the relationship between the sample pattern and the ideal pattern, and it effectively avoids the ambiguity of the result (the correlation is greater than or equal to 0.7).

Therefore, using the principle of maximum correlation is more persuasive and more intuitive to accept.

6. Conclusion

SVNCSs are a generalization of SVNSs, INSs, and CSs. We start to introduce some concepts of correlation coefficient of SVNCSs and discuss the properties. We also propose the decision method by using the weighted correlation coefficient of SVNCSs and give applications of correlation coefficient of SVNCSs in pattern recognition. Finally we give examples to explain the effectiveness of the method and give comparative analysis to show the superiority of the method.

Data Availability

The data of this manuscript is available, and every one can free access the data supporting the conclusion of the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (11661053, 11771198) and supported by the Provincial Natural Science Foundation of Jiangxi, China (20181BAB201003).

References

[1] L. A. Zadeh, "Fuzzy sets," *Information and Computation*, vol. 8, pp. 338–353, 1965.

[2] I. B. Turksen, "Interval valued fuzzy sets based on normal forms," *Fuzzy Sets and Systems*, vol. 20, no. 2, pp. 191–210, 1986.

[3] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.

[4] K. Atanassov and G. Gargov, "Interval valued intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 31, no. 3, pp. 343–349, 1989.

[5] D. Dubois and H. Prade, "Recent literature: fuzzy sets and systems," *Fuzzy Sets and Systems*, vol. 22, no. 3, pp. 325–330, 1987.

[6] R. R. Yager, "On the theory of bags," *International Journal of General Systems: Methodology, Applications, Education*, vol. 13, no. 1, pp. 23–37, 1987.

[7] V. Torra, "Hesitant fuzzy sets," *International Journal of Intelligent Systems*, vol. 25, no. 6, pp. 529–539, 2010.

[8] Y. B. Jun, C. S. Kim, and K. O. Yang, "Cubic sets," *Annals of Fuzzy Mathematics and Informatics*, vol. 4, no. 1, pp. 83–98, 2012.

[9] S. S. Ahn, Y. H. Kim, and J. M. Ko, "Cubic subalgebras and filters of CI-algebras," *Honam Mathematical Journal*, vol. 36, no. 1, pp. 43–54, 2014.

[10] T. Senapati, C. S. Kim, M. Bhowmik, and M. Pal, "Cubic subalgebras and cubic closed ideals of B-algebras," *Fuzzy Information and Engineering*, vol. 7, no. 2, pp. 129–149, 2015.

[11] Y. B. Jun and K. J. Lee, "Closed cubic ideals and cubic circ-subalgebras in BCK/BCI-algebras," *Applied Mathematical Sciences*, vol. 4, no. 65-68, pp. 3395–3402, 2010.

[12] Y. B. Jun, K. J. Lee, and M. S. Kang, "Cubic structures applied to ideals of BCI-algebras," *Computers and Mathematics with Applications*, vol. 62, no. 9, pp. 3334–3342, 2011.

[13] M. Akram, N. Yaqoob, and M. Gulistan, "Cubic KU-subalgebras," *International Journal of Pure and Applied Mathematics*, vol. 89, no. 5, pp. 659–665, 2013.

[14] N. Yaqoob, S. M. Mostafa, and M. A. Ansari, "On Cubic KU-Ideals of KU-Algebras," *ISRN Algebra*, vol. 2013, Article ID 935905, pp. 1–10, 2014.

[15] Y. B. Jun and A. Khan, "Cubic ideals in semigroups," *Honam Mathematical Journal*, vol. 35, no. 4, pp. 607–623, 2013.

[16] F. Smarandache, "Neutrosophic set, a generalisation of the intuitionistic fuzzy sets," *EEE International Conference on Granular Computing. IEEE*, 2006.

[17] S. Pramanik, P. Biswas, and B. C. Giri, "Hybrid vector similarity measures and their applications to multi-attribute decision making under neutrosophic environment," *Neural Computing and Applications*, vol. 28, no. 5, pp. 1163–1176, 2017.

[18] S. Broumi and F. Smarandache, "Correlation coefficient of interval neutrosophic set," *Applied Mechanics and Materials*, vol. 436, pp. 511–517, 2013.

[19] H. D. Cheng and Y. Guo, "A New neutrosophic approach to image thresholding," *New Mathematics and Natural Computation*, vol. 4, no. 3, pp. 291–308, 2008.

[20] Y. Guo and H. D. Cheng, "New neutrosophic approach to image segmentation," *Pattern Recognition*, vol. 42, no. 5, pp. 587–595, 2009.

[21] A. Kharal, "A neutrosophic multi-criteria decision making method," *New Mathematics and Natural Computation*, vol. 10, no. 2, pp. 143–162, 2014.

[22] J. J. Peng, J. Q. Wang, X. H. Wu, J. Wang, and X. H. Chen, "Multi-valued Neutrosophic Sets and Power Aggregation Operators with Their Applications in Multi-criteria Group Decision-making Problems," *International Journal of Computational Intelligence Systems*, vol. 8, no. 2, pp. 345–363, 2015.

[23] H. Zhang, J. Wang, and X. Chen, "An outranking approach for multi-criteria decision-making problems with interval-valued neutrosophic sets," *Applied Soft Computing*, vol. 25, pp. 336–346, 2014.

[24] J. J. Peng, J. Q. Wang, J. Wang, H. Y. Zhang, and X. H. Chen, "Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems," *International Journal of Systems Science*, vol. 47, no. 10, pp. 2342–2358, 2016.

[25] H. B. Wang, F. Smarandache, Y. Q. Zhang et al., "Interval eutrosophic sets and logic: theory and applications in computing," *Computer Science*, vol. 65, no. 4, p. 87, 2005.

[26] F. Smarandache, "Single valued neutrosophic sets," *Review of the Air Force Academy*, vol. 10, pp. 9–14, 2013.

[27] R. Şahin and A. Küçük, "Subsethood measure for single valued neutrosophic sets," *Journal of Intelligent and Fuzzy Systems*, vol. 29, no. 2, pp. 525–530, 2015.

[28] J. Ye, "An extended TOPSIS method for multiple attribute group decision making based on single valued neutrosophic linguistic numbers," *Journal of Intelligent and Fuzzy Systems*, vol. 28, no. 1, pp. 247–255, 2015.

[29] P. Biswas, S. Pramanik, and B. C. Giri, "TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment," *Neural Computing and Applications*, vol. 27, no. 3, pp. 727–737, 2016.

[30] P. Biswas, S. Pramanik, and B. C. Giri, "Value and ambiguity index based ranking method of single-valued trapezoidal

neutrosophic numbers and its application to multi-attribute decision making," *Neutrosophic Sets and Systems*, vol. 12, pp. 127–138, 2016.

[31] P. Biswas, S. Pramanik, and B. C. Giri, "Aggregation of triangular fuzzy neutrosophic set information and its application to multi-attribute decision making," *Neutrosophic Sets and Systems*, vol. 12, pp. 20–40, 2016.

[32] F. Smarandache and S. Pramanik, *New Trends in Neutrosophic Theory and Applications*, Artificial Intelligence, 2016.

[33] R. Şahin and P. Liu, "Maximizing deviation method for neutrosophic multiple attribute decision making with incomplete weight information," *Neural Computing and Applications*, vol. 27, no. 7, pp. 1–13, 2016.

[34] R. Şahin, "Cross-entropy measure on interval neutrosophic sets and its applications in multicriteria decision making," *Neural Computing and Applications*, vol. 28, no. 5, pp. 1177–1187, 2017.

[35] R. Şahin and P. Liu, "Possibility-induced simplified neutrosophic aggregation operators and their application to multi-criteria group decision-making," *Journal of Experimental and Theoretical Artificial Intelligence*, vol. 29, no. 4, pp. 769–785, 2016.

[36] P. Biswas, S. Pramanik, and B. C. Giri, "Multi-attribute group decision making based on expected value of neutrosophic trapezoidal numbers," *New Trends in Neutrosophic Theory and Applications*, pp. 105–124, 2015.

[37] R. Sahin, "Multi-criteria neutrosophic decision making method based on score and accuracy functions under neutrosophic environment," *Computer Science*, pp. 1–9, 2014.

[38] P. Biswas, S. Pramanik, and B. C. Giri, "GRA Method of Multiple Attribute Decision Making with Single Valued Neutrosophic Hesitant Fuzzy Set Information," *New Trends in Neutrosophic Theory and Applications*, pp. 55–63, 2016.

[39] P. Biswas, S. Pramanik, and B. C. Giri, "Some Distance Measures of Single Valued Neutrosophic Hesitant Fuzzy Sets and Their Applications to Multiple Attribute Decision Making Making," *New Trends in Neutrosophic Theory and Applications*, pp. 27–34, 2016.

[40] R. Şahin and P. Liu, "Distance and similarity measures for multiple attribute decision making with single-valued neutrosophic hesitant fuzzy information," *New Trends in Neutrosophic Theory and Applications*, pp. 35–54, 2016.

[41] R. Şahin and P. Liu, "Correlation coefficient of single-valued neutrosophic hesitant fuzzy sets and its applications in decision making," *Neural Computing and Applications*, vol. 28, no. 6, pp. 1387–1395, 2017.

[42] J. Ye, "Multiple-attribute decision-making method under a single-valued neutrosophic hesitant fuzzy environment," *Journal of Intelligent Systems*, vol. 24, no. 1, pp. 23–36, 2014.

[43] Y. B. Jun, F. Smarandache, and C. S. Kim, "Neutrosophic Cubic Sets," *New Mathematics and Natural Computation*, vol. 13, no. 1, pp. 41–54, 2017.

[44] D. Banerjee, B. C. Giri, and S. Pramanik, "GRA for multi attribute decision making in neutrosophic cubic set environment," *Neutrosophic Sets and Systems*, vol. 15, pp. 60–69, 2017.

[45] S. Pramanik, S. Dalapati, and S. Alam, "Neutrosophic cubic MCGDM method based on similarity measure," *Neutrosophic Sets and Systems*, vol. 16, pp. 44–56, 2017.

[46] Z. Lu and J. Ye, "Cosine measures of neutrosophic cubic sets for multiple attribute decision-making," *Symmetry*, vol. 9, no. 121, pp. 1–10, 2017.

[47] S. Pramanik, P. P. Dey, B. C. Giri, and F. Smarandache, "An extended TOPSIS for multi-attribute decision making problems with neutrosophic cubic information," *Neutrosophic Sets and Systems*, vol. 17, pp. 20–28, 2017.

[48] J. Zhan, M. Khan, M. Gulistan, and A. Ali, "Applications of neutrosophic cubic sets in multi-criteria decision-making," *International Journal for Uncertainty Quantification*, vol. 7, no. 5, pp. 377–394, 2017.

[49] J. Ye, "Linguistic neutrosophic cubic numbers and their multiple attribute decision-making method," *Information*, vol. 8, no. 3, 2017.

[50] M. Alia, I. Delib, and F. Smarandache, "The theory of neutrosophic cubic sets and their applications in pattern recognition," *Journal of Intelligent and Fuzzy Systems*, vol. 30, no. 4, pp. 1–7, 2016.

[51] P. Bonizzoni, G. D. Vedova, R. Dondi et al., "Correlation clustering and consensus clustering," *International Conference on Algorithms and Computation, Springer-Verlag*, vol. 3827, no. 5, pp. 226–235, 2005.

[52] H. P. Kriegel, P. Kroger, E. Schubert et al., "A general framework for increasing the robustness of PCA-based correlation clustering algorithms," *International Conference on Scientific and Statistical Database Management, Springer-Verlag*, vol. 5069, pp. 418–435, 2008.

[53] D. G. Park, Y. C. Kwun, and J. H. Park, "Correlation coefficient of interval-valued intuitionistic fuzzy sets and its application to multiple attribute group decision making problems," *Mathematical and Computer Modelling*, vol. 50, no. 9-10, pp. 1279–1293, 2009.

[54] G. W. Wei, H. J. Wang, and R. Lin, "Application of correlation coefficient to interval-valued intuitionistic fuzzy multiple attribute decision-making with incomplete weight information," *Knowledge and Information Systems*, vol. 26, no. 2, pp. 337–349, 2011.

[55] J. Ye, "Multicriteria fuzzy decision-making method using entropy weights-based correlation coefficients of interval-valued intuitionistic fuzzy sets," *Applied Mathematical Modelling*, vol. 34, no. 12, pp. 3864–3870, 2010.

[56] D. A. Chiang and N. P. Lin, "Correlation of fuzzy sets," *Fuzzy Sets and Systems*, vol. 102, no. 2, pp. 221–226, 1999.

[57] S. T. Liu and C. Kao, "Fuzzy measures for correlation coefficient of fuzzy numbers," *Fuzzy Sets and Systems*, vol. 128, no. 2, pp. 267–275, 2002.

[58] G. J. Wang and X. P. Li, "Correlation and information energy of interval-valued fuzzy numbers," *Fuzzy Sets and Systems*, vol. 103, no. 1, pp. 169–175, 1999.

[59] T. Gerstenkorn and J. Mako, "Correlation of intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 44, no. 1, pp. 39–43, 1991.

[60] N. Chen, Z. Xu, and M. Xia, "Correlation coefficients of hesitant fuzzy sets and their applications to clustering analysis," *Applied Mathematical Modelling*, vol. 37, no. 4, pp. 2197–2211, 2013.

[61] Z. Xu and M. Xia, "On distance and correlation measures of hesitant fuzzy information," *International Journal of Intelligent Systems*, vol. 26, no. 5, pp. 410–425, 2011.

[62] L. Wang, M. Ni, and L. Zhu, "Correlation Measures of Dual Hesitant Fuzzy Sets," *Journal of Applied Mathematics*, vol. 2013, no. 4, Article ID 593739, pp. 1–12, 2013.

[63] J. Ye, "Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment," *International Journal of General Systems*, vol. 42, no. 4, pp. 386–394, 2013.

[64] R. Şahin and P. Liu, "Correlation coefficient of single-valued neutrosophic hesitant fuzzy sets and its applications in decision making," *Neural Computing and Applications*, pp. 1–9, 2016.

[65] J. H. Yang and M. S. Yang, "A control chart pattern recognition system using a statistical correlation coefficient method," *Computers and Industrial Engineering*, vol. 48, no. 2, pp. 205–221, 2005.

[66] G. J. Klir and B. Yuan, "Fuzzy sets and fuzzy logic: Theory and applications, prentice-hall, upper saddle river," *Fuzzy Sets and Fuzzy Logic: Theory and Applications*, no. 1, pp. 283–287, 1995.

[67] J. J. Buckley, *Fuzzy Probability and Statistics (Studies in Fuzziness and Soft Computing)*, vol. 96, Springer, New York, NY, USA, 2006.

[68] F. Smarandache, "A unifying field in logics: neutrosophic logic," *Multiple-Valued Logic*, vol. 8, no. 3, pp. 385–438, 2002.