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Research Article

A Novel TOPSIS Method Based on Improved Grey Relational Analysis for Multiattribute Decision-Making Problem

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Multiattribute decision-making (MADM) problem is difficult to assess because of the large number of attribute indices and the diversity of data distribution. Based on the understanding of data dispersion degree, a new grey TOPSIS method for MADM is studied. The main idea of this paper is to redefine the grey relational analysis through the dispersion of data distribution and redesign the TOPSIS by using the improved grey relational analysis. As a classical multiattribute decision analysis method, traditional TOPSIS does not consider the data distribution of the degree of dispersion and aggregation when it is compared with the optimal and worst alternative solutions. In view of the limitations of traditional TOPSIS, this paper has made two major improvements to TOPSIS. Firstly, the new grey relational analysis is applied to evaluate the grey positive relational degree between each alternative and the optimal solution and compute the grey negative relational degree between each alternative and the worst solution. Secondly, the weights of every attribute index about the optimal and worst solutions are put forward based upon the distance standard deviation and the average distance. Finally, the comprehensive grey TOPSIS is utilized to analyze the ranking of weapon selection problem. The numerical results verify the feasibility of the improved grey relational analysis and also highlight the practicability of the grey comprehensive TOPSIS.

1. Introduction

In recent years, with the development of society, the research on multiattribute decision-making (MADM) is becoming more and more concerned about its simplicity, validity, and accuracy [1–9]. Generally speaking, multiattribute decision-making problem often involves complicated external environment and many different attributes. The TOPSIS (techniques for order preference by similarity to ideal solution) is a useful and powerful method for dealing with multiattribute decision-making (MADM) problems which is presented by Hwang and Yoon [10]. The classical TOPSIS approach is based on the suggestion of the chosen alternative which requires consideration of the distance between the positive ideal solution and the negative ideal solution. The most suitable alternative should be closest to the positive ideal solution and the farthest from the negative ideal solution. At present, with the deepening realization about TOPSIS, some extended TOPSIS techniques have been used widely for MADM problems to overcome the vagueness in the judgments made by the assessors [11–21]. In order to overcome the problem of index weight calculation, AHP is the main auxiliary method of TOPSIS, and fuzzy sets are introduced together to facilitate the quantification of the two pairs of indicators [11–13]. Some researchers have improved TOPSIS by using the intuitionistic fuzzy number, triangular fuzzy number, vague set, intuitionistic fuzzy entropy, and other uncertainty tools, which can solve MADM problems with intuitionistic fuzzy environment or linguistic fuzziness [7, 14–18].

In fact, in the process of multiattribute decision-making with TOPSIS, there are two main problems to solve. On the one hand, the weights of different indexes need to be determined when the alternatives are compared with the optimal or worst schemes. As a relatively simple method, analytic hierarchy process (AHP) has been successfully

applied to the weight calculation of TOPSIS [11–13, 19], but subjective interference also exists due to expert scoring and pairwise comparison. The technique of analytic network process (ANP) is used to obtain the relative weights to reduce the number of pairwise comparison on the basis of AHP [20]. So the ANP is the development and supplement of AHP. In addition, the method of comprehensive evaluation, principal component analysis, and other methods are all important methods to calculate weights, which are applied in TOPSIS [21–23]. On the other hand, the optimal or worst value for every attribute should be selected, and a quantitative measure method should be constructed to compare with each other. Fuzzy mathematics provides a quantitative calculation method for describing uncertain things [11–15, 18, 19], but there is a lot of uncertainty and subjectivity in the designing of fuzzy sets. Professor Deng proposed the grey system theory which is one of powerful methods for data analysis with partly known and partly unknown information in uncertain environment [24, 25]. Distance standard deviation is an important numerical characteristic which reflects the distribution of data. Very recently, it is worth noting that distance standard deviation is introduced into the grey confidence interval estimation for small samples [26]. This study shows that distance standard deviation contributes to describe the degree of dispersion between different samples for grey system. However, in the grey relational analysis, the value of distance standard deviation has not been explored. In view of this, this paper will introduce the distance standard deviation factor in the grey relational analysis, so as to take full account of the numerical characteristics of different series of data.

Based on the deepening realization about the generality of multiattribute decision-making problems and the core of TOPSIS method, part of the work has been completed in the following. In Section 2, the sample distance standard deviation is defined between different system behavior sequences, and then the improved grey relational analysis based on sample distance standard deviation is constructed, which paved the way to further discussion. In Section 3, the innovative new TOPSIS method is proposed. The degree of aggregation between different data is served as an important decision factor for TOPSIS method. The grey relational analysis is improved on the basis of the distance standard deviation and applied to quantify the correlation between different sequence data. The distance standard deviation is also used to calculate the weights between the sequence data and the optimal scheme or the worst case. Section 4 gives three applications of the developed approach, and the numerical steps and results are clearly shown. The last section summarizes the whole paper.

2. The Improved Grey Relational Analysis Based on Dispersion Degree

At present, the grey relational analysis has been applied more commonly to many scientific decision-making problems, such as computer science, engineering and other related fields. It provides a tool to evaluate grey relational degree based on distances between reference sequence and comparative sequence. In fact, only considering the maximum and minimum distance between data is not enough. Generally speaking, the sample aggregation characteristics reflect the inherent character of data, which should be considered in the calculation of grey relational degree. The numerical characteristics of the attribute data distribution can reflect the degree of dispersion of the data distribution. If the degree of dispersion of the data is small, the distribution is relatively concentrated, and this attribute is relatively important. And the standard deviation of sample distance of data coincides with this characteristic. Therefore, the sample distance standard deviation as an important concept that embodies the degree of data distribution density will be brought into the calculation of grey relational degree.

Definition 1. For the given system behavior sequence is as follows:

$$\begin{aligned} X_0 &= \left(x_0(1), x_0(2), \ldots, x_0(n)\right), \\ X_1 &= \left(x_1(1), x_1(2), \ldots, x_1(n)\right), \\ &\ldots, \\ X_i &= \left(x_i(1), x_i(2), \ldots, x_i(n)\right), \\ &\ldots, \\ X_m &= \left(x_m(1), x_m(2), \ldots, x_m(n)\right). \end{aligned} \tag{1}$$

The distance between $x_0(k)$ and $x_i(k)$ is defined as follows:

$$d\left(x_0(k), x_i(k)\right) = \left|x_0(k) - x_i(k)\right|. \tag{2}$$

For $\xi \in (0, 1)$, and for every i, j, let

$$\gamma\left(x_0(k), x_i(k)\right) = \begin{cases} \dfrac{\xi \max_i s\left(X_0, X_i\right)}{\left|x_0(k) - x_i(k)\right| + \xi \max_i s\left(X_0, X_i\right)}, & \text{if } \min_i s\left(X_0, X_i\right) > 0 \text{ and } s\left(X_0, X_i\right) \neq s\left(X_0, X_j\right) \\ \eta^{\alpha[\max_k d(x_0(k)-x_i(k)) - \min_k d(x_0(k)-x_i(k))]} e^{-\beta d(x_0(k), x_i(k))}, & \text{if } \min_i s\left(X_0, X_i\right) = 0 \text{ or } s\left(X_0, X_i\right) = s\left(X_0, X_j\right), \end{cases} \tag{3}$$

$$\gamma\left(X_0, X_i\right) = \frac{1}{n}\sum_{k=1}^{n} \gamma\left(x_0(k), x_i(k)\right), \tag{4}$$

where $s(X_0, X_i) = \sqrt{(1/(n-1)) \sum_{k=1}^{n} (d(x_0(k), x_i(k)) - m(X_0, X_i))^2}$ denotes the sample distance standard deviation between X_0 and X_i, and $m(X_0, X_i) = (1/n) \sum_{k=1}^{n} d(x_0(k), x_i(k))$ is defined as the average distance between X_0 and X_i.

Then $\gamma(X_0, X_i)$ is called grey relational degree with sample distance standard deviation, in which ξ and η are called distinguishing coefficient, $\eta \in (0, 1), \alpha, \beta \in (0, 1]$, and ξ's value satisfies the following requirements:

$$\xi = \begin{cases} \dfrac{\max_i s(X_0, X_i)}{\min_i s(X_0, X_i) + \max_i s(X_0, X_i)}, & \text{if } \min_i s(X_0, X_i) > 0 \\ \text{constant, and constant} \in (0, 1), & \text{if } \min_i s(X_0, X_i) = 0. \end{cases} \quad (5)$$

Theorem 2. *The improved grey relational degree $\gamma(X_0, X_i)$ with sample distance standard deviation $s(X_0, X_i)$ satisfies the following three axioms of grey correlation:*

(1) *The property of normality:* $0 < \gamma(X_0, X_i) \leq 1$.

(2) *The property of closeness: the greater the $\gamma(X_0, X_i)$, the closer of X_0 to X_i.*

(3) *The property of pair symmetry:* $\gamma(X_i, X_j) = \gamma(X_j, X_i)$.

Proof. It is obvious that $s(X_0, X_i) \geq 0$ for system behavior sequence $X_0, X_1, \ldots, X_i, \ldots, X_m$. So the theorem can be proved in two cases.

(1) When $\min_i s(X_0, X_i) = 0$ or $s(X_0, X_i) = s(X_0, X_j)$, from (3), we can receive the result that $\gamma(x_0(k), x_i(k)) = \eta^{\alpha[\max_k d(x_0(k)-x_i(k)) - \min_k d(x_0(k)-x_i(k))]} e^{-\beta d(x_0(k), x_i(k))}, 0 < \gamma(x_0(k), x_i(k)) \leq 1$ for every k, then in these circumstances, we can obtain the result that $0 < \gamma(X_0, X_i) \leq 1$ by (4). $\gamma(X_0, X_i) = 1$, if and only if $X_0 = X_i$.

For system behavior sequences X_0 and X_i, $\eta^{\alpha[\max_k d(x_0(k)-x_i(k)) - \min_k d(x_0(k)-x_i(k))]}$ is a constant. Because $e^{-\beta d(x_0(k), x_i(k))}$ is monotonically decreasing and $\gamma(x_0(k), x_i(k))$ is monotonically decreasing, the property of closeness is received.

At last, because of $|x_i(k) - x_j(k)| = |x_j(k) - x_i(k)|$, the property of pair symmetry is obvious.

(2) If $\min_i s(X_0, X_i) > 0$ and $s(X_0, X_i) \neq s(X_0, X_j)$, then $\max_i s(X_0, X_i) \geq \min_i s(X_0, X_i) > 0$. So, $0 < \gamma(X_0, X_i) < 1$ by (3). Because $\min_i s(X_0, X_i) > 0$, we can receive the result that $X_0 \neq X_i$ for every i, then $\gamma(X_0, X_i) \neq 1$ in this case.

Because $\max_i s(X_0, X_i)$ and $\min_i s(X_0, X_i)$ are constant which determined by system behavior sequence $X_0, X_1, \ldots, X_i, \ldots, X_m$. It is obvious that the closeness is established.

Because $|x_i(k) - x_j(k)| = |x_j(k) - x_i(k)|$, the property of pair symmetry can be received. □

Remark 3. The new grey relational analysis takes full advantage of the inherent regularity of data distribution and introduces the dispersion of data distribution into them. The distance standard deviation is more likely to reflect the overall dispersion of the data distribution than the distance.

Remark 4. In the case of $\min_i s(X_0, X_i) = 0$ or $s(X_0, X_i) = s(X_0, X_j)$, that is, there are two system behavior sequence X_0 and X_i that are parallel or overlapped, the distance between X_0 and X_i is fully considered by (3). Expressly if two system behavior sequence data are parallel, then the smaller the distance is, the greater the similarity is, and the greater the grey relational degree is.

3. The Novel TOPSIS with Improved Grey Relational Analysis to Solve Multiattribute Decision-Making Problem

This section tries to improve traditional TOPSIS with new grey relational analysis mentioned above. The degree of dispersion and aggregation between different data will serve as an important decision factor for TOPSIS. The above-mentioned grey relational analysis will be used to calculate the grey relational degree between the alternative scheme and the optimal or worst case scheme. At the same time, the standard deviation of distance will be taken as the decisive factor of weight calculation in order to determine the weight of each index for MADM problems.

Step 1. Standardization of the value of original decision matrix $X = (x_{ij})_{n\times m}$: this will help eliminating the influence of dimension and magnitude in data processing. The original decision matrix $X = (x_{ij})_{n\times m}$ can be expressed as follows:

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix}_{n\times m}, \quad (6)$$

where x_{ij} is the jth attribute value of the ith evaluation object. For multiattribute decision-making problem, the normalized decision matrix is $Y = (y_{ij})_{n\times m}$ after standardization of $X = (x_{ij})_{n\times m}$.

For the benefit attribute

$$y_{ij} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}. \quad (7)$$

For the cost attribute

$$y_{ij} = \frac{\max_i x_{ij} - x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}. \quad (8)$$

Step 2. To pick out the optimal and worst alternatives, the optimal alternative is denoted as $\overline{X} = (\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_m) \in R^m$, where

$$\overline{x}_j = \begin{cases} \max_i x_{ij}, & \text{if } x_{ij} \text{ is benefit attribute} \\ \min_i x_{ij}, & \text{if } x_{ij} \text{ is cost attribute,} \end{cases} \quad (9)$$

$$j = 1, 2, \ldots, m.$$

The worst alternative denoted as $\underline{X} = (\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_m) \in R^m$, where

$$\underline{x}_j = \begin{cases} \max_i x_{ij}, & \text{if } x_{ij} \text{ is cost attribute} \\ \min_i x_{ij}, & \text{if } x_{ij} \text{ is benefit attribute,} \end{cases} \quad j = 1, 2, \ldots, m. \tag{10}$$

For the normalized decision matrix $Y = (y_{ij})_{n\times m}$, the normalized optimal alternative can be denoted as $\overline{Y} = (\overline{y}_1, \overline{y}_2, \ldots, \overline{y}_m) \in R^m$, and the normalized worst alternative can be denoted as $\underline{Y} = (\underline{y}_1, \underline{y}_2, \ldots, \underline{y}_m) \in R^m$. It is not difficult to calculate that $\overline{Y} = (1, 1, \ldots, 1) \in R^m$, and $\underline{Y} = (0, 0, \ldots, 0) \in R^m$ by (7) and (8), respectively.

Step 3. To calculate the sample distance standard deviation for different attribute, the symbol s_{1j} used to denote the sample distance standard deviation between the jth attribute vector and the jth attribute optimal value, where

$$s_{1j} = \frac{1}{n-1}\sqrt{\sum_{i=1}^{n}\left(d\left(y_{ij}, \overline{y}_j\right) - m_{1j}\right)^2}, \quad j = 1, 2, \ldots, m, \tag{11}$$

$$m_{1j} = \frac{1}{n}\sum_{i=1}^{n} d\left(y_{ij}, \overline{y}_j\right), \quad j = 1, 2, \ldots, m. \tag{12}$$

The symbol s_{2j} is used to denote the sample distance standard deviation between the jth attribute vector and the jth attribute worst value, where

$$s_{2j} = \frac{1}{n-1}\sqrt{\sum_{i=1}^{n}\left(d\left(y_{ij}, \underline{y}_j\right) - m_{2j}\right)^2}, \quad j = 1, 2, \ldots, m, \tag{13}$$

$$m_{2j} = \frac{1}{n}\sum_{i=1}^{n} d\left(y_{ij}, \underline{y}_j\right), \quad j = 1, 2, \ldots, m. \tag{14}$$

Step 4. To compute the grey relational degree r_{ij}^+ and r_{ij}^-. There are some different cases based on (3).

(1) When $\min_j s_{1j} > 0$ and $s_{1j} \neq s_{1k}$ for every k, $k = 1, 2, \ldots, m$, the symbol r_{ij}^+ is the grey positive relational degree between y_{ij} and $\overline{y}_j$, where

$$r_{ij}^+ = \frac{\xi \max_j s_{1j}}{\left|y_{ij} - \overline{y}_j\right| + \xi \max_j s_{1j}}. \tag{15}$$

At the same case, the symbol r_{ij}^- is the grey negative relational degree between y_{ij} and $\underline{y}_j$, where

$$r_{ij}^- = \frac{\xi \max_j s_{2j}}{\left|y_{ij} - \underline{y}_j\right| + \xi \max_j s_{2j}}. \tag{16}$$

(2) When $\min_j s_{1j} = 0$ or $s_{1j} = s_{1k}$, for every k, $k = 1, 2, \ldots, m$, the grey positive relational degree r_{ij}^+ can be computed as follows:

$$r_{ij}^+ = \eta^{\alpha[\max_j d(y_{ij}, \overline{y}_j) - \min_j d(y_{ij}, \overline{y}_j)]} e^{-\beta d(y_{ij}, \overline{y}_j)}. \tag{17}$$

And the grey negative relational degree r_{ij}^- between y_{ij} and $\underline{y}_j$ can be received as follows:

$$r_{ij}^- = \eta^{\alpha[\max_j d(y_{ij}, \underline{y}_j) - \min_j d(y_{ij}, \underline{y}_j)]} e^{-\beta d(y_{ij}, \underline{y}_j)}. \tag{18}$$

Step 5. To determine weights $\overline{w}_j$ and $\underline{w}_j$ for the grey positive relational degree and grey negative relational degree, similar to the above, there are two cases below.

(1) When $\min_j s_{1j} > 0$, $s_{1j} \neq s_{1k}$, $\min_j s_{2j} > 0$, and $s_{2j} \neq s_{2k}$ for every k, $k = 1, 2, \ldots, m$, $\overline{w}_j$ and $\underline{w}_j$ can be obtained as follows:

$$\overline{w}_j = \frac{1/s_{1j}}{\sum_{j=1}^{m}\left(1/s_{1j}\right)}, \quad j = 1, 2, \ldots, m, \tag{19}$$

$$\underline{w}_j = \frac{1/s_{2j}}{\sum_{j=1}^{m}\left(1/s_{2j}\right)}, \quad j = 1, 2, \ldots, m, \tag{20}$$

where the sample distance standard deviation s_{1j} and s_{2j} can be determined by (11) and (13).

(2) Otherwise, the weights $\overline{w}_j$ and $\underline{w}_j$ can be calculated as follows:

$$\overline{w}_j = \frac{1/m_{1j}}{\sum_{j=1}^{m}\left(1/m_{1j}\right)}, \quad j = 1, 2, \ldots, m, \tag{21}$$

$$\underline{w}_j = \frac{1/m_{2j}}{\sum_{j=1}^{m}\left(1/m_{2j}\right)}, \quad j = 1, 2, \ldots, m, \tag{22}$$

where the average distance m_{1j} and m_{2j} can be determined by (12) and (14).

The smaller the distance standard deviation of attribute data or the average distance is, the stronger the regularity of attribute data is, and the bigger its weight is. On the contrary, the larger the distance standard deviation of attribute data or the average distance is, the larger the distribution range of attribute data is, the weaker the regularity is, and the smaller the weight is. It is obvious to find that the greater the $\overline{w}_j$ is, the closer alternative attribute value and the optimal alternative attribute value are. Similarly, the weight $\underline{w}_j$ has the same nature. The greater the $\underline{w}_j$ is, the closer alternative attribute value and the worst value are.

Step 6. To calculate the grey TOPSIS grade, the grey positive relational grade R_i^+ between the ith evaluation alternative and the optimal solution can be received as follows:

$$R_i^+ = \sum_{j=1}^{m} \overline{w}_j r_{ij}^+, \quad i = 1, 2, \ldots, n. \tag{23}$$

The grey negative relational grade R_i^- between the ith evaluation alternative and the worst solution can be received as follows:

$$R_i^- = \sum_{j=1}^{m} \underline{w}_j r_{ij}^-, \quad i = 1, 2, \ldots, n. \tag{24}$$

And the comprehensive grey TOPSIS evaluation result R_i is defined as follows:

$$R_i = \frac{R_i^+}{R_i^+ + R_i^-}, \quad i = 1, 2, \ldots, n. \tag{25}$$

It is obviously that the comprehensive grey TOPSIS evaluation result R_i reflects the closeness between alternative and positive ideal solution and negative ideal solution. The larger the R_i^+ is, the closer R_i is to positive ideal solution, and the smaller the R_i^- is, the farther R_i is to negative ideal solution. The reason behind this is that after the first dimensionless standardized processing of the original data, the improved TOPSIS method is used for evaluation, and the obtained grey relational degree and weights have the function of adaptive adjustment with the change of data. On the one hand, the grey positive relational degree can reflect the closeness between the data and the same attribute data of the positive ideal solution, while the grey negative relational degree can reflect the closeness between the data and the same attribute data of the negative ideal solution. On the other hand, the weights of different attributes can feedback the importance of the attributes based on the numerical characteristics of the attribute data.

4. Numerical Results

In this section, three examples will be used to demonstrate the effectiveness of the methods proposed above. The first example shows that the improved grey relational analysis is more comprehensive than the traditional grey relational analysis, which can avoid some unreasonable results. The second one is used to illustrate the rationality of the improved grey relational analysis. The novel TOPSIS with improved grey relational analysis will be applied for assessment of weapon system in the second example. In addition, Example 3 will illustrate that the method of this paper can avoid the occurrence of the rank reversal phenomenon, which is superior to the traditional TOPSIS method.

Example 1. The following example will try to solve the grey relational degree with improved method in this paper. Take four system behavior sequences for example, $X_1 = \{0.1, 0.9, 0.1, 0.9, 0.1\}$, $X_2 = \{1, 0, 1, 0, 1\}$, $X_3 = \{0.2, 1, 0.2, 1, 0.2\}$, and $X_4 = \{0.9, 0, 0.9, 0, 0.9\}$. Figure 1 illustrates the changing trend curves for different system behavior sequences.

Solution

Step 1. Because the data of four system behavior sequences are in a relatively small range $[0, 1]$, the data preprocessing can be ignored.

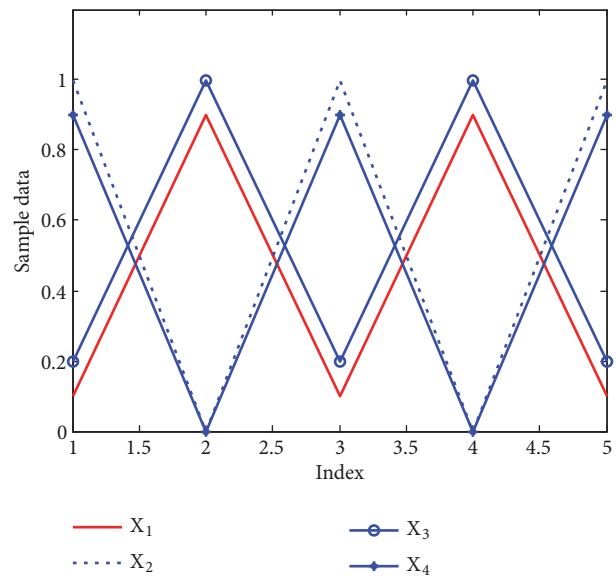

Figure 1: Four system behavior sequences.

According to (2), compute the distance $d(x_1(k), x_i(k)) = |x_1(k) - x_i(k)|$, respectively, $(k = 1, 2, \ldots, 5)$. The symbol $d(X_1, X_i) = (d(x_1(1), x_i(1)), d(x_1(2), x_i(2)), \ldots, d(x_1(5), x_i(5)))$ is used to represent the distance vector between X_1 and X_i, $i = 2, 3, 4$.

$$\begin{aligned} d(X_1, X_2) &= (0.9, 0.9, 0.9, 0.9, 0.9), \\ d(X_1, X_3) &= (0.1, 0.1, 0.1, 0.1, 0.1), \\ d(X_1, X_4) &= (0.8, 0.9, 0.8, 0.9, 0.8). \end{aligned} \tag{26}$$

Step 2. Compute the average distance $m(X_1, X_i)$ and the sample distance standard deviation $s(X_1, X_i)$.

$$\begin{aligned} m(X_1, X_2) &= 0.9, \\ m(X_1, X_3) &= 0.1, \\ m(X_1, X_4) &= 0.84, \\ s(X_1, X_2) &= 0, \\ s(X_1, X_3) &= 0, \\ s(X_1, X_4) &= 0.0548. \end{aligned} \tag{27}$$

Step 3. According to (3) and (4), the improved grey relational degree $\gamma(X_1, X_i)$ based on discrete degree of data in statistics can be solved as follows:

$$\begin{aligned} \gamma(X_1, X_2) &= 0.6376, \\ \gamma(X_1, X_3) &= 0.9512, \\ \gamma(X_1, X_4) &= 0.6500. \end{aligned} \tag{28}$$

The results show that $\gamma(X_1, X_3) > \gamma(X_1, X_4) > \gamma(X_1, X_2)$, which indicates X_1 is more relevant to X_3 compared with

Table 1: The comparison between the improved and classical grey relational analysis.

Grey relational degree	$\gamma(X_1, X_2)$	$\gamma(X_1, X_3)$	$\gamma(X_1, X_4)$
The classical grey relational analysis	0.4074	1	0.4270
The improved grey relational analysis	0.6376	0.9512	0.6500

Table 2: The original multiattribute decision data [27].

	$M_{\max}$	$M'_{\max}$	$O_{\max}$	$H_{\max}$	$R_{\max}$	N	P	T	$H_{\min}$	G_0	$R_{\min}$
X_1	6	750	6	24	100	8	0.75	20	0.3	1000	3
X_2	5.5	2300	5	27	90	8	0.8	15	0.025	1000	5
X_3	4.4	1200	5	27	75	6	0.76	20	0.025	1664	5
X_4	3	420	1	24.5	32	1	0.75	40	1	2375	8
X_5	2	400	2	5	12	3	0.75	10	0.5	220	1
X_6	2.2	400	2	3	8	3	0.8	10.4	0.05	85	1
X_7	2.2	410	2	3	8	3	0.7	10	0.05	85	0.5

others. The two sets of data X_1 and X_3 are parallel and no-overlap, so they can be considered different, and the grey relational degree cannot be valued at one. Obviously, the valuing and sorting of the improved grey relational analysis results are superior to the classical method shown in Table 1. In this example, some parameters are taken as follows: $\xi = 0.5$, $\eta = 0.8$, $\alpha = 0.5$, and $\beta = 0.5$.

Example 2. In this example, seven surface-to-air missile weapon system alternatives $X_1, X_2, \ldots, X_7$ will be evaluated by grey TOPSIS method in the following. The multiattribute weapon system is evaluated from the combination of traditional grey relational analysis and TOPSIS in reference paper [27], but no convincing calculation index weight is given. In order to obtain the best selection of weapon systems, [28] developed fuzzy analytic hierarchy process by entropy weight to evaluate the weight. Metin, Serkan, and Nevzat [29] utilized AHP and TOPSIS to assess weapon systems. AHP is more difficult because of the comparison between the two factors, then this paper from the distribution of different indicators to consider the dispersion degree and facilitate the given weight. A good surface-to-air missile weapon system depends on a lot of attributes, such as the maximum speed of missile ($M_{\max}$), the maximum speed of target ($M'_{\max}$), the maximum overload of target ($O_{\max}$), the highest boundary of killing range ($H_{\max}$), the farthest boundary of killing range ($R_{\max}$), the number of targets that can simultaneity be shot by one weapon system (N), the single shot kill probability of missiles (P), the reaction time of missile weapon system (T), the lowest boundary of killing range ($H_{\min}$), the launching weight of missiles (G_0), and the nearest boundary of killing range ($R_{\min}$) as listed in Table 2 [27]. Based on the above facts, the optimal selection of multiple weapons is very important and complicated for evaluation and assessment of weapon system.

Solution

Step 1. The normalized decision matrix $Y = (y_{ij})_{n\times m}$ can be calculated by (7) and (8) as follows:

$$Y = \begin{bmatrix} 1.0000 & 0.1842 & 1.0000 & 0.8750 & 1.0000 & 1.0000 & 0.5000 & 0.6667 & 0.7179 & 0.6004 & 0.6667 \\ 0.8750 & 1.0000 & 0.8000 & 1.0000 & 0.8913 & 1.0000 & 1.0000 & 0.8333 & 1.0000 & 0.6004 & 0.4000 \\ 0.6000 & 0.4211 & 0.8000 & 1.0000 & 0.7283 & 0.7143 & 0.6000 & 0.6667 & 1.0000 & 0.3105 & 0.4000 \\ 0.2500 & 0.0105 & 0 & 0.8958 & 0.2609 & 0 & 0.5000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2000 & 0.0833 & 0.0435 & 0.2857 & 0.5000 & 1.0000 & 0.5128 & 0.9410 & 0.9333 \\ 0.0500 & 0 & 0.2000 & 0 & 0 & 0.2857 & 1.0000 & 0.9867 & 0.9744 & 1.0000 & 0.9333 \\ 0.0500 & 0.0053 & 0.2000 & 0 & 0 & 0.2857 & 0 & 1.0000 & 0.9744 & 1.0000 & 1.0000 \end{bmatrix} \tag{29}$$

Step 2. Use (9) and (10) to determine the optimal alternative $\overline{Y}$ and worst alternative $\underline{Y}$.

$$\begin{aligned} \overline{Y} &= (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1), \\ \underline{Y} &= (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0). \end{aligned} \tag{30}$$

Step 3. The average distance and distance standard deviation are calculated, respectively, by (11)-(14).

$$\begin{aligned} M_1 &= (0.5964, 0.7684, 0.5429, 0.4494, 0.5823, 0.4898, 0.4143, 0.2638, 0.2601, 0.3639, 0.3810), \\ M_2 &= (0.4036, 0.2316, 0.4571, 0.5506, 0.4177, 0.5102, 0.5857, 0.7362, 0.7399, 0.6361, 0.6190), \\ S_1 &= (0.4189, 0.3733, 0.3952, 0.4921, 0.4422, 0.3943, 0.3436, 0.3566, 0.3752, 0.3810, 0.3706), \\ S_2 &= (0.4679, 0.6896, 0.4059, 0.5041, 0.4766, 0.3949, 0.3904, 0.6225, 0.6399, 0.4812, 0.4511), \end{aligned} \tag{31}$$

where $M_1 = (m_{11}, \ldots, m_{1,11})$, $M_2 = (m_{21}, \ldots, m_{2,11})$, $S_1 = (s_{11}, \ldots, s_{1,11})$, $S_2 = (s_{21}, \ldots, s_{2,11})$.

Step 4. To determine the indicator weights,

$$\begin{aligned} \overline{w} &= (0.0849, 0.0952, 0.0900, 0.0722, 0.0804, 0.0902, 0.1035, 0.0997, 0.0947, 0.0933, 0.0959), \\ \underline{w} &= (0.0942, 0.0639, 0.1086, 0.0874, 0.0925, 0.1116, 0.1129, 0.0708, 0.0689, 0.0916, 0.0977), \end{aligned} \tag{32}$$

where $\overline{w} = (\overline{w}_1, \ldots, \overline{w}_{11})$, $\underline{w} = (\underline{w}_1, \ldots, \underline{w}_{11})$.

Step 5. The grey positive relational degree r_{ij}^{+} and grey negative relational degree r_{ij}^{-} can be obtained by (15) and (16), $i = 1, 2, \ldots, 7$; $j = 1, 2, \ldots, 11$. Then, we can get the grey positive relational grade R_i^{+} and grey negative relational grade R_i^{-}.

$$\begin{aligned} R_1^{+} &= 0.6370, \\ R_2^{+} &= 0.7606, \\ R_3^{+} &= 0.5232, \\ R_4^{+} &= 0.2856, \\ R_5^{+} &= 0.4550, \\ R_6^{+} &= 0.5826, \\ R_7^{+} &= 0.5248, \\ R_1^{-} &= 0.3835, \\ R_2^{-} &= 0.3510, \\ R_3^{-} &= 0.4171, \\ R_4^{+} &= 0.8114, \\ R_5^{-} &= 0.6250, \\ R_6^{-} &= 0.6078, \\ R_7^{-} &= 0.6838. \end{aligned} \tag{33}$$

Step 6. At last, the comprehensive grey TOPSIS evaluation results can be derived as follows:

$$\begin{aligned} R_1 &= 0.6242, \\ R_2 &= 0.6842, \\ R_3 &= 0.5564, \\ R_4 &= 0.2604, \\ R_5 &= 0.4213, \\ R_6 &= 0.4894, \\ R_7 &= 0.4342. \end{aligned} \tag{34}$$

For the sake of convenience, the data in Table 3 retains three significant digits. Obviously, the presented method in this paper is consistent with the sorting results of the reference method [27] and TOPSIS. According to the comprehensive grey TOPSIS evaluation results, the order is $R_2, R_1, R_3, R_6, R_7, R_5, R_4$. In general, the ranking results of the methods are consistent, but locally, the change of evaluation values is different by Figure 2. The attribute value of the single shot kill probability of missiles (P) of X_5, X_6, and X_7 is relatively close in Table 2. And the weights $\overline{w}_7 = 0.1035$ and $\underline{w}_7 = 0.1129$ are the biggest in the same kind of attribute index weights. And the weights of attribute indexes with large difference are relatively small. These lead to a close ranking of X_5, X_6, and X_7.

Example 3. For the multiattribute missile selection problem, this example will delete or add alternative to evaluate whether there is a rank reversal phenomenon based on the original data of Example 2. The traditional TOPSIS method usually uses Euclidean distance to measure, and the weights of different attributes are fixed, which easily causes the phenomenon of rank reversal [30]. In the decision-making process of the proposed method, when the alternatives are compared with

Table 3: The comparison between the presented method and reference method.

quantization results	R_1	R_2	R_3	R_4	R_5	R_6	R_7
The presented method	0.624	0.684	0.556	0.260	0.421	0.489	0.434
The reference method [27]	0.638	0.716	0.586	0.281	0.425	0.474	0.431
TOPSIS	0.689	0.787	0.632	0.274	0.442	0.497	0.451

Table 4: The ranking results of the comprehensive grey TOPSIS evaluation in this paper.

	Normalized decision matrix											evaluation results	Rank
	$M_{\max}$	$M'_{\max}$	$O_{\max}$	$H_{\max}$	$R_{\max}$	N	P	T	$H_{\min}$	G_0	$R_{\min}$		
Drop X_7 out of alternative set													
X_1	1	0.1842	1	0.875	1.0000	1	0	0.6667	0.7179	0.6004	0.7143	0.5557	2
X_2	0.875	1	0.8	1	0.8913	1	1	0.8333	1	0.6004	0.4286	0.6387	1
X_3	0.6	0.4211	0.8	1	0.7283	0.7143	0.2	0.667	1	0.3105	0.4286	0.4982	3
X_4	0.25	0.0105	0	0.8958	0.2609	0	0	0	0	0	0	0.2339	6
X_5	0	0	0.2	0.0833	0.0435	0.2857	0	1	0.5128	0.941	1	0.3908	5
X_6	0.05	0	0.2	0	0	0.2857	1	0.9867	0.9744	1	1	0.4698	4
Add a new alternative X_8 to the original alternative set													
X_1	1	0.1842	1	0.875	1	1	0.5	0.6667	0.7179	0.6004	0.6667	0.6145	2
X_2	0.875	1	0.8	1	0.8913	1	1	0.8333	1	0.6004	0.4	0.6733	1
X_3	0.6	0.4211	0.8	1	0.7283	0.7143	0.6	0.6667	1	0.3105	0.4	0.5467	3
X_4	0.25	0.0105	0	0.8958	0.2609	0	0.5	0	0	0	0	0.2577	8
X_5	0	0	0.2	0.0833	0.0435	0.2857	0.5	1	0.5128	0.941	0.9333	0.4134	7
X_6	0.05	0	0.2	0	0	0.2857	1	0.9867	0.9744	1	0.9333	0.4794	5
X_7	0.05	0.0053	0.2	0	0	0.2857	0	1	0.9744	1	1	0.4301	6
X_8	0.05	0	0.22	0	0	0.3571	1	1	0.9744	1	1	0.4940	4

the positive ideal solution and the negative ideal solution, the attributes have two sets of weights, and the weights can be dynamically adjusted with the change of the density of data distribution. Now we drop X_7 out of alternative set based on the second example. Then we add a new alternative $X_8 = \{2.2, 400, 2.1, 3, 8, 3.5, 0.8, 10, 0.05, 85, 0.5\}$ to the original set of Example 2. The main results are shown in Table 4.

In the case of dropping X_7 out of alternative set, the indicator weights are updated to the following results:

$$\begin{aligned} \overline{w} &= (0.0884, 0.0955, 0.0908, 0.0803, 0.0855, 0.0900, 0.0758, 0.1019, 0.0953, 0.0994, 0.0971), \\ \underline{w} &= (0.0907, 0.0916, 0.0910, 0.0895, 0.0903, 0.0909, 0.0886, 0.0922, 0.0915, 0.0920, 0.0917). \end{aligned} \tag{35}$$

In the second case, the weights $\overline{w}_j$ and $\underline{w}_j$ for the grey positive relational degree and grey negative relational degree can be determined as follows:

$$\begin{aligned} \overline{w} &= (0.0849, 0.0974, 0.0922, 0.0698, 0.0795, 0.0937, 0.0988, 0.1008, 0.0969, 0.0921, 0.0939), \\ \underline{w} &= (0.0923, 0.0642, 0.1150, 0.0940, 0.0896, 0.1264, 0.1022, 0.0697, 0.0690, 0.0865, 0.0911). \end{aligned} \tag{36}$$

The above weights are two sets of weights given after considering both positive and negative ideal solutions, which can be adjusted automatically according to the characteristics of data distribution.

Based on Example 2, the ranking results of the original alternative set are

$$R_2 \succ R_1 \succ R_3 \succ R_6 \succ R_7 \succ R_5 \succ R_4. \tag{37}$$

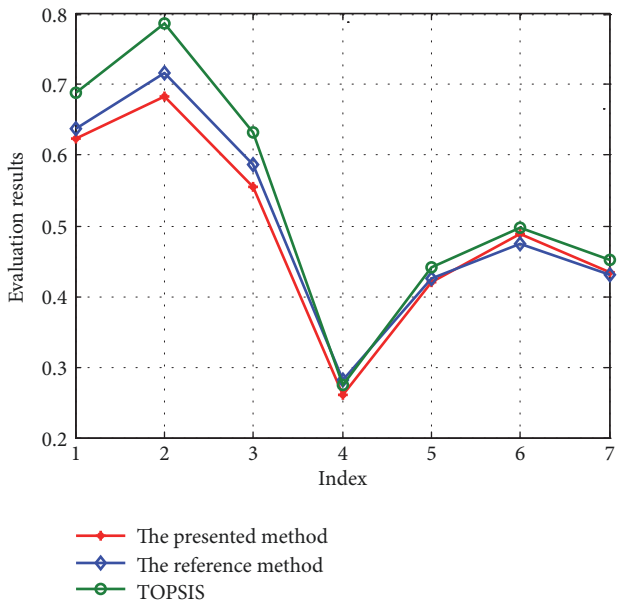

Figure 2: Evaluation results and trend.

Similarly, we can obtain the following results with dropping X_7 out of alternative set:

$$R_2 \succ R_1 \succ R_3 \succ R_6 \succ R_5 \succ R_4. \tag{38}$$

Then for the last case, we can receive the following ranking:

$$R_2 \succ R_1 \succ R_3 \succ R_8 \succ R_6 \succ R_7 \succ R_5 \succ R_4. \tag{39}$$

It is not difficult to find that the ranking of the original alternatives is unchanged under the two different circumstances, so there is no rank reversal. We believe that no rank reversal is a normal phenomenon, so the novel TOPSIS method in the paper is more in line with people's thinking logic.

Since the improved TOPSIS method in this paper has two sets of attribute weights during the evaluation process, the weights can be dynamically adjusted with the change of data distribution density no matter in reducing the evaluation objects or increasing the evaluation objects. The final result of the evaluation presented to us is that the original data will not have the phenomenon of rank reversal. Correspondingly, the traditional TOPSIS method is prone to rank reversal because the attribute weights cannot be adjusted dynamically. The specific information under the two different circumstances is shown in Table 4.

5. Conclusions

The traditional TOPSIS method is limited because it only takes the distance between the data into account, without introducing the degree of data dispersion. In addition, the classical grey relational analysis method does not study the aggregation and dispersion degree of different series of data. Based on this, a novel grey TOPSIS method developed from an improved grey relational analysis and a new indicator weight determination method, which well evaluated different objects. The improved grey relational analysis method makes full use of the discrete degree of the different sequence data and considers the overall distribution of the data, so as to avoid the shortage of the maximum and minimum distance of data. At the same time, the concept of sample distance standard deviation is proposed to solve the problem of different indicator weights determination. And there examples have successfully tested the effectiveness of the proposed method. Especially, the ranking of multiple weapons with MADM typical features verifies the feasibility for practical MADM problem. In the process of solving MADM, the present method in this paper overcomes the rank reversal problem of classical TOPSIS. It is worth noting that the proposed TOPSIS method depends on quantitative test data. As a future work, qualitative data and semantic fuzziness will also be introduced into the improved TOPSIS method to solve more complex multiattribute decision problems. In addition, we will reveal the necessity of introducing grey relational analysis into MADM problems further.

Data Availability

Data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] G. T. Temur, "A novel multi attribute decision making approach for location decision under high uncertainty," *Applied Soft Computing*, vol. 40, pp. 674–682, 2016.

[2] H. Bayram and R. Şahin, "A simulation based multi-attribute group decision making technique with decision constraints," *Applied Soft Computing*, vol. 49, pp. 629–640, 2016.

[3] Y. Zhang, W. Wang, and A. Bernard, "Embedding multi-attribute decision making into evolutionary optimization to solve the many-objective combinatorial optimization problems," *The Journal of Grey System*, vol. 28, no. 3, pp. 124–143, 2016.

[4] Q. Pang, H. Wang, and Z. Xu, "Probabilistic linguistic term sets in multi-attribute group decision making," *Information Sciences*, vol. 369, pp. 128–143, 2016.

[5] A. Baykasoğu, K. Subulan, and F. S. Karaslan, "A new fuzzy linear assignment method for multi-attribute decision making with an application to spare parts inventory classification," *Applied Soft Computing*, vol. 42, pp. 1–17, 2016.

[6] D. Xie, Y. Lu, C. Gu, and F. Li, "Scheme design considering network topology and multi-attribute decision-making for under frequency load shedding," *International Journal of Electrical Power & Energy Systems*, vol. 88, pp. 42–54, 2017.

[7] Y. He, H. Chen, Z. He, and L. Zhou, "Multi-attribute decision making based on neutral averaging operators for intuitionistic fuzzy information," *Applied Soft Computing*, vol. 27, pp. 64–76, 2015.

[8] S. Singh and M. S. Dasgupta, "Evaluation of research on CO2 trans-critical work recovery expander using multi attribute decision making methods," *Renewable & Sustainable Energy Reviews*, vol. 59, pp. 119–129, 2016.

[9] J. Qin, X. Liu, and W. Pedrycz, "Frank aggregation operators and their application to hesitant fuzzy multiple attribute decision making," *Applied Soft Computing*, vol. 41, pp. 428–452, 2016.

[10] C. L. Hwang and K. Yoon, *Multiple Attribute Decision Making: Methods and Applications*, vol. 186, Springer, Heidelberg, Germany, 1981.

[11] M. P. Amiri, "Project selection for oil-fields development by using the AHP and fuzzy TOPSIS methods," *Expert Systems with Applications*, vol. 37, no. 9, pp. 6218–6224, 2010.

[12] C.-C. Sun, "A performance evaluation model by integrating fuzzy AHP and fuzzy TOPSIS methods," *Expert Systems with Applications*, vol. 37, no. 12, pp. 7745–7754, 2010.

[13] T. Paksoy, N. Y. Pehlivan, and C. Kahraman, "Organizational strategy development in distribution channel management using fuzzy AHP and hierarchical fuzzy TOPSIS," *Expert Systems with Applications*, vol. 39, no. 3, pp. 2822–2841, 2012.

[14] Z.-L. Yue, "TOPSIS-based group decision-making methodology in intuitionistic fuzzy setting," *Information Sciences*, vol. 277, pp. 141–153, 2014.

[15] T. Wang, J. Liu, J. Li, and C. Niu, "An integrating OWA–TOPSIS framework in intuitionistic fuzzy settings for multiple attribute decision making," *Computers & Industrial Engineering*, vol. 98, pp. 185–194, 2016.

[16] Z. Pei, "A note on the TOPSIS method in MADM problems with linguistic evaluations," *Applied Soft Computing*, vol. 36, article no. 3048, pp. 24–35, 2015.

[17] S. Zhou, W. Liu, and W. Chang, "An improved TOPSIS with weighted hesitant vague information," *Chaos, Solitons & Fractals*, vol. 89, pp. 47–53, 2016.

[18] D. Joshi and S. Kumar, "Intuitionistic fuzzy entropy and distance measure based TOPSIS method for multi-criteria decision making," *Egyptian Informatics Journal*, vol. 15, no. 2, pp. 97–104, 2014.

[19] F. Torfi, R. Z. Farahani, and S. Rezapour, "Fuzzy AHP to determine the relative weights of evaluation criteria and Fuzzy TOPSIS to rank the alternatives," *Applied Soft Computing*, vol. 10, no. 2, pp. 520–528, 2010.

[20] H. J. Shyur, "COTS evaluation using modified TOPSIS and ANP," *Applied Mathematics and Computation*, vol. 177, no. 1, pp. 251–259, 2006.

[21] J. Li, J. Zhou, and Y. Zhang, "Comprehensive evaluation of energy efficiency based on TOPSIS for protocols of collaborative wireless sensor networks," *International Journal of Distributed Sensor Networks*, vol. 2015, Article ID 640736, 13 pages, 2015.

[22] L.-I. Tong, C.-H. Wang, and H.-C. Chen, "Optimization of multiple responses using principal component analysis and technique for order preference by similarity to ideal solution," *The International Journal of Advanced Manufacturing Technology*, vol. 27, no. 3-4, pp. 407–414, 2005.

[23] Z. Zhang, Y. Wang, and Z. Wang, "A grey TOPSIS method based on weighted relational coefficient," *The Journal of Grey System*, vol. 26, no. 2, pp. 112–123, 2014.

[24] D. Ju-Long, "Control problems of grey systems," *Systems & Control Letters*, vol. 1, no. 5, pp. 288–294, 1982.

[25] L. Sifeng, Y. Yingjie, W. Lifeng et al., *Grey System Theory and Its Application*, Science Press, Beijing, China, 7th edition, 2014.

[26] Y. Wenguang, W. Yunjie, and W. Jianmin, "Grey confidence interval estimation of small samples based on numerical characteristics," *The Journal of Grey System*, vol. 29, no. 2, pp. 113–124, 2017.

[27] H. Gu and B. Song, "Study on effectiveness evaluation of weapon systems based on grey relational analysis and TOPSIS," *Journal of Systems Engineering and Electronics*, vol. 20, no. 1, pp. 106–111, 2009.

[28] M. Dağdeviren, S. Yavuz, and N. Kilinç, "Weapon selection using the AHP and TOPSIS methods under fuzzy environment," *Expert Systems with Applications*, vol. 36, no. 4, pp. 8143–8151, 2009.

[29] D.-L. Mon, C.-H. Cheng, and J.-C. Lin, "Evaluating weapon system using fuzzy analytic hierarchy process based on entropy weight," *Fuzzy Sets and Systems*, vol. 62, no. 2, pp. 127–134, 1994.

[30] Y.-M. Wang and Y. Luo, "On rank reversal in decision analysis," *Mathematical and Computer Modelling*, vol. 49, no. 5-6, pp. 1221–1229, 2009.