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Research Article

The Coarse-Position-Free Coarse-Time Positioning Method for BDS Receiver

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The Coarse-Time Positioning method is important for the quick positioning of the BeiDou Navigation Satellite System (BDS) navigation receiver in the weak signal environment. The coarse position estimation is the key technology of the Coarse-Time Positioning (CTP) for the BDS navigation receiver without coarse position assistance. In this paper, a Coarse-Position-Free Coarse-Time Positioning method based on mixed-type code phase ambiguity resolution is proposed. In this CTP method, in order to estimate the approximate position, the coarse position estimation method based on the mixed-type code phase ambiguity search is used. This method does not require any additional external auxiliary position information to complete the coarse position estimation. Based on the real observation data and dynamic simulation observation data, the test experiment is designed. According to the error of coarse position estimation, the positioning accuracy, and the success rate of the coarse-time solution, the Coarse-Position-Free Coarse-Time Positioning (CPFCTP) method of the BDS receiver based on the mixed-type code phase ambiguity resolution is evaluated. The experimental results show that, when the coarse-time deviation is within 30 s, the CPFCTP method has a success rate of over 97%. When the coarse-time deviation is within 5 s, the success rate of positioning is 100%. At the same time, the experimental results also show that, for the situation that the navigation signals of some BeiDou satellites are completely occluded, the CPFCTP method proposed in this paper can obtain the correct code phase ambiguity and still maintain a high success rate of CTP at the same time.

1. Introduction

The Coarse-Time Positioning (CTP) method of satellite navigation receiver can reduce the Time-to-First-Fix (TTFF) of the navigation receiver, and the repositioning time after the navigation signal is lost under high dynamic condition and can improve the availability [1] of the receiver positioning service under the condition of weak signal environment. Before the frame synchronization of the navigation signal, it can directly use the code phase observation to perform the rapid positioning and the navigation positioning solution, thus saving the steps that traditional navigation receivers require to obtain the complete pseudorange observation. In this way, the TTFF within 1s [2] can be obtained, and the CTP method is also the core technology of the A-BDS receiver. However, in traditional CTP, the coarse position of the receiver shall be obtained in advance. For the roaming A-BDS receiver, this method cannot guarantee the reliable coarse position information, so, how to estimate the coarse position has become a difficult problem in the CTP method of A-BDS receiver [2–4]. In the weak signal conditions such as in urban canyons and dense forests as well as in the presence of occlusion environment, the quality of navigation signal declines rapidly, which will bring a great challenge to the A-BDS receiver to complete the coarse position estimation and rapid positioning [5].

At present, a variety of Global Navigation Satellite Systems (GNSSs) are developing rapidly. By the end of 2018, the BDS will complete the construction of the basic global constellation system and will firstly cover the countries along the Belt and Road in response to the Chinese government's One Belt One Road initiative where the Belt and Road refers to the "Silk Road Economic Belt" and the "21st Century Maritime Silk Road". With the progress of BDS global system construction, it is urgent to improve the navigation ability of BeiDou navigation receiver under conditions such as weak signal, occlusion environment, and incomplete navigation information. In these cases, the BeiDou navigation receiver with assisted navigation function can show better adaptability. The traditional assisted satellite navigation receiver can obtain the coarse position, approximate time, and assisted ephemeris data through communication connection [6], thus providing effective support information for navigation receivers to complete positioning in various adverse environments. In general, coarse-time synchronization [7] can be achieved through mobile communication networks, which is a typical method for obtaining approximate time information by mobile positioning terminals. With the assistance of the coarse-time information obtained from the mobile communication networks, the receiver can start the process of the CTP. The solution of the CTP requires the receiver to obtain the current approximate position. However, the acquisition of approximate position often requires the communication base station to provide additional positioning function for the mobile terminal integrated with the module of the BeiDou navigation receiver, which increases the construction cost of positioning assistance system and reduces the availability of the BeiDou navigation receiver in the adverse environment. Therefore, the coarse position estimation is a key step in the CTP of BeiDou navigation receiver.

A lot of researches have been carried out on the CTP without coarse position assistance in the literature. In literature [8], a positioning algorithm based on wide area grid search is proposed. The computational complexity of this algorithm is very large. Literature [9, 10] obtained the receiver's coarse position based on the Doppler aided coarse positioning. This method is only suitable for stationary or low dynamic satellite navigation receivers. When the receiver's speed exceeds a certain limit, the error of coarse positioning will become big, which leads to the incorrectness of subsequent pseudorange recovery. Literature [2] improves the rapid positioning method of the Doppler based coarse positioning, mainly by adding constraint information; however, this method does not fundamentally solve the failure problem of the Doppler based coarse positioning under dynamic conditions. In literature [11], it is proposed to screen the coarse position estimation results based on the residual information of satellite clock with the aid of high-precision clock. In literature [4], it is proposed to perform coarse position estimation and CTP method based on 3-satellite code phase ambiguity search combined with elevation auxiliary information. This method is suitable for users on the ground or near the ground. However, this method requires auxiliary elevation information to estimate the coarse position, and there is a risk of positioning failure for receivers that lack the external position assistance information. In order to solve the problem of Coarse-Position-Free Coarse-Time Positioning (CPFCTP) of BeiDou navigation receiver, based on mixed-type code phase ambiguity resolution, this paper proposed a new CTP resolution method without coarse position. This method does not need to obtain coarse position by external auxiliary positioning system and is based on the existing critical information of the satellite navigation receiver to complete CTP with no coarse position.

The structure of this paper is organized as follows. In Section 2, the CTP method with no coarse position assistance and the traditional CTP method are compared, and a quick coarse position estimation method with no coarse position assistance was given. This method is based on mixed-type code phase ambiguity search. In Section 3, we evaluate the CTP method of BeiDou navigation receiver with no coarse position assistance based on mixed-type code phase ambiguity resolution, and we show the corresponding statistic of positioning error and the success rate of CTP. Based on the real and simulated experimental data, the results under different observation conditions are compared, and the positioning accuracy and success rate of the CTP method with no coarse position assistance are analyzed. Finally, the conclusion was given in Section 4.

2. The Coarse-Time Positioning Method for BeiDou Navigation Receiver Based on Quick Coarse Position Estimation

2.1. The Coarse-Time Positioning Method for BeiDou Navigation Receiver. When A-BDS receiver is in urban canyons and dense forests, it is often difficult to decode data from navigation messages, especially critical data for navigation resolution [12] such as the time of signal transmission and broadcast ephemeris. Then, the traditional positioning solution will be completely ineffective. For the navigation signal from the BeiDou navigation satellites with low received signal energy level, although it is difficult for the BeiDou navigation receiver to complete bit synchronization, frame synchronization, and navigation message decoding, it can extract different types of code phase observation by the acquisition and tracking of the navigation signal, such as sub-1 ms code phase observation, sub-2 ms code phase observation, and sub-20 ms code phase observation.

Here the sub-1 ms, sub-2 ms, and sub-20 ms observations are different types of partial pseudorange observation, that is, code phase observation, where the symbol ms represents the time unit of milliseconds, and the 1 ms in the symbol sub-1 ms represents one type of length unit of the code phase observation, which represents the distance traveled by light in a vacuum for 1 millisecond, which is approximately 300 km. The 2 ms in the symbol sub-2 ms represents one type of length unit of the code phase observation, which represents the distance traveled by light for 2 milliseconds in vacuum, approximately 600 km. The 20 ms in the symbol sub-20 ms represents one type of length unit of code phase observation, which represents the distance traveled by light in a vacuum for 20 milliseconds, approximately 6000 km. The sub-1 ms observation represents the section of the complete pseudorange observation measured below 1 millisecond, and its length is between 0 m and 300 km. The sub-2 ms observation represents the section of the complete pseudorange observation measured below 2 milliseconds, and its length is between 0 m and 600 km. The sub-20 ms observation represents the section of the complete pseudorange observation measured below 20 milliseconds, and its length is between 0 m and 6000 km.

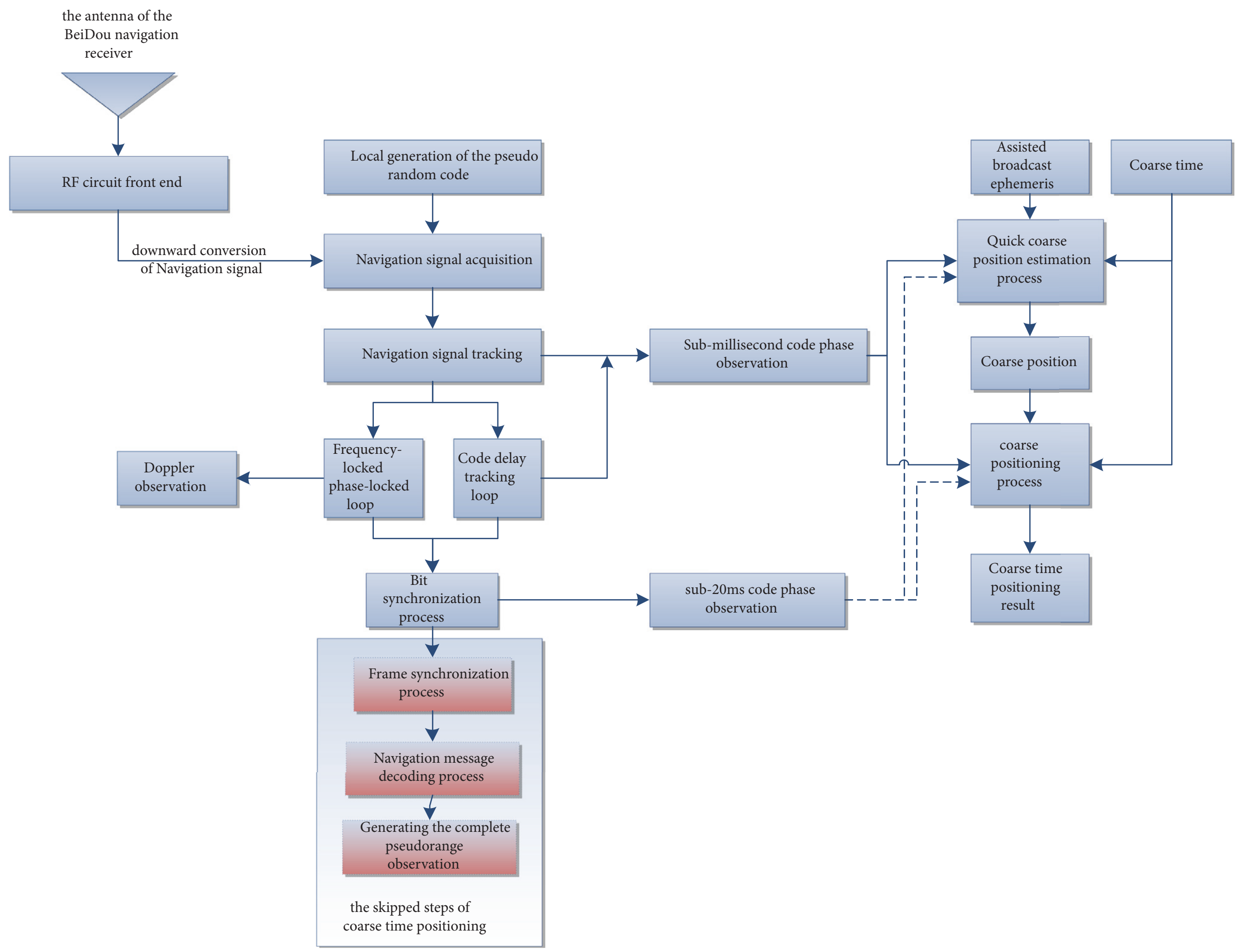

Figure 1: The CPFCTP process based on different types of code phase observation.

By collecting the code phase observation generated by the BeiDou navigation receiver, the position solution [13] can be directly carried out, thus completing the CTP process of the BeiDou navigation receiver. The CTP process based on the code phase observation below milliseconds is shown in Figure 1. In Figure 1, when the bit synchronization of the navigation signal is completed, the code phase observation of sub-20 ms can be obtained. The CTP method using this observation is relatively simple [1, 4], and under the condition of weak signal propagation environment, it is usually easier to obtain code phase observation of below 1 millisecond. When the navigation signal propagation environment is relatively poor, the quality of signal from navigation satellite with different observation angle varies greatly, so it often happens that the current visible satellite with navigation signal of high received energy level can obtain sub-20 ms code phase observation quickly, while for the satellite with navigation signal of low received energy level, the length of the time needed to generate the sub-1 ms code phase observation is already relatively long.

For the BDS system, there are usually three frequencies of navigation signal, namely, B1I, B2I, and B3I. The ranging code period of the three frequencies is 1 ms [14, 15]. For the convenience of description, only the B1I navigation signal is studied in this paper. The initial coarse position should be obtained first, and the coarse positioning process can be completed by using the code phase observation; then the precondition of coarse-time resolution would be satisfied.

In order to complete the CTP, the BDS receiver usually needs to obtain the following assistance information from external communication connection: the current valid satellite ephemeris data and the current time information with the precision of several seconds, the code phase observation, and the coarse position of the BeiDou receiver with the precision of tens of kilometers, in which the current time data can also be obtained from the high-precision real-time clock of the navigation receiver; the coarse position can be obtained by external base station or map matching positioning. But the expense of the extra positioning system for obtaining the coarse position is relatively high. This paper mainly describes how to use the BeiDou navigation receiver's own data to obtain the approximate position.

According to the principle of coarse position estimation, based on the currently valid satellite ephemeris data, the

Table 1: Different types of code phase observations generated by BeiDou receiver at different positioning stages.

positioning stages	BeiDou MEO satellite	BeiDou IGSO satellite	BeiDou GEO satellite
Signal acquisition	Sub-1 ms code phase	Sub-1 ms code phase	Sub-1 ms code phase
Bit synchronization	Sub-20 ms code phase	Sub-20 ms code phase	Sub-2 ms code phase
Frame synchronization	Complete pseudorange	Complete pseudorange	Complete pseudorange

current time information with the precision of several seconds, the code phase observation, and the coarse position of the navigation receiver with the precision of tens of kilometers, the relative integer ambiguity of the code phase observation can be estimated and then the complete pseudorange observation can be reconstructed.

The calculation method for generating complete pseudorange observation is described in (1) ~ (2) [16, 17]:

$$\begin{aligned} \mathrm{Pr}_r^s &= \mathrm{T}_{pr} \cdot c = \left(\left(T_r + \delta t_r\right) - \left(T_s + \delta t_s\right)\right) \cdot c \\ &= \left(T_r - T_s\right) \cdot c + \left(\delta t_r - \delta t_s\right) \cdot c, \end{aligned} \tag{1}$$

$$T_s = t_{tow} + \left(30 n_w + n_b\right) \cdot 20ms + \left(n_c + \frac{CP}{C_0}\right) \cdot 1ms, \tag{2}$$

where Pr_r^s is the complete pseudorange observation from the receiver r to the satellite s; T_r and T_s, respectively, are the time of reception and time of transmission of the navigation signal; δt_r is the receiver clock bias; and δt_s is the satellite clock bias. T_{pr} is the propagation time of the navigation signal, t_{tow} is the time of week [14], and t_{tow} can be used to calculate the time of the week in seconds. For the BeiDou D1 navigation message, TOW is defined as the number of seconds that have occurred since the last Sunday, 00:00:00 of BDT. The TOW count occurs at the leading edge of preamble first bit of the subframe. For the BeiDou D2 navigation message, TOW count starts from zero at 00:00:00 of BDT on every Sunday. In format D2, TOW refers to the leading edge of preamble first bit in subframe 1 of each frame; n_w is the number of full words of the received navigation message; n_b is the full number of bits in the current word of the navigation message, and a full word contains 30 bits, each of which has a duration of 20 milliseconds; n_c is the number of the integrated C/A codes in the current bit of the navigation message; ms represents the unit of time in milliseconds; and CP is the current code phase observation in the C/A code.

For the BeiDou navigation satellites with different orbit types, the navigation receiver can obtain different types of code phase observation at different processing stages such as acquisition and tracking of navigation signal. The detailed information is shown in Table 1.

In the BeiDou navigation receiver's CTP method, it is necessary to calculate the receiver's 3D position, common bias, and coarse time simultaneously, at which it is no longer dependent on the time of week information from the navigation message. The bit synchronization and frame synchronization step in navigation signal processing can be skipped simultaneously.

The observation equation of CTP is shown in (3) [2]

$$\begin{aligned} \rho_r^i &= \rho_r^{i,N} \cdot L_r^{i,ms} + \rho_r^{i,f} + \varepsilon_r^i = c \cdot \left(t_r^i - t_t^i\right) \\ &= \left\| \mathbf{s}^i\left(t_t^i\right) - \mathbf{r}\left(t_r^i\right) \right\| + b + \delta_r^i + \varepsilon_r^i, \end{aligned} \tag{3}$$

where ρ_r^i is the pseudorange observation from BDS receive r to navigation satellite i; $L_r^{i,ms}$ is the length of 1-ms code phase or 20-ms code phase; $\rho_r^{i,N}$ is the integer ambiguity of the code phase observation; $\rho_r^{i,f}$ is the code phase observation; ε_r^i is the pseudorange observation residuals; c is the speed of light; t_t^i is the transmission time of the navigation signal; t_r^i is the time when the navigation signal is received; $\mathbf{s}^i(t_t^i)$ is the position of satellite i at the time of signal transmission; $\mathbf{r}(t_r^i)$ is the position of receiver r when the navigation signal is received; b is the common bias, mainly including the clock bias and hardware delay of the BeiDou receiver, and its unit is meters; and δ_r^i are errors such as atmospheric delay and multipath effect.

The predicted pseudorange calculated by the BeiDou receiver based on the coarse position and coarse time is $\tilde{\rho}_r^i$, $\delta\rho_r^i$ is the prediction residuals based on the observation equation of CTP, $\delta\mathbf{r}$ is the correction of position $\mathbf{r}$, and δt_c is the correction of coarse time t_c. For the A-BDS receiver, the CTP equation [18, 19] is shown in

$$\delta\rho_r^i = \rho_r^i - \tilde{\rho}_r^i = -\mathbf{e}_r^i \cdot \delta\mathbf{r} + V_r^i \cdot \delta t_c + b + \varepsilon_r^i, \tag{4}$$

where $\mathbf{e}_r^i$ is the unit vector of the direction from the approximate position of the BeiDou receiver r to the position of satellite i that can be calculated with the approximate time; V_r^i is the projection of the change rate of the pseudorange in the line of sight direction between the coarse position of receiver r and the position of satellite i that can be calculated with the approximate time; it is usually calculated on the basis of approximate time and broadcast ephemeris data, as shown in (5); b is the common bias, mainly including the clock bias error and hardware delay error of the BeiDou receiver; and ε_r^i is the residual of the observation.

$$V_r^i = \mathbf{e}_r^i \cdot \mathbf{v}^i - \delta \dot{t}^i, \tag{5}$$

where V_r^i is the pseudorange change rate and $\mathbf{v}^i$ and $\delta\dot{t}^i$, respectively, are the velocity and clock drift of BeiDou satellite i. For m in-view BeiDou satellites, the matrix representation

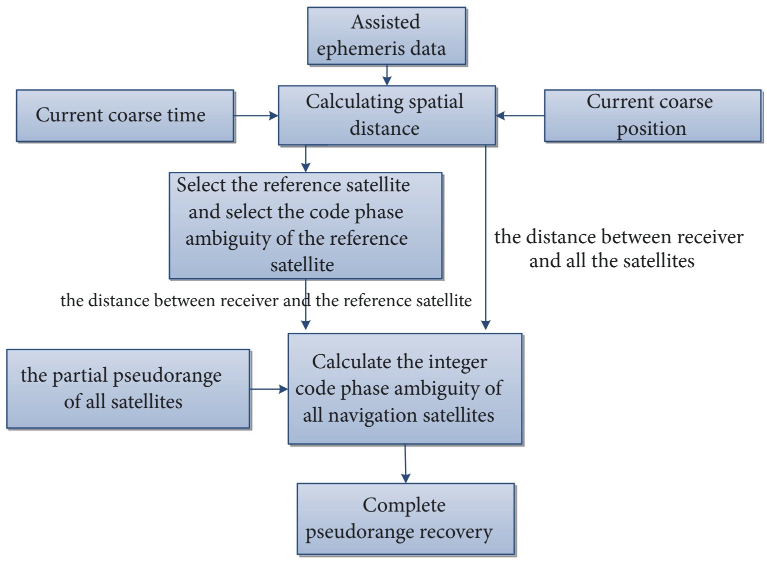

Figure 2: Relative code phase ambiguity calculation method with known coarse position and coarse time.

of the equation of CTP method in the ECEF coordinate system is shown in (6) ~ (8) [19]:

$$\Delta \boldsymbol{\rho} = \begin{bmatrix} -e_{rx}^{(1)} & -e_{ry}^{(1)} & -e_{rz}^{(1)} & 1 & V_r^{(1)} \\ -e_{rx}^{(2)} & -e_{ry}^{(2)} & -e_{rz}^{(2)} & 1 & V_r^{(2)} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -e_{rx}^{(m)} & -e_{ry}^{(m)} & -e_{rz}^{(m)} & 1 & V_r^{(m)} \end{bmatrix} \begin{bmatrix} \delta r_x \\ \delta r_y \\ \delta r_z \\ \delta t_r \\ \delta t_c \end{bmatrix}, \quad (6)$$

$$\Delta \mathbf{X} = \begin{bmatrix} \delta r_x \\ \delta r_y \\ \delta r_z \\ \delta t_r \\ \delta t_c \end{bmatrix}, \quad (7)$$

$$\mathbf{A} = \begin{bmatrix} -e_{rx}^{(1)} & -e_{ry}^{(1)} & -e_{rz}^{(1)} & 1 & V_r^{(1)} \\ -e_{rx}^{(2)} & -e_{ry}^{(2)} & -e_{rz}^{(2)} & 1 & V_r^{(2)} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -e_{rx}^{(m)} & -e_{ry}^{(m)} & -e_{rz}^{(m)} & 1 & V_r^{(m)} \end{bmatrix}, \quad (8)$$

where $\mathbf{A}$ is the design matrix; $\Delta\mathbf{X}$ is the state correction vector of the receiver; δr_x is the correction value of positioning on X axis in ECEF coordinate system; δr_y is the correction value of positioning on Y axis in ECEF coordinate system; δr_z is the correction value of the positioning on Z axis of the ECEF coordinate system; δt_r is the correction value of receiver clock bias error; δt_c is the correction value of the coarse time of the navigation receiver; $\Delta\boldsymbol{\rho}$ is the system of observation error equations expressed by linearization, $\begin{bmatrix} e_{rx}^{(m)} & e_{ry}^{(m)} & e_{rz}^{(m)} \end{bmatrix}$ is the unit vector on the direction of the in-view satellite m, and $V_r^{(m)}$ is the change rate of the pseudorange from receiver r to satellite m.

Normally, the standard positioning can be carried out so long as the condition of 4 in-view navigation satellites is satisfied. At least 5 in-view navigation satellites are needed in order to solve the CTP equation. This is because the coarse time t_c also needs to be obtained in the solution of the CTP equation at the same time. If the condition of the CTP is satisfied, (9) can be used to carry out the resolution of coarse positioning equation.

$$\mathbf{X} = \Delta\mathbf{X} + \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\Delta\boldsymbol{\rho} \quad (9)$$

When the assistance information, such as the coarse position of the navigation receiver and the current coarse time, is obtained, the relative code phase ambiguity can be obtained through the method of single difference of the pseudorange between 2 satellites to eliminate the influence of common bias on the basis of the reference satellite with the highest elevation angle, and then the complete pseudorange observation can be recovered. The calculation method of relative code phase ambiguity is shown in Figure 2 when the coarse position and coarse time of the navigation receiver are obtained.

The method of calculating the range of the propagation time delay between navigation satellite and the receiver is shown in Figure 3. In the figure, the Earth can be approximately regarded as a perfect sphere when the accuracy of estimation allows this. The range of pseudorange from the ground user to the satellite can be obtained based on the approximate value of the Earth radius and the satellite orbit. The minimum pseudorange $d_{\min}$ can be obtained when the ground user is at the subsatellite point; the maximum pseudorange value $d_{\max}$ can be obtained when the connection line between the ground user and the satellite is approximately tangent to the Earth sphere.

The position of the navigation satellite can be obtained by using the actual broadcast ephemeris of the navigation

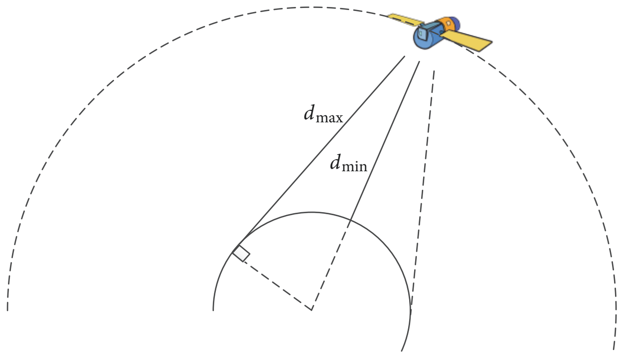

Figure 3: Schematic diagram of calculation of propagation delay range between navigation satellite and receiver.

satellite, and the approximate distance between the navigation satellite and the ground user can be calculated based on the position of the navigation satellite, which is also the approximate propagation distance of the navigation signal from the navigation satellite to the ground user. A rough navigation signal propagation time can be obtained by this approximate propagation distance of the navigation signal. According to the statistical results of the distance between the navigation satellite and the ground user calculated based on the actual broadcast ephemeris of the BeiDou satellites in the year of 2018, the average propagation time delay from the BeiDou MEO satellite to the ground user is about 80 ms; the propagation time delay from the BeiDou GEO/IGSO satellite to the ground user is about 126 ms; for the terrestrial users, the propagation time delay range of the BeiDou GEO/IGSO navigation satellite to terrestrial users is about 118 ms~140 ms, and the PRN numbers of the BeiDou GEO/IGSO navigation satellites are 1~10 and 13; the propagation time delay range from the BeiDou MEO navigation satellite to terrestrial users is about 71 ms~91 ms, and the PRN numbers of the BeiDou MEO navigation satellites are 11, 12, and 14.

When the BeiDou receiver is in the environment where the energy level of the received navigation signal changes dramatically, different types of code phase observation are usually obtained for different navigation satellites at the initial positioning stage. For navigation satellites with quick bit synchronization, the code phase observation of sub-2 ms or sub-20 ms can be obtained. But for navigation satellites with low received navigation signal energy level, only sub-1 ms code phase observations are available. Therefore, in the calculation of CTP, it is often necessary to deal with the mixed-type code phase observations and to generate the relative ambiguity of the mixed-type code phase observation according to the assistance information such as coarse position and coarse time.

2.2. The Coarse Position Estimation Method Based on Mixed-Type Code Phase Ambiguity Resolution. The core of CTP method without coarse position assistance is the coarse position estimation, and the coarse position estimation method, based on code phase ambiguity resolution, can obtain the coarse position, which can meet the precision requirement of CTP resolution. Assuming that, for all in-view BeiDou navigation satellites, only code phase lock is completed, the BeiDou navigation receiver obtains sub-1 ms code phase observation or sub-20 ms code phase observation. In this case, use 3 BeiDou navigation satellites with elevation constraints or 4 BeiDou navigation satellites to estimate the coarse position, of which, the code phase integer ambiguity combination number that needs to be calculated is still a large value. In order to obtain a smaller integer ambiguity search space of code phase, the constraint conditions can be added to the search space of code phase ambiguity by means of intersatellite distance constraint. The coarse position estimation method based on mixed-type code phase ambiguity resolution is shown in Figure 4.

The coarse positioning method based on mixed-type code phase ambiguity resolution includes the following steps:

(1) Choose several 4-satellite combinations participating in the coarse position estimation; each combination must satisfy the following conditions: for 2 of the BeiDou navigation satellites, the receiver has obtained sub-20 ms code phase observation; for the other visible satellites, the receiver has only obtained sub-1 ms code phase observation; and the GDOP value of the 4-satellite combination is relatively small.

For 3 or 4 visible BeiDou satellites of any 4-satellite combination, if the corresponding sub-20 ms code phase observations have been obtained, then the size of the integer ambiguity search space of the corresponding mixed-type code phase observations is already small and no further study is needed. Therefore, the research objective of this paper is mainly solving the following problem: For the code phase observation of 4 satellites, only the sub-20 ms code phase observations corresponding to 2 visible BeiDou satellites are obtained. In this case, how to estimate the approximate position of the receiver based on the existing mixed-type code phase observation is a problem worth studying. If this problem is solved, the time consumption of the positioning for the satellite navigation receiver will be shorter, and the receiver will be more adaptable to the harsh navigation signal propagation environment.

(2) Choose one combination with the smallest search space of code phase ambiguity from several 4-satellite combination candidates, and generate corresponding integer code phase ambiguity search space, wherein the compression method based on intersatellite distance is used to obtain a smaller code phase ambiguity search space.

(3) Iterate over the search space of integer code phase ambiguity corresponding to the selected 4-satellite combination, estimate the coarse position on the basis of each ambiguity combination candidate, respectively, and calculate the RSS residuals according to the estimation results. Finally, choose the most likely integer code phase ambiguity according to the minimum value of the RSS residual, and the result of the coarse position solution corresponding to this combination is the final coarse positioning result.

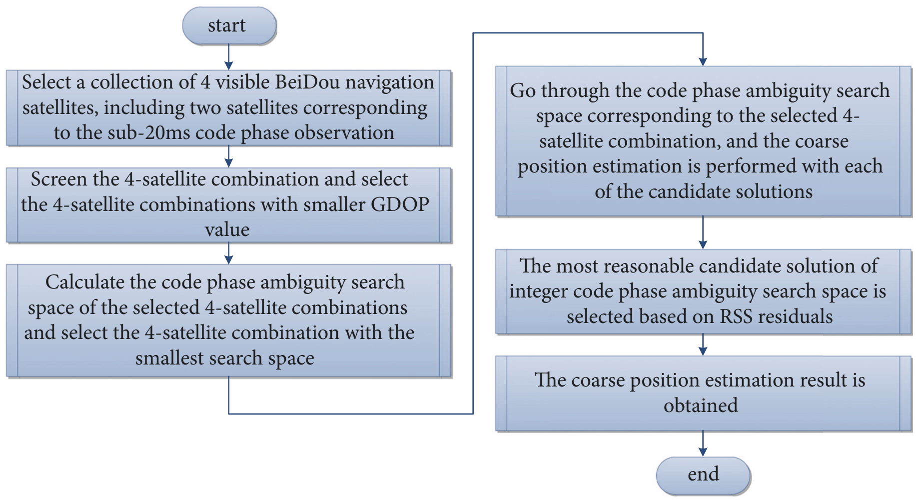

FIGURE 4: A coarse position estimation method based on the mixed-type code phase ambiguity resolution.

The statistic of RSS residual is used when searching for correct integer code phase ambiguity, and the calculation method of RSS residual statistic is shown in (10) ~ (14):

$$RSS = \min\left(RSS1, RSS2, RSS3, RSS4\right), \tag{10}$$

$$RSS1 = \operatorname{mod}\left(\left\|\mathbf{s}_i - \mathbf{u}\right\| + c \cdot \delta t_u - \rho_i^{frac} - c \cdot \delta t_i, L_{i,ms}\right), \tag{11}$$

$$RSS2 = \operatorname{mod}\left(\left\|\mathbf{s}_i - \mathbf{u}\right\| + c \cdot \delta t_u, L_{i,ms}\right) - \rho_i^{frac} - c \cdot \delta t_i, \tag{12}$$

$$RSS3 = \operatorname{mod}\left(\left\|\mathbf{s}_i - \mathbf{u}\right\| + c \cdot \delta t_u - c \cdot \delta t_i, L_{i,ms}\right) - \rho_i^{frac}, \tag{13}$$

$$RSS4 = \operatorname{mod}\left(\left\|\mathbf{s}_i - \mathbf{u}\right\| + c \cdot \delta t_u - \rho_i^{frac}, L_{i,ms}\right) - c \cdot \delta t_i, \tag{14}$$

where $\mathbf{s}_i$ is the position of satellite i; $\mathbf{u}$ is the position of receiver; δt_u is the receiver clock bias; $L_{i,ms}$ is the length of one unit of code phase ambiguity corresponding to satellite i; ρ_i^{frac} is a code phase observation corresponding to satellite i; c is the speed of light in the vacuum; δt_i is the satellite clock bias of satellite i; $\operatorname{mod}(\cdot,\cdot)$ is the modulus operator; and min is the minimum operator.

When all the 4 navigation satellites have sub-20 ms code phase observations, the corresponding search space of the integer code phase ambiguity is usually small. But under the condition of low received navigation signal energy level, it is usually difficult to obtain sub-20 ms code phase observation of 4 navigation satellites. In contrast, the coarse position estimation method based on the mixed-type code phase ambiguity resolution has many advantages under the condition of low received navigation signal energy level or with less number of receiver channels. When the navigation receiver obtains only 2 sub-20 ms code phase observations corresponding to 2 satellites and 2 satellites corresponding sub-1 ms code phase observations, the method can be used to perform the CTP without coarse position. Compared with the need for 4 satellites to simultaneously obtain sub-20 ms code phase observations, the TTFF can be reduced. Compared with the coarse position estimation method based on elevation hypothesis with 3 in-view satellites, the method proposed in this paper does not need the assistance of elevation information and does not need to use the average projection of multiple satellites to calculate the initial position. When conducting a coarse ambiguity search, the initial position of the receiver in the ECEF coordinate system is just set to $[0, 0, 0]$.

Three satellites are selected from the 4 BeiDou satellites participating in the coarse position estimation. The 3 satellites include 1 satellite corresponding to sub-20 ms code phase observation and the 4th BeiDou navigation satellite corresponding to sub-20 ms code phase observation. For navigation satellites which can obtain sub-20 ms code phase observations, the corresponding sub-1 ms code phase observations can also be obtained at the same time. There are 3 maximum intersatellite pseudorange difference constraints among the 3 BeiDou satellites, assuming that the maximum intersatellite pseudorange difference constraints of the 3 satellites are sorted from small to large, expressed as $[x, y, z]$, and the maximum pseudorange difference obtained by (15) is integer because of the upward rounding operation. In order to simplify the calculation, the part below millisecond is ignored when comparing the pseudorange difference between satellites; only the integer millisecond part is considered. Under the maximum pseudorange difference constraint, the calculation process of all possible integer ambiguity combinations of 3 satellites is described as follows.

Assume that the 3 navigation satellites are i, j, and k, the maximum pseudorange difference between satellite i and satellite j is x, the maximum pseudorange difference between satellite i and satellite k is y, and the maximum pseudorange difference between satellite j and satellite k is z;

n is the representation of the integer millisecond pseudorange difference between satellite i and satellite j.

The method of calculating the maximum value of the intersatellite pseudorange difference is shown in (15) ~ (19):

$$\Delta\rho_{\max}^{ij} = ceil\left(\rho_j - \rho_i + d\right), \tag{15}$$

$$\alpha = \arcsin\left(\frac{R_e}{R_j}\right), \tag{16}$$

$$\beta = \arccos\left(\frac{\left(R_j^2 + L_{ij}^2 - R_i^2\right)}{R_j}\right) - \alpha, \tag{17}$$

$$\rho_j = \sqrt{R_j^2 - R_e^2}, \tag{18}$$

$$\rho_i = \sqrt{\rho_j^2 + L_{ij}^2 - 2\rho_j L_{ij}\cos\beta}, \tag{19}$$

where $ceil$ is the upward rounding function; $\Delta\rho_{\max}^{ij}$ is the maximum integer pseudorange difference between satellite i and satellite j; ρ_i and ρ_j, respectively, are the distance from receiver to satellite i and satellite j; R_e is the Earth radius; L_{ij} is the distance between satellite i and satellite j; R_i and R_j, respectively, are the distance from satellite i and satellite j to the Earth center; d is the error compensation for calculating maximum intersatellite pseudorange difference, and the calculation process of d is shown in Appendix A.

For all the 3 navigation satellites with sub-1 ms code phase observations, if satellite i is BeiDou MEO satellite, then set the integer code phase ambiguity of satellite i as 80 ms; if satellite i is BeiDou GEO/IGSO satellite, set the integer code phase ambiguity of satellite i as 120 ms. Based on the above settings, the search space of integer code phase ambiguity corresponding to satellite j and satellite k can be obtained by the method described in Appendix B.

The coarse position calculation using the code phase observation of 4 BeiDou satellites has the following advantages: it can meet the requirements of at least 4 in-view navigation satellites for the coarse position calculation, and no additional auxiliary conditions are required, such as the elevation information assistance of the BDS receiver or using the base station of mobile communication to assist positioning. Assuming that the sub-20 ms code phase observations corresponding to 2 of the 4 visible navigation satellites have been obtained, by introducing the sub-20 ms code phase observations, on the basis of the code phase ambiguity search space corresponding to 3 navigation satellites, a code phase ambiguity search space corresponding to the 4th navigation satellite can be added further, which will not cause a larger expansion of the search space. At the same time, it can meet the need of 4 in-view navigation satellites when estimating the coarse position; otherwise, we must use other auxiliary information to estimate the approximate position, for example, using the method of 3-satellite coarse position estimation based on elevation assistance. Compared with the 3-satellite coarse position estimation method with elevation assistance, the coarse position estimation method based on 4-satellite observations does not require a prior elevation information, so the method can be applied to a wider range of application scenarios.

The indexes of 4 BeiDou navigation satellites involved in the coarse position estimation process are i, j, k, and m. First, assume that satellite i and satellite j correspond to sub-20 ms code phase observations, where the satellite i and satellite j must be BeiDou MEO/IGSO satellites, because if the satellites i and j are BeiDou GEO satellites, then the corresponding code phase observations are sub-2 ms code phase observations, which will cause the search space of code phase ambiguity to expand too much, resulting in more time consuming of the coarse position estimation process based on code phase ambiguity search and, then, it will lose the practical significance of quick coarse position estimation method.

The generation method of the integer code phase ambiguity search space for 4 BeiDou navigation satellites is the extension of the code phase ambiguity search space generation method based on 3 BeiDou satellites. First, choose a BeiDou satellite$i$, whose corresponding sub-20 ms code phase observations and sub-1 ms code phase observations can be obtained at the current epoch. When the satellite i is a BeiDou MEO satellite, set the default code phase ambiguity as 80 ms; the corresponding 20 ms code phase ambiguity value is 4, and the corresponding 1 ms code phase ambiguity value is 80. When satellite i is a BeiDou IGSO satellite, set the default code phase ambiguity as 120 ms, where the 20 ms code phase integer ambiguity is 6 and the 1 ms code phase integer ambiguity is 120. Then, choose a BeiDou navigation satellite j, both of whose corresponding sub-20 ms code phase observations and sub-1 ms code phase observations can be obtained at current epoch. For the BeiDou satellite k and m participating in the coarse position estimation, only the corresponding sub-1 ms code phase observations can be obtained at present. When generating the search space of code phase integer ambiguity for 4 navigation satellites, the maximum difference of the integer ambiguity between 2 satellites with sub-20 ms code phase observations is 1. By setting the 20 ms code phase integer ambiguity of the 1st navigation satellite, the search space of the 20 ms code phase integer ambiguity for the 2nd navigation satellite can be generated. The size of code phase ambiguity search space is s, $s = 1$ *or* 3.

Attention should be focused here, in the generation of code phase ambiguity search space for 4 BeiDou satellites, the firstly selected 2 satellites with sub-20 ms code phase observations are not necessary satellite i and j. According to (16) ~ (20), the maximum pseudorange difference between 4 BeiDou satellites can be obtained, then the maximum pseudorange difference is converted to the integer code phase ambiguity, during which the maximum pseudorange difference between 2 satellites with sub-20 ms code phase observation is converted to the constraint condition of the integer 20 ms code phase ambiguity, and the other maximum pseudorange difference values are converted to the constraint condition of integer 1 ms code phase ambiguity.

3. Results and Discussions

Based on the actual observation data of the BeiDou receiver and the simulated observation data of the dynamic carriers, for the CTP method with no coarse position assistance proposed in this paper, the corresponding practical performance is evaluated. The experimental data in this paper are as follows: the BeiDou broadcast ephemeris and the BeiDou observations of the static station are from the IGS-MGEX project, and the precise ephemeris data are from the iGMAS project, of which the precise ephemeris data is used in the software simulation of the observation for the dynamic carriers.

The configuration of the computer environment for the prototype program based on CTP with no coarse position assistance is as follows: the operating system is 64 bit WIN7 SP1, the processor is Intel (R) Core (TM) i5-8250U @ 1.60 GHz, and the software development environment is 64 bit Matlab R2017a. The observation simulation software is the self-developed simulation software for the BeiDou receiver. The simulation software is implemented in the Matlab language, which is convenient for the configuration and modification of different simulation scenarios.

Code phase observations are generated from real pseudorange observations or simulated pseudorange observations by removing the corresponding integer code phase ambiguity. For example, for sub-20 ms code phase observation, it can be obtained from the complete pseudorange by removing the integer 20 ms code phase ambiguity. For the BDS system, the code phase observations are sub-1 ms code phase observations, sub-2 ms code phase observations, and sub-20 ms code phase observations. The corresponding ambiguity values are 1 ms code phase integer ambiguity, 2 ms code phase integer ambiguity, and 20 ms code phase integer ambiguity.

In order to analyze the performance of the CTP method with no coarse position assistance under the dynamic condition, we designed the simulation experiments of the code phase observation for the navigation receiver and used the observation simulation software to generate the code phase observation for the BeiDou receiver under dynamic conditions and the corresponding pseudorange observation.

The basic principle of observation software simulation for the BeiDou receiver is realized by software programming according to the mathematical model and principle of GNSS orbit determination and positioning, and its key model parameters, system errors, random errors are realized by using the data products (precise satellite orbit, precise clock bias, ionospheric parameters, etc.) provided by the iGMAS project and IGS-MGEX project. The general flow of BDS code phase observation and pseudorange observation simulation using the iGMAS project data product and IGS-MGEX project data is shown in Figure 6 of Appendix C. In Figure 6, the ionospheric error is simulated by using the data of the ionospheric model in RINEX format file provided by the IGS-MGEX project. This method not only has the advantages of the conventional software simulation methods, but also makes the observations obtained through the software simulation more close to the real observations. It can also be said that it has the advantages [20] of some hardware simulation methods.

In order to evaluate the performance of CTP with no coarse position assistance based on mixed-type code phase ambiguity resolution, the coarse position estimation accuracy, CTP accuracy, and positioning success rate are evaluated, respectively. For the evaluation of CTP accuracy, two evaluation indexes are used, which are the Circular Error Probable (CEP) and Spherical Error Probable (SEP). They can be used to evaluate the horizontal positioning accuracy and the overall positioning accuracy, respectively. The calculation methods of CEP and SEP are described in formula (20) ~ (23) [21]:

$$CEP = \begin{cases} 0.562\sigma_E + 0.615\sigma_N & \sigma_E \geq \sigma_N \\ 0.615\sigma_E + 0.562\sigma_N & \sigma_E < \sigma_N, \end{cases} \tag{20}$$

$$SEP = \overline{\sigma} \cdot \left(1 - \frac{d}{9}\right)^{3/2}, \tag{21}$$

$$\overline{\sigma}^2 = {\sigma_E}^2 + {\sigma_N}^2 + {\sigma_U}^2, \tag{22}$$

$$d = \frac{2\left({\sigma_E}^4 + {\sigma_N}^4 + {\sigma_U}^4\right)}{\overline{\sigma}^4}, \tag{23}$$

where σ_E, σ_N, and σ_U, respectively, are the Root Mean Square (RMS) error in the east direction, north direction, and up direction of the ENU coordinate system.

3.1. Performance Evaluation of Coarse Position Estimation Method for BeiDou Navigation Receiver. Using the observation data of the IGS static station JFNG, we tested and evaluated the coarse position estimation method based on mixed-type code phase ambiguity resolution. The date of the observation is February 6, 2018, and the sampling interval is 30s. The pseudorange observation of BeiDou B1 frequency and the code phase observation generated by the corresponding simulation method are used to in the test. Among them, the PRN numbers of BeiDou navigation satellite are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 13. The coarse-time deviation is set to 5s, 10s, 15s, 20s, 25s, and 30s, respectively. The sub-20 ms code phase observations corresponding to PRN 6 and 9 BDS satellites are selected as the first two sub-20 ms code phase observations obtained at the current epoch, and the sub-1 ms code phase observations corresponding to other navigation satellites have been obtained. At all the epochs, the initial position is set to $[0, 0, 0]$, and then the coarse position estimation is carried out based on the 4-satellite code phase ambiguity search method. The error statistic of RMS (95%) can be obtained by collecting coarse position estimation results for a period of time (about 3 hours), and the error statistics on the X, Y, and Z axis and 3D directions of the ECEF coordinate system are collected, respectively. At the same time, the success rate of the CTP method without coarse position assistance is calculated, and the result is shown in Table 2.

It can be seen from Table 2 that with the increase of coarse-time deviation, the positioning error of the 4-satellite coarse positioning method proposed in this paper is gradually increasing, and with the development of the timekeeping system of satellite navigation receiver, generally, the coarse-time deviation condition within 30 s is satisfied in the practical application. The 3D RMS error obtained by using the

Table 2: Test 1 results of the coarse position estimation method based on mixed-type code phase ambiguity resolution (unit: m).

coarse time deviation	Error Type				
	RMS error in the X-axis direction	RMS error in the Y-axis direction	RMS error in the Z-axis direction	3-D RMS error	CTP success rate
5s	884	2669	1217	3065	100%
10s	1813	5367	2439	6169	99%
15s	2741	8061	3661	9270	98.6%
20s	3710	10781	4895	12409	98%
25s	4700	13504	6136	15561	97.3%
30s	5652	16234	7363	18701	97%

Table 3: Test 2 results of the coarse position estimation method based on mixed-type code phase ambiguity resolution (unit: m).

coarse time deviation	Error Type				
	RMS error in the X-axis direction	RMS error in the Y-axis direction	RMS error in the Z-axis direction	3-D RMS error	CTP success rate
5s	884	2669	1217	3065	100%
10s	1813	5367	2439	6169	99%
15s	2741	8061	3661	9270	98.6%
20s	3710	10781	4895	12409	98%
25s	4700	13504	6136	15561	97.3%
30s	5652	16234	7363	18701	97%

4-satellite coarse positioning method in this paper is less than 20 km, and the success rate of coarse positioning is over 97%. Under the condition of 5s coarse-time deviation, the success rate of CTP is 100% and the coarse positioning error is about 3 km.

Set the coarse-time deviation as 5s, 10s, 15s, 20s, 25s, and 30s, respectively, and then choose sub-20 ms code phase observations corresponding to the PRN 7 and 13 BDS satellites as the first two code phase observations obtained at the current epoch; the sub-1 ms code phase observations corresponding to other satellites have been obtained. At all the epochs, the initial solution position is set to $[0, 0, 0]$, and then the coarse position estimation is carried out based on the 4-satellite code phase ambiguity search method. The error statistic of RMS (95%) can be obtained by collecting coarse position estimation results for a period of time (about 3 hours), and the error statistics on the X, Y, and Z axis and 3D directions of the ECEF coordinate system are, respectively, calculated. At the same time, the success rate of the CTP method with no coarse position assistance is calculated, and the result is shown in Table 3.

From the statistical results of coarse positioning in Table 3, it can be seen that with the increase of coarse-time deviation, the positioning error of the 4-satellite coarse positioning method proposed in this paper is gradually increasing, and compared with the coarse position estimation results in Table 2, we can see that, of the 4 satellites involved in the coarse position estimation, if the 2 satellites corresponding to the 20 ms code phase observations are different, the error of the coarse position estimation and the success rate of the coarse positioning are barely affected. This is because, in the case of the same coarse-time deviation, for these 2 different mixed-type code phase observation combinations, the same code phase ambiguity resolution success rate can be obtained based on the CPFCTP method proposed in this paper. After the code phase ambiguity is resolved successfully, the recovered full pseudorange observation of the coarse positioning stage can be obtained. At this time, for the above two tests, the obtained complete pseudorange observations of the coarse positioning stage are all the same, because the real pseudorange observations of these two tests are in fact identical, and all of them are obtained from the same simulation experiment in which only the ambiguities of the code phase observation are different. Therefore, the errors of the results obtained by performing the coarse position estimation based on these recovered pseudorange observations should be the same. This is consistent with the statistical results of the coarse positioning errors in Tables 2 and 3.

3.2. Coarse-Position-Free Coarse-Time Positioning Method Based on Mixed-Type Code Phase Ambiguity Resolution. The date of the software simulation test is October 13, 2016. By using the precise satellite orbit, precise clock bias error, and ionospheric parameters of the same day, the simulated observations of the BeiDou receiver are generated. Then, the simulated code phase observation is processed using the method of CTP with no coarse position. During the CTP, the broadcast ephemeris data is used, which is from IGS-MGEX project. In the test process of CTP method, the sub-1 ms code phase observations and sub-20 ms code phase observations are used for CTP in each epoch to test the actual performance of positioning resolution with no coarse position at each epoch. In this test, the coarse-time deviation is set to 20s, and

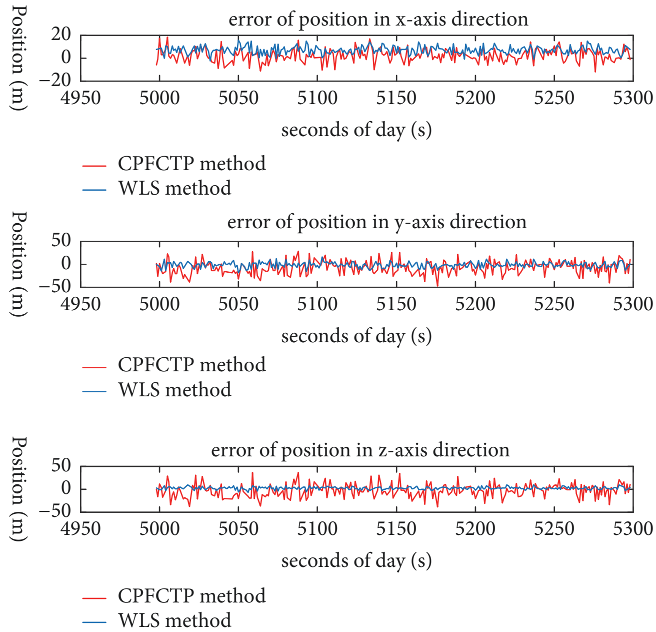

Figure 5: Comparison of positioning error between CPFCTP method and WLS method.

Table 4: Result error of CPFCTP method of the BeiDou navigation receiver (unit: m).

Error type	X	Y	Z	CEP	SEP
result error	4.2	10.6	9.2	6.5	13.6

Table 5: Errors in the results of the standard positioning method of the BeiDou receiver (unit: m).

Error type	X	Y	Z	CEP	SEP
result error	7.5	6.0	3.2	5.99	9.3

the method proposed in this paper can be used to calculate the CTP with no initial coarse position. At the stage of coarse position estimation, the sub-20 ms code phase observations corresponding to the PRN 7 and 9 BDS satellites are used in this test at each epoch, as the first 2 sub-20 ms code phase observations obtained at the current epoch. The sub-1 ms code phase observations of the other BeiDou satellites have been obtained. The positioning errors of the CPFCTP method on the X, Y, and Z axis of ECEF coordinate system are shown in Figure 5. The statistical results of CTP errors in this test are shown in Table 4. At this point, the 3D error of coarse positioning is about 14408 meters, and the success rate of coarse positioning is about 99.6%.

The weighted least square (WLS) method is used to deal with the complete pseudorange observations generated by the same observation simulation experiment. The positioning error of the standard WLS positioning method is tested here, and the results are shown in Figure 5. The error statistics of standard positioning method are shown in Table 5.

By comparing the results in Tables 4 and 5, it can be seen that the accuracy of CTP results is slightly lower than that of the WLS positioning method based on complete pseudorange observations. The accuracy of coarse positioning results is about 10 meters, which can meet the needs of real-time navigation. The horizontal positioning accuracy of CTP method is similar to that of standard standalone positioning method based on WLS, but in vertical direction, the positioning accuracy of the CTP method is much lower than that of the standard positioning method based on WLS.

Using the software simulation test method, set the visible satellites number as 3, 6, 7, 8, 9, 10, 11, 13, and 14, and set the invisible satellites number as 1, 2, 4, and 5 in the simulation test. Simulate the test scene with signal occlusion, and set the coarse-time deviation of the current epoch as 20s. The time length of the whole test is about 3 hours. Among them, in each epoch, the method proposed in this paper is used to calculate the solution of CTP with no coarse position, and set the initial position of each epoch as [0, 0, 0]. At the stage of coarse position estimation, in this test, the sub-20 ms code phase observations corresponding to the PRN 7 and 9 BDS satellites are used in each epoch and regarded as the first two sub-20 ms code phase observations obtained at the current epoch, and the sub-1 ms code phase observations corresponding to other navigation satellites have been obtained. The statistical results of the coarse position estimation error of the test experiment are shown in Table 6. At this point, the 3D error of the coarse position estimation is about 14408 meters, and the success rate of coarse positioning is 99.6%. In the result of CTP in this test, the CEP value is 5.4m and the SEP is 13 m.

From the statistical results of coarse positioning errors in Table 6, it can be seen that in the scenario with signal occlusion, as long as the configuration of visible satellite constellation can still satisfy the conditions of coarse position estimation and CTP resolution, the success rate of CTP will not be reduced. Compared with the error results of coarse position estimation in Table 3, the error of coarse position

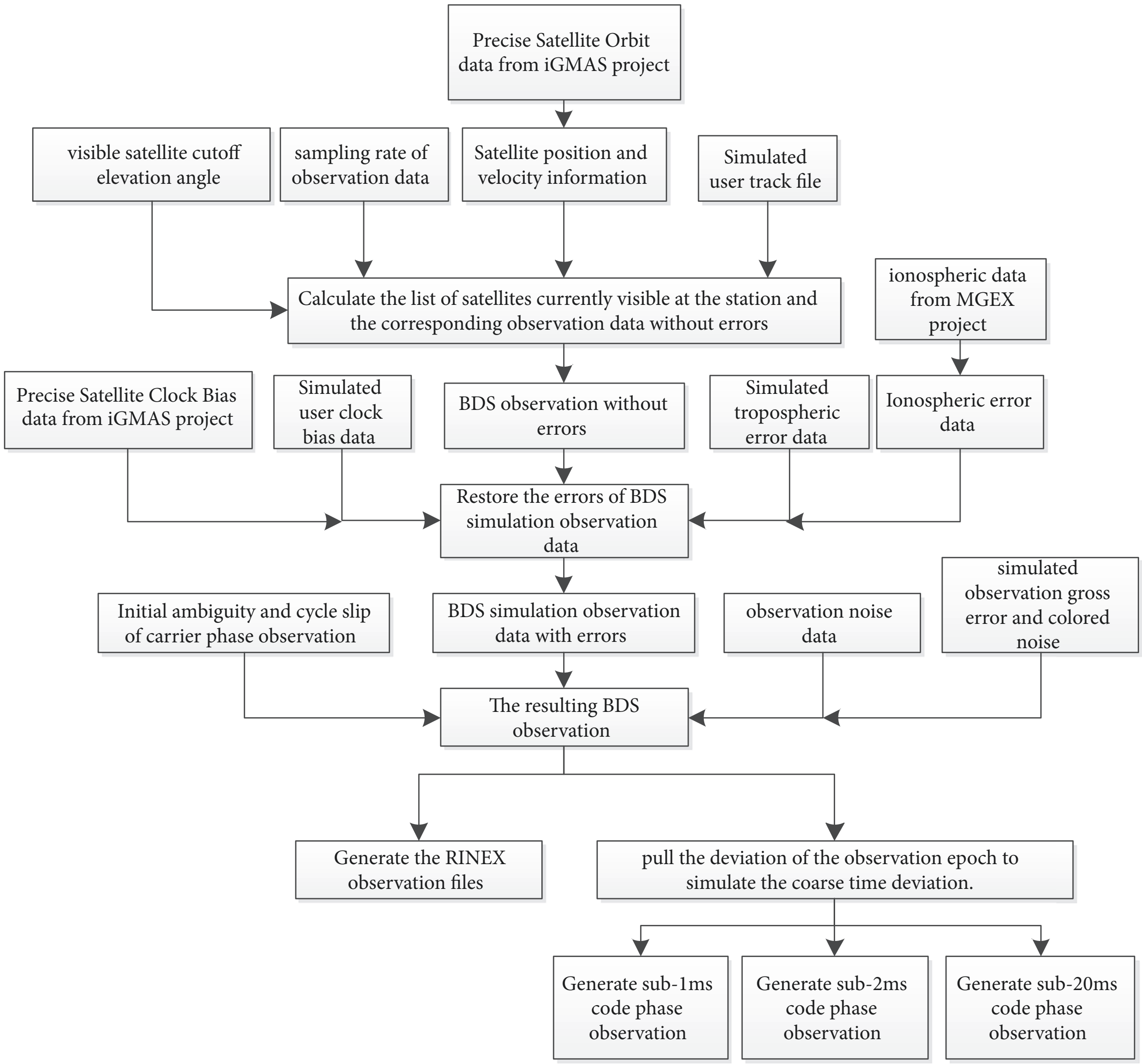

Figure 6: BDS observation simulation process using data products from the iGMAS project and data from IGS-MGEX project.

Table 6: Test results of coarse position estimation method based on mixed-type code phase ambiguity resolution in signal occlusion scene (unit: m).

Coarse time deviation	Error type				CTP success rate
	X	Y	Z	3-D	
20s	4806	11597	7071	14408	99.6%

estimation is larger. For the 3D positioning error that still meets the condition of coarse-time resolution, the success rate of positioning does not decrease.

4. Conclusion

This paper analyzes the related problems of the CTP for the BeiDou receiver when there is no coarse position assistance, and proposes a Coarse-Position-Free Coarse-Time Positioning (CPFCTP) method based on the mixed-type code phase ambiguity resolution. Through theoretical analysis and test analysis, using actual observation data and software simulation testing data, the accuracy and positioning success rate of the CTP method with no coarse position assistance are evaluated. The main work of this paper is as follows:

(1) The BeiDou receiver can perform coarse position estimation and the CTP directly based on the code phase observations of 4 satellites, which does not need any auxiliary initial position assistance and does not need the elevation information of the receiver. The CPFCTP method can obtain the solution of the coarse time with a relatively high success rate when the coarse-time deviation is within 30s. This method has a success rate of 97% or more. When the coarse-time deviation is within 5 s, the positioning success rate is 100%.

(2) The positioning accuracy of the Coarse-Time Positioning method is close to the positioning accuracy of the standard standalone positioning method based on the complete pseudorange observations. The positioning accuracy of

the CPFCTP method for the BeiDou receiver is close to that of the standard standalone positioning method, but slightly lower than the latter. However, the positioning accuracy of the proposed CPFCTP method is still on the order of 10 meters, which meets the actual needs of real-time navigation.

(3) In the case that some of the navigation signals are completely occluded, the correct code phase ambiguity resolution can be performed using the CPFCTP method, and the accuracy of the coarse position estimation is slightly reduced but still satisfies the condition of the CTP resolution, of which the positioning success rate is basically the same as that of this method when all the navigation signals are available.

The coarse position estimation method based on the mixed-type code phase ambiguity resolution proposed in this paper also has some shortcomings. For example, in order to obtain the correct relative code phase ambiguity combination of 4 satellites, this method requires a larger search space of the code phase ambiguity candidate than that of the 3-satellites code phase ambiguity search method. Considering the increased performance of the BeiDou receiver's data processing device, the added overhead can bring the benefit of the direct use of 4 satellites' observations in the stage of coarse positioning; therefore, this method has no requirements for the initial coarse position, without the need for additional auxiliary information, such as elevation assumptions or additional positioning assistance systems. In addition, the prototype program of this paper is only implemented on the PC platform, and the dynamic standalone positioning experiment is only implemented on the software simulation platform; therefore, the error statistics of the Coarse-Time Positioning experiments conducted in this paper can only reflect the performance of the proposed CPFCTP method under the simulation conditions. However, these results can also provide guidelines for the future work. In order to evaluate the actual positioning error statistic of the CPFCTP method in the actual scene, our next step is to port the prototype program to the embedded platform of the BeiDou receiver.

For the BeiDou GEO satellites, the sub-2 ms code phase observation corresponding to the GEO satellite can be obtained after completing the bit synchronization step of the navigation signal; however, this type of observation does not contribute much to search space compression of the code phase ambiguity. Compared with that of the GPS system, the number of sub-20 ms code phase observations of the BDS system during the bit synchronization step is limited. However, if the navigation message transmission rate of the GEO satellites and their better observation condition in the Asia-Pacific region can be utilized, a better BDS solution can be provided for solving the CTP problem with no coarse position assistance. This additional information can provide some inspiration for future work.

Appendix

A.

In order to estimate the error compensation parameter d, the following statements mainly analyze the influence of the Earth radius error on the intersatellite pseudorange difference. The orbital height of the BeiDou MEO satellite is about 21,528 km, the orbital height of the BeiDou GEO/IGSO satellite is about 35,786 km, the length of the Earth's long semiaxis is about 6,378 km, the length of the Earth's short semiaxis is about 6,357 km, and the change of the Earth's radius varies from 0 m to 22 km.

For the intersatellite pseudorange difference between two BeiDou satellites, the partial derivative of the maximum intersatellite pseudorange difference $\Delta\rho_{\max}^{ij}$ to the Earth's radius R_e can be calculated based on the calculation method of the maximum intersatellite pseudorange difference shown in (15) ~ (19).

$$\begin{aligned}\frac{\partial\Delta\rho_{\max}^{ij}}{\partial R_e} &= \frac{\left(\rho_j-\rho_i\right)R_e}{\rho_i\rho_j} + \frac{\rho_j L_{ij}\sin\left(\beta\right)}{\rho_i R_j\sqrt{1-\left(R_e/R_j\right)^2}} \\ &\quad - \frac{R_e L_{ij}\cos\left(\beta\right)}{\rho_i\rho_j} \\ &\le \frac{\left(\rho_j-\rho_i\right)R_e}{\rho_i\rho_j} + \frac{\rho_j L_{ij}\sin\left(\beta\right)}{\rho_i R_j\sqrt{1-\left(R_e/R_j\right)^2}} \\ &\le \frac{\left(\rho_j-\rho_i\right)R_e}{\rho_i\rho_j} + \frac{\rho_j \left(R_i-R_e\right)}{\rho_i R_j\sqrt{1-\left(R_e/R_j\right)^2}}\end{aligned} \tag{A.1}$$

According to the navigation signal propagation time delay range of the BeiDou satellites of different orbit types described in Section 2.1, the navigation signal propagation time delay range of the BeiDou GEO/IGSO satellites to ground users is about 118 ms~140 ms. The maximum value of the signal propagation time delay is about 140 ms, the minimum value is about 118 ms, and the maximum intersatellite difference of signal propagation time delay from two different BeiDou GEO/IGSO satellites to ground users is about 22 ms. The Earth's radius is about 6,300 km, which is equivalent to the distance that light travels through a vacuum for about 22 ms. For the intersatellite pseudorange difference between the two BeiDou GEO/IGSO satellites, based on the calculation method of the partial derivative of the maximum intersatellite pseudorange difference $\Delta\rho_{\max}^{ij}$ to the Earth's radius R_e shown in (A.1), the following relationship can be obtained in which the expressions of signal propagation time delay in the unit of milliseconds are used to represent the corresponding propagation distance of the navigation signal.

$$\begin{aligned}\frac{\partial\Delta\rho_{\max}^{ij}}{\partial R_e} &\le \frac{\left(\rho_j-\rho_i\right)R_e}{\rho_i\rho_j} + \frac{\rho_j \left(R_i-R_e\right)}{\rho_i R_j\sqrt{1-\left(R_e/R_j\right)^2}} \\ &\le \frac{22\cdot 22}{118\cdot 118} + \frac{140\left(140-22\right)}{118\cdot 140\sqrt{1-\left(22/140\right)^2}} \\ &\le 1.04,\end{aligned} \tag{A.2}$$

$$\frac{\partial \Delta\rho_{\max}^{ij}}{\partial R_e} \geq -\frac{R_e L_{ij}\cos(\beta)}{\rho_i\rho_j} \geq -\frac{R_e L_{ij}\cos(\beta)}{\rho_i\rho_j} \geq -\frac{2R_e\rho_j}{\rho_i\rho_j} \geq -\frac{2\cdot 22}{140} \geq -0.3 \quad \text{(A.3)}$$

According to the navigation signal propagation time delay range of the BeiDou satellites of different orbit types described in Section 2.1, the navigation signal propagation time delay range of the BeiDou MEO satellites to ground users is about 71 ms~91 ms. The maximum value of the signal propagation time delay is about 91 ms, the minimum is about 71 ms, and the maximum intersatellite difference of signal propagation time delay from two different BeiDou MEO satellites to ground users is about 20 ms. The Earth's radius is about 6,300 km, which is equivalent to the distance that light travels through a vacuum for about 22 ms. For the intersatellite pseudorange difference between two BeiDou MEO satellites, based on the calculation method of the partial derivative of the maximum intersatellite pseudorange difference $\Delta\rho_{\max}^{ij}$ to the Earth's radius R_e shown in (A.1), the following relationship can be obtained in which the expressions of signal propagation time in unit of milliseconds are used to represent the corresponding distance of navigation signal propagation.

$$\frac{\partial \Delta\rho_{\max}^{ij}}{\partial R_e} \leq \frac{(\rho_j-\rho_i)R_e}{\rho_i\rho_j} + \frac{\rho_j(R_i-R_e)}{\rho_i R_j\sqrt{1-(R_e/R_j)^2}} \leq \frac{20\cdot 22}{71\cdot 71} + \frac{91(91-22)}{71\cdot 91\sqrt{1-(22/91)^2}} \leq 1.08, \quad \text{(A.4)}$$

$$\frac{\partial \Delta\rho_{\max}^{ij}}{\partial R_e} \geq -\frac{R_e L_{ij}\cos(\beta)}{\rho_i\rho_j} \geq -\frac{R_e L_{ij}\cos(\beta)}{\rho_i\rho_j} \geq -\frac{2R_e\rho_j}{\rho_i\rho_j} \geq -\frac{2\cdot 22}{91} \geq -0.48 \quad \text{(A.5)}$$

According to the navigation signal propagation time delay range of the BeiDou satellites of different orbit types described in Section 2.1, the navigation signal propagation time delay range of the BeiDou satellites to ground users is about 71 ms~140 ms. The maximum value of the signal propagation time delay is about 140 ms, the minimum is about 71 ms, and the maximum intersatellite difference of signal propagation time delay from the BeiDou MEO satellite and the BeiDou GEO/IGSO satellite to ground users is about 69 ms. The Earth's radius is about 6,300 km, which is equivalent to the distance that light travels through a vacuum for about 22 ms. For the intersatellite pseudorange difference between the BeiDou MEO satellite and the BeiDou GEO/IGSO satellite, based on the calculation method of the partial derivative of the maximum intersatellite pseudorange difference $\Delta\rho_{\max}^{ij}$ to the Earth's radius R_e shown in (A.1), the following relationship can be obtained in which the expressions of signal propagation time in unit of milliseconds are used to represent the corresponding propagation distance of navigation signal.

$$\frac{\partial \Delta\rho_{\max}^{ij}}{\partial R_e} \leq \frac{(\rho_j-\rho_i)R_e}{\rho_i\rho_j} + \frac{\rho_j(R_i-R_e)}{\rho_i R_j\sqrt{1-(R_e/R_j)^2}} \leq \frac{(140-71)\cdot 22}{71\cdot 140} + \frac{140(140-22)}{71\cdot 140\sqrt{1-(22/140)^2}} \leq 1.83, \quad \text{(A.6)}$$

$$\frac{\partial \Delta\rho_{\max}^{ij}}{\partial R_e} \geq -\frac{R_e L_{ij}\cos(\beta)}{\rho_i\rho_j} \geq -\frac{R_e L_{ij}\cos(\beta)}{\rho_i\rho_j} \geq -\frac{2R_e\rho_j}{\rho_i\rho_j} \geq -\frac{2\cdot 22}{140} \geq -0.3 \quad \text{(A.7)}$$

According to the results of formula (A.2)~(A.7), the maximum ratio of the error of $\Delta\rho_{\max}^{ij}$ and the Earth radius error is less than 2. Since the maximum error of the Earth's radius is about 22 km, the error of $\Delta\rho_{\max}^{ij}$ is less than 44 km. So the maximum error of the intersatellite pseudorange difference for the BDS system is less than 44 km, which is much smaller than the length of 1-ms code phase observation (about 300 km in length). Here, the error compensation value d for calculating the maximum intersatellite pseudorange difference of the BDS system can be set to 0.5 ms (about 150 km in length).

B.

The pseudocode for generating the integer code phase ambiguity search space of 3 BeiDou satellites is shown in Pseudocode 1. In the ambiguity search method proposed in this paper, the code phase observations corresponding to the 3 BeiDou satellites are sub-1 ms code phase observations. The type of solution in the code phase ambiguity candidate set is 1-ms integer code phase ambiguity, wherein the reference satellite simultaneously has the corresponding sub-20 ms code phase observation. It is easy to convert the 1-ms integer code phase ambiguity into a 20-ms integer code phase ambiguity.

Among them, *PrCodeInt(idp, 1)* is the code phase ambiguity of the navigation satellite i at the epoch *idp*, and *PrCodeInt(idp, 2)* is the code phase ambiguity corresponding to the navigation satellite j at the epoch *idp*, and *PrCodeInt(idp, 3)* is the code phase ambiguity corresponding to the satellite k at the epoch *idp*.

PrDiffCs represents all of the maximum pseudorange difference between any two pairs of pseudorange observations corresponding to 3 navigation satellites, where *PrDiffCs(1)* represents the maximum pseudorange difference between satellite i and satellite j, *PrDiffCs(2)* represents the maximum pseudorange difference between satellite i and k, *PrDiffCs(3)* represents the maximum pseudorange difference between the satellite j and k, *xSol* represents the pseudorange difference

```
idp = 1;
for xSol = -PrDiffCs(1) : PrDiffCs(1)
    if abs(xSol) <= (PrDiffCs(3) - PrDiffCs(2))
        for ySol = -PrDiffCs(2) : PrDiffCs(2)
            PrCodeInt(idp, 2) = PrCodeInt(idp, 1) + xSol;
            PrCodeInt(idp, 3) = PrCodeInt(idp, 1) + ySol;
            idp = idp + 1;
        end
    elseif abs(xSol) > (PrDiffCs(3) - PrDiffCs(2))
        if xSol > 0
            for ySol = -(PrDiffCs(3) - xSol)    : PrDiffCs(2)
                PrCodeInt(idp, 2) = PrCodeInt(idp, 1) + xSol;
                PrCodeInt(idp, 3) = PrCodeInt(idp, 1) + ySol;
                idp = idp + 1;
            end
        elseif xSol < 0
            for ySol = -PrDiffCs(2) : (PrDiffCs(3) + xSol)
                PrCodeInt(idp, 2) = PrCodeInt(idp, 1) + xSol;
                PrCodeInt(idp, 3) = PrCodeInt(idp, 1) + ySol;
                idp = idp + 1;
            end
        end
    end
end
```

Pseudocode 1: Pseudo code for generating the 3-satellite code phase ambiguity search space.

between the satellites i and j at the current time, and *ySol* represents the pseudorange difference between the satellites i and k.

The size of the code phase ambiguity search space obtained using the code phase ambiguity space search method listed in Pseudocode 1 is as shown in (B.1) [4].

$$\varphi = \begin{cases} 2\left(\sum_{i=1}^{z-y}(2y+1) + \sum_{i=z-y+1}^{x}(y+z-i+1)\right) + (2y+1), & x > z-y \\ 2\sum_{i=1}^{x}(2y+1) + (2y+1), & x \le z-y \end{cases} \quad \text{(B.1)}$$

C.

See Figure 6.

Abbreviations

AR: Ambiguity resolution
A-BDS: Assisted BDS
BDS: BeiDou Navigation Satellite System
BDT: BeiDou Navigation Satellite System Time
CEP: Circular Error Probable
CPFCTP: Coarse-Position-Free Coarse-Time Positioning
CPA: Coarse position assistance
CPAR: Code phase ambiguity resolution
CTP: Coarse-Time Positioning
ECEF: Earth Centered Earth Fixed
GEO: Geostationary Earth orbit
GNSS: Global Navigation Satellite System
GPS: Global Positioning System
ICD: Interface control document
IGS: International GNSS Service
IGSO: Inclined geostationary satellite orbit
IGS-MGEX: International GNSS Service Multi-GNSS Experiment
iGMAS: International GNSS Monitoring & Assessment System
LEO: Low Earth orbit
MEO: Medium Earth orbit
PNT: Positioning, navigation, timing
PRN: Pseudorandom number
RAC: Radial, along, cross
RMS: Root Mean Square
RSS: Root Sum of Squares
SP3: Standard product #3
SEP: Spherical Error Probable
TOF: Time of flight

TOR: Time of reception
TOT: Time of transmission
TOW: Time of week
TTFF: Time-to-First-Fix
URE: User range error
WLS: Weighted least square
3D: 3-Dimensional.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Disclosure

The affiliation "School of Space Information, Space Engineering University" is the new name of the college, whose original name was the "Department of Information Equipment in the Academy of Equipment".

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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References

[1] J. Shourang, *Study on data processing technology of accuracy and availability augmenting for global navigation satellite system [Doctoral thesis]*, National University of Defense Technology, Changsha, China, 2016.

[2] H.-W. Chen, H.-S. Wang, Y.-T. Chiang, and F.-R. Chang, "A new coarse-time GPS positioning algorithm using combined Doppler and code-phase measurements," *GPS Solutions*, vol. 18, no. 4, pp. 541–551, 2014.

[3] S. Jing, B. Xu, L. Yong, and G. Sun, "Doppler-aided rapid positioning method for BDS receivers," *IEEE Electronics Letters*, vol. 51, no. 25, pp. 2139–2141, 2015.

[4] S. Jing, B. Xu, W. Liu, and G. Sun, "A millisecond integer ambiguity search method based on the inter-satellite distance limit for coarse-time GPS positioning," *GPS Solutions*, vol. 21, no. 1, pp. 23–29, 2017.

[5] C. Song, F.-X. Wang, and Z.-W. Zhuang, "Method for assisted-GPS positioning based on ambiguity resolution in weak signal environment," *Journal on Communication*, vol. 30, no. 9, pp. 89–101, 2009.

[6] S. Cheng, *Research on the key techniques of assisted GPS positioning system [Doctoral thesis]*, National University of Defense Technology, Changsha, China, 2009.

[7] W. Peng, *Research on satellite navigation receivers positioning technology based on elevation and clock bias assisted [Doctoral thesis]*, National University of Defense Technology, Changsha, China, 2013.

[8] N. Sirola, "Exhaustive global grid search in computing receiver position from modular satellite range measurements," *Journal of Physics: Conference Series*, vol. 52, pp. 73–82, 2006.

[9] S. Jinhai, W. Feng, L. Jinhai, and Y. Yuepeng, "A novel algorithm for rapidly positioning using Doppler-aided RangeFit," *Journal of Harbin Engineering University*, vol. 32, no. 10, pp. 1351–1354, 2011.

[10] N. Othieno and S. Gleason, "Combined Doppler and time free positioning technique for low dynamics receivers," in *Proceedings of the 2012 IEEE/ION Position, Location and Navigation Symposium, PLANS 2012*, pp. 60–65, 2012.

[11] P. Wu, X. Ouyang, W. Liu, and F. Wang, "A positioning method of bds receiver under weak signal circumstances based on compressed fractional-step," *Tehnički Vjesnik*, vol. 21, no. 1, pp. 43–48, 2014.

[12] Z. Huang, *A-BDS coarse-time navigation and its application in indoor positioning [Master thesis]*, Information Engineering University, Zhengzhou, China, 2015.

[13] F. van Diggelen, *A-GPS: Assisted GPS, GNSS, and SBAS*, Artech House, Boston, Mass, USA, 2009.

[14] China Satellite Navigation Office, "BeiDou navigation satellite system signal in space interface control document open service signal B3I (version 1.0)," 2018.

[15] China Satellite Navigation Office, "BeiDou navigation satellite system signal in space interface control document open service signal (version 2.1)," 2016.

[16] S. Peng, D. Zhao, Z. Huang, Y. Tian, H. Wu, and C. Zhu, "Research and implementation of BDS/GPS coarse time navigation algorithm," in *China Satellite Navigation Conference (CSNC) 2016 Proceedings*, vol. I, pp. 227–240, 2016.

[17] X. Gang, *Principles of GPS and Receiver Design*, Publishing House of Electronics Industry, Beijing, China, 2009.

[18] F. Van Diggelen and C. Abraham, "Coarse-time AGPS; computing TOW from pseudorange measurements, and the effect on HDOP," in *Proceedings of the 20th International Technical Meeting of the Satellite Division of The Institute of Navigation 2007 ION GNSS 2007*, pp. 357–367, 2007.

[19] D. Akopian and J. Syrjarinne, "A fast positioning method without navigation data decoding for assisted GPS receivers," *IEEE Transactions on Vehicular Technology*, vol. 58, no. 8, pp. 4640–4645, 2009.

[20] C.-L. Kuang, X. Zou, and T. Li, "High precise GPS observation simulation using IGS data products," *Journal of System Simulation*, vol. 19, no. 12, pp. 2857–2863, 2007.

[21] Z. Shifeng, Y. Huabo, and C. Hong, *Inertial Guidance Weapon Precision Analysis and Evaluation*, NUDT Press, Changsha, China, 2008.