

Research Article

A Hybrid Approach for Project Crashing Optimization Strategy with Risk Consideration: A Case Study for an EPC Project

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Received 30 August 2018; Revised 1 December 2018; Accepted 9 January 2019; Published 17 January 2019

Academic Editor: Alessandro Contento

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This study aims to develop and provide a comprehensive evaluation strategy for schedule-related variations and time-cost analysis for an engineering–procurement–construction (EPC) project. Time-cost analysis is an important aspect of project scheduling, particularly in long-term and costly EPC projects. In this study, a hybrid method is proposed for the time-cost optimization strategy evaluation of a project. Monte Carlo simulation is applied to determine contingency plans and realize the effective management of estimated schedule uncertainties. A mathematical integer linear programming optimization model coded using CPLEX is developed to assess appropriate strategies for project execution under time and cost constraints. A set of project evaluation optimization models considering risk and project crash plan and the relationship between crash cost and delay penalty is also developed for assessing project feasibility. The correlation between project risk and crashing strategy has seldom been evaluated simultaneously in previous research. This work fills this research gap by quantifying the feasibility of a project, with combined data on risk, schedule, and cost as evaluation indicators. It allows project managers to consider management issues and strategies before they implement projects. A practical example with numerical applications is presented to illustrate the contribution of the decision-making support mechanism, and several managerial insights are provided.

1. Introduction

Construction projects are being implemented with diversified contents and shortened plant construction time due to the rapid growth of the competitive modern market and the need for quickly profitable investments. Projects often fail to meet the original target schedule because of uncertainties. According to Coppendale [1] and Chatzoglou and Macaulay [2], on the average, only 10% to 15% of large projects are completed on time, and the rest are delayed. Yang and Teng [3] reported that construction projects are naturally uncertain in terms of activity duration and thus cause an indeterminate project completion schedule. Scheduling is possibly the most common issue in project planning and control because delays cause many problems between project owners and contractors. Delays in project completion

aggravate the cost burden of projects. Delay claims for equitable adjustments can amount to millions of dollars. Thus, scheduling and cost risk analysis are crucial. Project managers must design and implement mitigating strategies to overcome the growing uncertainty faced by projects. Project risk management, a systematic risk assessment method for project implementation, provides a platform for owners and contractors to manage risks and communicate. The scope of actual project risk management includes environment safety and health [4], schedules [5–7], and costs [8–10]. Project managers must formulate strategies for overcoming or avoiding the occurrence of uncertainties so that projects remain on track.

Schedule and cost are the two most important indicators in project practice. When project resources are limited, an alternative relationship exists between schedule and cost.

Necessary crashing schedules must be planned in advance to complete projects on time. However, the completion times of projects are typically random variables due to numerous activities and unfixed project execution times. The first problem to be solved in the randomness of projects is estimating the project completion time. All issues, including project scheduling, project resource investment optimization, project cost, and project risk assessment, should be based on the expected project completion time. Most studies on project deadlines are limited to the development of a project crashing strategy rather than comprehensively evaluating project risk and crashing strategy simultaneously. Bromilow's log-log time-cost (BTC) model was proposed in the 1970s to estimate project duration, but different parameter estimates are required for different project types. The parameters of the BTC model have no guarantee and are invariant over time [11]. Given this background, this study explores the probability of a potential project risk affecting the project completion schedule and the practical consideration of the time-cost trade-off. The probability of the project completion schedule by contract under risks is discussed through risk analysis, and the activities located on the critical path are identified through schedule sensitivity analysis. The relationship between activity time and crash cost is transformed into a mathematical model. The model is solved through linear programming to calculate the optimal crash cost. A set of project evaluation models considering risk and project crash plan and the relationship between crash cost and delay penalty is developed by using risk, cost, and schedule as indicators for assessing project feasibility. This study provides project managers a reference for management and strategy building in the bidding stage and increases the probability of projects being completed within the target time. To protect their interests, owners generally specify the required delay penalties in their contracts. If a project fails to meet the deadline, the project manager shall use proper management skills to implement schedule and cost control while considering the cost incurred by project crash plan and delay penalties. Balance between these two aspects should be achieved to complete a project successfully in accordance with the objectives and quality set by the plan.

This study applies the methods of Monte Carlo simulation using Primavera Risk Analysis software and integer linear programming coded using IBM ILOG CPLEX. These methods combine probabilistic activity duration with systematic delay analysis procedures to predict the overall project delay and estimate the additional cost brought about by a crash plan under risk consideration. This paper is structured as follows. Section 2 describes relevant studies and research issues. Section 3 provides the framework of the schedule risk analysis methodology and the formulation of the mathematical linear programming model. Section 4 focuses on the models proposed in Section 3 and presents an actual case study to illustrate the applications of the proposed models for evaluating project strategies. Section 5 presents the conclusions and discussions.

2. Background

2.1. Schedule Method-CPM/PERT. The traditional critical path method (CPM) has been widely used in the construction industry for schedule analysis and project planning since the 1950s. The critical path represents the longest and most inflexible chain of activities in the overall project. The total float time for activities located in the critical path is zero. A project may contain several important critical paths. The backwardness of any activity in critical paths affects the completion of the entire project after a project starts. Hulett [6] stated that CPM is a traditional and widely accepted approach for scheduling, and it is essential for developing the logic of a project and managing daily project tasks. However, CPM does not consider risk or uncertainty [3, 12, 13]. CPM scheduling is accurate only when every activity begins as planned and consumes the same amount of time as estimated. Managers understand that projects do not always go according to plan and therefore require frequent status reviews. Given that projects generally do not proceed as planned, CPM can only serve as the beginning of project schedule management. Project managers should understand key reservations about standard CPM and should know how to perform a schedule risk analysis to obtain information that is crucial to project success before they embark on projects.

The program evaluation and review technique (PERT) in conjunction with CPM was developed in the late 1950s. Network planning technology using classical CPM/PERT as the core has been widely applied in project management. PERT uses a three-point estimate (optimistic, most likely, and pessimistic estimates) of activity duration to represent the lack of certainty in duration estimates [6].

- (1) Optimistic time (t_o): refers to the minimum possible time required to accomplish a task under the assumption that everything proceeds better than is normally expected.
- (2) Pessimistic time (t_p): refers to the maximum possible time required to accomplish a task under the assumption that everything goes wrong.
- (3) Most likely time (t_m): refers to the estimated time required to accomplish a task under the assumption that everything proceeds normally. This duration is more likely to occur than the others.

The completion time for project is expressed in formulas (1) and (2). The completion time (T) for project is the maximum of all the completed paths, that is, the completion time of the critical path. The formula is shown below, where $P(j)$ refers to the set of all jobs on path j , t_i refers to the average time (or mean time) of activity i on path j , and T_j refers to the completion time of path j .

$$T = \max(T_j) \quad (1)$$

$$T_j = \sum_{i \in P(j)} t_i, \quad \text{where } i = 1, 2, 3, \dots, n \quad j = 1, 2, 3, \dots, m \quad (2)$$

However, PERT might underestimate the schedule risk because it ignores important risks at the merge points applied

to the path when multiple paths and merge points exist [6]. Omid et al. [7] stated that PERT considers only one critical path, and it does not consider paths close to the critical one. Nearly all schedules for actual projects possess multiple paths. Thus, the Monte Carlo simulation method is applied in this study to overcome the shortcomings of PERT, because it can correctly compute risks at the merge points and represent the duration with a realistic probability distribution. This method is a preferred and reliable solution [6, 9, 12].

2.2. Monte Carlo Simulation. Hulett [6] stated that CPM scheduling tools, which include manual and software-based systems, cannot handle the uncertainty that exists in the real world regarding project activity durations because these tools assume that activity durations are with certainty as single-point numbers. A stochastic risk analysis technique called Monte Carlo simulation can be applied to evaluate project uncertainties [14–17]. Monte Carlo simulation is suitable for determining the project completion date because the date is determined by the uncertainty in the duration of many activities that have already been linked logically in the CPM schedule [6]. Monte Carlo simulation is a statistical sampling technique that operates with random components as input variables subject to uncertainties and presents a set of results in terms of probabilities after several iterations [18, 19]. As Covert [20] stated, a probability density function (PDF) is used to define the probability distributions for continuous distributions, which can be expressed in terms of the mathematical formula of $f_y(x)$, where $f_y(x)$ is the PDF defined over the range, x . Any point estimate (c) has some probability to be sufficient or to be exceeded. The probability that an estimate will be exceeded (i.e., overrun) is the risk, and the probability that the estimate will be sufficient is the opportunity. Therefore, the risk is the integral of the PDF from the point estimate, c , to infinity (∞), which can be expressed as formula (3). Opportunity represents the area under the curve from $-\infty$ to c , which is expressed as formula (4).

$$\begin{aligned} \text{Risk} &= \int_c^{\infty} f_y(x) dx = 1 - \int_{-\infty}^c f_y(x) dx \\ &= 1 - F_y(c) \end{aligned} \quad (3)$$

$$\text{Opportunity} = \int_{-\infty}^c f_y(x) dx = F_y(c) \quad (4)$$

A triangular distribution model is frequently used in project risk analyses, and three-point scenarios are applied in the analyses [13, 15, 21–23]. The parameters of a triangular distribution are estimated using the lowest possible value (L), the highest possible value (H), and the most likely value (M). In Monte Carlo simulation, each project activity has a respective range and a pattern of duration possibilities. Figure 1 is the typical triangular distribution [20].

$$\text{Triangular distribution: } f_y(x; L, M, H) = T(L, M, H) \quad (5)$$

where L is the lowest possible value (optimistic value)

M is the most likely value

H is the highest possible value (pessimistic value)

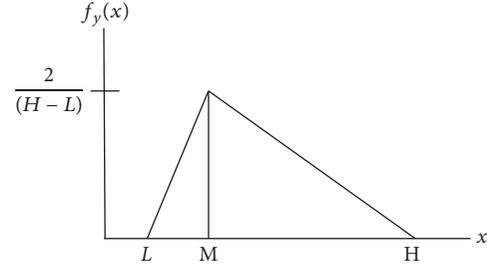


FIGURE 1: Triangular distribution [20].

The PDF of the triangular distribution $T(L, M, H)$ is

$$f_y(x) = \begin{cases} \frac{2(x-L)}{(H-L)(M-L)} & \text{if } L \leq x < M \\ \frac{2(H-x)}{(H-L)(H-M)} & \text{if } M \leq x \leq H \end{cases} \quad (6)$$

Formulas (7), (8), and (9) combine the optimistic, pessimistic, and most likely time to estimate the average time, variance, and standard deviation of project activity i , respectively [6].

$$\text{Triangular average time } (t_i) = \frac{(t_o + t_m + t_p)}{3} \quad (7)$$

$$\begin{aligned} \text{Triangular variance } (\mu_i) \\ = \frac{(t_p - t_m)^2 + (t_p - t_m) \times (t_m - t_o) + (t_m - t_o)^2}{18} \end{aligned} \quad (8)$$

$$\text{Triangular standard deviation } (\sigma_i) = \sqrt{\sum \mu_i}, \quad (9)$$

where $i = 1, 2, 3, \dots, n$

Experts familiar with the project tasks should accomplish them to provide estimates in the workshop. If these uncertainties are identified early in the project, plans that minimize or prevent risks can be formulated. Project managers can accurately and confidently estimate the overall completion time for the project under consideration by dealing with a range of probable durations. The results of the Monte Carlo simulation show the logical consequences of a particular set of risk assumptions, which can include the range estimates of durations, resource variations, and correlations among project categories. Monte Carlo simulation provides quantitative results for decision-making and determines the key risk factors that can make planned activities meet the scheduled milestones.

2.3. Mathematical Methods for Time-Cost Trade-Off Analysis. The time-cost trade-off problem has been studied since the 1960s and is considered as a difficult combinatorial problem [24, 25]. The solving process of mathematical programming involves converting the relationship between activity time and crash cost in the network diagram into a mathematical model and then using linear programming, integer programming, or dynamic programming to solve the model.

Kelley [26] used parametric linear programming to determine an optimal schedule. Butcher [27] assumed that activity time and direct cost were in irregular forms and used dynamic programming to obtain the shortest completion schedule when the direct cost was known. Perera [28] constructed three constraint models of crashing workload, scheduled completion, and network diagram loop and utilized linear programming to solve the optimal crashing scheduling. Russel and Caselton [29] applied dynamic programming to analyze the two-dimensional problem of resolving the start time of each activity in the unit and the buffer time for entering the next unit. They constructed a mathematical analysis model to obtain the shortest schedule. Reda [30] constructed a minimum-cost linear programming model that can calculate a specific construction period for an engineering project on the basis of the relationship among activity constraints. Moselhi and EI-Rayes [31] adopted the model of Russell and Caselton to construct a resource scheduling model that considers the direct and indirect costs of activities in accordance with the principle of minimum total project cost. Burns et al. [32] combined linear and integer programming to solve the trade-off between construction schedule and cost. Feng et al. [33] used a hybrid approach to minimize construction project time and cost simultaneously through a combination of simulation and mathematical algorithms. Sakellaropoulos and Chassiakos [34] proposed the incorporation of parameters describing the actual project into the mathematical model, followed by an analysis of the time and cost of project scheduling. Moussourakis and Haksever [35] presented a mixed integer programming model that minimizes the total cost subject to a project deadline or the project completion time subject to a budget constraint for various types of activity cost functions. Chassiakos et al. [36] utilized an integer linear programming model to obtain an optimal project time-cost curve that considers all activity time-cost alternatives simultaneously. Liberatore and Bruce [37] used a hybrid mathematical model to analyze the time and cost of a project. Mokhtari et al. [38] developed a hybrid approach for the stochastic time-cost trade-off problem to improve project completion probability in a specified deadline from a risky value to a confident probability through simulation and a mathematical program. Gonen [39] proposed a linear programming approach for budget allocation and demonstrated the budget constraint method, including sensitivity analysis. Sato and Hirao [40] used a mathematical modeling approach to analyze the trade-off problem between budgets and critical risks. Ghaffari et al. [41] employed a fuzzy linear programming model to assess project risks on the basis of project life cycles. Zeng et al. [42] used a stochastic optimization model to establish the total expected travel time cost. Dupont et al. [43] used a mixed integer linear programming model to show profit-and-loss targets. Atan and Eren [44] established mixed integer linear methods for several leveling objectives by using a heuristic algorithm.

Although the time-cost trade-off issue appears to have no unique optimum solution, mathematical programming provides a correct calculated solution. Therefore, this study uses the results of risk sensitivity analysis from Monte

Carlo simulation to analyze activities in critical paths and adopts IBM ILOG CPLEX optimizing software [45, 46] for mathematical programming to solve the issue wherein the crash cost and delay penalty are considered. A time-cost trade-off analysis is performed by calculating the relationship between optimal project completion time and crash cost in consideration of risks through the constructed mathematical programming model.

2.4. Research Objectives and Issues. This study aims to quantify the feasibility of a project by using data on risk, schedule, and cost as evaluation indicators. The following issues are raised to fulfill the objectives.

- (1) How can project execution proceed in consideration of project schedule risk management and crashing strategy?

Comprehensive stepwise workflows for schedule risk management and time-cost trade-off analysis are proposed in this study to conduct a quantitative analysis that would aid management in decision-making and performing proactive actions.

- (2) How can project risk and the probability of a target schedule for project completion be considered simultaneously?

A quantitative risk analysis is performed to provide a numerical estimate of the sensitive schedule risk effect on the project. Monte Carlo simulation, through statistical distribution functions, can be used to compute the probability of project schedule completion under a dynamic situation.

- (3) How can project schedule and cost be optimized under risk consideration?

The most sensitive schedule risk activities on the critical path in the project are identified by Monte Carlo simulation method and then an integer linear programming method is used to derive the optimal solution according to the proposed models. Quantifying the project risks can address the gap in time-cost trade-off issues that may arise in reality.

- (4) How can an optimal selection between crash cost and delay penalty be achieved?

The total crash cost under different scenarios can be determined through a proposed model solved by an integer linear programming method. Afterward, the relationship between crash cost and delay penalty can be compared to attain the optimal selection for project execution. When the total cost of the crash plan is higher than the upper limit of delay penalty, the project manager may stop the crash plan and pay the delay penalty to minimize the total project cost.

3. Methodology

3.1. Schedule Risk Analysis. Schedule risk analysis is a project management method for assessing the risk of a baseline

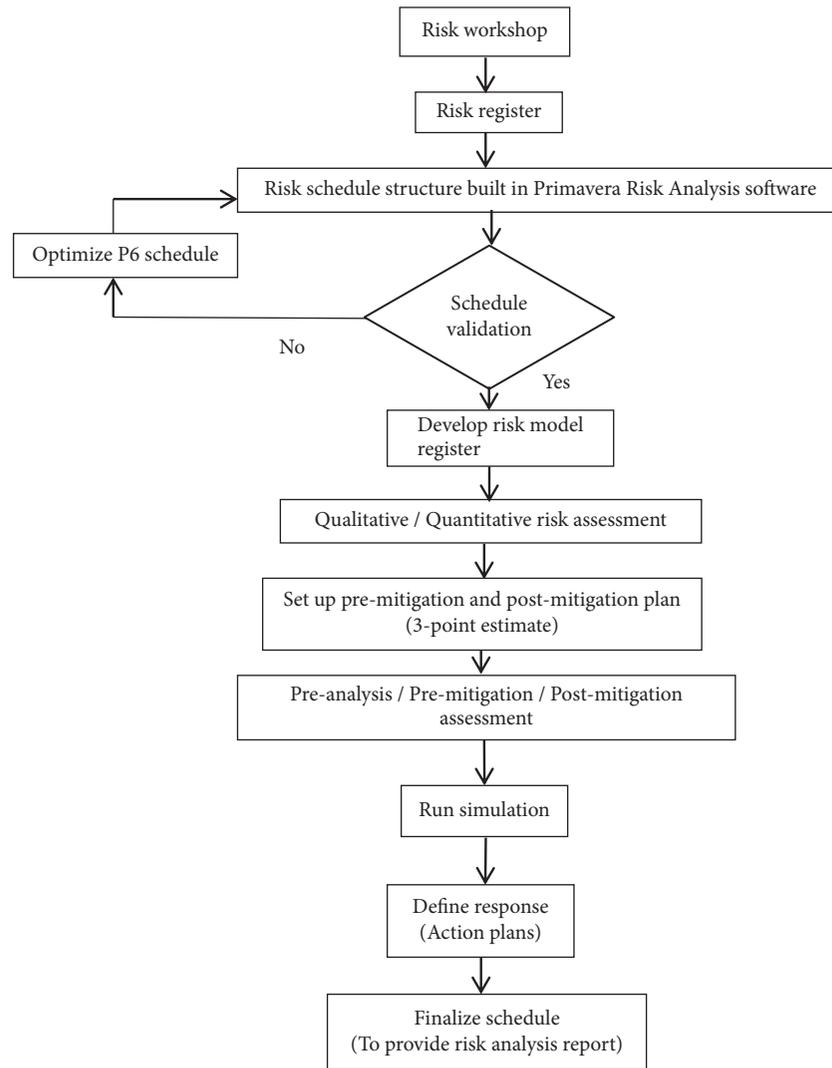


FIGURE 2: Project schedule risk analysis framework.

schedule and forecasting the impact of time on project objectives [47]. The processes of risk analysis include (1) planning risk management, (2) identifying potential risks, (3) qualifying and quantifying potential risk probability and impacts on a schedule, (4) combining information to determine the probability of schedule completion, (5) defining risk responses, and (6) monitoring and controlling action plans. The project schedule risk analysis framework for this study, which mainly comprises 10 steps, and its workflow are illustrated in Figure 2.

Step 1. A risk workshop is convened by the project manager, and experts who are experienced in executing EPC projects are invited. Workshops provide a good environment for sharing information and having a cross-disciplinary discussion. The objectives of a risk workshop are as follows: (1) to verify and analyze a project schedule, (2) to identify risk items, (3) to define risk basic information, (4) to evaluate risk impact and probability, (5) to develop a mitigation plan, (6) to decide

on risk response strategies, and (7) to estimate risk control results.

Step 2. A risk register is used to record all required data for each risk item, such as risk type, status, mapping result, risk impact level and frequency, mitigation action, and expected risk result after mitigation.

Step 3. Primavera P6 (revision 8.3) and Primavera Risk Analysis (revision 8.7) software programs are used to build the risk schedule structure.

Step 4. Schedule validation is conducted. Prior to conducting the schedule risk analysis, the maturity and readiness of the project should be verified to avoid the factors that influence risk assessment, such as logic errors, open-ended activities, negative lags, and start-to-finish links. Meanwhile, unnecessary constraints should be removed. Schedule validation may also increase the reliability of risk assessment.

Step 5. A risk model is developed. A risk model development comprises two parts: risk identification/assessment/response and risk mapping.

Step 6. After potential risks have been discussed with the owner, project experts, and department staff, all of the risks are recorded in the risk register, together with other details, such as probability (P), impact (I), and scoring ($\text{Risk} = P * I$) indicated by the risk matrix, for risk premitigation and risk postmitigation plans. This process is called qualitative risk analysis. Afterward, quantitative risk analysis involves identifying and calculating the effects of risks, determining the probability and impact for cost and schedule mitigation, and implementing response actions. This process is time and labor intensive and requires all participants to exert effort. However, the assessment results are useful for subsequent schedule risk assessments. All risks in project activities should be identified upon the completion of the potential risk assessment. Afterward, risk assessment software can be used to import the impacts and probabilities of the risks for simulation and predict the completion date of the project.

Step 7. Premitigation and postmitigation plans are developed to analyze the identified risks and model different scenarios. Risk ranges are established in workshops with more than 20 participants rather than through interviews with one or only a few participants.

Stage 1. Preanalysis check: a preliminary verification is conducted to identify risk-sensitive activities and their influences on the total project schedule given the uncertainty of the project activities. These activities are prioritized in the subsequent risk analysis and control. In the preanalysis process, the remaining duration of different activities is determined after a discussion, and a three-point estimate is commonly adopted to distinguish optimistic, most likely, and pessimistic activity periods.

Stage 2. Premitigation check: in addition to the uncertainty of the activity itself, the effects of the risks are considered.

Stage 3. Postmitigation check: in addition to the uncertainty of the activity itself and the effects of risks, risk mitigation effects are considered.

Step 8. Monte Carlo simulation is then performed via the Primavera Risk Analysis software program to determine the effects of the aforementioned risks on the schedule.

Step 9. The risk responses are defined. The action owners monitor and control the action plans.

Step 10. The schedule is finalized. A risk analysis report is provided to monitor all potential risks.

After schedule risk analysis, the sensitive schedule risk items on the critical path can be identified through Monte Carlo simulation. The activities on the critical path can be further analyzed by a proposed mathematical model to

obtain the optimal solution for the project time-cost trade-off strategy.

3.2. Model Description and Formulation. The most critical path is selected from Monte Carlo simulation, which corresponds to the path with the maximum schedule sensitivity index. On the basis of the results of the sensitivity analysis, the activities on the critical path are analyzed. After a schedule network diagram is created through an analysis of precedence relationships and the activity time of each node, the implementation time of the activities on the critical path is calculated. Afterward, a three-point estimate of activity duration to represent the lack of certainty in the duration estimates is conducted through discussion meetings among stakeholders in the workshop. Accordingly, a mathematical linear programming model is proposed to determine the relationship among project completion schedule, project crash cost, delay penalty, and total project cost. The time-cost trade-off for the best schedule and crash cost of the project can be determined. The workflow is illustrated in Figure 3.

3.2.1. Model 1: Integer Linear Programming Model for Project Duration. A mathematical linear programming model developed from the precedence relationship and activity time of each node can be used to solve and determine the project completion time. The constructed model indicates that every activity should be initiated after the finish time of the prior activity and ensures that the time to finish the project is as short as possible. The network concept diagram is shown in Figure 4.

I and J are the sets. I is a set of nodes, p is the number of nodes, and each $i \in I$. J is a set of activities, q is the number of activities, and each $j \in J$. The decision variable N_i represents the starting time of node i , and N_p is the starting time of the last node, that is, the completion time of the project. The parameter d_j represents the duration of activity j , and $(i-1)$ represents the node prior to the node i ($i = 1, 2, 3, \dots, n$).

An integer linear programming model for project duration is established with mathematical formulas (10)-(13) to calculate the minimum project duration under normal operating conditions.

$$\text{Minimize } N_p - N_1, \quad (10)$$

$$\text{Subject to } N_1 \geq 0, \quad (11)$$

$$N_i \geq N_{(i-1)} + d_j \quad \forall i \in I, \forall j \in J, \quad (12)$$

$$N_i \in Z^+ \quad \forall i \in I. \quad (13)$$

Formula (10) is an objective function that requires the project duration to be minimized under normal operating conditions. Formulas (11) and (12) are constraints that indicate that the starting time of each node must not be preceded by that of the prior neighboring node and the activity duration between the two nodes, with the starting time of the initial node larger than or equal to 0. Formula (13) is a decision variable constraint that indicates that the starting time of each node of the decision should be a positive integer larger than or equal to zero.

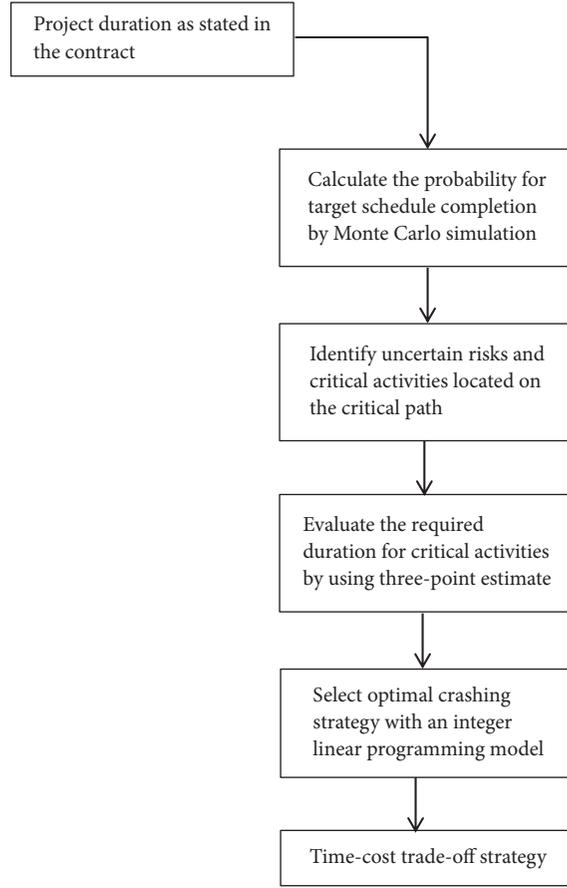


FIGURE 3: Time-cost trade-off analysis framework.

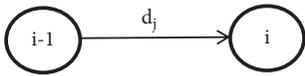


FIGURE 4: Network concept diagram.

3.2.2. *Model 2: Integer Linear Programming Model for Project Crash Plan.* The shortest time for project completion under normal operating conditions is calculated by Model 1. A cost decision variable is added to Model 2 in order to develop a comprehensive trade-off crash plan. This is done by balancing the crash plan with certain activities and considering the variable factors of crash cost and delay penalty when the shortest time to finish the project under normal operating conditions cannot meet the stated duration in the contract. However, when the project crash plan is conducted, the crash cost of the activity may increase with the crash time. Additional time allotted for compression increases the crash cost with the required amount of input resources. Thus, the crash cost becomes an incremental piecewise linear function type with the crash time of different segments.

K is a set of different segments for crash time, k is the number of segments for each activity, and each $k \in K$. An integer linear programming model for the project crash plan is expressed in mathematical formulas (14)-(24).

$$\begin{aligned} \text{Minimize } & (L - S) \times CP \\ & + \sum_{k \in K} \sum_{j \in J} [C_j^k (T_j^k - Y_j^k M_j^{k-1})] + F_j \end{aligned} \quad (14)$$

$$\forall_j \in J, \forall_k \in K,$$

$$\text{Subject to } N_i \geq 0, \quad (15)$$

$$N_i \geq N_{(i-1)} + d_j - \sum_{k=K} T_j^k \quad (16)$$

$$\forall_i \in I, \forall_j \in J, \forall_k \in K,$$

$$\sum_{k=K} T_j^k \leq D_j \quad \forall_j \in J, \quad (17)$$

$$\sum_{k=K} Y_j^k = 1 \quad \forall_j \in J, \quad (18)$$

$$\text{and } Y_j^k \in \{0, 1\} \quad \forall_j \in J, \forall_k \in K, \quad (19)$$

$$T_j^k \geq O_j^k Y_j^k \quad \forall_j \in J, \forall_k \in K, \quad (20)$$

$$T_j^k \leq M_j^k Y_j^k \quad \forall_j \in J, \forall_k \in K, \quad (21)$$

$$N_i \in Z^+ \quad \forall_i \in I, \quad (22)$$

$$T_j^k \in Z^+ \quad \forall_j \in J, \forall_k \in K, \quad (23)$$

$$L \geq S. \quad (24)$$

Decision variable T_j^k represents the crash time of activity j at segment k , which is an integer variable and is represented in terms of days. Decision variable Y_j^k indicates whether the work segment k of activity j crash time is selected, which is a 0 or 1 integer variable. In addition, C_j^k is the crash cost per unit time of T_j^k ; F_j represents the fixed cost of activity j ; D_j denotes the difference between the most likely time or pessimistic time or the average time of activity j and the optimistic time, that is, the allowable upper limit of the crash duration for the activity j ; M_j^k represents the upper limit of work segment k for activity j crash time; and O_j^k represents the lower limit of work segment k for activity j crash time; CP indicates the delay penalty cost per unit time if the deadline cannot be met; L is the project duration after crashing; S indicates the agreed project duration in the contract.

The mathematical models are expressed in formulas (14)-(24). Formula (14) is an objective function that requires minimizing the total project crash cost. Formulas (15) and (16) are constraints stipulating that the starting time of each node must not be earlier than that of the forward node plus the time of the activity between the two nodes minus the required activity crash time between the two nodes, with the starting time of the initial node being larger than or equal to zero. Formula (17) indicates that the crash time of activity j for all segments should not be larger than the difference between the optimistic time or most likely time or pessimistic time minus its expected time, that is, the upper limit of the crash time of each activity. Formulas (18) and (19) represent the integer variable which is 0 or 1. If the work segment k of activity j crash time is not conducting the crash plan, then the integer variable is specified as 0. If it is conducting the crash plan, then the integer variable is specified as 1. Formula (20) indicates that the crash time of activity j at segment k shall be larger than or equal to the lower limit of activity j crash time at segment k , and formula (21) indicates that the crash time of activity j at segment k shall be less than or equal to the upper limit of activity j crash time at segment k . Formulas (22) and (23) are decision variable constraints that indicate that the starting time of each node in the decision should be a positive integer larger than or equal to zero, and the crash time of each activity at any segment should be a positive integer larger than or equal to zero. Formula (24) indicates that the project duration after crashing should be larger than or equal to the agreed project duration in the contract.

4. Case Study

4.1. Project Background. A case study that uses Monte Carlo simulation for quantitative risk analysis and integer linear programming for time-cost trade-off is described below. The applied case is an EPC project for a high-value-added fertilizer plant that consists of raw material tanks, reactors, a nitric phosphate scrubber, handling conveying systems,

and warehouse units. This plant, located in Taiwan, is a joint venture based on a 30%/70% share between a foreign company and a local government-based company. The total land area for this project is approximately 45,000 m², with the required utility units to be developed. The requested schedule completion for this project is 976 days with a liquidated damage charging at 0.1% of the contract price per day of delay according to the contract condition. The estimated budget for this EPC project is 53,000,000 USD, and the ceiling penalty is 10% of the contract amount.

4.2. Monte Carlo Simulation and Analysis. A series of symposia (workshops) is held to provide project duration information for the optimistic, most likely, and pessimistic ranges of each risk item. Participants should understand the background, constraints, and key features of the project, its relation to nearby residents, and its interaction with government organizations. A person without related engineering experience cannot make estimates from various angles. Primavera Risk Analysis R8.7 and Primavera P6 R8.3 are applied to calculate the possible project duration. A simulation with 3,000 iterations is performed to compute the duration estimate, and Latin hypercube sampling is used as the simulation method (Hulett 2009).

4.2.1. Preanalysis. The preanalysis check is the preliminary verification means used to determine risk-sensitive items with a maximum impact on the total project schedule. Risk-sensitive items should be emphasized in the follow-up risk analysis. Sensitivity analysis can determine which of the most important inputs have the greatest impact on the outputs and can reflect the correlation between activity and project schedule duration during the simulation. Schedule sensitivity also reflects the risks in the activities and their relationships in schedule logic. The result can be presented in a tornado graph, which is easy to read (Figure 5). A total of 679 items are included in the activities under this case project. The seven most sensitive items with great impact on each activity are selected as the major items for the subsequent risk analysis.

4.2.2. Premitigation. Figure 6 shows the expected schedule by considering the uncertainty of the current planned project schedule and the impact of the most sensitive risk items before taking mitigation actions. This project has an 80% probability to be completed in 1,138 days and 100% probability to be completed in 1,185 days. Compared with the planned schedule that this case initially required, 976 days, the completion probability of the scheduled project is only nearly 1%.

4.2.3. Postmitigation. Figure 7 shows the expected schedule by considering the uncertainty of the current planned project schedule, the impact of the most sensitive risk items, and the effect of risk disposal actions. This project has an 80% probability to be completed in 1,079 days and a 100% probability to be completed in 1,123 days. According to practical engineering experiences, most companies consider adopting the 80th percentile as a conservative and prudent planning schedule target (Hulett 2009; Mulcahy 2010). Hence, we need

TABLE 1: Monte Carlo simulation results.

Description	Deterministic probability (%)	Std. Dev.	P80 (days)	P100 (days)
Pre-analysis	55	4.27	979	988
Pre-mitigation	< 1	24.96	1,138	1,185
Post-mitigation	< 1	26.56	1,079	1,123

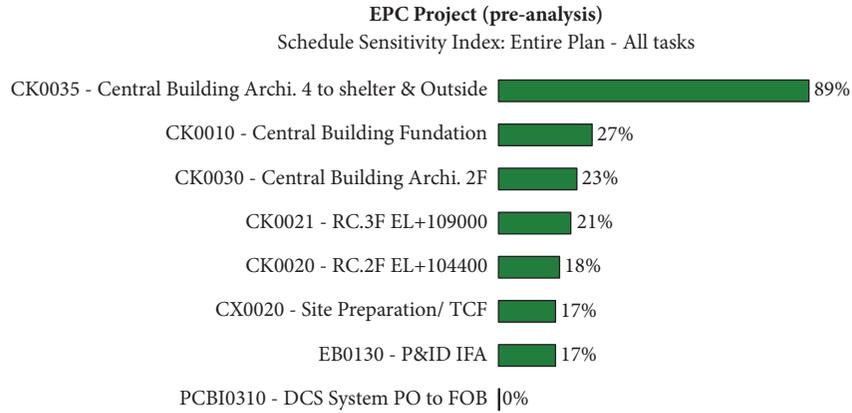


FIGURE 5: Schedule sensitivity of activity.

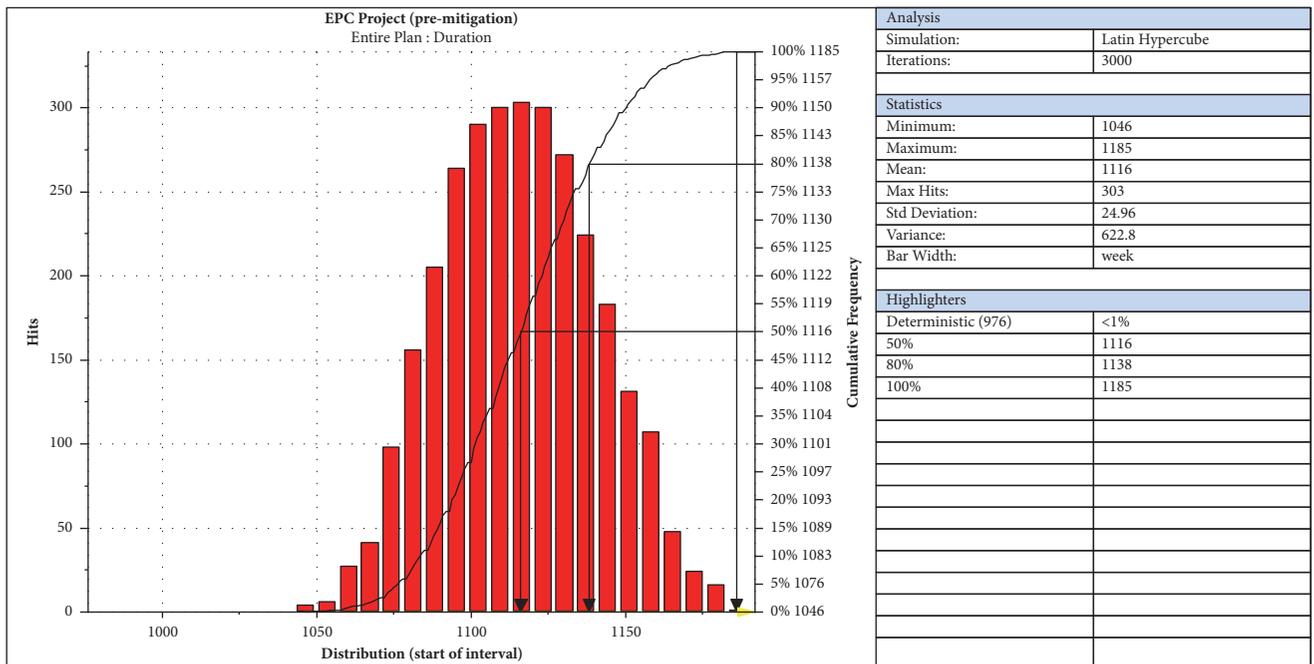


FIGURE 6: Probability distribution chart for premitigation simulation.

a schedule contingency of 103 days (976-1,079) to achieve a conservative 80th percentile level of certainty after conducting risk treatment and considering the planned mitigation actions.

4.2.4. *Distribution Analyzer.* Table 1 summarizes the Monte Carlo simulation results, which provide a schedule comparison for this case project. Terms P80 and P100 represent

probabilities of 80% and 100%, respectively. The probability of finishing the project within the requested schedule (976 days) becomes less than 1% after considering the uncertainty of the current planned schedule and the impact of the risk items. However, this project has an 80% probability to be completed in 1,079 days when risk response actions are adopted. Figure 8 presents a comparison diagram for the comprehensiveness of the schedule risk analysis with a probability placement

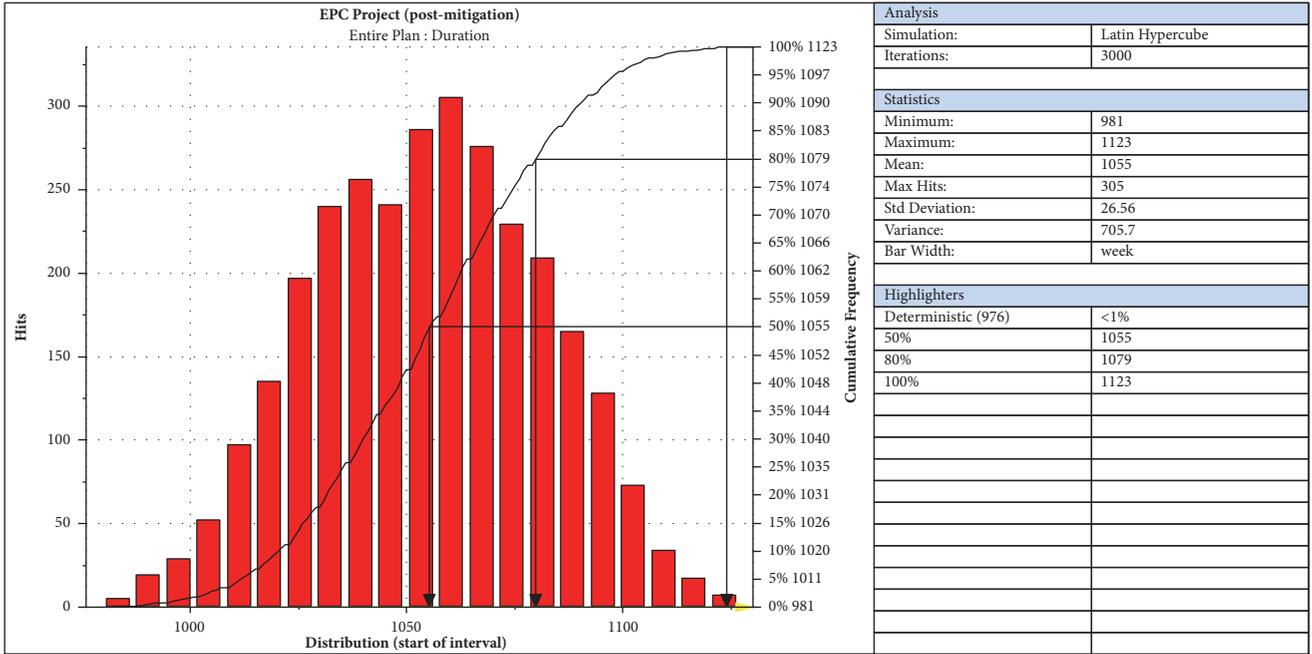


FIGURE 7: Probability distribution chart for postmitigation simulation.

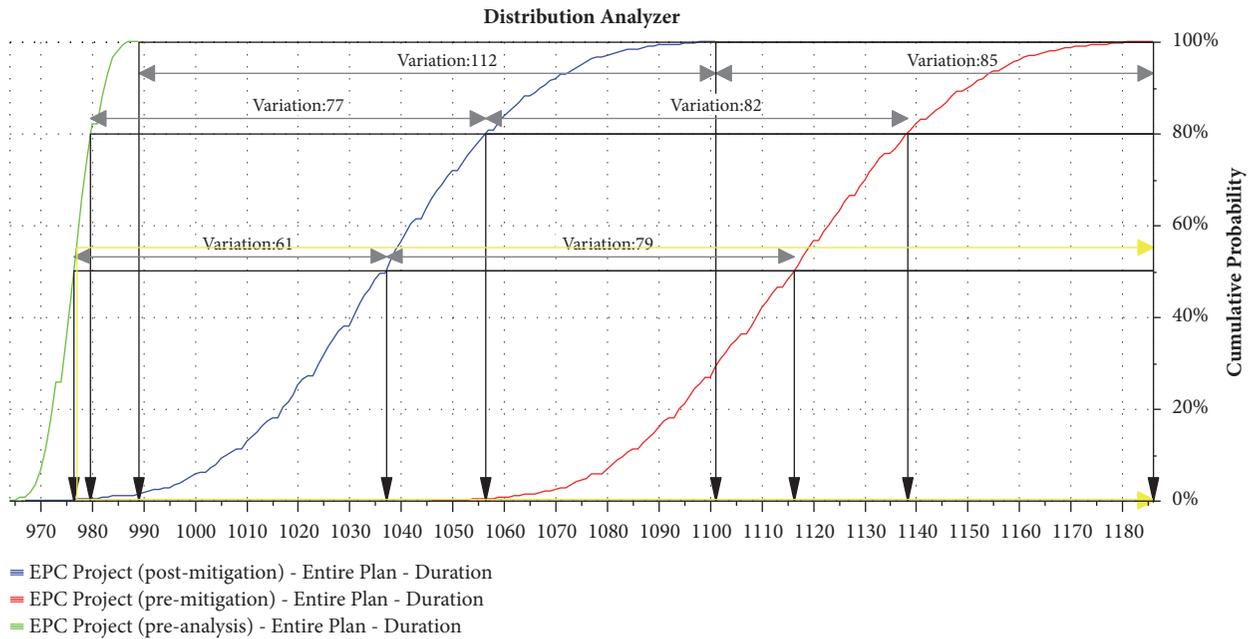


FIGURE 8: Project schedule comparison.

analysis provided for different completion duration. It shows the current estimated project schedule (preanalysis), the impact of risk occurrence on the project (premitigation), and the schedule after introducing risk response behaviors (postmitigation).

According to the result of the risk analysis, the most possible duration for project completion within P80 for the risk postmitigation result is 1079 days, which exceeds the

contract requested duration (976 days) and the allowable delay period (100 days). Therefore, a proper crash plan is necessary. In the next section, we will use the method of integer linear programming to develop the optimal solution of the project crash plan. Work activities on the critical path are made into network diagrams. The relationship between the activity time and crash cost needed for each work activity in the network diagram is converted into a mathematical

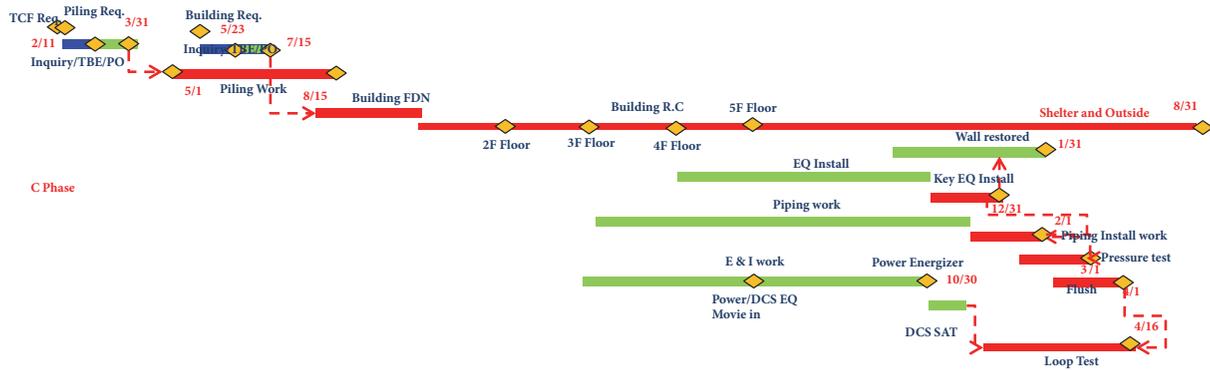


FIGURE 9: Schedule network bar chart.

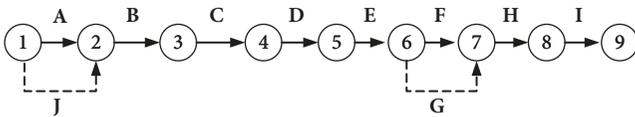


FIGURE 10: Critical path network diagram.

model, which is solved by integer linear programming. We expect to determine the additional engineering cost in accordance with the project duration as stated in the contract and also consider the relationship between delay penalty and crash cost.

4.3. Integer Linear Programming Modeling and Analysis

4.3.1. Schedule Network Analysis. Work activities and a network logic diagram with activity durations on the critical path can be obtained from schedule network analysis. As shown in the schedule sensitivity index (Figure 5), construction plays an important role in the entire plan, especially in the activity “CK0035 central building arch. 4 to a shelter and outside,” which has 89% schedule sensitivity. Figure 9 provides a bar chart illustrating the critical path of the case project. The activities located in the longest bar of Figure 9 are defined as the most critical items of the case project. These critical items and correlative activities are used to make a critical path network diagram shown in Figure 10. The completion time of the case project under different paths can be solved based on the precedence relationships of each node and its activity time in the network diagram. Nine nodes and ten activities exist.

4.3.2. Model Numerical Application. Table 2 shows the input data of the case analysis. The duration of activity G is longer than that of activity F, though both can be conducted simultaneously. G is defined as one activity located in the critical path for subsequent analysis. Similarly, activity A and activity J can be conducted at the same period. The completed duration of activity A is longer than that of activity J. A is defined as one activity located in the critical path for subsequent

analysis. Hence, the critical path is A+B+C+D+E+G+H+I after analysis. Table 3 shows the different crash cost for different crash duration of each activity. Three segments for crash durations (1-10 days as segment 1, 11-20 days as segment 2, and ≥ 21 days as segment 3) are proposed with different crash costs for each activity. For example, the crash cost (10 days x 5,000USD/day + 6 days x 5,500USD/day) shall be paid when B is crashed by 16 days.

Table 4 shows the calculation results of project duration under different scenarios by applying the integer linear programming Model 1. Under optimistic time, the project can be completed in advance within the contractual deadline. Hence, we will not discuss this case. But under pessimistic time, the project will be overdue for 146 days (1,122 days–976 days). As mentioned in Section 4.1, the ceiling penalty is 10% of the contract amount, so the maximum delay penalty cost accepted by the contract is converted into 100 days. Hence, the next step is to focus on finding the optimum solution for crash cost and crash schedule in the cases of average time, most likely time, and pessimistic time.

Tables 5–7 show the calculation results of project crash plans by applying integer linear programming Model 2. With an integer linear programming technique, the overall project cost can be reduced using less expensive resources, and project planners can adjust the resource selection to shorten the project duration. Table 5 indicates that when the project is completed under average time, the optimum project duration meets the requested date, resulting in no delay penalty with a crash cost of 133,500 USD. Analysis shows that the delay penalty cost is 53,000 USD/day (0.1% of the contract price per day of delay), and the crash cost of several activities, such as J, B, C, and I, is much lower than the delay penalty cost per day. Hence, after the calculation of the integer optimization model via CPLEX R12.6.2 software, the crash plan for the average time case is to crash activity B for 10 days, activity C for 13 days, and activity I for 6 days to meet the contracted deadline with the lowest total cost. The required crash days of each activity for the average time case after analysis fall in the reasonable range as indicated in the crash time constraint of Table 2.

TABLE 2: Input data for case analysis.

Activity	Subsequent activity	o*	Duration (days) m*	p*	Average time (days) $t = (to+tm+tp)/3$	Crash time constraint (days) (days)	Crash time constraint (t - o) (m - o) (days)	Crash time constraint (p - o) (days)	Fixed cost (1000USD)
J (Site Preparation)	B	57	69	76	67	10	12	19	58
A (Design and Subcontracting)	B	107	119	131	119	12	12	24	2,453
B (Piling)	C	95	117	127	113	18	22	32	605
C (Foundation)	D	73	89	97	86	13	16	24	384
D (RC 2F)	E	59	73	79	71	11	14	20	1,500
E (RC 3F)	F, G	59	72	78	70	11	13	19	1,500
F (RC 4F)	H	58	72	79	70	12	14	21	1,500
G (Arch. 2F)	H	79	92	98	90	11	13	19	3,339
H (Arch. 3F)	I	59	72	78	70	11	13	19	3,339
I (Arch. 4F to shelter & Outside)	- (FINISH)	326	398	434	386	60	72	108	3,339

Note*: o: optimistic; m: most likely; p: pessimistic.

TABLE 3: Different crash cost for different crash duration.

Activity	Subsequent activity	Crash Cost (1000USD/DAY) ($k = \text{segment 1}$) *	Crash Cost (1000USD/DAY) ($k = \text{segment 2}$) *	Crash Cost (1000USD/DAY) ($k = \text{segment 3}$) *
J (Site Preparation)	B	0.5	1	2
A (Design and Subcontracting)	B	10	25	30
B (Piling)	C	5	5.5	8
C (Foundation)	D	4	4.5	7
D (RC 2F)	E	10	20	-
E (RC 3F)	E, G	10	20	-
F (RC 4F)	H	10	20	-
G (Arch. 2F)	H	20	30	40
H (Arch. 3F)	I	25	35	45
I (Arch. 4F to shelter &Outside)	- (FINISH)	5	8.5	10

Note*: segment 1: crash duration 1-10 days.
 Segment 2: crash duration 11-20 days.
 Segment 3: crash duration ≥ 21 days.

TABLE 4: Project duration in case of different expected times.

Activity	Average time (days)		Optimistic time (days)		Pessimistic Time (days)		Most likely time (days)	
	Start time	Project duration	Start time	Project duration	Start time	Project duration	Start time	Project duration
0	0		0		0		0	
A	119		107		131		119	
B	232		202		258		236	
C	318		275		355		325	
D	389	1005	334	857	434	1122	398	1032
E	459		393		512		470	
G	549		472		610		562	
H	619		531		688		634	
I	1005		857		1122		1032	

TABLE 5: Optimum project duration and crash plan in case of average time.

Node	Start time (days)	Activity	Crash time (days)	Crash cost (USD)	Project crash cost (USD)	Project delay penalty cost (USD)
1	0	A	0	0		
2	119	B	10	50,000		
3	222	C	13	53,500		
4	295	D	0	0		
5	366	E	0	0	133,500	0
6	436	G	0	0		
7	526	H	0	0		
8	596	I	6	30,000		
9	976					

Table 6 shows that when the project is completed by considering the case of the most likely time, the optimum project duration meets the requested date, resulting in no delay penalty with a crash cost of 306,000 USD. After the calculation of the integer optimization model via CPLEX software, the crash plan for the most likely time case is to crash activity B for 22 days, activity C for 16 days, and activity I for 18 days to meet the contracted deadline with the lowest total cost. The required crash days of each activity for the most

likely time case after analysis fall in the reasonable range as indicated in the crash time constraint of Table 2.

Table 7 shows that when the project is completed by considering the case of pessimistic time, the optimum project duration meets the requested date, resulting in no delay penalty with a crash cost of 1,149,000 USD. After the calculation of the integer optimization model via CPLEX software, the crash plan for the pessimistic time case is to crash activity A for 10 days, activity B for 32 days, activity C for 24 days,

TABLE 6: Optimum project duration and crash plan in case of most likely time.

Node	Start time (days)	Activity	Crash time (days)	Crash cost (USD)	Project crash cost (USD)	Project delay penalty cost (USD)
1	0	A	0	0		
2	119	B	22	121,000		
3	214	C	16	67,000		
4	287	D	0	0		
5	360	E	0	0	306,000	0
6	432	G	0	0		
7	524	H	0	0		
8	596	I	18	118,000		
9	976					

TABLE 7: Optimum project duration and crash plan in case of pessimistic time.

Node	Start time (days)	Activity	Crash time (days)	Crash cost (USD)	Project crash cost (USD)	Project delay penalty cost (USD)
1	0	A	10	100,000		
2	121	B	32	201,000		
3	216	C	24	113,000		
4	289	D	10	100,000		
5	358	E	10	100,000	1,149,000	0
6	426	G	0	0		
7	524	H	0	0		
8	602	I	60	535,000		
9	976					

TABLE 8: Analysis of crash cost and delay penalty cost in average time case.

Duration (days)	976	977	978	979	980	981
Overdue time (days)	0	1	2	3	4	5
Crash cost (USD)	133,500	128,500	123,500	118,500	113,500	108,500
Penalty cost (USD)	0	53,000	106,000	159,000	212,000	265,000
Total extra cost to pay (USD)	133,500	181,500	229,500	277,500	325,500	373,500

activity D for 10 days, activity E for 10 days, and activity I for 60 days to meet the contracted deadline with the lowest total cost. The required crash days of each activity for the pessimistic time case after analysis fall in the reasonable range as indicated in the crash time constraint of Table 2.

4.3.3. Crash Plan Feasibility. In the case of average time, we set parameter L (project duration after crashing) to 976-981 and analyze the differences in the crash cost and delay penalty cost of the project. As shown in Table 8 and Figure 11, the slope of the decline of the crash cost with many overdue days is less steep than the slope of the rise of the total extra cost. When this project has longer duration, the gap between crash cost and total extra cost is getting bigger and delay penalty cost is increased accordingly. Given that the delay penalty cost of this case is high, it is suggested that the crash plan be conducted to make the project schedule completion meet the contracted deadline. Any extension of the project duration increases the total extra cost for this case project. However, from the viewpoint of total project cost minimized, if the

cost of crash activities is higher than the delay penalty, then conducting the crash plan becomes an unsuitable option. Hence, optimizing the relationship between crash cost and crash time to achieve the best solution is worthy of further evaluation.

After the several analyses in this study, the company can find the appropriate execution option for each activity so that the project can be completed by a desired deadline with the minimum cost. The company can make a final decision with support data to approach this case project with confidence.

5. Discussion and Conclusion

Various risks and uncertainties exist in EPC projects. They lead to projects not completed within time and cost limits. In the construction industry, contractors always use previous execution experiences to estimate project durations and costs, which lead to many risks during project execution. This study presents the frameworks of project schedule risk analysis and time-cost trade-off strategy. It introduces a hybrid method for solving time-cost trade-off problems based on Monte

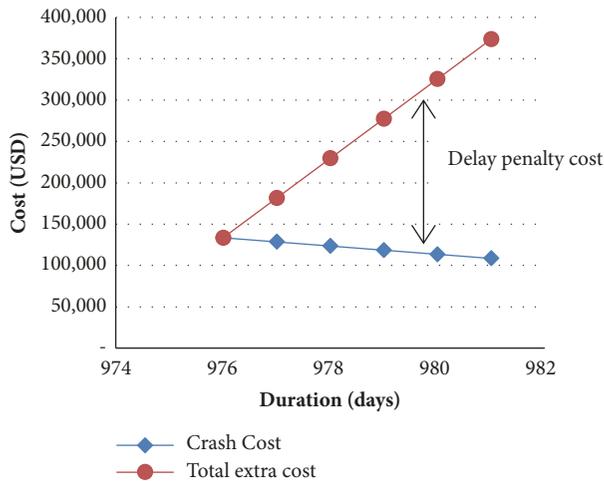


FIGURE 11: Illustration for crash cost and delay penalty cost in average time case.

Carlo simulation and integer linear programming. Monte Carlo simulation is initially used to develop the probability distribution of the possible project completion duration in different scenarios by considering potential risks, and integer linear programming is applied to determine the crashing strategy for time-cost optimization.

Monte Carlo simulation can be used for the engineering schedule risk analysis to obtain the most probable project completion time by identification of risk factors and sensitivity analysis. Qualitative and quantitative risk analyses should be conducted for all potential risk items during the quotation phase. As in the case study, the project completion duration specified in the contract, which is 976 days, turns out to be an impossible requirement because the simulation results validate that the total project duration is 1,079 days with 80% probability and 1,123 days with 100% probability after considering the risk mitigation effects. On the basis of the risk analysis result, a contractor might propose a reasonable schedule to the owner to avoid the fine caused by schedule delay. But the owner would not accept it and would disqualify the contractor from the bidding. On the other hand, paying delay penalties will result in serious damage to the corporation's reputation, so companies are not inclined to choose this option. Therefore, contractors must determine crashing strategies and assess whether to approach this project. This study introduces a new hybrid method to provide a simple tool to evaluate project execution under a crashing strategy and risk consideration. The proposed hybrid method selects the most critical path using a schedule sensitivity index from Monte Carlo simulation results and then uses a mathematical integer linear programming optimization method to solve the constructed model for the selected path. This model can be effectively applied in a practical EPC project for assessment during a bidding stage and may help project planners to manage the project completion time accurately from a risky amount to a desirable predefined value. This approach will also allow managers to understand the trade-off between project execution time and cost under crashing strategies

and risk consideration. It enables managers to optimize their decision-making reference while approaching a project.

This study (1) provides comprehensive plans for project schedule risk analysis and time-cost trade-off strategy, (2) identifies and understands the critical path and near critical paths through a real case study, (3) proposes a hybrid approach with Monte Carlo simulation and mathematical integer linear programming to solve problems related to project scheduling, (4) constructs a mathematical model of the project crashing strategy coded by CPLEX to determine the optimal project completion schedule and minimize the total cost, (5) analyzes the relationship between project crash cost and delay penalty, and (6) provides reference points to project budgeting under crashing strategy and risk consideration.

Data Availability

All data are provided by the author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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