Research Article

Genetic Approach for Multiobjective Optimization of Epicyclical Gear Train

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The determination of optimal design of the planetary gear train with a lightweight, a short center distance, and a high efficiency is an important issue in the preliminary design of power transmission systems. Conventional and traditional methods have been widely used in optimization. They are deterministic and limited to solve some mechanical problems with several variables and constraints. Therefore, some optimization methods have been developed, such as the nonconventional method, the genetic algorithm (GA). This paper describes a multiobjective optimization for the epicyclical gear train system using the GA. It is aimed to obtain the optimal dimensions for epicyclical gear components like a module, number of teeth, the tooth width, the shaft diameter of the gears, and a performed efficiency under the variation of operating mode of PGT system. The problem is formulated under the satisfaction of assembly and balance constraints, bending strength, contact strength of teeth, and other dimension conditions. The mathematical model and all steps of the GA are presented in detail.

1. Introduction

Gear trains are used in most types of machinery and vehicles for power transmission. Epicyclical gear train takes a very significant place among the gear transmissions which are used in many branches of industry such as automobile, aerospace, machine tools, and turbines. It has several advantages like a smaller envelope size than a parallel shaft for the same power, low weight, coaxial shafts resulting in more compact installation, and high transmission ratio speed and efficiency [1].

The design of the planetary gear train is highly complicated and specific. This complexity leads to many design variables, mathematical formulations, constraints, and many influencing factors. Using conventional or traditional optimization techniques to solve a design problem, they could not provide an optimum result according to the complex shape and geometry of gears and many factors. Therefore, a stochastic approach genetic algorithm (GA) is applied to solve this problem and to obtain a satisfactory result as well

[2, 3]. Therefore, many approaches for planetary gear optimization, especially a minimal weight or volume and a performed efficiency, have been proposed in the literature.

Yan and Lai [4] have described a concept of elementary gear trains, which has been developed to solve the conjugate surface equations. This concept shows the solid modeling of the proposed elementary planetary gear trains. Rao [5] has used the principles of the genetic process to test isomorphism as well as to know relatively the characteristics of speed ratios and transmission efficiency, with the same number of links and degrees of freedom. Mundo [6] has studied the model of the planetary gear train with non-circular gear geometries by illustrating the example of a bicycle where the efforts of pedaling have been minimized. Rosic et al. [7] have described the genetic algorithm applied to a multiobjective optimization of a planetary gear train, especially the efficiency and the axial distance. However, this approach gives a quantitative description of the compromise between efficiency and size. Stefanovic and Milovancevic [8] have presented an original model for multicriteria

optimization of the planetary gear trains. A mathematical model for optimization is defined by the variables, objective functions, and conditions required for the proper functioning of a system. Volume, mass, efficiency, and production costs describe the objective function. Chen et al. [9] used a differential evolution algorithm to optimize a two-stage planetary gear train. They have proved an optimum design of the planetary gear train with minimum volume and weight. Höhn et al. [10] presented a study on lightweight design of planetary gear transmission. To achieve their objective, different normal modules are used for each transmission stage which is also the best way to reduce the tooth-load factors or to increase efficiency. Dilawer et al. [11] described an optimal design of the compound of the epicyclic gear train with the load analysis which was carried out in the gear trains by different modules and power levels. In the study of Akhila and Reddy [12], the main aim was to design and analyze a three-stage epicyclic planetary reduction gear unit used for a flight vehicle. Daoudi and Boudi [13] have proposed a methodology for the shape optimization of the epicyclic gear train system based on parametric shape optimization. Their objective was to design a lighter gearbox with respect to kinematic and dynamic behavior. Harsha and Rao [14] used the genetic algorithm to optimize the volume function of two stages of the planetary gear train. The variables were tooth number of the sun gear, module, tooth width, and modification coefficient of the sun gear for each stage. The software MATLAB is used with the satisfaction of the strength and geometric constraints to give results. Mei et al. [15] adopted the genetic algorithm in the entire process for the purpose of reducing the variation of transmission errors in the meshing process by the modification of the tooth profile of spur gear. Hüseyin Filiz et al. [16] applied the GA for the minimization of kinetic energy. Allowable bending stress and allowable contact stress are considered as design constraints. Minimum teeth number for a given pressure angle, center distance, recommendation on the face width, and limitations on teeth ratios are considered as geometrical and kinematical constraints. Kissling and Bae [17] have described a new optimization tool, based on KISSsys software, which permits to layout automatically a complete gearbox using functions for the layout of gear stages, shaft dimensions and bearings, lifetime, safety factors, and the total ratio. This tool is used to generate a complete set of different variants of gear reducers. The main results are displayed in 3D graphics, showing weight, costs, and efficiency of the different variants. Adarsh Kumar et al. [18] presented several optimization techniques of epicyclic gear train to reduce load failures in design improvement and optimization of the products. Daoudi and Boudi [19] have illustrated a comparison between two advanced optimization algorithms known as genetic algorithm (GA) and particle swarm optimization (PSO) in order to find the optimal combination of design parameters for minimum volume of planetary gear train. They have presented another work about the minimization of the weight and the center distance of one pair of spur gears [20]. Nandeppagoudar et al. [21] redesigned the three stages of planetary gearbox by reducing one stage which will be suitable for machine tool

application where the space and load constraints are the prime objectives. Analytical and numerical results are compared to see the effectiveness of modified planetary gearbox.

The objective of this study stems from the idea of providing an optimal design of the epicyclic gear train by the genetic algorithm approach. It would offer a global optimization of the geometrical parameters of the epicyclic gear train system as a module, number of teeth, tooth width, and shaft diameter, in order to obtain light weight, short center distance, and maximal efficiency. In addition, the specificity of the present work is that it takes into account the variation of the gear ratio or the operating modes of the PGT.

2. Genetic Algorithm

Originally developed by Holland, the genetic algorithm (GA) is a robust technique based on the natural selection and genetic production mechanism. This algorithm works with a group of possible solutions within a search space instead of a single solution, as described in gradient optimization methods [22, 23].

Genetic algorithm is referred to as a search method of the optimal solution to simulate Darwin's genetic selection and biological evolution process. In fact, it is a series of random iterations and evolutionary computations which simulate the process of selection, crossover, and mutation occurred in natural selection and population genetic as described in Figure 1. The good quality of the solution is gradually maintained and combined according to the survival of the fittest and through crossover and mutation, while continually producing better individuals and out of bad individuals [2, 24]. Through the generational produce and optimizing the individual, the whole group evolves forward and constantly approaches to the optimal solution.

3. Problem Definition

The model of the application was a single stage of an epicyclic gear train type I with three planets as shown in Figure 2. The objective of this work has been the minimization of the weight, the center distance, and the maximization of the efficiency.

Many factors and parameters like torque, material, tooth width, input speed, module, and others have been applied to design optimization as described in Table 1. The product of the transversal load factors and the application factors for bending stress and contact stress has been considered to be equal to 1 just to simplify the calculation.

3.1. Objective Function. The objective function is a quantity to be minimized or maximized by exploring a search space under the imposed constraints. In this paper, three objective functions have been defined as follows:

- (1) $F_1(b, m, Z_s, Z_p, d'_s, d'_p)$: minimization of the weight
- (2) $F_2(m, Z_s, Z_p)$: minimisation of the center distance
- (3) $F_3(\eta_{PGT})$: maximization of efficiency

FIGURE 1: Flowchart of GA for solving the problem.

FIGURE 2: Simple epicyclical gear drive type I [1].

TABLE 1: Description of parameters and variables [25, 26].

Parameters	Symbol	Coefficient value
Tooth width	b	[12 120]
Module	m	[2 20]
Tooth number of sun gear	Z_s	[17 30]
Tooth number of planet gear	Z_p	[17 50]
Diameter of sun shaft	d'_s	[10 30]
Diameter of planet shaft	d'_p	[10 40]
Input power	P	7.5
Input speed	n	1500
Density material of standard steel	ρ	7850
Helix angle factor	Z_β	1
Elastic coefficient	Z_E	189.98
Contact ratio coefficient	Z_ϵ	0.94
Product of load factor	K_i	1.75
Tooth form factor	Y_{Fa}	2
Stress concentration coefficient	Y_{Sa}	1.73
Contact ratio coefficient	Y_ϵ	1.08
Allowable contact stress	$[\sigma_{c_{adm}}]$	1100
Allowable bending stress	$[\sigma_{b_{adm}}]$	460
Allowable shearing stress of shafts	γ	30.67

The total weight of the planetary gear train has been presented by the envelope of the three components: sun gear, planet gear, and ring gear. For the number of teeth, just the number of teeth of the sun and the planet will be considered as variables. The number of teeth Z_r is generated by the linear equation of Z_s and Z_p , equation (13). Equation (1) defines the expression of the weight function:

$$F_1 = \frac{\pi}{4} \rho \cdot b [d_s^2 + n_p \cdot d_p^2 + d_r^2 - (d'_s + n_p \cdot d'_p)^2]; \quad d = m \cdot Z. \quad (1)$$

The minimization of the center distance determines smaller gears which would require less material and cost to make and less space to operate it. The following equation presents the center distance of the epicyclical gear train system [27]:

$$F_2 = \frac{m}{2} (Z_s + Z_p). \quad (2)$$

The last objective function is the efficiency, which is more detailed in the part bellow and defined as follows:

$$F_3 = \eta_{PGT}. \quad (3)$$

3.2. Constraint Functions. This optimization has been formulated with the satisfaction of many constraints. The bending strength, the contact strength, and the dynamic factor K_V were calculated according to method B of the standard ISO [28].

(i) Dimensional constraint:

$$\begin{aligned} \text{number of teeth: } Z_s &\geq Z_p, \\ \text{width coefficient of tooth: } 6 &< k \leq 12; b = m \cdot k. \end{aligned} \quad (4)$$

(ii) Balance and assembly condition for PGT:

$$\begin{aligned} \frac{Z_p + Z_s}{n_p} &= \text{integer number}, \\ Z_p + 2 &< (Z_p + Z_s) \times \sin\left(\frac{180}{n_p}\right), \\ Z_r &= 2 \times Z_p + Z_s; \\ a &= \frac{(d_s + d_p)}{2} = \frac{(d_r - d_p)}{2}. \end{aligned} \quad (5)$$

(iii) Bending strength of teeth:

$$\sigma_b = Y_{Fa} Y_{Sa} Y_{\epsilon} \frac{2KT}{bZm^2} K_A K_V K_{F\alpha} K_{F\beta} \leq [\sigma_{b_{adm}}]. \quad (6)$$

(iv) Contact strength of teeth: The expression of the contact stress in equation (7) represents the wear load of gears between planet and sun gears and between planet and ring gears:

$$\sigma_c = Z_\beta Z_H Z_E Z_\epsilon \sqrt{\frac{2T(I_{pgt} \pm 1)}{bZ^2 m^2 I_{pgt}}} K_A K_V K_{H\alpha} K_{H\beta} \leq [\sigma_{c_{adm}}]. \quad (7)$$

The condition of shaft diameters is defined in equation (8). The planet shaft is considered the most solicited in this study to simplify the calculation:

$$d_p'^3 \geq \frac{4.97e6 \cdot P}{n' \cdot \gamma}, \quad n' = \frac{n}{a}. \quad (8)$$

Therefore, the previous expression shows that eight constraints must be introduced in the objective functions. The syntax of the constraints in the genetic algorithm has been presented by the function $g_i(x)$, where i presents the number of all constraints; in our case, $i = 1, \dots, 8$. The symbol x presents the variable vector. A feasible design must satisfy all constraints where all $g_i(x)$ function values must be less than or equal to zero. All constraints considered in the above problem are inequal, except for the last one. Then, the constraints can be reformulated as shown in the following equations:

$$g_1(x) = 6 - \frac{b}{m}, \quad (9)$$

$$g_2(x) = \frac{b}{m} - 12, \quad (10)$$

$$g_3(x) = Z_p + 2 - (Z_p + Z_s) \times \sin\left(\frac{180}{n_p}\right), \quad (11)$$

$$g_4(x) = Y_{Fa} Y_{Sa} Y_{\epsilon} \frac{2KT}{bZm^2} K_A K_V K_{F\alpha} K_{F\beta} - [\sigma_{b_{adm}}], \quad (12)$$

$$g_5(x) = Z_\beta Z_H Z_E Z_\epsilon \sqrt{\frac{2T(i \pm 1)}{bZ^2 m^2 i}} K_A K_V K_{H\alpha} K_{H\beta} - [\sigma_{c_{adm}}], \quad (13)$$

$$g_6(x) = \frac{4.97e6 \cdot P}{n' \cdot \gamma} - d_p'^3, \quad (14)$$

$$g_7(x) = \frac{Z_p + Z_s}{n_p} - \text{integer number}, \quad (15)$$

$$g_8(x) = Z_p - Z_s. \quad (16)$$

3.3. Binary Coding in GA. Amount of iterations and variables in a function are coded in bit patterns as shown in Table 2. Generally, binary-coded chromosomes having 1's and 0's were used. Genetic operators could be used due to this coding using the following formula [22]:

$$2^l \geq \frac{x(i)_{\max} - x(i)_{\min}}{\epsilon} + 1. \quad (17)$$

Here, l represents the string length for all variables, $x(i)_{\min}$ and $x(i)_{\max}$ represents respectively lower and upper boundary of variable, and ϵ is the variable step. When all string lengths of each variable are determined, the chromosome is produced by the concatenation of the previous as defined in Table 2.

3.4. Efficiency Analysis. The planetary gear train is a system with one configuration and many combinations. Indeed, it can have the case where at least one of the parts, planetary, ring, or the planet carrier, is fixed and the two others are either the input or the output. In this case, there are six different operating modes or transmission ratios and a different expression of transmission efficiency [29, 30]. Figure 3 presents the basic circuit of the transmission relation diagram of the simple planetary gear train (SPGT) with 1 DOF, where i, j , and k represent, respectively, the sun, ring, and planet carrier, and the fixed sign indicates which part of PGT was blocked [31, 32].

The transmission relation diagram of SPGT with one degree of freedom (1 DOF) has six basic cases, and their efficiency expression is given in Table 3. Equation (18) describes the basic meshing efficiency between the driver and the driven gears. It is used in the contact ratio range $1 < \epsilon_\alpha < 2$, μ presents the friction coefficient with a constant value of 0.05 and Z_{ij} is the teeth number of the gear i and j , respectively. In this paper, the value of the basic transmission coefficient has been $\eta_{PGT}^0 = 0.95$ [33]:

$$\eta_{PGT}^0 = 1 - \pi \cdot \mu \left(\frac{1}{Z_j} \pm \frac{1}{Z_i} \right) \cdot (\epsilon_1^2 + \epsilon_2^2 - \epsilon_1 - \epsilon_2 + 1). \quad (18)$$

Based on the kinematic equation of the gear system, the Willis formula, the torque equation, and the power balance equation have been obtained, as expressed in equations (19)–(21) [34]:

TABLE 2: Coding of variables.

Design variable vectors	Vectors	Randomized binary digits	String length
Tooth width ($\varepsilon = 0.01$)	X(1)	111110101	9
Module ($\varepsilon = 0.1$)	X(2)	1011011	7
Number of teeth_sun ($\varepsilon = 1$)	X(3)	1101	4
Number of teeth_planets ($\varepsilon = 1$)	X(4)	101011	6
Shaft diameter_sun ($\varepsilon = 1$)	X(5)	11111	5
Shaft diameter_planet ($\varepsilon = 1$)	X(6)	1110100	7
A single chromosome		1111101011011011110110101111111110100	38

FIGURE 3: Basic circuit transmission relation diagram.

$$\omega_i - \tau \cdot \omega_j + (\tau - 1) \cdot \omega_k = 0; \quad \tau = \pm \frac{Z_j}{Z_i}, \quad (19)$$

$$\eta_{\text{PGT}} = \frac{(1 - \tau) \cdot \omega_k \cdot \eta_{j(i-k)} + \tau \cdot \omega_j \cdot \eta_{k(i-j)}}{\omega_i}. \quad (25)$$

where τ represents the basic gear ratio between j and i gear and the $\mp$ sign represents the internal and external gear pairs. As identified, i, j , and k were s, r , and pc , respectively. $i = s$, $j = r$, and $k = pc$, and based on the equation, the following equations were obtained:

$$T_s + T_{pc} + T_r = 0, \quad (20)$$

$$P_s + P_{pc} + P_r = 0. \quad (21)$$

Firstly, assuming the ring gear j is fixed, the efficiency $\eta_{j(i-k)}$ can be calculated by using equation (22). Then, assuming the planet carrier k is fixed, the efficiency $\eta_{k(i-j)}$ can be calculated by using equation (23). When assuming the sun gear j is fixed, ω_k can be obtained by formula (17) as shown in equation (24):

$$\eta_{j(i-k)} = -\frac{T_k \cdot \omega_k}{T_i \cdot \omega_i}, \quad (22)$$

$$\eta_{k(i-j)} = \frac{T_j \cdot \omega_j}{T_j \cdot \omega_j}, \quad (23)$$

$$\omega_k = \frac{1}{1 - \tau} \omega_i. \quad (24)$$

Table 2 summarizes the basic gear transmission diagram and efficiency in the six cases of the SPGT. For the complex planetary gear transmission, the equivalent efficiency η_{PGT} is expressed in equation (25) by using equations (19), (20), and (24):

4. Results and Discussion

4.1. Results for Optimization of the Weight and Center Distance. Table 4 shows the best fitness of the weight and the center distance for each configuration of the epicyclical gear train, as well as the optimal design parameters for each transmission ratio variation. The first mode gives the lighter weight and the minimal center distance for the PGT system when it was a speed reducer than modes two, three, and five. However, for the case when the epicyclical gear train is used as a speed multiplier, the optimal solution has been presented at the 6th with a value of 0.342 kg and 39 for the weight and the center distance, respectively.

For both operations of the PGT as speed reducer or speed multiplier, the center distance was the same 39 mm, but for the best weight, there is a margin of a difference of 66 g.

For the design parameters, the big size has been found in the second mode with tooth number of the sun gear of 30 and with a module of 2.75 mm in the 4th mode. For the rest of the modes, the value of the tooth number of the sun and planet gears was the same with a value of 21 and 18, respectively. The shaft diameter of the sun and planet gears has been found to be equal, respectively, 20 mm and 17 mm for the first mode and 20 mm and 11 mm for the last mode.

The average of the tooth width was 26.69 mm in different transmission ratio, but the best one was in the first mode with a value of 24 mm in the case of the PGT as a speed reducer and 26 mm in the 6th mode of PGT as a speed multiplier.

TABLE 3: SPGT basic gear transmission diagram and efficiency.

Mode	Transmission diagram	τ range	Transmission ratio	Efficiency
1		$\tau < 0$ or $\tau > 0$	$I_{\text{pgt1}} = \tau$	η_{ij}^0
2		$\tau < 0$ or $\tau > 1$ $\tau > 0$	$I_{\text{pgt2}} = 1 - \tau$	$(\tau \cdot \eta_{ij}^0 - 1) / (\tau - 1)$ $(\tau \cdot \eta_{ij}^0 - 1) / (\tau - 1)$
3		$\tau < 0$ or $\tau > 0$	$I_{\text{pgt3}} = 1/\tau$	η_{ij}^0
4		$\tau < 1$ $\tau > 1$	$I_{\text{pgt4}} = (\tau - 1)/\tau$	$(\tau - \eta_{ij}^0) / (\tau - 1)$ $(\tau \cdot \eta_{ij}^0 - 1) / (\eta_{ij}^0 (\tau - 1))$
5		$\tau < 0$ or $\tau < 1$ $0 < \tau < 1$	$I_{\text{pgt5}} = 1/(1 - \tau)$	$(\eta_{ij}^0 (\tau - 1)) / (\tau - \eta_{ij}^0)$ $(\eta_{ij}^0 (\tau - 1)) / (\tau - \eta_{ij}^0)$
6		$\tau < 1$ $\tau > 1$	$I_{\text{pgt6}} = \tau/(\tau - 1)$	$(\eta_{ij}^0 (\tau - 1)) / (\tau \cdot \eta_{ij}^0 - 1)$ $(\tau - 1) / (\tau - \eta_{ij}^0)$

TABLE 4: Optimization results of the epicyclical gear train design.

Function variables	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
Number of teeth of the sun gear	21	30	21	27	21	21
Number of teeth of the planet gear	18	18	18	18	18	18
Number of teeth of the ring gear	57	66	57	63	57	57
Module	2	2	2	2.75	2	2
Shaft diameter (sun)	20	30	18	18	23	20
Shaft diameter (planet)	17	17	21	21	20	11
Width coefficient of tooth	12.00	12.17	12.50	12.14	13.70	13.01
Tooth width	24.00	24.33	25.00	33.38	27.40	26.03
Transmission ratio	-2.71	-0.45	0.26	3.34	0.73	1.37
Weight of PGT	0.276	0.312	0.280	0.664	0.375	0.342
Center distance	39	39	39	61.88	39	39

FIGURE 4: Variation of the optimum fitness for the weight with different transmission ratios.

FIGURE 5: Best solution of the center distance in six cases of the PGT system.

The figures below show the variation of the fitness functions during generations and transmission ratio variation. The first and the third mode converge rapidly to the final solution of the minimal weight than the rest configuration of the epicyclic gear train (see Figure 4). For the center distance, mode 3 is the faster to give the final solution than the rest mode; the 2nd mode takes more time to converge to the final solution, and it was the last for the weight and the center distance functions (see Figure 5).

A Pareto front, presented as a plot of the weight and the center distance of the planetary gear train, gives a quantitative description of the compromise between weight and size in both cases of the operation mode of the epicyclic gear train system as a speed reducer and a speed multiplier (see Figures 6 and 7).

FIGURE 6: Pareto front of the GA multiobjective optimization of the epicyclic gear train case of a speed multiplier.

FIGURE 7: Pareto front of the GA multiobjective optimization of the epicyclic gear train case of a speed reducer.

TABLE 5: Optimization results of the epicyclic gear train efficiency by the GA.

Types	1	2	3	4	5	6
Range	-2.71	-2.2	-2.71	-2.33	-2.71	-2.71
Transmission ratio	-2.71	-0.45	0.26	3.34	0.73	1.37
Efficiency %	95.00	96.56	95.00	98.49	96.29	98.60

FIGURE 8: Optimum results for efficiency in each type of PGT.

4.2. Results for Optimization of the Efficiency. As mentioned beforehand in analysis efficiency, a planetary gear train is a system with one configuration and many combinations. Therefore, six results of the efficiency were obtained as shown in Table 5 and mapped in Figure 8. This diversity of values depends on the change of the operating mode of the epicyclic gear train. The best value of efficiency was given by the last type of PGT, the case when the ring gear was the input, the planet carrier the output, and the ring fixed, with a percentage of 98.60%. However, when the sun gear was fixed and the input was the ring, the efficiency of PGT decreased by 3.60% compared to the best one.

5. Conclusions

Gears are the most important elements of the power transmission system. This paper presents one of the powerful

metaheuristic approaches to optimization of the epicyclic gear system with the spur gear by means of the genetic algorithm. The objective of this optimization has focused on the minimization of the hall weight of the PGT, minimization of center distance, and maximization of the efficiency. The latter has been accomplished using GA under dimensional conditions of gears, balance or assembly constraint of PGT, bending strength of teeth, contact strength of teeth, and torsional strength of the gear shaft. The design variables to be optimized were the module, the number of teeth for both the sun and planet gears, the tooth width, and the shaft diameter of the gears, especially sun and planet gears.

The objective functions were analyzed according to the six cases of the PGT operation. The fitness function of the weight was found to be equal to 0.276 kg in mode 1 of the PGT as a speed reducer and equal to 0.342 kg for the 6th mode of the PGT as a speed multiplier. The value of the center distance was 39 mm for two operations of the PGT as a speed reducer and a speed multiplier. The module was found to be equal to 2 mm in several modes. The teeth number of the sun and planet gears was found to be equal to 21 and 18, respectively. The optimal tooth width values were found to be equal to 24.00 mm and 26.03 mm in the first and the last mode, respectively. The transmission efficiency was very important in the last type of planetary gear train system with the percentage of 98.60%, the case when the planet carrier was the output, the ring the input, and the sun gear fixed.

In addition, the change of the operating mode of the epicyclic gear train system has been a great impact on the variable of optimization and in the final solutions, which is the added value of this paper compared to the previous research quoted in the literature.

In light of this study, GA is an efficient approach for the optimization of complex problems with many variables, multiobjective functions, and constraints. It also provides a better result in a short time compared with other traditional optimization.

Nomenclature

a :	Center distance (mm)
b :	Tooth width of gears (mm)
d_s , d_p , and d_r :	Primitive diameter of sun, planet, and ring gears (mm)
d'_s and d'_p :	Shaft diameter of the sun and planet gears (mm)
i , j , and k :	The sun, the ring, and the planet carrier
I_{pgt} :	Global gear ratio PGT
K :	Coefficient of tooth length
K_A :	Application factor
$K_{F\alpha}$:	Transverse load factor (bending stress)
$K_{F\beta}$:	Face load factor (bending stress)
$K_{H\alpha}$:	Transverse load factor (contact stress)
$K_{H\beta}$:	Face load factor (contact stress)
K_v :	Dynamic factor
m :	Module (mm)
n :	Input speed (rpm)

n_p :	Number of planets
W :	Total weight of the epicyclic gear train (kg)
$g_i(x)$:	Constraint functions
$x(i)_{\min}$ and $x(i)_{\max}$:	Lower and upper boundaries of variables
P_s , P_{pc} and P_r :	Power of sun, planet carrier, and ring gear (kw)
T_s , T_{pc} , and T_r :	Torque of sun, planet carrier, and ring gear (N-mm)
Z_s , Z_p , and Z_r :	Number of teeth on sun, planet, and ring gears
Y_{Fa} :	Tooth form factor
Y_{Sa} :	Stress concentration coefficient
Y_ϵ :	Contact ratio coefficient
ω_s , ω_p , and ω_k :	Angular velocity of sun, planet carrier, and ring ($\text{rad}\cdot\text{s}^{-1}$)
τ :	Basic transmission ratio of PGT
ρ :	Density material of the gear ($\text{kg}\cdot\text{m}^{-3}$)
y :	Tooth form factor
σ :	Gear material strength ($\text{N}\cdot\text{mm}^{-2}$)
γ :	Allowable shearing stress of shafts (Mpa)
η_{PGT} :	Efficiency of planetary gear train
$\pm$:	Internal and external gear pairs
PGT:	Planetary gear train
1 DOF:	One degree of freedom
GA:	Genetic algorithm.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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