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Research Article

Marginal Cost Pricing Analysis on Tradable Credits in Traffic Engineering

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This paper tries to explore a more applicable tradable credit scheme for managing network mobility from the angle of marginal cost pricing. The classic mathematical model-Cobweb model is used to analyze the stability of credit price. It is found that credit price is not always convergent in the trading market. It will show convergence, divergence, two-period simple behaviors, and even more complex dynamic behaviors, such as cycle movements and chaos. Considering the applicability and public goods character of tradable credits scheme, one public pricing mechanism- marginal cost pricing is explored. Analytical investigations and the numerical simulation of a particular case with linear demand and supply indicate that marginal cost pricing is an effective, sustainable, and socially feasible manner in managing the demand for car travel.

1. Introduction

Congestion pricing is the most widely known tool in alleviating congestion. Today it is being practised worldwide in a variety of forms, such as cordon-based pricing in Singapore, London, and Stockholm and tolls on non-HOV (high occupancy vehicle) lanes in the US. Despite its appeal, congestion pricing has its limitations in practice. The major defect is the perceived unfairness [1, 2]. In order to overcome the unfairness of Pigouvian tolls, various ideas have been investigated, which include negative tolls [3, 4], Pareto-improving pricing schemes without revenue rebate [5, 6], Pareto-improving and revenue-neutral road pricing schemes [7, 8], and Pareto-improving congestion pricing cum revenue refunding schemes [9]. Unfortunately, although revenue redistribution and the Pareto-improvement schemes are effective, the government plays a role as an objectionable toll collector [10]. Therefore, travelers will be resistant as long as the policy has something to do with tolls.

Considering the resistance to congestion tolls, tradable credits schemes have been proposed in the transportation literatures. As a quantity-based control method, tradable credits scheme is regarded as one of the most feasible alternative tools of congestion pricing, which is equitable for travelers. More importantly, the government casts off the role as an objectionable toll collector. It borrows the concept of carbon emissions permits in environmental science, which can be traced back to Dales [11] for the purpose of attaining water quality targets. Recently, Yang and Wang [10] made a wonderful development in tradable credit schemes analysis which proved the existence and uniqueness of the equilibrium link flow pattern with fixed or elastic demand in a general network context. Based on Yang and Wang [10], various extensions are made by researchers.

Nie [12] analyzed the effects of transaction costs on traveler behavior. Wang et al. [13] explored the tradable credits for congestion management with heterogeneous users. Wu et al. [14] developed a more equitable congestion pricing and

tradable credits scheme by considering the effect of income on travelers. Wang et al. (2013) discussed the trial and error method for optimal tradable credit schemes. Xiao et al. [15] and Tian et al. [16] applied the credits scheme into bottleneck congestion management. He et al. [17] investigated the effects of tradable credit schemes on Cournot-Nash (CN) players and Wardrop-equilibrium (WE) players on the network. Nie and Yin [18] designed a new tradable credit scheme for managing commuters' rush hour choices. Wang et al. [19–21] introduced tradable credits into continuous network design. Gao and Hu (2014) compared the optimal tradable credits scheme and congestion pricing scheme in managing congestion. Li and Gao [22] discussed managing rush hour congestion with lane reversal and tradable credits.

Bao et al. [23] studied travelers' loss aversion behavior under a given tradable credits scheme. Zhu et al. [24] explored the multiclass network equilibrium problem under a tradable credits scheme. Gao et al. [25] discussed the tradable credits and transit investment optimization in a two modes traffic network. In particular, more reviews on tradable credits schemes, we can refer to Fan and Jiang [26], Grant-Muller and Xu [27], and Dogterom and Dijst [28].

Indubitably, quantity-based tradable credits schemes are economically attractive because of their ability to reach a certain reduction goal at minimised aggregate costs. Compared to traditional road pricing measures, quantity-based tradable credits schemes enable participants to financially benefit from the system and ensure that money circulates among participants instead of flowing to regulating authorities. However, it may be not smooth in practical applications. Every traveler would be a participant in the credits market. But market equilibrium conditions may be difficult to reach in the free trading market because of the imperfect information of the market and traveler's bounded rationality or incomplete rationality. Thus it is difficult to prevent speculators profiting from hoarding credits and selling them for a high price in busy periods. Under such circumstances, tradable credits may be inefficient compared with congestion pricing and the expected results will be discounted. Therefore, it brings a new question; i.e., is there a way that could ensure the fairness and efficiency at the same time? Gao et al. [29] had a try on a price-based tradable credits scheme. They found that the price-based tradable credits scheme is workable in managing congestion. In this paper, we try to make further efforts to discuss the price-based tradable credits in managing congestion. A public pricing tradable credit scheme is proposed to overcome the inefficiency of tradable credit scheme in practical application. In the tradable credits scheme, travelers can buy or sell credits according to their individual travel needs. The government does not interfere in the market as a buyer or seller but acts as a manager to monitor the system and determine the uniform credit price.

For the remainder, in Section 2, the stability of market is discussed. A price-based tradable credits scheme is proposed in Section 3. Section 4 validates the rationality of the new tradable credits scheme by a simple example. Finally, Section 5 concludes the paper and gives some extensions for future study.

2. Stability Analysis of Market-Based Credit Price with Cobweb Models

In 2013, Ye and Yang analyzed the price and flow dynamics under a tradable credit scheme when the credits can be traded in a free market. They found that the length of the time horizon of the credit scheme is critical for the system performances especially in terms of stability and convergence. In this paper, we will analyze stability of market-based credit price from the classic economic angle.

For the quantity-based tradable credits scheme, its price is likely to show a great fluctuation in the credits market if credit price is totally dominated by the free market. In this section, based on the Cobweb models in economics, we analyze the stability of market-based credit price. The cobweb model or cobweb theory is a classic economic model in explaining why prices might be subject to periodic fluctuations in certain types of markets. It is the classical model in dynamic economics analysis. It discusses the trend of supply and demand and the price dynamics in a market of a non-storable good that takes one time unit to produce. The most well-known instance is that it explained the agricultural price dynamics (Ezekiel, 1938). Due to the time lag between production decisions and realization (sowing and harvest), producers form price expectations and undertake production decisions one time period ahead, based on current and past experience [30, 31].

In tradable credits scheme, the generalized cobweb model can be written as follows:

$$X_t = f\left(p_{t-1}\right) \tag{1}$$

$$p_t = p\left(X_t\right) \tag{2}$$

where t is a time unit and X_t is the total amounts of trading credits in the tth period, i.e., the number of credits which are supplied by the travelers who wants to acquire revenue by selling credits. $f(p_{t-1})$ is the supply function of credits and $p(X_t)$ denotes the demand function. Equation (1) denotes that the amounts of trading credits in the tth period are determined by the credit price in the $(t-1)$th period. The amounts of trading credits and the demand function in the tth period determines current credit price and the current price determines the amounts of trading credits in the $(t+1)$th period and so forth.

2.1. Analysis of Linear Cobweb Models. In the credits market, any traveler could be a seller or a purchaser. The transaction process among travelers is a black box. Therefore, it is difficult to determine the function form between the amounts of trading credits and the credit price. In order to analyze the stability of credit price in free market, for the sake of simplicity, we suppose the supply function and demand functions are all linear. In fact, within the early cobweb model, demand and supply schedules are linear. As Dieci and Westerhoff put it in 2009, "Despite such a simple setup, this model provides a qualitative explanation for the cyclical tendencies observed in many commodity markets." Therefore, in order

to give a qualitative explanation for the stability of market-based tradable credit price, the supply function and demand functions are written as follows:

$$X_t = a + b \cdot p_{t-1} \tag{3}$$

$$p_t = \frac{c}{d} - \frac{X_t}{d} \tag{4}$$

where a, b, c, and d represent structure parameters. Combining (3) and (4), the equilibrium condition can be written as follows:

$$c - d \cdot p_t = a + b \cdot p_{t-1} \tag{5}$$

Accordingly, the following difference equation with credit price is obtained:

$$dp_t + bp_{t-1} = c - a \tag{6}$$

Suppose the initial credit price is p_0. Based on (6) and the initial credit price p_0, the credit price in the tth period can be written as follows:

$$p_t = \frac{c-a}{b+d} + \left(p_0 - \frac{c-a}{b+d}\right)\left(-\frac{b}{d}\right)^t \tag{7}$$

In (7), credit price (p_t) will show convergence, divergence, and two-period simple behaviors when $p_0 \neq (c-a)/(b+d)$. Otherwise, credit price will be a constant when $p_0 = (c-a)/(b+d)$.

(i) Credit price (p_t) would converge to $(c-a)/(b+d)$ when $b < d$. As shown in (7), if $b < d$, when $t \longrightarrow +\infty$, $(-b/d)^t \longrightarrow 0$ and $p_t \longrightarrow (c-a)/(b+d)$.

(ii) Credit price will swing between p_0 and $2(c-a)/d - p_0$ when $b = d$. In (7), if $b = d$, when $t \longrightarrow +\infty$,

$$\left(-\frac{b}{d}\right)^t = \begin{cases} 1, & t \text{ is the odd.} \\ -1, & t \text{ is the even.} \end{cases}$$
$$\text{and } p_t = \begin{cases} p_0, & t \text{ is the odd.} \\ \dfrac{2(c-a)}{b+d} - p_0, & t \text{ is the even.} \end{cases} \tag{8}$$

(iii) Credit price will change with t's changing and its track is divergent when $b > d$. In (7), if $b > d$, when $t \longrightarrow +\infty$, $(-b/d)^t \longrightarrow \infty$ and $p_t \longrightarrow \infty$.

The above three cases can be illustrated as follows.

Figure 1 illustrates the case (i); i.e., credit price (p_t) would converges to $(c-a)/(b+d)$ when $b < d$.

In Figure 1, the initial credit price is p_0 and the quantity of credits supplied is $a + bp_0$ (the abscissa of point B). However, the demand is $c - dp_0$ (the abscissa of point A). Supply exceeds demand and the difference value is $a + (b+d)p_0 - c$. Under such circumstances, because credits cannot be stored, all distributed credits must be consumed and credit price will fall to $(c - a - bp_0)/d$ (the ordinate of point C). The government will guide the distribution of credits in the next period according to the credit price at point C. Therefore, based on the credit price at point C, the government will distribute a certain amounts of credits which correspond to point D. Now the demand on credits is at point C. Supply falls short of demand in the market and the D-value is CD. Correspondingly, credit price will rise. The above process will be repetitive unless credit price converges to point H (i.e., the intersection point of demand curve and supply curve). Credit price shows the following regularities: fall-rise-fall-rise and the convergence price is $(c-a)/(b+d)$.

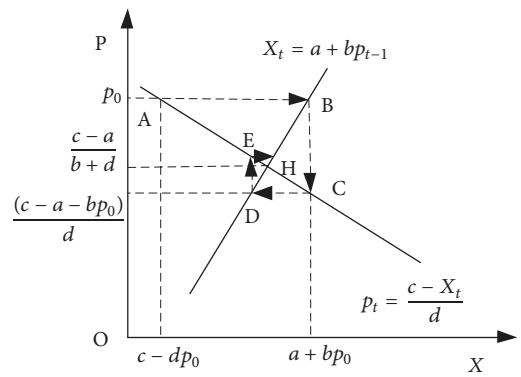

Figure 1: Credit price is convergent when b<d.

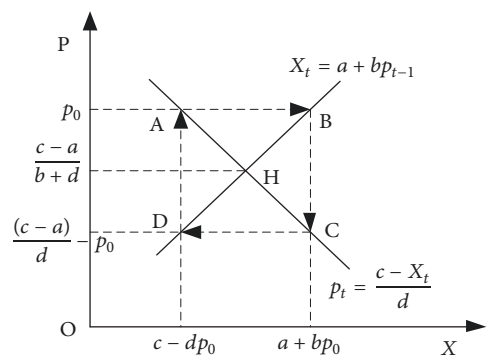

Figure 2: Credit price swings between two values when b=d.

Figure 2 illustrates the case (ii). As shown in Figure 2, the quantity of credits supplied by producers is $a + bp_0$ (the abscissa of point B) with the initial credit price p_0. However, the demand of credits is $c - dp_0$ at this point. Supply exceeds demand and the difference value is $a+2\cdot b\cdot p_0-c$. Therefore, the credit price dropped from p_0 to $(c-a)/d - p_0$ (the ordinate of point D). Unfortunately, supply does not meet the demand, which leads to the credit price returns to the original point p_0. In the next circles, this process will be repeated and credit price (p_t) will swing between p_0 and $(c-a)/d - p_0$.

In the same way, Figure 3 shows that credit price will be away from the equilibrium point farther and farther when the slope of supply-curve is smaller than the absolute value of the demand-curve slope (i.e., $b > d$).

2.2. Discussion of Nonlinear Cobweb Models. The above linear cobweb models demonstrated the instability of credit price. However, it has poor practicality, and the possible range of dynamic outcomes is basically restricted to three types of evolution around the equilibrium price. In fact, the demand

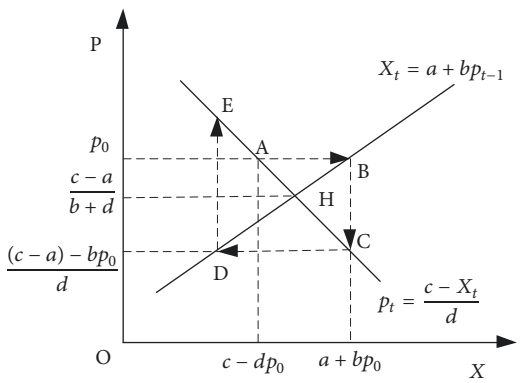

Figure 3: Credit price is divergent when b>d.

and supply schedules are more likely to be nonlinear and the dynamics outcomes are more diverse. In the last 30 years, the growing popularity of nonlinear dynamics in economic analysis has brought about a renewed interest in cobweb models, and the basic setup has been extended or modified so as to consider nonlinear demand and supply curves together with different adaptive expectations schemes. In particular, Chiarella [32] introduced a fairly general nonlinear supply function into the traditional cobweb model under adaptive expectations. He found that in its locally unstable region the cobweb model exhibits a regime of period doubling followed by a chaotic regime. Hommes [33] analyzed the price-quantity dynamics of the cobweb model with adaptive expectations and nonlinear supply and demand curves. They proved that chaotic dynamical behaviour can occur, even if both the supply and demand curves are monotonic. Later, some researchers (such as Brock and Hommes [34], Goeree and Hommes [35], Branch [36], and Chiarella and He [37]) proposed that agents transform between different available prediction rules, relying on certain fitness measures. Risk aversion and time-varying second moment elements are incorporated into the basic cobweb model by Boussard [38] and Chiarella et al. [39]. Dieci and Westerhoff [30] discussed the steady-state properties and the dynamic behavior of a generalization of the classical cobweb model with market interactions. More recently, researchers discussed the price fluctuations in different economics field by extending the model in a different direction [31, 40–43].

In a word, whether the supply and demand curves are linear or nonlinear, the stability of credit price cannot be ensured in the free market. Even though the credit price is stable, according to Yang and Wang [10], credit price is not unique when an all-or-nothing flow pattern occurs in the network. As is well known, the unstable credit price will lead to the credits market fluctuations. Speculators take chance to profit from hoarding credits and selling them for a high price. In this case, tradable credits scheme may lose its sense. Therefore, it is necessary to form a stable or uniform credit price. Considering this, in Section 3 we try to explore a price-based tradable credits scheme from the angle of public pricing.

3. A Price-Based Tradable Credits Scheme

Suppose there exists a network $G = (N, A)$. N denotes the set of nodes in the network and a set A of direct links. W represents the set of all OD pairs and a set R_w of routes between an OD pair $w \in W$ in the network. The OD demand between w is represented by d_w, $w \in W$, and v_a denotes the flow on link $a \in A$.

3.1. Credits Distribution. The government is the direct "producer," and they cannot gain revenue from credits. Credits are uniformly distributed to every traveler for free daily. K is the total number of credits distributed by the government. D represents the total OD demands in the network. k is the number of credits of every traveler in a uniform distribution scheme, which can be written as follows:

$$k = \frac{K}{D} \tag{9}$$

3.2. Credits Charge and Trading Credits. Credits charge is link-specific, which is denoted by κ_a, $a \in A$. Suppose credits charge is nonnegative.

Travelers trade with each other on an online platform (such as Alibaba and Amazon) in a fixed price which is determined by the government. The platform will display the total trading volumes of the system and the remaining quantity of every user in real time. So, for every traveler, the platform is transparent and the information is complete. The government supervises the running of the platform. Suppose the transaction is low enough that it can be ignored. From the economic point of view, some travelers play the "two-faced" role, i.e., "producer" or "consumer," and they will swing between the two roles with the changing of the credit price. The government is neither a seller nor a purchaser, but a price maker. The government cannot reap revenue from the tradable market but supervises the running of the market.

3.3. Credit Pricing. Tradable credits scheme serves every traveler in the city. It has very strong social attributes. Tradable credit is actually a kind of public goods which are produced and distributed by the government. Therefore, the government could implement public pricing on tradable credit scheme. Public pricing is common in public service field, such as energy, water supply, transportation (rail way and urban public transport) and so on. The research of public pricing can be traced back to 1870s. In 1873, the Railway and Canal Commission discussed the optimal public pricing problem. They thought that marginal cost pricing is the optimal pricing rule. In 30s and 40s of 20th century, marginal cost pricing had become the dominant principle in public pricing field. The typical researcher is Hotelling [44]. He proposed that public goods and service's price should be marginal cost. In common with any other theoretical principle, the principle of marginal cost pricing is not in practice to be followed absolutely and at all events but is a principle that is to be followed insofar as this is compatible with other desirable objectives and from which deviations of greater or lesser magnitude are to be desired when conflicting

objectives are considered [45]. Therefore, in this paper, we will implement marginal cost pricing on tradable credits scheme.

3.3.1. The Sense Analysis of Marginal Cost Pricing on Tradable Credits. In order to obtain benefits from credits, some travelers will become "producers" by giving up the original routes and choosing the routes with less credits charge. Obviously, credits' supply quantity will rise when the credit price increases. On the other hand, in order to satisfy the travel demand, some travelers will become "consumers" by purchasing credits from "producers." On the contrary, demand of credits will fall with the rising of the credit price. Figure 4 illustrates the supply and demand. In Figure 4, the horizontal axis denotes the supply of credits and the ordinate represents the credit price. The shape of supply curve in Figure 4 is explainable. With the increasing of credit price, more people will transfer to the routes with less credits charge, which leads to these routes' congestion. According to the time cost function (for example, the BPR (Bureau of Public Road)), the growth of time cost is nonlinear, which confirms the supply curve in Figure 4.

In Figure 4, the benefits of "consumers" can be denoted by the area under the demand curve while the total cost can be represented by the area under supply curve. If the supply of credits is X_1, the social welfare equals to the difference between the benefits of "consumers" and the social cost (i.e., the area of ACDB). This is because under the tradable credits scheme it is revenue-neutral for the government, and the benefits of "producers" offset the expenditure of "consumers". It is easy to find that the area of ACDB is not the maximum in Figure 4. If the supply of credits is X_2, excessive credits are supplied. The social welfare is equal to the difference between the areas of AHB and EHF. It is also not the maximum. Therefore, we have reason to believe that the social welfare will reach maximum at point H.

It can also be verified from the marginal point of view. According to the general market-based rules, social welfare will be the maximum when the cost of marginal product equals to the benefits obtained by consuming the product. In Figure 4, when the supply quantity is X_1, "producers" supply an additional unit credit and the price (p_1) that "consumers" want to pay will be greater than the production cost (p_2), which means the supply of credits cannot satisfy the needs of "consumers." Credit price will rise, and more travelers will become "producers." However, when the supply quantity is X_2, "producers" supply an additional unit credit and the price (p_4) that "consumers" want to pay will be lower than the production cost (p_3), which shows that supply exceeds demand. Meanwhile, with the increasing of supply, more travelers will transfer to the routes with less credits charge, which leads to congestion. The cost of "producers" will increase accordingly. Credit price will fall. Only when the supply quantity is X_H does the supply meet the demand and the cost of marginal credit equals the benefits obtained by consuming the credit.

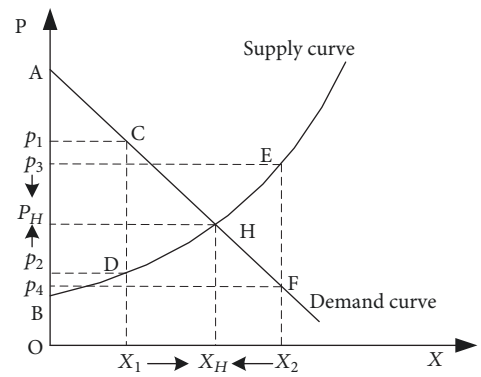

Figure 4: Marginal cost pricing for credit.

3.3.2. Marginal Cost Pricing with Mathematical Method. Based on Figure 4, the rationality of marginal pricing can also be demonstrated by the mathematical method. In order to have a better analysis on marginal cost pricing, the following basic assumptions are made in this paper:

(1) Suppose the total travel demand is fixed in a period.

(2) Suppose all travelers are homogenous.

(3) The transaction fee is such that it is low enough to be negligible.

(4) All taxpayers (travelers) are eligible to receive free credits.

(i) Definition of "Producer" with a Tradable Credit Scheme. If a traveler has redundant credits when he/she finishes the trip, we call the traveler the "producer," and the redundant credits are his/her supply quantity (output). It can be represented by the following inequality:

$$k - \sum_{a \in A} \kappa_a \delta_{a,r} \geq 0, \quad r \in R_w, \ w \in W \tag{10}$$

where inequality (10) denotes the total credits charge on route r is less than or equal to the amount of initial distribution. The routes which meet the inequality (10) can be denoted by a set R'_w. $\delta_{a,r}$ is 1 if route r uses link a and 0 otherwise. Then the total supply of credits on the network can be written as follows:

$$S = \sum_{w \in W} \sum_{r \in R_w{}'} \left(k - \left(\sum_{a \in A} \kappa_a \delta_{a,r} \right) \right) f_r{}' \tag{11}$$

where f'_r denotes the equilibrium flow on route r after a tradable credits scheme is implemented.

(ii) Definition of "Consumer" with a Tradable Credit Scheme. Similarly, if the amount of initial distribution is less than the total charge on route r, travelers on this route need to purchase credits to finish their trips. We call these travelers "consumers." The demand for credits of a "consumer" on route r can be represented as

$$\sum_{a \in A} \kappa_a \delta_{a,r} - k \geq 0, \quad r \in R_w, \ w \in W \tag{12}$$

The total demand for credits can be written as

$$X = \sum_{w\in W}\sum_{r\in R_w''}\left(\left(\sum_{a\in A}\kappa_a\delta_{a,r}\right)-k\right)f_r' \tag{13}$$

where R_w'' denotes the set of routes which satisfy inequality (11).

According to Yang and Wang [10], the equilibrium credit price is positive only when all the issued credits are consumed. However, under the price-based tradable credits scheme, credits may be residual after trading. The redundant credits can be written as

$$Q = K - \sum_{a\in A}\kappa_a v_a \tag{14}$$

Q denotes the surplus credits.

(iii) Model and Solution. Besides the above analysis, to facilitate the presentation of the essential ideas, the following basic assumptions are made in this paper:

The demand is a function of credits price, which can be written as $X = f(p)$, and its inverse function can be written as follows:

$$p = P(X) \tag{15}$$

where $P(X)$ is continuously differentiable.

Suppose c is the total "production costs" of "producers." Here, "producers" are the government and the travelers with superfluous credits (i.e., sellers of credits). Therefore, production costs include two parts: the government service cost for credits distribution, platform construction and operation, and the "production costs" of sellers. For simplicity, suppose the government service cost is a constant$M$. For the "production costs" of sellers, it mainly contains the increased time cost and fuel consumption that travelers transfer from the original routes to the new routes after a tradable credit scheme is implemented. It is explicable. The network will reach user equilibrium when there is no any traffic management measure. After a tradable credit scheme is implemented, the network will reach new user equilibrium. In this process, some travelers would change their routes to obtain credits benefits, and the price they pay is the increased time cost and fuel consumption. We call the increased time cost and fuel consumption the "production costs" of sellers.

Before the implementation of tradable credits schemes, suppose the route's equilibrium cost is u_w between OD pair w. If some two routes have flows, they exist in the following relationship:

$$u_w = \sum_{a\in A}t_a(v_a)\delta_{a,r_i} = \sum_{a\in A}t_a(v_a)\delta_{a,r_j}, \quad \text{if } f_{r_i}>0,\ f_{r_j}>0 \text{ and } r_i, r_j\in R_w \tag{16}$$

where v_a is the flow on link a and $t_a(v_a)$ is the time cost when the network is equilibrium.

After a tradable credits scheme is implemented, if there is no flow transfer among routes, two routes with different credits charge will satisfy the following inequality:

$$\sum_{a\in A}t_a(v_a)\delta_{a,r_i} + p\sum_{a\in A}\kappa_a\delta_{a,r_i} \geq \sum_{a\in A}t_a(v_a)\delta_{a,r_j} + p\sum_{a\in A}\kappa_a\delta_{a,r_j}, \quad r_i, r_j\in R_w,\ \text{if } i\neq j \tag{17}$$

Therefore, travelers will transfer to the routes with less credits charge. Accordingly, these routes' costs will increase until the network reaches equilibrium. Suppose the equilibrium cost is u_w', and (16) will reappear, which can be written as follows:

$$u_w' = \sum_{a\in A}t_a'(v_a')\delta_{a,r_i} + p\sum_{a\in A}\kappa_a\delta_{a,r_i} = \sum_{a\in A}t_a'(v_a')\delta_{a,r_j} + p\sum_{a\in A}\kappa_a\delta_{a,r_j}, \quad \text{if } f_{r_i}>0,\ f_{r_j}>0 \text{ and } r_i, r_j\in R_w \tag{18}$$

where v_a' and $t_a'(v_a')$ denote the flow and cost on link a, respectively, when the network is equilibrium with a tradable credits scheme.

According to (16), (17), and (18), when the network is equilibrium, the time costs of two routes with different credits charge have the following relationship:

$$\sum_{a\in A}t_a'(v_a')\delta_{a,r_j} \geq \sum_{a\in A}t_a'(v_a')\delta_{a,r_i}, \quad \text{if } f_{r_i}>0,\ f_{r_j}>0 \text{ and } r_i\in R_w',\ r_j\in R_w,\ r_i\neq r_j \tag{19}$$

We call these travelers who transfer to the routes with higher time cost the "producers" and call the time cost difference between before and after the implementation of a tradable credits scheme the "production costs." It can be written as follows:

$$\Delta t_r = \sum_{a\in A}t_a'(v_a')\delta_{a,r_j} - \sum_{a\in A}t_a(v_a)\delta_{a,r_j}, \quad r_j\in R_w' \tag{20}$$

Therefore, the total "production costs" can be written as follows:

$$\Delta T = \alpha\sum_{r\in R_w'}\Delta t_r f_r' \tag{21}$$

where α denotes the value of time. The link flow and path flow have the following relationship:

$$v_a = \sum_{r\in R_w} f_r\cdot\delta_{a,r}, \quad \forall a\in A \tag{22}$$

According to (13) and (15), the demand and "production costs" are both functions of path flow. Combined with (22), "production costs" can be represented by the demand.

$$\Delta T = h(X) \tag{23}$$

where $h(X)$ is continuously differentiable.

Therefore, the total "production costs" can be written as

$$c(X) = M + \Delta T = M + h(X) \tag{24}$$

In (24), it includes the "production costs" and the government service cost.

Suppose the government aims to maximize the social welfare which is equal to the difference between the social benefits and the social cost. Under a tradable credits scheme, the government's revenue is neutral. Travelers' benefits include two parts: one is the benefits of "consumers"; the other is the benefits of "producers." So, the social benefits can be written as

$$b_s = \int_0^X p(x)\,dx - p \cdot Q \tag{25}$$

where the first term on the right hand side is the benefits of "consumers" and the second term is the loss of "producers" due to excessive credits production.

Based on (24) and (25), social welfare can be represented as follows:

$$SW = \int_0^X p(x)\,dx - p(X) \cdot Q - M - h(X) \tag{26}$$

According to the above assumptions, social welfare is the maximum when it meets the following conditions:

$$\frac{dSW}{dX} = p(X) - h'(X) - p'(X) \cdot Q = 0 \tag{27}$$

According to (27), the credit price can be written as follows:

$$p(X) = h'(X) + p'(X) \cdot Q \tag{28}$$

It is worth noting that, in (28), the credit price includes two parts. $h'(X)$ is the marginal costs of "producer". $p'(X) \cdot Q$ is the marginal loss of "producers" due to excessive credits production

4. A Simple Numerical Example

In order to intuitively and clearly show the stability of credit price and marginal cost pricing for readers, in this section, we give a mini and simple network instead of a large and complex network.

As shown in Figure 5, there are three links connecting a single O-D pair. The travel demand is 10. A total amount of 30 credits is uniformly distributed to all travelers, and the credit charge for using link 1, link 2, and link 3 is 4, 2, and 1 respectively, thus, $K = 30$, $\kappa = (4, 2, 1)^{\mathrm{T}}$. Let

$$\begin{aligned} t_1(v_1) &= v_1 + 3, \\ t_2(v_2) &= 2v_2 + 1, \\ t_3(v_3) &= v_3 + 6, \\ k &= 3 \end{aligned} \tag{29}$$

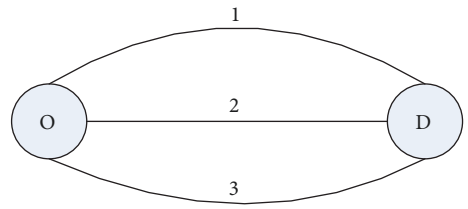

Figure 5: A simple network.

4.1. Stability Analysis of the Credit Price. After the tradable credits scheme being implemented, the user equilibrium is given as follows:

$$\begin{aligned} v_1' + 3 + 4p &= 2v_2' + 1 + 2p, \\ v_1' + 3 + 4p &= v_3' + 6 + p, \\ v_1' + v_2' + v_3' &= 10. \end{aligned} \tag{30}$$

Then

$$\begin{aligned} v_1' &= \frac{24 - 8p}{5}, \\ v_2' &= \frac{17 + p}{5}, \\ v_3' &= \frac{9 + 7p}{5} \end{aligned} \tag{31}$$

(i) Demand Function. According to the previous analysis, travelers on route 1 are the "consumers." The demand function can be written as follows:

$$X_t = (\kappa_1 - k)\, v_1' \tag{32}$$

Introducing $k = 3$ and $\kappa_1 = 4$ into (32), we get

$$X_t = \frac{24 - 8p_{t-1}}{5} \tag{33}$$

(ii) Supply Function. Travelers on routes 2 and 3 are the "producers." The quantity of production can be written as

$$S_t = (k - \kappa_2) \cdot v_2' + (k - \kappa_3) \cdot v_3' \tag{34}$$

Introducing $k = 3$, $\kappa_2 = 2$, and $\kappa_3 = 1$ into (34) and letting $X_t = S_t$, we get

$$p_t = \frac{X_t}{3} - \frac{7}{3} \tag{35}$$

(iii) Stability Analysis of Credit Price. According to (3) and (4), we know that $a = 24/5$, $b = -8/5$, $c = 7$, and $d = -3$. Based on the analysis of linear Cobweb models in Section 2.2, credit price will change with t's changing and its track is divergent when $b > d$. Obviously, here $b > d$, and credit price is divergent. Therefore, the credit price is unstable and the market-based pricing will lose efficiency in this example.

4.2. Marginal Cost Pricing

(i) Network Equilibrium without a Tradable Credits Scheme. The user equilibrium is given as follows:

$$\begin{aligned} v_1 + 3 &= 2v_2 + 1, \\ v_1 + 3 &= v_3 + 6, \\ v_1 + v_2 + v_3 &= 10. \end{aligned} \tag{36}$$

The unique solution is $v_1 = 24/5$, $v_2 = 17/5$, $v_3 = 9/5$, and $t_1 = t_2 = t_3 = 39/5$.

(ii) Marginal Cost Pricing with a Tradable Credits Scheme

(a) "Production Cost." Based on the above analysis, according to the (21), the sellers' "production costs" can be written as follows:

$$\Delta T = t_2'\left(v_2'\right) - t_2\left(v_2\right) + t_3'\left(v_3'\right) - t_3\left(v_3\right) \tag{37}$$

Combining with (32) and introducing v_1', v_2', v_3', t_2, and t_3 into (37), the following is obtained:

$$h(X) = \frac{115}{64}X^2 - \frac{119}{8}X + 30 \tag{38}$$

Combining (14) and (32), we obtain

$$p \cdot Q = \frac{115}{64}X^2 - \frac{149}{8}X \tag{39}$$

(b) Social Welfare and Credit Price. According to (26), social welfare can be written as follows:

$$SW = \int_0^X \left(3 - \frac{5}{8}x\right) dx - M - \frac{250}{64}X^2 + \frac{268}{8}X + \frac{423}{5} \tag{40}$$

Then

$$\frac{dSW}{dX} = 3 - \frac{5}{8}X - \frac{115}{16}X + \frac{268}{8} = 0 \tag{41}$$

$X \approx 4.672$, $p \approx 0.08$, $v_1 = 4.672$, $v_2 = 3.416$, $v_3 = 1.912$, and $Q = 2.568$. In this case, social welfare reaches the "maximum" when the credit price is 0.08.

What I need to say here is that the actual traffic network is complex. In the complex traffic network, the user equilibrium before and after tradable credits being implemented can be represented by the Beckmann model, which can be solved by the Frank-wolf algorithm. According to the definition on marginal cost, it is the derivative of production cost. So the marginal cost pricing could be easily obtained by deriving (26) with respect to independent variable.

5. Discussion and Future Research

This paper explores the tradable credits scheme from the angle of public pricing. Firstly, based on the Cobweb model in economics, we analyze the stability of market-based credit price. It is found that credit price is not always convergent in the credits market environment. In the linear Cobweb models, the possible range of dynamic outcomes is basically restricted to three types of evolution around the equilibrium price. Credit price is stable if the slope of supply curve is greater than the absolute value of the demand-curve slope. Credit price will swing like a pendulum between two values when the absolute value of the supply-curve slope is equal to that of the demand-curve slope, and it is divergent when the supply-curve slope is less than the absolute value of the demand-curve slope. When the demand and supply schedules are non-linear, the possible range of dynamic outcomes would show more complex dynamic behaviors, such as cycle movements and chaos.

Therefore, considering the inefficiency of market-based tradable credits schemes in practice, and in its role as a public good, we try to explore a price-based tradable credits scheme, i.e., a public pricing mechanism-marginal cost pricing scheme. Like quantity-based tradable credits scheme, the price-based tradable credits scheme enables participants to financially benefit from the system and ensure that money circulates among participants instead of flowing to regulating authorities. It offers revenue-neutral incentives for mobility and environmental quality. Unlike quantity-based tradable credits scheme, in application, as well as congestion pricing, it is more practical and efficient. More importantly, the uniform credit price prevents speculators profiting from hoarding credits and selling them for a high price in busy periods. Unlike quantity-based tradable credits scheme, the government will interfere the credits market by setting the credit price in the price-based tradable credits scheme. Actually it is understandable and has certain rationality. This is because the tradable credits scheme has some similarities with some public service (e.g., the public traffic service). They all have some nonprofit characters. Most importantly, the government is only a price maker, not a seller or buyer; i.e., money circulates among participants instead of flowing to the government. It is revenue-neutral for the government.

Beyond this preliminary research, there are some potentially interesting topics for further study. Under the price-based tradable credits schemes, in real-size network, the number of routes may be huge, which will waste a lot of time if we list them one by one. In view of this, in future research, we could eliminate the invalid routes based on traveler habits and behavior. The specific comparison with quantity-based tradable credits schemes also need to be studied in future. The analysis on social welfare maximization and minimum system cost by jointing the optimal price of credits and the optimal credits charge scheme. Furthermore, the current modeling of credit schemes applies only to homogeneous users but it could also be extended to the heterogeneous case. Besides the above interesting topics, the multimodal, elastic demand, and stochastic demand are also should be explored in future.

It is worth noting that although marginal cost pricing gives the optimal solutions for public service corporations under the ideal environment, it has serious weaknesses. As Coase [46] said, "It did not take into account the stimulus

to correct forecasting of having a subsequent market test whether consumers were willing to pay the total cost; it ignored the probable effects on the administrative structure, with state enterprise superceding private enterprise and centralized operations superceding decentralized operations; it failed to take into account the misallocation of resources resulting from the additional taxation necessitated by the subsidies." Therefore, in future research, we can analyze the public pricing from two angles. One is the fairness targeted public pricing theory. The other one is the efficiency- oriented public pricing theory.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper. The [DATA TYPE] data used to support the findings of this study are included within the article.

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