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Research Article

Adaptive Output Feedback Stabilization of Random Nonlinear Systems with Unmodeled Dynamics Driven by Colored Noise

Mingyue Cui

School of Mathematics and Informational Science, Yantai University, Yantai, Shandong Province 264005, China

Correspondence should be addressed to Mingyue Cui; mingyuecmy@126.com

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This paper focuses on the problem of adaptive output feedback stabilization for random nonlinear systems with unmodeled dynamics and uncertain nonlinear functions driven by colored noise. Under the assumption of unmodeled dynamics having enough stability margin, an adaptive output feedback stabilization controller is designed based on a reduced-order observer such that the state of the closed-loop system has an asymptotic gain in the 2-th moment (AG-2-M) and the mean square of the output can be made arbitrarily small by tuning parameters. A simulation example is used to illustrate the effectiveness of the control scheme.

1. Introduction

During the past decades, the control problem for systems with unmodeled dynamics mainly caused by simplifications in modeling process has received much attention. To deal with the unmodeled dynamics, an available dynamic signal is constructed to bound the unmodeled dynamics by exploiting prior information on the system in [1]. By the aid of small-gain technique in [2, 3], the unmodeled dynamics were treated by a worst-case design on the basis of small-gain arguments in [4]. By introducing input-to-state stability for time-varying control systems, sufficient conditions for global stabilization of triangular systems are given in [5]. For time-varying control systems, various equivalent characterizations of the nonuniform in time input-to-state stability (ISS) property are established in [6]. By using a suitable extension of the small-gain theorem, a uniform input-to-state stabilization controller is given in [7]. For MIMO nonlinear systems, a robust adaptive observer was given in [8]. By introducing K-filters and dynamic signal, [9] designed an adaptive output feedback controller such that the closed-loop control system is semiglobally uniformly ultimately bounded. By combining small-gain theorem and changing supply function techniques, [10] proposed a robust adaptive output feedback controller. However, most of the existing references mainly focus on the deterministic case.

With the development of stochastic theory [11–15], many researchers pay great attention to the control problem of stochastic differential equations (SDEs) in the presence of unmodeled dynamics. Under the assumption of unmodeled dynamics having enough stability margin, [16] presented an output feedback controller based on minimal-order observer. In [17], a unifying solution to stochastic adaptive output feedback stabilization was presented by introducing dynamic signal and changing supply function technique. Also, some other control strategies such as adaptive neural networks control [18–20], adaptive fuzzy control [21, 22], and adaptive tracking control [23] were proposed.

Since the stochastic disturbance is more reasonably described as colored noise than white noise in practical engineering, the models described by SDEs may not suit very well. In [24], a theoretical framework on stability of random nonlinear systems where stochastic disturbance is colored noise and applications to controller design are given. For a class of random nonlinear systems with unmodeled dynamics, under the assumption of unmodeled dynamics having enough stability margin, [25] designed a feedback stabilization controller by backstepping method. Until now, to our knowledge, there are no many results on adaptive output feedback control for random nonlinear systems with unmodeled dynamics and uncertain nonlinear functions in the literature. In this paper, motivated by [16, 17, 24, 25], the purpose of this paper is to solve this problem under some milder assumptions. The main work consists of the following aspects.

(i) Because it is difficult to deal with Hessian terms caused by Wiener process, the stochastic disturbance is regarded as colored noise whose second-order moment is bounded in this paper. Distinguished from the existing stochastic analysis method, the method of ordinary differential equations is used to analyze the stability of the closed-loop system.

(ii) For easier-to-implement and more reliable practical purposes, a reduced-order observer and 1-dimension adaptive law are introduced, which lead to a simple controller. The state of the closed-loop system has an asymptotic gain in the 2-th moment (AG-2-M) and the mean square of the output can be regulated to an arbitrarily small neighborhood of the origin.

The paper is organized as follows: in Section 2, the mathematical preparation is given and the problem is formulated. Section 3 gives the observer-based backstepping adaptive controller design procedure. The main result is presented in Section 4. A simulation example is included in Section 5 to illustrate effectiveness of the proposed design method. Section 6 concludes the paper.

Notations: the following notations are used throughout the paper. For a vector x, x^T denotes its transpose; for a matrix X, $\lambda_{\min}(X)$ and $\lambda_{\max}(X)$ denote its smallest and largest eigenvalue, respectively; $|\cdot|$ denotes the usual Euclidean norm of "$\cdot$"; E denotes the mathematical expectation; $\mathbb{R}_+$ denotes the set of all nonnegative real numbers; $\mathbb{R}^n$ denotes the real n-dimensional space; $\mathbb{R}^{n\times m}$ denotes the real $n \times m$ matrix space; C^i denotes the set of all functions with continuous i-th partial derivative. For simplicity, sometimes the arguments of functions are dropped.

2. Mathematical Preliminaries and Problem Formulation

2.1. Mathematical Preliminaries. Consider the following random nonlinear affine system:

$$\begin{aligned} \dot{x} &= f(x,t) + g(x,t)\,\xi(t), \\ x(t_0) &= x_0, \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state of system, $\xi(t) \in \mathbb{R}^m$ is a stochastic process, and the underlying complete probability space is taken to be the quartet $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$ with a filtration $\mathcal{F}_t$ satisfying the usual condition (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all P-null sets). Both functions $f : \mathbb{R}^n \times \mathbb{R}_+ \longrightarrow \mathbb{R}^n$ and $g : \mathbb{R}^n \times \mathbb{R}_+ \longrightarrow \mathbb{R}^{n\times m}$ are locally Lipschitz in x piecewise continuous in t; i.e., for each $R > 0$, there exists a constant L_R such that

$$\begin{aligned} &\left|f(x_1,t) - f(x_2,t)\right| + \left|g(x_1,t) - g(x_2,t)\right| \\ &\quad \le L_R\left|x_2 - x_1\right| \end{aligned} \tag{2}$$

for any $t \in \mathbb{R}_+$ and $x_1, x_2 \in U_R = \{x : |x| \le R\}$, $x_1 \neq x_2$. Moreover, $f(0,t)$ and $g(0,t)$ are bounded a.s. Process ξ is a $\mathcal{F}_t$-adapted process and piecewise continuous, and there exists a positive constant K such that

$$\sup_{t_0 \le s \le t} E\left|\xi(s)\right|^2 \le K, \quad \forall t \ge t_0. \tag{3}$$

The following definition, criterion, and inequality are represented now for the stability analysis.

Definition 1 ([24]). The state of system (1) has an asymptotic gain in the m-th moment (AG-m-M) if there exists a function $\gamma(\cdot)$ of class $\mathcal{K}$ such that, for any $x_0 \in R^n$,

$$\lim_{t\longrightarrow\infty} E\left|x(t)\right|^m \le \gamma\left(\delta_K\right), \tag{4}$$

where $\delta_K = \delta_1 K + \delta_2$ with $\delta_1 > 0, \delta_2 \ge 0$.

Lemma 2. *For system (1) with conditions (3), if there exist parameters $a_1 > 0$, $a_2 > 0$, $c > 0$, $\delta_1 > 0$, $\delta_2 \ge 0$ and a function $V \in C^1$ such that*

$$a_1 |x|^m \le V(x) \le a_2 |x|^m, \tag{5}$$

$$\dot{V}(x(t)) \le -cV(x(t)) + \delta_1 \left|\xi(t)\right|^2 + \delta_2, \tag{6}$$

then system (1) has a unique global solution, and the state of system has an AG-m-M.

Proof. Following the same lines as the proof of [24, Theorem 3], the result can be obtained. □

Lemma 3 ([26]). *Consider the continuous functions $k(t)$, $h(t)$, and they are integrable over every finite interval. If a continuous function $y(t)$ satisfies the inequality*

$$\dot{y}(t) \le k(t)\,y(t) + h(t), \quad \forall t \ge t_0, \tag{7}$$

then

$$y(t) \le y(t_0)\,e^{\int_{t_0}^{t} k(s)ds} + \int_{t_0}^{t} e^{\int_{s}^{t} k(u)du} h(s)\,ds, \quad \forall t \ge t_0. \tag{8}$$

2.2. Problem Formulation. Consider the following system

$$\begin{aligned} \dot{\chi} &= q(\chi, x_1) + p(\chi, x_1)\,\xi_0, \\ \dot{x}_1 &= x_2 + \phi_1(x,\chi) + \Omega_1^T(x_1)\,\xi, \\ &\vdots \\ \dot{x}_i &= x_{i+1} + \phi_i(x,\chi) + \Omega_i^T(x_1)\,\xi, \\ &\vdots \\ \dot{x}_n &= u + \phi_n(x,\chi) + \Omega_n^T(x_1)\,\xi, \\ y &= x_1, \end{aligned} \tag{9}$$

where $x = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$, $\chi \in \mathbb{R}^{n_0}$, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the state, the unmodeled dynamics, the input and the output of system, respectively. $\xi_0 \in \mathbb{R}^{m_0}$ and $\xi \in \mathbb{R}^m$ are $\mathcal{F}_t$-adapted stochastic processes. In this paper, it is assumed that only the output y can be measured and uncertain functions q, p, ϕ_i, and Ω_i are smooth.

The objective in this paper is to design an adaptive output feedback controller such that the state of the closed-loop system has an AG-m-M, and the mean square of the output can be regulated to an arbitrarily small neighborhood of the origin.

Throughout the paper, the following assumptions are made on system (9).

Assumption 4. $\xi_0 \in \mathbb{R}^{m_0}$ and $\xi \in \mathbb{R}^m$ are $\mathcal{F}_t$-adapted processes and piecewise continuous, and there exists a positive constant K such that

$$\begin{gathered}\sup_{t_0\le s\le t} E\left|\xi_0(s)\right|^2 \le K,\\ \sup_{t_0\le s\le t} E\left|\xi(s)\right|^2 \le K,\\ \sup_{t_0\le s\le t} E\left|\xi(s)\right|^4 \le K,\\ \forall t \ge t_0.\end{gathered} \tag{10}$$

Assumption 5. For the χ-system, there exists a function $V_0 \in C^1$, a $\mathcal{K}_\infty$-function $\delta(\cdot)$, a smooth function $\delta_0(\cdot) : \delta_0(0) = 0$, and constants $\underline{a} > 0$, $\overline{a} > 0$, $c_0 > 0$, $\gamma_0 > 0$, $d_0 > 0$ such that

$$\begin{gathered}\underline{a}\left|\chi\right|^2 \le V_0(\chi) \le \overline{a}\left|\chi\right|^2,\\ \frac{\partial V_0}{\partial\chi}q(\chi,x_1) + \frac{1}{4d_0}\left\|\frac{\partial V_0}{\partial\chi}p(\chi,x_1)\right\|^2\\ \le -c_0V_0(\chi) - \gamma_0\delta(|\chi|) + y\delta_0(y).\end{gathered} \tag{11}$$

Assumption 6. For each $1 \le i \le n$, there exits an unknown positive constant θ_i such that

$$\begin{gathered}\left|\phi_i(x,\chi)\right| \le \theta_i\left(\delta_i(y) + \gamma_i\pi(|\chi|)\right),\\ \left|\Omega_i(x_1)\right| \le \theta_i\psi_i(y),\end{gathered} \tag{12}$$

where δ_i, ψ_i are known nonnegative smooth functions and $\delta_i(0) = \psi_i(0) = 0$ are assumed.

Assumption 7. There exists a constant $\gamma > 0$ such that

$$\delta(\mu) \ge \gamma\pi^2(\mu), \quad \forall\mu \in [0,\infty). \tag{13}$$

Remark 8. Assumption 5 describes the dynamical behavior of the unobservable state χ. It has some stability margin with respect to the unmodeled dynamics. The term $y\delta_0(y)$ imposes restrictions on the influence of state x on the stability of the unobservable state χ.

Remark 9. In Assumption 6, using the identity $f(x)-f(x_0) = (\int_0^1(df/d\lambda)|_{\lambda=\alpha(x-x_0)+x_0}d\alpha)(x-x_0) = \overline{f}(x,x_0)(x-x_0)$ to $\delta_i(y)$, there exist smooth functions $\overline{\delta}_i$ such that

$$\delta_i(y) = y\overline{\delta}_i(y), \tag{14}$$

which will be frequently used in the subsequent sections.

Remark 10. Assumption 7 depicts the connection between the stability margin of the unobservable state χ and the unmodeled dynamics.

3. Controller Design

3.1. Reduced-Order Observer Design. To counteract the unavailable state x_i $(i = 1,2,\ldots,n)$, a reduced-order partial-state observer is introduced as follows:

$$\begin{gathered}\dot{\hat{x}}_i = \hat{x}_{i+1} + k_{i+1}y - k_i\left(\hat{x}_1 + k_1y\right),\\ i = 1,\ldots,n-2,\\ \dot{\hat{x}}_{n-1} = u - k_{n-1}\left(\hat{x}_1 + k_1y\right),\end{gathered} \tag{15}$$

where $k = (k_1,k_2,\ldots,k_{n-1})^T$ is chosen such that

$$A_0 = \begin{bmatrix} -k & \begin{matrix} I_{n-2} \\ 0\cdots0 \end{matrix}\end{bmatrix} \tag{16}$$

is asymptotically stable. For each $i = 1,2,\ldots,n-1$, denote the observer error as

$$\varepsilon_i = \frac{1}{\theta^*}\left(x_{i+1} - \hat{x}_i - k_iy\right), \tag{17}$$

where $\theta^* = \max\{1,\theta_1,\ldots,\theta_n\}$. From (9), (15), and (17), one has

$$\begin{aligned}\dot{\varepsilon}_i &= \frac{1}{\theta^*}\Big(x_{i+2} + \phi_{i+1} + \Omega_{i+1}^T\xi\\ &\quad - \left(\hat{x}_{i+1} + k_{i+1}y - k_i\left(\hat{x}_1 + k_1y\right)\right)\\ &\quad - k_i\left(x_2 + \phi_1 + \Omega_1^T\xi\right)\Big) = \left(\varepsilon_{i+1} - k_i\varepsilon_1\right) + \frac{1}{\theta^*}\big(\phi_{i+1}\\ &\quad - k_i\phi_1\big) + \frac{1}{\theta^*}\left(\Omega_{i+1} - k_i\Omega_1\right)^T\xi\end{aligned} \tag{18}$$

with $x_{n+1} = u$, $\hat{x}_n = u$, and $k_n = 0$, whose compact from is

$$\dot{\varepsilon} = A_0\varepsilon + \Delta(y,\chi) + \Omega^T(y)\xi, \tag{19}$$

where $\Delta(y,\chi) = (1/\theta^*)(\phi_2 - k_1\phi_1,\ldots,\phi_n - k_{n-1}\phi_1)^T$ and $\Omega(y) = (1/\theta^*)(\Omega_2 - k_1\Omega_1,\ldots,\Omega_n - k_{n-1}\Omega_1)$.

From Assumption 4, we have

$$\begin{aligned}\left|\Delta(y,\chi)\right|^2 &= \sum_{i=1}^{n-1}\left(\frac{1}{\theta^*}\left(\phi_{i+1} - k_i\phi_1\right)\right)^2 \le \sum_{i=1}^{n-1}\big(\delta_{i+1}(y)\\ &\quad + k_i\delta_1(y) + \gamma_{i+1}\pi(|\chi|) + k_i\gamma_1\pi(|\chi|)\big)^2\\ &\le \sum_{i=1}^{n-1}2\left(\delta_{i+1}(y) + k_i\delta_1(y)\right)^2 + \sum_{i=1}^{n-1}2\left(\gamma_{i+1} + k_i\gamma_1\right)^2\\ &\quad \cdot\pi^2(|\chi|) \le y^2\Phi_1(y) + a\delta(\chi),\end{aligned} \tag{20}$$

where $\Phi_1(y) = \sum_{i=1}^{n-1} 2(\overline{\delta}_{i+1}(y) + k_i\overline{\delta}_1(y))^2$ and $a = \sum_{i=1}^{n-1}(2/\gamma)(\gamma_{i+1} + k_i\gamma_1)^2$.

$$\begin{aligned} |\Omega(y)|^2 &= \sum_{i=1}^{n-1}\left|\frac{1}{\theta^*}(\Omega_{i+1} - k_i\Omega_1)\right|^2 \\ &\le \sum_{i=1}^{n-1}(\psi_{i+1}(y) + k_i\psi_1(y))^2 = y^2\Psi_1(y), \end{aligned} \tag{21}$$

where $\Psi_1(y) = \sum_{i=1}^{n-1}(\overline{\psi}_{i+1}(y) + k_i\overline{\psi}_1(y))^2$. Since A_0 is stable, there exists a symmetric positive definite matrix P such that $PA_0 + A_0^TP = -2I_{n-1}$. Along the solution of (19), differentiating the quadratic function $V_\varepsilon = (1/2)\varepsilon^TP\varepsilon$ yields

$$\begin{aligned} \dot{V}_\varepsilon &= -\varepsilon^T\varepsilon + \varepsilon^TP\left(\Delta(y,\chi) + \Omega^T(y)\xi\right) \\ &\le -\varepsilon^T\varepsilon + \frac{1}{2}\varepsilon^T\varepsilon + \frac{1}{2}|P|^2\left(|\Delta(y,\chi)| + |\Omega(y)||\xi|\right)^2 \\ &\le -\frac{1}{2}\varepsilon^T\varepsilon + |P|^2\Phi_1(y)y^2 + a|P|^2\delta(\chi) \\ &\quad + |P|^2\Psi_1(y)y^2\xi^2 \\ &\le -\frac{1}{2}\varepsilon^T\varepsilon + |P|^2\Phi_1(y)y^2 + a|P|^2\delta(\chi) \\ &\quad + \frac{1}{4d_\varepsilon}|P|^4\Psi_1^2(y)y^4 + d_\varepsilon\xi^4, \end{aligned} \tag{22}$$

where d_ε is a design parameter.

3.2. Adaptive Backstepping Controller Design. From (9) and (17), the derivative of output is represented as

$$\dot{y} = \theta^*\varepsilon_1 + \hat{x}_1 + k_1y + \phi_1(x,\chi) + \Omega_1^T(y)\xi, \tag{23}$$

which, together with (9), (15), and (19), consists of the following system

$$\begin{aligned} \dot{\chi} &= q(\chi,y) + p(\chi,y)\xi_0, \\ \dot{\varepsilon} &= A_0\varepsilon + \Delta(y,\chi) + \Omega^T\xi, \\ \dot{y} &= \hat{x}_1 + \theta^*\varepsilon_1 + k_1y + \phi_1(x,\chi) + \Omega_1^T(y)\xi, \\ \dot{\hat{x}}_i &= \hat{x}_{i+1} + k_{i+1}y - k_i(\hat{x}_1 + k_1y), \\ &\qquad i = 1,\dots,n-2, \\ \dot{\hat{x}}_{n-1} &= u - k_{n-1}(\hat{x}_1 + k_1y). \end{aligned} \tag{24}$$

An adaptive backstepping controller based on the $(y, \hat{x}_1, \dots, \hat{x}_{n-1})$–system of (24) will be developed.

Step 1. Introduce the first two error variables

$$\begin{aligned} z_1 &= y, \\ z_2 &= \hat{x}_1 - \alpha_1, \end{aligned} \tag{25}$$

where α_1 will be given in later. Consider the Lyapunov function candidate

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2r_\theta}\tilde{\theta}^2, \tag{26}$$

where $\tilde{\theta} = \theta - \hat{\theta}$, $\hat{\theta}$ is the estimate of $\theta = \max\{\theta^*, \theta^{*2}\}$. In view of (24) and (25), the derivative of V_1 satisfies

$$\begin{aligned} \dot{V}_1 &\le y\left(z_2 + \alpha_1 + \theta^*\varepsilon_1 + k_1y + \phi_1(x,\chi) + \Omega_1^T(y)\xi\right) \\ &\quad - \frac{1}{r_\theta}\tilde{\theta}\dot{\hat{\theta}}. \end{aligned} \tag{27}$$

Applying Young's inequality, one has

$$y\theta^*\varepsilon_1 \le r_1y^2\theta + \frac{1}{4r_1}\varepsilon_1^2, \tag{28}$$

where $r_1 > 0$ is a design parameter. By Assumption 4, one has

$$\begin{aligned} y\phi_1(x,\chi) &\le |y|\theta_1\left(\delta_1(y) + \gamma_1\pi(|\chi|)\right) \\ &\le \frac{1}{2\epsilon_1}y^2\theta_1^2 + \frac{\epsilon_1}{2}\left(\delta_1(y) + \gamma_1\pi_1(|\chi|)\right)^2 \\ &\le \frac{1}{2\epsilon_1}y^2\theta + \epsilon_1y^2\overline{\delta}_1^2(y) + \epsilon_1\frac{\gamma_1^2}{\gamma}\delta(|\chi|), \end{aligned} \tag{29}$$

$$\begin{aligned} y\Omega_1^T(y)\xi &\le |y|\theta_1\psi_1(y)|\xi| \\ &\le \frac{1}{4d_1}y^2\theta_1\psi_1^2(y) + d_1|\xi|^2 \\ &\le \frac{1}{4d_1}y^2\psi_1^2(y)\theta + d_1|\xi|^2, \end{aligned} \tag{30}$$

where $\epsilon_1 > 0$ and d_1 are design parameters. Substituting (28)-(30) into (27), one has

$$\begin{aligned} \dot{V}_1 &\le yz_2 + y\left(\alpha_1 + k_1y + \epsilon_1y\overline{\delta}_1^2(y) + \omega_1\hat{\theta}\right) \\ &\quad + \epsilon_1\frac{\gamma_1^2}{\gamma}\delta(|\chi|) + d_1|\xi|^2 + \tilde{\theta}\left(\omega_1y - \frac{1}{r_\theta}\dot{\hat{\theta}}\right) \\ &\quad + \frac{1}{4r_1}\varepsilon_1^2, \end{aligned} \tag{31}$$

where $\omega_1 = r_1y + (1/2\epsilon_1)y + (1/4d_1)y\psi_1^2(y)$. The stabilizing function α_1 and the tuning function τ_1 are designed as

$$\begin{aligned} \alpha_1 &= -c_1z_1 - k_1y - \omega_1\hat{\theta} - \beta(y), \\ \tau_1 &= -\sigma\hat{\theta} + \omega_1y, \end{aligned} \tag{32}$$

where $c_1 > 0$ is a design parameter and $\beta(y)$ will be chosen in later. Then

$$\begin{aligned} \dot{V}_1 &\le -c_1z_1^2 + y\left(\epsilon_1y\overline{\delta}_1^2(y) - \beta(y)\right) + z_1z_2 \\ &\quad + \epsilon_1\frac{\gamma_1^2}{\gamma}\delta(|\chi|) + d_1|\xi|^2 + \sigma\tilde{\theta}\hat{\theta} + \tilde{\theta}\left(\tau_1 - \frac{1}{r_\theta}\dot{\hat{\theta}}\right) \\ &\quad + \frac{1}{4r_1}\varepsilon_1^2. \end{aligned} \tag{33}$$

Step $i = 2, \ldots, n$. Introduce the coordinate change

$$z_{i+1} = \hat{x}_i - \alpha_i\left(y, \hat{x}_1, \ldots, \hat{x}_{i-1}, \hat{\theta}\right) \tag{34}$$

with $z_{n+1} = 0$ and $\hat{x}_n = u$. The differential of z_i is given as follows:

$$\begin{aligned}
\dot{z}_i &= \hat{x}_i + k_i y - k_{i-1}\left(\hat{x}_1 + k_1 y\right) \\
&\quad - \frac{\partial \alpha_{i-1}}{\partial y}\left(\hat{x}_1 + \theta^* \varepsilon_1 + k_1 y + \phi_1(x, \chi) + \Omega_1^T(y)\xi\right) \\
&\quad - \sum_{j=1}^{i-2} \frac{\partial \alpha_{i-1}}{\partial \hat{x}_j}\left(\hat{x}_{j+1} + k_{j+1} y - k_j\left(\hat{x}_1 + k_1 y\right)\right) \\
&\quad - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} \\
&= z_{i+1} + \alpha_i + \eta_i \\
&\quad - \frac{\partial \alpha_{i-1}}{\partial y}\left(\theta^* \varepsilon_1 + \phi_1(x, \chi) + \Omega_1^T(y)\xi\right) - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}},
\end{aligned} \tag{35}$$

where $\eta_i = k_i y - k_{i-1}(\hat{x}_1 + k_1 y) - (\partial\alpha_{i-1}/\partial y)(\hat{x}_1 + k_1 y) - \sum_{j=1}^{i-2}(\partial\alpha_{i-1}/\partial\hat{x}_j)(\hat{x}_{j+1} + k_{j+1} y - k_j(\hat{x}_1 + k_1 y))$. Assume that one has designed smooth function α_j, τ_j $(j = 2, \ldots, i-1)$ such that the following inequality holds for $V_{i-1} = V_{i-2} + (1/4)z_{i-1}^2$,

$$\begin{aligned}
\dot{V}_{i-1} &\leq -\sum_{j=1}^{i-1} c_j z_j^2 + y\left(\sum_{j=1}^{i-1} \epsilon_j y \bar{\delta}_1^2(y) - \beta(y)\right) + z_{i-1} z_i \\
&\quad + \sum_{j=1}^{i-1} \epsilon_j \frac{\gamma_1^2}{\gamma}\delta(|\chi|) + \sum_{j=1}^{i-1} d_j |\xi|^2 \\
&\quad + \left(\sum_{j=1}^{i-2} z_{j+1}\frac{\partial \alpha_j}{\partial \hat{\theta}}\right)\left(r_\theta \tau_{i-1} - \dot{\hat{\theta}}\right) + \sigma\tilde{\theta}\hat{\theta} \\
&\quad + \tilde{\theta}\left(\tau_{i-1} - \frac{1}{r_\theta}\dot{\hat{\theta}}\right) + \sum_{j=1}^{i-1}\frac{1}{4r_j}\varepsilon_1^2,
\end{aligned} \tag{36}$$

where $\epsilon_j > 0$, $d_j > 0$, $r_j > 0$ $(2 \leq j \leq i-1)$ are design parameter. In the following, we will prove that (36) holds for the i-th Lyapunov function candidate

$$V_i = V_{i-1} + \frac{1}{4}z_i^2. \tag{37}$$

The derivative of V_i satisfies

$$\begin{aligned}
\dot{V}_i &\leq -\sum_{j=1}^{i-1} c_j z_j^2 + y\left(\sum_{j=1}^{i-1} \epsilon_j y \bar{\delta}_1^2(y) - \beta(y)\right) + z_{i-1} z_i \\
&\quad + \sum_{j=1}^{i-1} \epsilon_j \frac{\gamma_1^2}{\gamma}\delta(|\chi|) + \sum_{j=1}^{i-1} d_j |\xi|^2 + \left(\sum_{j=1}^{i-2} z_{j+1}\frac{\partial \alpha_j}{\partial \hat{\theta}}\right) \\
&\quad \cdot \left(r_\theta \tau_{i-1} - \dot{\hat{\theta}}\right) + \sigma\tilde{\theta}\hat{\theta} + \tilde{\theta}\left(\tau_{i-1} - \frac{1}{r_\theta}\dot{\hat{\theta}}\right) + \sum_{j=1}^{i-1}\frac{1}{4r_j}\varepsilon_1^2 \\
&\quad + z_i\Bigg(z_{i+1} + \alpha_i + \eta_i \\
&\quad - \frac{\partial \alpha_{i-1}}{\alpha y}\left(\theta^* \varepsilon_1 + \phi_1(x, \chi) + \Omega_1^T(y)\xi\right) - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}\Bigg).
\end{aligned} \tag{38}$$

Applying Young's inequality, one has

$$z_i \frac{\partial \alpha_{i-1}}{\alpha y}\theta^* \varepsilon_1 \leq r_i z_i^2 \left|\frac{\partial \alpha_{i-1}}{\alpha y}\right|^2 \theta + \frac{1}{4r_i}\varepsilon_1^2, \tag{39}$$

where $r_i > 0$ is a design parameter. By Assumption 4, one has

$$\begin{aligned}
z_i \frac{\partial \alpha_{i-1}}{\alpha y}\phi_1(x, \chi) &\leq |z_i|\left|\frac{\partial \alpha_{i-1}}{\alpha y}\right|\theta_1\left(\delta_1(y) + \gamma_1 \pi(\chi)\right) \\
&\leq \frac{1}{2\epsilon_i} z_i^2 \left|\frac{\partial \alpha_{i-1}}{\alpha y}\right|^2 \theta_1^2 \\
&\quad + \frac{\epsilon_i}{2}\left(y\bar{\delta}_1(y) + \gamma_1 \pi(\chi)\right)^2 \\
&\leq \frac{1}{2\epsilon_i} z_i^2 \left|\frac{\partial \alpha_{i-1}}{\alpha y}\right|^2 \theta + \epsilon_i y^2 \bar{\delta}_1^2(y) \\
&\quad + \epsilon_i \frac{\gamma_1^2}{\gamma}\delta(\chi),
\end{aligned} \tag{40}$$

$$\begin{aligned}
z_i \frac{\partial \alpha_{i-1}}{\alpha y}\Omega_1^T(y)\xi &\leq |z_i|\left|\frac{\partial \alpha_{i-1}}{\alpha y}\right|\theta_1 \psi_1(y)|\xi| \\
&\leq \frac{1}{4d_i} z_i^2 \left|\frac{\partial \alpha_{i-1}}{\alpha y}\right|^2 \theta_1^2 \psi_1^2(y) + d_i|\xi|^2 \\
&\leq \frac{1}{4d_i} z_i^2 \left|\frac{\partial \alpha_{i-1}}{\alpha y}\right|^2 \theta \psi_1^2(y) + d_i|\xi|^2,
\end{aligned} \tag{41}$$

where $\epsilon_i > 0$ and $d_i > 0$ are design parameters. With (39)-(41), it is obtained that

$$\begin{aligned}
\dot{V}_i &\leq -\sum_{j=1}^{i-1} c_j z_j^2 + y\left(\sum_{j=1}^{i-1} \epsilon_j y \bar{\delta}_1^2(y) - \beta(y)\right) + z_{i-1} z_i \\
&\quad + \sum_{j=1}^{i-1} \epsilon_j \frac{\gamma_1^2}{\gamma}\delta(|\chi|) + \sum_{j=1}^{i-1} d_j |\xi|^2 \\
&\quad + \left(\sum_{j=1}^{i-2} z_{j+1}\frac{\partial \alpha_j}{\partial \hat{\theta}}\right)\left(r_\theta \tau_{i-1} - \dot{\hat{\theta}}\right) + \sigma\tilde{\theta}\hat{\theta} \\
&\quad + \tilde{\theta}\left(\tau_{i-1} - \frac{1}{r_\theta}\dot{\hat{\theta}}\right) + \sum_{j=1}^{i-1}\frac{1}{4r_j}\varepsilon_1^2
\end{aligned}$$

$$+ z_i\left(z_{i+1} + \alpha_i + \eta_i\right) + r_i z_i^2\left|\frac{\partial \alpha_{i-1}}{\alpha y}\right|^2 \theta + \frac{1}{4r_i}\varepsilon_1^2 + \frac{1}{2\epsilon_i} z_i^2\left|\frac{\partial \alpha_{i-1}}{\alpha y}\right|^2 \theta + \epsilon_i y^2 \overline{\delta}_1^2(y) + \epsilon_i \frac{\gamma_1^2}{\gamma}\delta(\chi) + \frac{1}{4d_i} z_i^2\left|\frac{\partial \alpha_{i-1}}{\alpha y}\right|^2 \theta \psi_1^2(y) + d_i|\xi|^2 - z_i\frac{\partial \alpha_{i-1}}{\partial \widehat{\theta}}\dot{\widehat{\theta}}. \tag{42}$$

By choosing the i-th tuning function as $\tau_i = \tau_{i-1} + \omega_i z_i$ with $\omega_i = z_i|\partial\alpha_{i-1}/\partial y|^2(r_i + 1/2\epsilon_i + (1/4d_i)\psi_1^2(y))$, one has

$$\begin{aligned}\dot{V}_i \le & -\sum_{j=1}^{i-1} c_j z_j^2 + y\left(\sum_{j=1}^{i}\epsilon_j y \overline{\delta}_1^2(y) - \beta(y)\right) + z_i z_{i-1} + \sum_{j=1}^{i}\epsilon_j\frac{\gamma_1^2}{\gamma}\delta(|\chi|) + \sum_{j=1}^{i} d_j|\xi|^2 + \left(\sum_{j=1}^{i-2} z_{j+1}\frac{\partial \alpha_j}{\partial \widehat{\theta}}\right)\left(r_\theta \tau_i \right.\\ & \left. - \dot{\widehat{\theta}}\right) + \sigma\widetilde{\theta}\widehat{\theta} + \widetilde{\theta}\left(\tau_i - \frac{1}{r_\theta}\dot{\widehat{\theta}}\right) + \sum_{j=1}^{i}\frac{1}{4r_j}\varepsilon_1^2 + z_i\left(z_{i-1}\right.\\ & \left. + \alpha_i + \eta_i + \omega_i\widehat{\theta} - \frac{\partial \alpha_{i-1}}{\partial \widehat{\theta}} r_\theta \tau_i - \sum_{j=1}^{i-2} z_{j+1}\frac{\partial \alpha_j}{\partial \widehat{\theta}} r_\theta \omega_i\right).\end{aligned} \tag{43}$$

Choose

$$\alpha_i = -z_{i-1} - c_i z_i - \eta_i - \omega_i\widehat{\theta} + \frac{\partial \alpha_{i-1}}{\partial \widehat{\theta}} r_\theta \tau_i + \sum_{j=1}^{i-2} z_{j+1}\frac{\partial \alpha_j}{\partial \widehat{\theta}} r_\theta \omega_i, \tag{44}$$

where $c_i > 0$ is a design parameter. Then

$$\begin{aligned}\dot{V}_i \le & -\sum_{j=1}^{i} c_j z_j^2 + y\left(\sum_{j=1}^{i}\epsilon_j y \overline{\delta}_1^2(y) - \beta(y)\right) + z_i z_{i+1} + \sum_{j=1}^{i}\epsilon_j\frac{\gamma_1^2}{\gamma}\delta(|\chi|) + \sum_{j=1}^{i} d_j|\xi|^2 \\ & + \left(\sum_{j=1}^{i-1} z_{j+1}\frac{\partial \alpha_j}{\partial \widehat{\theta}}\right)\left(r_\theta\tau_i - \dot{\widehat{\theta}}\right) + \sigma\widetilde{\theta}\widehat{\theta} + \widetilde{\theta}\left(\tau_i - \frac{1}{r_\theta}\dot{\widehat{\theta}}\right) + \sum_{j=1}^{i}\frac{1}{4r_j}\varepsilon_1^2.\end{aligned} \tag{45}$$

At the end of the recursive procedure, the control law and adaptive law are chosen as

$$u = \alpha_n\left(y, \widehat{x}_1, \ldots, \widehat{x}_{n-1}, \widehat{\theta}\right), \quad \dot{\widehat{\theta}} = r_\theta \tau_n. \tag{46}$$

By (45) and (46), one gets

$$\dot{V}_n \le -\sum_{j=1}^{n} c_j z_j^2 + y\left(\sum_{j=1}^{n}\epsilon_j y \overline{\delta}_1^2(y) - \beta(y)\right) + \sum_{j=1}^{n}\epsilon_j\frac{\gamma_1^2}{\gamma}\delta(|\chi|) + \sum_{j=1}^{n} d_j|\xi|^2 + \sigma\widetilde{\theta}\widehat{\theta} + \sum_{j=1}^{n}\frac{1}{4r_j}\varepsilon_1^2. \tag{47}$$

The Lyapunov function for the whole system is

$$V = \varepsilon_0 V_0 + r_\varepsilon V_\varepsilon + V_n. \tag{48}$$

Then, by Assumption 6, (22), and (47), we obtain

$$\begin{aligned}\dot{V} \le & -c_0\varepsilon_0 V_0(\chi) - \sum_{j=1}^{n} c_j z_j^2 + y\left(r_\varepsilon|P|^2\Phi_1(y)y + \frac{r_\varepsilon}{4d_\varepsilon}|P|^4\Psi_1^2(y)y^3 + \varepsilon_0\delta_0(y) + \sum_{j=1}^{n}\epsilon_j y\overline{\delta}_1^2(y) - \beta(y)\right) \\ & + \left(r_\varepsilon a|P|^2 + \sum_{j=1}^{n}\epsilon_j\frac{\gamma_1^2}{\gamma} - \varepsilon_0\gamma_0\right)\delta(|\chi|) + d_0\xi_0^2 + r_\varepsilon d_\varepsilon|\xi|^4 + \sum_{j=1}^{n} d_j|\xi|^2 + \sigma\widetilde{\theta}\widehat{\theta} - \frac{r_\varepsilon}{2}|\varepsilon|^2 + \sum_{j=1}^{n}\frac{1}{4r_j}\varepsilon_1^2.\end{aligned} \tag{49}$$

By choosing

$$\begin{aligned} & r_\varepsilon \ge \sum_{j=1}^{n}\frac{1}{r_j}, \\ & \varepsilon_0 \ge \frac{1}{\gamma_0}\left(r_\varepsilon a|P|^2 + \sum_{j=1}^{n}\epsilon_j\frac{\gamma_1^2}{\gamma}\right), \\ & \beta(y) = r_\varepsilon|P|^2\Phi_1(y)y + \frac{r_\varepsilon}{4d_\varepsilon}|P|^4\Psi_1^2(y)y^3 + \varepsilon_0\delta_0(y) + \sum_{j=1}^{n}\epsilon_j y\overline{\delta}_1^2(y),\end{aligned} \tag{50}$$

it is obtained that

$$\begin{aligned}\dot{V} & \le -c_0\varepsilon_0 V_0(\chi) - \sum_{j=1}^{n} c_j z_j^2 - \frac{r_\varepsilon}{4}|\varepsilon|^2 - \frac{1}{2}\sigma\widetilde{\theta}^2 + d_0\xi_0^2 + r_\varepsilon d_\varepsilon|\xi|^4 + \sum_{j=1}^{n} d_j|\xi|^2 + \frac{1}{2}\sigma\theta^2 \\ & \le -cV_e + d_0\xi_0^2 + r_\varepsilon d_\varepsilon|\xi|^4 + \sum_{j=1}^{n} d_j|\xi|^2 + \frac{1}{2}\sigma\theta^2,\end{aligned} \tag{51}$$

where $c = \min\{c_0, 2c_1, \ldots, 2c_n, 1/2\lambda_{\max}(P), r_\theta\sigma\}$.

Remark 11. Because the system of this paper contains the colored noise and uncertain nonlinear functions (see (9)), the small-gain technique in [1–7] which is applied to deterministic nonlinear system is not applicable any more. Because the system of this paper is not an Itô type stochastic differential equation and the changing supply function technique in [17] cannot deal with the colored noise, the unmodeled dynamics is assumed to have enough stability margin motivated by [16] in this paper

Remark 12. Different from the deterministic nonlinear systems in [1–10], a class of random nonlinear systems driven by colored noise is considered in this paper. The stochastic disturbance is regarded as colored noise other than the white noise in [16, 17] which is more reasonable. Different from [25] only considering the problem of the state feedback stabilization, an adaptive output feedback stabilization controller is designed in this paper.

4. Stability Analysis

Theorem 13. *For the random system (9), under Assumptions 4–7, the control law and adaptive law (46), the closed-loop system has a unique solution on $[t_0,\infty)$, and the state of the closed-loop system has an AG-2-M. Furthermore, the output $y(t)$ satisfies*

$$\lim_{t\longrightarrow\infty} E|y|^2 \le \frac{2\delta_K}{c}, \tag{52}$$

where $\delta_K = (d_0 + r_\varepsilon d_\varepsilon + \sum_{j=1}^{n} d_j)K + (1/2)\sigma\theta^2$ and the right-hand can be made small enough by tuning parameters.

Proof. Define $\Xi = (\chi, \varepsilon^T, z_1, \ldots, z_n, \tilde{\theta})^T$. From the definition of V, V satisfies

$$a_1|\Xi|^2 \le V \le a_2|\Xi|^2, \tag{53}$$

with $a_1 = (1/2)\min\{2\varepsilon_0\underline{a}, r_\varepsilon\lambda_{\min}(P), 1, 1/r_\theta\}$ and $a_2 = (1/2)\max\{2\varepsilon_0\overline{a}, r_\varepsilon\lambda_{\max}(P), 1, 1/r_\theta\}$. Since the functions of the closed-loop system satisfy the local Lipschitz condition, from Lemma 2, (53), and (51), then the closed-loop system has a unique solution on $[t_0,\infty)$, and the state of the closed-loop system has an AG-2-M.

Furthermore, by defining $v(t) = EV(t)$, from (51), one has

$$\begin{aligned} \dot{v}(t) &= E\dot{V}(t) \\ &\le -cv(t) + d_0E|\xi_0|^2 + r_\varepsilon d_\varepsilon E|\xi|^4 + \sum_{j=1}^{n} d_jE|\xi|^2 \\ &\quad + \frac{1}{2}\sigma\theta^2. \end{aligned} \tag{54}$$

By Lemma 3 and (3), it is obtained that

$$v(t) \le |v(0)|e^{-c(t-t_0)} + \delta_K, \tag{55}$$

which together with (48) implies

$$E|y|^2 \le 2EV_e(t) \le 2V_e(0)e^{-c(t-t_0)} + \frac{2\delta_K}{c}, \tag{56}$$

which leads to (52).

Noting $c = \min\{c_0, 2c_1, \ldots, 2c_n, 1/2\lambda_{\max}(P), r_\theta\sigma\}$ and $\delta_K = (d_0+r_\varepsilon d_\varepsilon+\sum_{j=1}^{n} d_j)K+(1/2)\sigma\theta^2$, it is clear that the right-hand sides of (52) can be made small enough by choosing $c_1, \ldots, c_n$ large enough and $d_1, \ldots, d_n$ small enough. □

Remark 14. By Chebyshev's inequality, for any $\varepsilon > 0$ and $\varepsilon_0 > 0$, there exists a moment $T > 0$ such that when $t > T$, $P\{|y(t)| > \varepsilon\} \le (1/\varepsilon^2)(\varepsilon_0 + 2\delta_K/c) \le \varepsilon'$, where ε' can be regulated to small enough, which implies the asymptotically stabilization in probability in some sense.

5. A Simulation Example

Consider the following nonlinear system

$$\begin{aligned} \dot{\chi} &= q(\chi, x_1) + p(\chi, x_1)\xi_0, \\ \dot{x}_1 &= x_2 + \phi_1(x,\chi) + \Omega_1^T(x_1)\xi, \\ \dot{x}_2 &= u + \phi_2(x,\chi) + \Omega_2^T(x_1)\xi, \\ y &= x_1, \end{aligned} \tag{57}$$

where $q(\chi, x_1) = -6\chi+2x_1+\tanh x_1$, $p(\chi, x_1) = \cos\chi+\sin x_1$, $\phi_1(x,\chi) = \theta_1(x_1+\chi)\sin x_2$, $\phi_2(x,\chi) = \theta_2\chi\cos x_2$, $\Omega_1^T(x_1) = [\theta_1x_1, 0]$, $\Omega_2^T(x_1) = [0, \theta_2x_1]$ with $\theta_1 = 0.3$, $\theta_2 = 0.2$.

For χ-subsystem of (57), by choosing the Lyapunov function $V_0(\chi) = \underline{a}|\chi|^2 = \overline{a}|\chi|^2 = (1/2)\chi^2$, one can verify

$$\begin{aligned} &\frac{\partial V_0}{\partial\chi}q(\chi, x_1) + \frac{1}{4d_0}\left\|\frac{\partial V_0}{\partial\chi}p(\chi, x_1)\right\|^2 \\ &\quad\le -V_0 - \left(\frac{9}{2} - \frac{1}{d_0}\right)\chi^2 + x_1^2, \end{aligned} \tag{58}$$

which implies that Assumption 5 holds for $d_0 = 1/2$, $c_0 = 1$, $\gamma_0 = 5/2$, $\delta(|\chi|) = |\chi|^2$, $\delta_0(y) = y$. By verifying Assumptions 6 and 7, it is easy to obtain that $\delta_1 = |y|$, $\delta_2 = 0$, $\pi = |\chi|$, $\gamma_1 = 1$, $\gamma_2 = 1$, $\gamma = 1$, $\psi_1 = |y|$ and $\psi_2 = |y|$.

The following observer is needed

$$\hat{x}_1 = u - k_1(\hat{x}_1 + k_1y), \tag{59}$$

where $k_1 > 0$ is a design parameter. By defining two error variables (25), the adaptive output feedback control law is given by the recursive design procedure in Section 3, i.e.,

$$\begin{aligned} \alpha_1 &= -c_1z_1 - k_1y - r_\varepsilon|P|^2\Phi_1(y)y \\ &\quad - \frac{r_\varepsilon}{4d_\varepsilon}|P|^4\Psi_1^2(y)y^3 - r_1y\hat{\theta} - \frac{1}{2\epsilon_1}y\hat{\theta} \\ &\quad - \frac{1}{4d_1}y\psi_1^2(y)\hat{\theta} - \epsilon_1y\overline{\delta}_1^2 - \epsilon_2y\overline{\delta}_1^2 - \varepsilon_0\delta_0, \\ \tau_1 &= \sigma\hat{\theta} + r_1y^2 + \frac{1}{2\epsilon_1}y^2 + \frac{1}{4d_1}y^2\psi_1^2(y), \end{aligned}$$

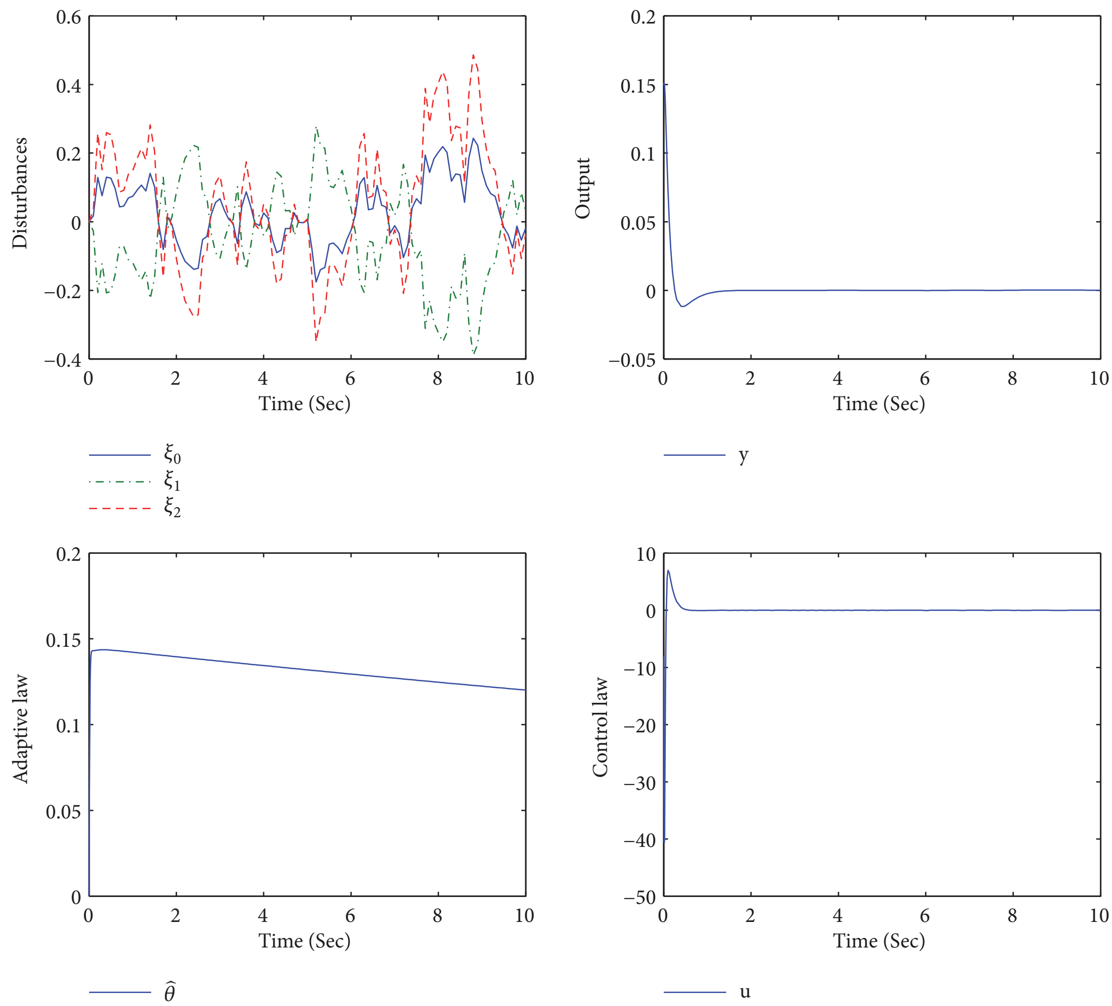

FIGURE 1: The response of closed-loop system.

$$\dot{\hat{\theta}} = r_\theta \tau_2$$
$$= r_\theta \left(\tau_1 + z_2^2 \left| \frac{\partial \alpha_1}{\alpha y} \right|^2 \left(r_2 + \frac{1}{2\epsilon_2} + \frac{1}{4d_2} \psi_1^2 (y) \right) \right),$$
$$u = \alpha_2$$
$$= -z_1 - c_2 z_2 - \eta_2$$
$$- z_2 \left| \frac{\partial \alpha_1}{\partial y} \right|^2 \left(r_2 + \frac{1}{2\epsilon_2} + \frac{1}{4d_2} \psi_1^2 (y) \right) \hat{\theta}$$
$$+ \frac{\partial \alpha_1}{\partial \hat{\theta}} r_\theta \tau_2,$$
$$\eta_2 = -k_1 (\hat{x}_1 + k_1 y) - \frac{\partial \alpha_1}{\partial y} (\hat{x}_1 + k_1 y), \tag{60}$$

where $P = 1/k_1$, $\Phi_1 = 2k_1^2$, $\Psi_1 = (1 + k_1)^2$ and $\overline{\delta}_1 = \text{sign}(y)$.

In the simulation, the disturbance ξ_i ($i = 0, 1, 2$) is produced by

$$\dot{\xi} (t) = -\xi (t) + b_i w (t), \qquad \xi_i (0) = 0, \tag{61}$$

where $b_1 = 0.5, b_1 = -0.8, b_1 = 1$ and $w(t) \in \mathbb{R}$ is a zero-mean white noise whose spectral function equals 1. Therefore, ξ_i is a zero-mean widely stationary process and $E\xi_i^2 = |b_i|/2$; thus, $K = 0.5$. Choose the initial values $\chi(0) = -0.3$, $x_1(0) = 0.15$, $x_2(0) = 0.2$ and the design parameters $k_1 = 3.7$, $d_\varepsilon = 0.5$, $r_1 = 1$, $\epsilon_1 = 0.3$, $d_1 = 0.5$, $r_2 = 1$, $\epsilon_2 = 0.5$, $d_2 = 0.2$, $c_1 = 0.5$, $c_2 = 0.4$, $r_\theta = 0.0075$, $\sigma = 2.5$, $\hat{x}_1(0) = 0$, $\hat{\theta}(0) = 0$, $r_\varepsilon = 1/r_1 + 1/r_2$, $\varepsilon_0 = (1/\gamma_0)(r_\varepsilon(2/\gamma)(\gamma_2 + k_1\gamma_1)^2|P|^2 + \sum_{j=1}^{2} \epsilon_j(\gamma_1^2/\gamma)) = (2/5)r_\varepsilon(1 + k_1)^2(1/k_1^2) + \epsilon_1 + \epsilon_2$.

Figure 1 shows the system responses of closed-loop system. From the results of simulation, the response of closed-loop system is all bounded and the output y can be practically regulated to zero.

6. Conclusion

In this paper, a class of random nonlinear systems with unmodeled dynamics and uncertain nonlinear functions is investigated. Based on a reduced-order observer, an adaptive output feedback stabilization controller is designed such that the mean square of the output can be made small enough by choosing the appropriate parameters. The method of ordinary differential equations is used to analyze the stability of the closed-loop system. A simulation example demonstrates the effectiveness of the proposed scheme.

There are other problems under current investigation such as the tracking control, the control based on stochastic small-gain technique, the control based on changing supply function technique, and their practical application. In addition, one can try to apply these methods to generalized triangular form systems driven by colored noise.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The author declares that they have no conflicts of interest.

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