Hindawi
Mathematical Problems in Engineering
Volume 2019, Article ID 9651489, 12 pages
https://doi.org/10.1155/2019/9651489

Research Article

A Time-Replacement Policy for Multistate Systems with Aging Components under Maintenance, from a Component Perspective

Chao-Hui Huang [1] **and Chun-Ho Wang** [2]

[1] *Department of Applied Science, R.O.C. Naval Academy, No. 669, Junxiao Rd., Zuoying District, Kaohsiung City 81345, Taiwan*
[2] *Department of Power Vehicle and Systems Engineering, Chung Cheng Institute of Technology, National Defense University, No. 75, Shiyuan Rd., Daxi Township, Taoyuan County 33551, Taiwan*

Correspondence should be addressed to Chun-Ho Wang; u8733803@gmail.com

Received 2 September 2018; Revised 10 January 2019; Accepted 5 February 2019; Published 18 March 2019

Academic Editor: Gen Q. Xu

This study aims for multistate systems (MSSs) with aging multistate components (MSCs) to construct a time-replacement policy and thereby determine the optimal time to replace the entire system. The nonhomogeneous continuous time Markov models (NHCTMMs) quantify the transition intensities among the degradation states of each component. The dynamic system state probabilities are therefore assessed using the established NHCTMMs. Solving NHCTMMs is rather complicated compared to homogeneous continuous time Markov models (HCTMMs) in determining reliability related performance indexes. Often, traditional mathematics cannot acquire accurate explicit expressions, in particular, for multiple components that are involved in designed system configuration. To overcome this difficulty, this study uses Markov reward models and the bound approximation approach to assess rewards of MSSs with MSCs, including such things as total maintenance costs and the benefits of the system staying in acceptable working states. Accordingly, we established a long-run expected benefit (LREB) per unit time, representing overall MSS performance through a lifetime, to determine the optimal time to replace the entire system, at which time the LREB values are maximized. Finally, a simulated case illustrates the practicability of the proposed approach.

1. Introduction

Conventional binary-state systems assume that component constituted system configurations involve either only perfect functioning or complete failure. The optimal system reliability design, maintenance policy, and repairing management are thereby determined. However, the design of modern devices tends to be of large scale and complicated; these devices normally confront different kinds of faults and errors, including damage, impacts, and aging factors, throughout their lifetimes. Various systems, such as computer server, telecommunication, and electricity distribution systems, become tolerant to these faults and errors. Even if a fault occurs, these systems continue working at an acceptable or degraded performance level. Accordingly, from being perfectly functioning, systems normally experience multiple intermediate states during the degradation process, before complete failure occurs. Confining systems to binary states can ignore the intrinsic multistate property of these systems and result in a biased evaluation of actual system performance. Therefore, constructing a multistate reliability theory can provide further insight into the complicated failure theory, lifetime prediction, and improvement of system reliability, which is an essential issue in both practical and theoretical aspects.

Multistate systems (MSSs) are regarded as failures when they degrade into unacceptable performance levels and cannot meet operational requirements. Preventive maintenance (PM) implementation is beneficial to sustain or improve system performance during the planning horizon. Incorporating imperfect maintenance theories into the stochastic process [1, 2] can properly imitate the status of MSS degradation. Although components can exhibit better performance after maintenance, satisfying system performance requirements as these components gradually age is usually necessary. Overly frequent maintenance not only consumes a tremendous amount of cost but also decreases the marginal effectiveness of system performance improvements and increases maintenance time. This phenomenon induces the essential issue of determining the time-replacement policy for MSSs.

This study extends the work of Wang and Huang [3] which aims for MSSs with aging multistate components (MSCs) to establish an optimal time-replacement policy, given the PM policy from the component perspective. Determining a PM policy from the component perspective has practical applications because the component can be monitored in real time, and its degradation or failure can cause the sudden failure of the system, resulting in catastrophic consequences. A recent case of this occurred when General Motors developed the On Star system, which can monitor car parts and alert drivers before the system needs to be repaired [4]. A performance index regarding the long-run expected benefit (LREB) per unit time for an MSS with aging MSCs was established to evaluate its long-run benefits. The optimal time to replace all the components in MSSs can be determined easily by maximizing the long-run expected benefit (LREB) values. Mathematically, the nonhomogeneous continuous time Markov reward models (NHCTMRMs) [5, 6] for all components and the system are established to evaluate the maintenance costs and benefits of a well-maintained system. Similarly, establishing NHCTMMs mainly calculates the instantaneous state probability distribution of MSSs and thereby determines system availability for MSSs with aging MSCs. Solving the NHCTMMs and NHCTMRMs is more complicated than solving the HCTMMs. This solving difficulty is significantly increased along with the system augmentations, such as the extension of components and their degradation states. Under this consideration, conventional mathematics cannot obtain explicit expressions of NHCTMM and NHCTMRM solutions. Often, using calculators as common mathematical tools, such as MATLAB or Mathcad, may induce the problem of inaccuracy [7–10]. Ding et al. [9] developed the bound approximation approach that can solve the NHCTMMs efficiently. The bound approximation approach partitions the system lifetime into multiple tiny intervals and sets the transition intensities within intervals as constant. The HCTMMs then find the instantaneous probability at the end of each time interval; via iterative procedure, the NHCTMMs can be solved efficiently. This study uses the bound approximation approach [9] to solve the nonlinear simultaneous differential equations corresponding to the established NHCTMMs and NHCTMRMs. The time-dependent LREB index values are then determined throughout lifetime. Maximizing the LREB index values can allow engineers to find the optimal time to replace an MSS given a specified PM policy that is implemented from the component perspective [11]. A simulated case involving execution of sensitivity analysis verifies the efficacy of the proposed approach.

2. Literature Review

In general, some indices were proposed in evaluation of dynamic system over time mainly related with system availability; these indices can fall into two categories with expense-oriented and benefit-oriented indices. These two categories commonly refer to index like "long-run average cost" [12, 13] and "long-run expected profit" [14, 15], respectively, despite other similar measures. Zhang and Wang [12] proposed a replacement policy for a deteriorating system with multifailure modes including nonrepairable failure (catastrophic failure) and k repairable failure modes. On the basis of a geometric process model to evaluate the aging effect and accumulative wear for a deteriorating system, they established an explicit expression of the long-run average cost (LRAC) per unit time of a system. The system is replaced whenever the number of system failures reaches N or a nonrepairable failure occurs. The optimal N^* is determined by minimizing the LRAC. Although the property of system deterioration is considered in the proposed replacement policy [11], their study is confined to a binary system. Wang and Zhang [16] extended the study of Zhang and Wang [12], except that the repair times after failure are assumed to be independent and identically distributed. Two replacement models were proposed: one is based on the limiting availability and the other is based on the LRAC rate of the system. By maximizing the limiting availability A(N) and minimizing the LRAC rate C(N), the optimal replacement policies N in both cases are determined. Chang et al. [17] proposed a replacement model given the system experiences including a type-I failure (minor), rectified by a minimal repair, and a type-II failure (catastrophic) that calls for a replacement. A long-run expected cost per unit time in operating the system, incorporating costs due to minimal repair and different forms of the replacement state derived, is a determinant to decide whether to repair or replace the system. The maintenance model for replacement policy in Chang et al. [17] is from the perspective of a binary system. Jain and Gupta [18] proposed an optimal replacement policy for a repairable system with multiple vacations and imperfect fault coverage. The reliability analysis of a repairable system consists of a single repair person who can take multiple vacations. A repair system considers two types of failure modes: major and minor. Maintenance policy involves perfect recovery of the system after a minor fault, whereas imperfect recovery with some probability occurs after a major fault. System lifetime and vacation time of the repair person are assumed to be exponentially distributed, whereas the repair time follows a general distribution. By assuming a geometric process for system working/vacation time, the supplementary variable technique and Laplace transform approach derive the reliability indices of the system. Accordingly, a replacement policy to maximize the expected profit after a long run time was proposed.

Liu and Huang [19] constructed an optimal replacement policy for a system consisting of a single multistate element. This study mainly considers scenarios in which the performance rate and transition intensities cannot be crisply determined because of a lack of accuracy or fluctuation of collected data. The optimal threshold value of degradation states related to replacing an element is determined to maximize its objective value of average benefit per unit time. Zhang et al. [20] studied a degenerately simple system with $k + 1$ states, including k failure states and one working state. A replacement policy N based on the number of system failures was applied given implementation of imperfect maintenance. The objective was to maximize the long-run expected profit per unit time. The explicit expression of the long-run expected profit per unit time was derived and the corresponding optimal solution was determined. Zhang and Wang [13] extended the study of Zhang et al. [20] and proposed a new replacement policy, T, based on the system age for a multistate degenerative simple system. The model assumes that the system after repair is not "as good as new," and the degeneration of the system is stochastic. The explicit expression of the long-run average cost per unit time is derived and minimized to determine an optimal replacement policy T^*. The long-run average cost is mathematically the same measure as the long-run expected cost referred to in Chang *et al.* [17]. Liu and Huang [14] developed an optimal replacement policy for an MSS with imperfect maintenance from the system perspective. There are some efforts aiming at determining the PM policy from the system perspective [9, 21, 22]. Once the system falls into an unacceptable state, the system fails and imperfect maintenance for all components is implemented without considering the states of the components. The established corrective maintenance model assumes that the components recover back to a perfect functioning state after maintenance, whereas degradation intensities increase with the amount of maintenance to characterize the impact of aging factors on components using a quasi-renewal process [15]. An LREB index per unit time associated with the number of MSS failures, denoted as N, was developed to determine the optimal MSS failure number N to replace a whole system. Liu and Huang [14] used the NHCTMMs to evaluate the instantaneous multistate probability distribution of aging components. Then, the stochastic process of MSSs was elucidated in terms of their multistate probability distribution, derived by a universal generating function [23]. The maintenance costs of components accounted for in Liu and Huang [14] are independent of their performances levels: no matter the degradation states of the components, maintenance costs are identical. Normally, components at low degradation states cost much more to be restored to perfect functioning than those at high degradation states in maintenance. Furthermore, the components may not need to be restored to perfect status. A rather simple system configuration was illustrated to obtain explicit expressions of system performances in the study.

Much of the previously discussed literatures aimed at binary systems to address the problem of replacement policy given imperfect maintenance from the system implementation perspective. Although some works address the replacement policy problems for multistate systems, the PM from the perspective of a system with binary components is given. Furthermore, in order to deduce the explicit expression of the MSS performance measure, a rather simple system configuration is given to illustrate the theoretical results. Practically, for a safety system, the PM policy from the component perspective can prevent a sudden system failure due to component degradation or failure, avoiding possible catastrophic consequences. In this regard, this study aims at MSSs with aging MSCs to establish an optimal time-replacement policy, given the implementation of a PM policy from a component perspective. The PM policy involves maintenance actions in the degradation states for aging MSCs. A performance index regarding the long-run expected benefit (LREB) per unit time for MSSs with aging MSCs determined the optimal time to replace an MSS by maximizing the LREB values throughout the lifetime. The term "benefit," a general designate, corresponds to the profit making with system staying at acceptable states capable of functioning properly with distinct performances in this paper. Alternatively, given other considerations, the term "benefit" could relate to other measures such as productivity and delivery rate in a manufacturing system.

3. NHCTMMs

3.1. NHCTMM for an Aging MSC. The Lisnianski and Levitin [24, 25] use the homogeneous continuous time Markov model to characterize the degradation process of individual MSC. This approach assumes that the transition intensity to the next state has no age effect only depending on the current state. It is inapplicable for cases that a component's degradation process does not only rely on the current state but also the age of the component. Taking the aging factors, the NHCTMMs are employed to obtain the stochastic behavior of individual aging components by transition intensities enhanced with age. For a repairable aging MSC, the component degrades to states with lower performance rate from the states with higher performance rate; conversely, it is restored to states with higher performance rate from the states with lower performance rate after appropriate maintenances. Figure 1 shows a typical state-transition diagram of a repairable aging MSC.

For component l, $l = 1, 2, \ldots, m$, the performance degradation is characterized by the stochastic process $\{G^l(t) \mid t \geq 0\}$. The intensity $\lambda^l_{ij}(t)$ of any transition from state i to state j, $j \in \{i-1, i-2, \ldots, 1\}$, is a monotonically increasing function associated with component age, while μ^l_{ji} of any transition from state j to state i is a repairing rate indicating the repair time is exponentially distributed. The corresponding Chapman-Kolmogorov differential equation of NHCTMM for this repairable aging MSC is expressed as

$$\frac{d}{dt} p^l_j(t) = \sum_{\substack{i=1\\ i\neq j}}^{K_l} p^l_i(t)\,\alpha^l_{ij}(t) - p^l_j(t) \sum_{\substack{i=1\\ i\neq j}}^{K_l} \alpha^l_{ji}(t), \qquad j = 1, 2, \ldots, K_l \tag{1}$$

where $p^l_i(t)$ and $p^l_j(t)$ are the instantaneous probabilities of states i and j occurring at instant t, respectively, for component l. $\alpha^l_{ij}(t)$ and $\alpha^l_{ji}(t)$ are time-varying transition intensities from state i to j and from j to i in terms of $\lambda^l_{ij}(\mathrm{t})$ and μ^l_{ji}, respectively. Equation (1) contains K_l nonlinear differential equations. Ideally, given the initial probability values of all states, the distribution of states for a component at instant t can be determined by solving the NHCTMM.

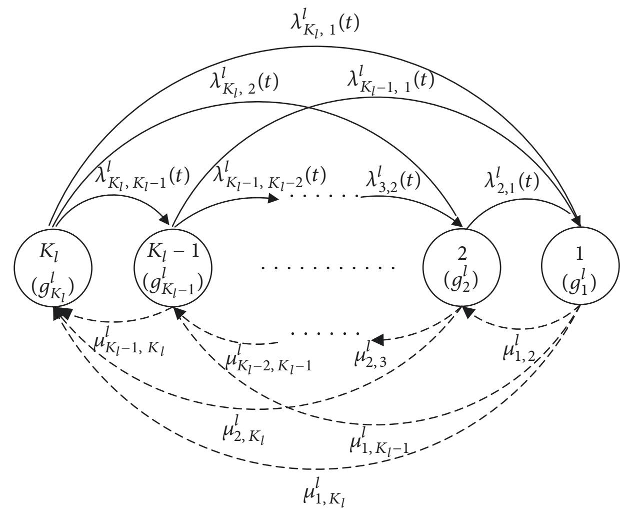

FIGURE 1: State-transition diagram of a component.

However, solving the NHCTMM is a complicated challenge [7–10].

3.2. The NHCTMM and NHCTMRM for the MSS with Aging MSCs. When the MSS consists of n aging MSCs, its performance rate is clearly determined by the performance rates of these MSCs. The state of the entire system is determined by the states of MSCs configured in the system. Let $L^n = \{g_1^1, \ldots, g^1_{K_1}\} \times \{g_1^2, \ldots, g^2_{K_2}\} \times \ldots \times \{g_1^n, \ldots, g^n_{K_n}\}$ be the space of possible combinations of performance rates for all the MSCs. Mapping the space of the components performance rate into the system's performance rate according to MSS structure function can derive the space of possible values of performance rates for the entire system $M = \{g_1, \ldots, g_K\}$. Mathematically, the space of performance rates for a MSS significantly increases with the augmentation of a MSS such as extension of components and their degradation states. Technically, the MSS states with identical performance rate are united into one state and thereby establish the Markov model to reduce the solving complexity of the established NHCTMMs without calculation inaccuracy. Accordingly, the MSS performance measures are determined. The Chapman-Kolmogorov differential equation of NHCTMM for a MSS is expressed as

$$\frac{d}{dt} p_j(t) = \sum_{\substack{i=1 \\ i\neq j}}^{K} p_i(t)\,\alpha_{ij}(t) - p_j(t) \sum_{\substack{i=1 \\ i\neq j}}^{K} \alpha_{ji}(t), \qquad j = 1, 2, \ldots, K \tag{2}$$

where $p_i(t)$ and $p_j(t)$ are the instantaneous probabilities of states i and j occurring at instant t, respectively. $\alpha_{ij}(t)$ and $\alpha_{ji}(t)$ are time-varying transition intensities from state i to j and from state j to i in terms of $\lambda^l_{ij}(t)$ and μ^l_{ji}, respectively, for a MSS. Equation (2) contains K nonlinear differential equations. Ideally, given the initial probability values of all MSS states, the MSS states probabilities distribution at instant t can be determined.

The nonhomogeneous Markov reward model [24, 26] can be used to effectively assess the reliability performances and cost associated measures for an aging MSC/MSS over its lifetime. Basically, the NHCTMRM is a Markov process with rewards; the reward matrix $\mathbf{r} = [r_{ij}]$ must be constructed according to varied indicators such as maintenance costs, revenue staying at the acceptable states, availability, and mean time to failure. The Chapman-Kolmogorov differential equation corresponding to NHCTMRM for a MSS is expressed as

$$\frac{dV_i(t)}{dt} = r_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{K} \alpha_{ij}(t)\, r_{ij} + \sum_{j=1}^{K} \alpha_{ij}(t)\, V_j(t), \qquad i = 1, 2, \ldots, K \tag{3}$$

where $V_i(t)$ and $V_j(t)$ are the total expected reward (TER) values accumulated until time t while in states i and j, respectively. Substituting the initial value $V_i(0) = 0$ into (3), the simultaneous nonlinear differential equations can be solved to derive the TER value for a MSS.

4. Bound Approximation Approach

Solving the NHCTMM to obtain the aging MSC/MSS performance indicators requires a lot of time. Often, the use of calculators embedded in the common mathematical tools, such as MATLAB or MATHCAD, may induce the problem of inaccuracy [8, 13]. The bound approximation approach [9] allows the determination of instantaneous state probabilities for an aging MSC/MSS. This approach divides the lifetime into multiple intervals and sets the failure rate during each interval to be a constant. The HCTMM is then used to find the instantaneous probability at the end of each time interval. As an example of an aging MSC, this numerical approach initially divides the component lifetime T into N time intervals. The duration of each time interval is

$\Delta t = T/N$. Then, two constants $\lambda_{ij}^{l(n-)}$ and $\lambda_{ij}^{l(n+)}$ are used to approximate the failure rate $\lambda_{ij}^{l}(t)$ in each time interval $t_n = [\Delta t \cdot (n-1), \Delta t \cdot n]$, $1 \leq n \leq N$, using the following equations:

$$\lambda_{ij}^{l(n-)} = \lambda_{ij}^{l}(\Delta t \cdot (n-1)) \tag{4}$$

$$\lambda_{ij}^{l(n+)} = \lambda_{ij}^{l}(\Delta t \cdot (n)) \tag{5}$$

where $\lambda_{ij}^{l(n-)}$ and $\lambda_{ij}^{l(n+)}$ represent the failure rate of the l^{th} component at the beginning and end of the n^{th} time interval. Equations (4) and (5) also give the lower and upper bounds of $\lambda_{ij}^{l}(t)$ in the n^{th} time interval. Using $\lambda_{ij}^{l(n-)}$ and $\lambda_{ij}^{l(n+)}$ to solve the component's NHCTMM, the state probabilities $P_j^{l(n-)}(\Delta t \cdot n)$ and $P_j^{l(n+)}(\Delta t \cdot n)$ at the end of time interval $t_n = [\Delta t \cdot (n-1), \Delta t \cdot n]$, $1 \leq n \leq N$, can be derived. These differential equations [24] are

$$\begin{aligned} \frac{dP_j^{l(n-)}(t)}{dt} &= \sum_{\substack{i=1\\ i\neq j}}^{K_l} P_i^{l(n-)}(t)\,\alpha_{ij}^{l(n-)}(t) \\ &\quad - P_j^{l(n-)}(t)\sum_{\substack{i=1\\ i\neq j}}^{K_l}\alpha_{ji}^{l(n-)}(t), \\ &\qquad j = 1,\ldots,K_l \end{aligned} \tag{6}$$

$$\begin{aligned} \frac{dP_j^{l(n+)}(t)}{dt} &= \sum_{\substack{i=1\\ i\neq j}}^{K_l} P_i^{l(n+)}(t)\,\alpha_{ij}^{l(n+)}(t) \\ &\quad - P_j^{l(n+)}(t)\sum_{\substack{i=1\\ i\neq j}}^{K_l}\alpha_{ji}^{l(n+)}(t), \\ &\qquad j = 1,\ldots,K_l \end{aligned} \tag{7}$$

At each time interval t_n, the lower bound $\lambda_{ij}^{l(n-)}$and upper bound $\lambda_{ij}^{l(n+)}$ of the failure rate are utilized to determine the lower bound and upper bound intensities $\alpha_{ij}^{l(n-)}(t)$ and $\alpha_{ij}^{l(n+)}(t)$ for transitions from state i to j. During the first time interval, the initial condition of the component is already known. Hence, given that the component is in state K at $t = 0$, the initial conditions for (6) and (7) during the first time interval $n = 1$ are as follows:

$$\begin{aligned} P_{K_l}^{l(1-)}(0) &= 1, \\ P_{K_l-1}^{l(1-)}(0) &= \ldots P_1^{l(1-)}(0) = 0 \end{aligned} \tag{8}$$

$$\begin{aligned} P_{K_l}^{l(1+)}(0) &= 1, \\ P_{K_l-1}^{l(1+)}(0) &= \ldots P_1^{l(1+)}(0) = 0 \end{aligned} \tag{9}$$

The initial conditions for t_n, $n = 2, 3, \ldots, N$, are defined by the following recurrence relations:

$$\begin{aligned} P_j^{l(n-)}(\Delta t \cdot (n-1)) &= P_j^{l((n-1)-)}(\Delta t \cdot (n-1)), \\ &\qquad j = 1, 2, \ldots, K_l \;\; n = 1, 2, \ldots, N \end{aligned} \tag{10}$$

$$\begin{aligned} P_j^{l(n+)}(\Delta t \cdot (n-1)) &= P_j^{l((n-1)+)}(\Delta t \cdot (n-1)), \\ &\qquad j = 1, 2, \ldots, K_l \;\; n = 1, 2, \ldots, N \end{aligned} \tag{11}$$

This means that the initial conditions for the next interval are defined by the solutions at the end of preceding time interval. By solving the NHCTMRM using the bound approximation approach, the lower bound $V_i^{l(n-)}$ and upper bound $V_i^{l(n+)}$ of the TER accumulated at each time interval $[\Delta t \cdot (n-1), \Delta t \cdot n]$ can be obtained from any state. The equations for the NHCTMRM are

$$\begin{aligned} \frac{dV_i^{l(n-)}(t)}{dt} &= r_{ii}^{l} + \sum_{\substack{j=1\\ j\neq i}}^{K_l}\alpha_{ij}^{l(n-)}(t)\,r_{ij}^{l} \\ &\quad + \sum_{j=1}^{K_l} a_{ij}^{l(n-)}(t)\,V_j^{l(n-)}(t), \\ &\qquad i = 1, 2, \ldots, K_l, \;\; n = 1, \ldots, N \end{aligned} \tag{12}$$

$$\begin{aligned} \frac{dV_i^{l(n+)}(t)}{dt} &= r_{ii}^{l} + \sum_{\substack{j=1\\ j\neq i}}^{K_l}\alpha_{ij}^{l(n+)}(t)\,r_{ij}^{l} \\ &\quad + \sum_{j=1}^{K_l} a_{ij}^{l(n+)}(t)\,V_j^{l(n+)}(t), \\ &\qquad i = 1, 2, \ldots, K_l, \;\; n = 1, \ldots, N \end{aligned} \tag{13}$$

For any state during each time interval, the initial reward is 0; that is,

$$\begin{aligned} V_i^{l(n-)}(0) &= V_i^{l(n+)}(0) = 0, \\ &\qquad i = 1, 2, \ldots, K_l, \;\; n = 1, \ldots, N \end{aligned} \tag{14}$$

Solving (12) and (13) under the initial condition (14) gives the lower and upper bounds of TER accumulated during each time interval for all states. Multiplying $V_i^{l(n-)}(\Delta t)$ and $V_i^{l(n+)}(\Delta t)$ by their corresponding state probabilities $P_i^{l(n-)}(\Delta t \cdot (n-1))$ and $P_i^{l(n+)}(\Delta t \cdot (n-1))$ gives the system's upper and lower mean reward values for any state during each time interval. The sum of all mean reward values for all states gives the component's overall lower reward bound $V^{l(n-)}$ and upper reward bound $V^{l(n+)}$ for any time interval. These bounds are calculated as

$$V^{l(n-)}(t) = \sum_{i=1}^{k_l} V_i^{l(n-)}(\Delta t) \cdot P_i^{l(n-)}(\Delta t \cdot (n-1)), \quad n = 1, \ldots, N \tag{15}$$

$$V^{l(n+)}(t) = \sum_{i=1}^{k_l} V_i^{l(n+)}(\Delta t) \cdot P_i^{l(n+)}(\Delta t \cdot (n-1)), \quad n = 1, \ldots, N \tag{16}$$

where $t = \Delta t \cdot (n-1)$. Finally, summing the TER over N time intervals gives the lower and upper bounds of TER over the component's lifetime:

$$TER^{l(-)}(t) = \sum_{n=1}^{N} V^{l(n-)}(t) \tag{17}$$

$$TER^{l(+)}(t) = \sum_{n=1}^{N} V^{l(n+)}(t) \tag{18}$$

The exact $TER^l(t)$ value falls somewhere between the lower and upper bound; that is, $TER^{l(-)}(t) \le TER^l(t) \le TER^{l(+)}(t)$. A more accurate $TER^l(t)$ value can be obtained by dividing T into smaller intervals.

5. Proposed Approach

This study aims for MSSs with aging MSCs to determine the time-replacement policy in which a PM from the component perspective is implemented. The calculation procedure integrates the multiple states of all components to obtain the distinctive states for MSSs and, therefore, to determine MSS performances that are established on the basis of nonhomogeneous Markov models. As mentioned previously, the bound approximation approach [9] can solve the simultaneous Chapman-Kolmogorov differential equations related to the established NHCTMMs. This study develops an LREB per unit time for MSSs with MSCs, taking into consideration maintenance costs, the benefits of a well-maintained system, and system loss due to maintenance. The establishment of the LREB index is as follows:

$$LREB(t) = \frac{V_a(t) - V_r(t) - C_r \cdot t_{rep} - C_{rep}}{t + t_{rep}} \tag{19}$$

where $LREB(t)$ represents the LREB of a system during t period; $V_a(t)$ represents the system benefit for staying at acceptable states during t period. The benefit mainly relates to the production rate of a system. $V_r(t)$ represents the total maintenance cost of a system during t time period. C_r represents the replacement expense per unit time; t_{rep} represents the time required to replace a system; C_{rep} represents material expense related to system replacement. Accordingly, the optimal replacement time is determined by maximizing the LREB values throughout a lifetime. A simulated system configuration with two distinctive PM policies elucidates the proposed approach. The following sections illustrate the model assumptions and calculation procedures.

5.1. Model Assumptions

(1) The aging MSCs of the system degrade from perfectly functioning to complete failure over multiple states of degradation.

(2) The failure rate of an individual component is an increasing function of time.

(3) Components at degradation states can be restored to previous better states by appropriate maintenance.

(4) Real-time monitoring of the system can identify the performance of individual components within the system.

(5) A specific PM policy is implemented on the aging MSCs.

(6) There are five maintenance activities:

    (1) No service or repair.
    (2) Minor service: enables restoration to state $j + 1$ from state j.
    (3) Major service: enables restoration to state $j + 2$ from state j.
    (4) Minor repair: enables restoration to state $j + 3$ from state j.
    (5) Major repair: enables restoration to state $j + 4$ from state j.

5.2. Calculation Procedure of LREB for Numerical Cases. The proposed approach is elucidated on the basis of a series-parallel system [11]. The structural function of flow transmission is used to model series and parallel links in the system. This simulated system contains three components. Components 1 and 2 are connected in parallel: both are connected to component 3 in series. Each component has five states possessing distinctive output performance, with state 5 being perfectly functioning and state 1 being complete failure. For example, in component 1, the output performances of its five states in descending order are 150, 100, 80, 50, and 0. Each individual component is initially in a perfectly functioning state. Hence, the initial probability of all the states is 0, except for state 5, which has a probability of 1. The minimum acceptable system performance (user demand) is set as 100. Figure 2 shows the system configuration with possible PM implementations at degradation states (the dotted lines) in state-space diagrams of MSCs. Table 1 lists the failure-rate functions of aging MSCs, whereas Table 2 presents the repair rates. Table 3 lists maintenance costs related to PM actions of components, whereas Table 4 presents the replacement-related parameters. Table 5 lists the benefits corresponding to system states with distinct performances.

The calculation procedure involved in the proposed approach has five steps.

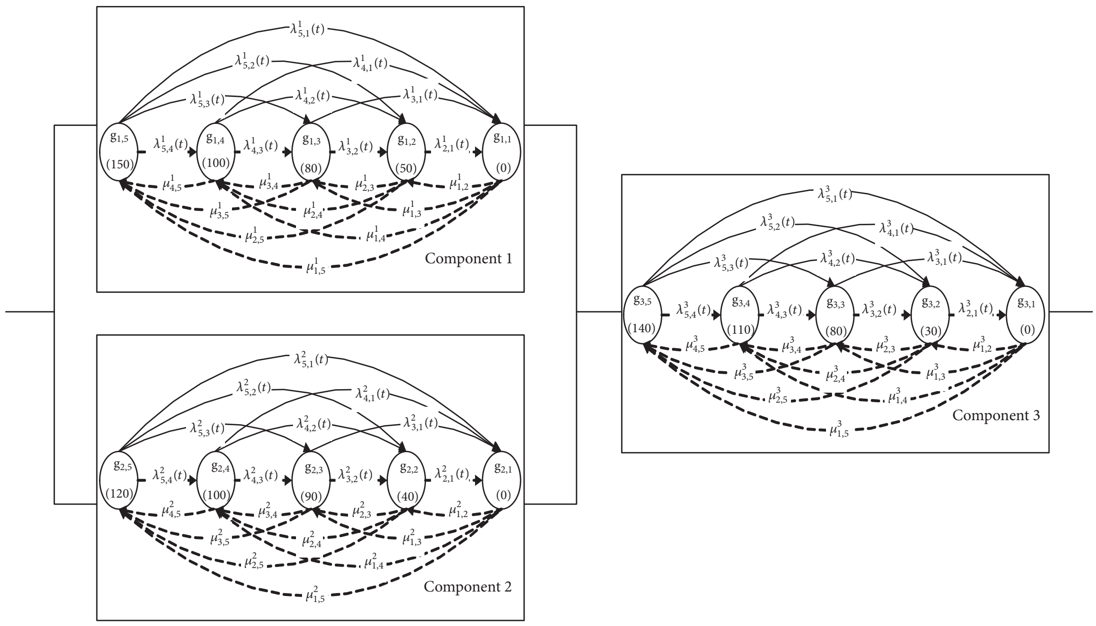

Figure 2: MSS configuration with MSC state-space diagrams.

Table 1: Failure-rate function of each component between states.

Failure rate	Component		
	1	2	3
$\lambda_{5,4}(t)$	0.24+0.07 t	0.24+0.07 t	0.34+0.14 t
$\lambda_{5,3}(t)$	0.18+0.04 t	0.18+0.04 t	0.28+0.08 t
$\lambda_{5,2}(t)$	0.14+0.02 t	0.14+0.02 t	0.24+0.04 t
$\lambda_{5,1}(t)$	0.12+0.01 t	0.12+0.01 t	0.22+0.02 t
$\lambda_{4,3}(t)$	0.26+0.08 t	0.26+0.08 t	0.36+0.16 t
$\lambda_{4,2}(t)$	0.20+0.05 t	0.20+0.05 t	0.30+0.10 t
$\lambda_{4,1}(t)$	0.16+0.03 t	0.16+0.03 t	0.26+0.06 t
$\lambda_{3,2}(t)$	0.28+0.09 t	0.28+0.09 t	0.38+0.18 t
$\lambda_{3,1}(t)$	0.22+0.06 t	0.22+0.06 t	0.32+0.12 t
$\lambda_{2,1}(t)$	0.30+0.10 t	0.30+0.10 t	0.40+0.20 t

Note: $\lambda_{i,j}$(t) is the failure rate at time t (hrs) of each component from state i to state j.

Step 1. Generate the possible states of the MSSs.

The proposed approach initially combines the multistate components constituting the system to form an MSS. For the simulated case, in total, 125 possible states are determined for this system, configuring three components each with five distinctive states with different performances.

Step 2. Obtain the reduced MSS.

Uniting the systems states with an identical performance from step 1 can obtain the reduced MSS. This process can significantly reduce the subsequent calculation complex without loss of system performance information. Accordingly, a total of ten distinctive performance states for this system are determined; the state-transition diagram of this reduced MSS is also constructed. Figure 3 displays the corresponding diagram.

Step 3. Determine the total maintenance cost of the system.

According to the state-transition diagrams for components 1–3 shown in Figure 2, the NHCTMMs and NHCTMRMs of three components are established, respectively. The reward matrix of NHCTMRMs is determined from Table 5. The bound approximation approach [9] then solves the nonlinear simultaneous differential equations related to the established NHCTMMs and NHCTMRMs. Solving NHCTMMs obtains instantaneous state probability distributions of three components, whereas solving NHCTMRMs obtains the total maintenance cost for five states regarding three components. The expected total maintenance costs for these three components are accordingly calculated. Summing up the total maintenance cost of three components derives the total maintenance cost for this system.

Step 4. Determine the MSS benefit.

Initially, the NHCTMMs and NHCTMRMs related to the reduced MSS are established, respectively, on the basis of the state-transition diagrams shown in Figure 3. The reward matrix of NHCTMRMs is determined from the benefit parameters in Table 5. The bound approximation approach [9] again solved the established NHCTMMs and NHCTMRMs; solving NHCTMMs obtains instantaneous probability distributions of ten states for reduced MSS. By using the obtained probability distributions, the expected total system benefit for this system is obtained after complete calculations of the NHCTMRMs.

Step 5. Determine the optimal time to replace the system.

First, using (19), the LREB values over the time horizon can be determined given that specific time can be obtained. Then the optimal time to replace the system is therefore determined by maximizing the LREB value. This study uses

Table 2: Repair rate of each component between states (per hour).

Component	Repair rate									
	$\mu_{1,5}$	$\mu_{1,4}$	$\mu_{2,5}$	$\mu_{1,3}$	$\mu_{2,4}$	$\mu_{3,5}$	$\mu_{1,2}$	$\mu_{2,3}$	$\mu_{3,4}$	$\mu_{4,5}$
1	0.125	0.320	0.335	0.410	0.425	0.440	0.455	0.470	0.485	0.500
2	0.080	0.245	0.260	0.275	0.290	0.305	0.350	0.365	0.380	0.395
3	0.065	0.095	0.110	0.140	0.155	0.170	0.185	0.200	0.215	0.230

Note: $\mu_{j,i}$ is the repair rate of each component from state j to state i.

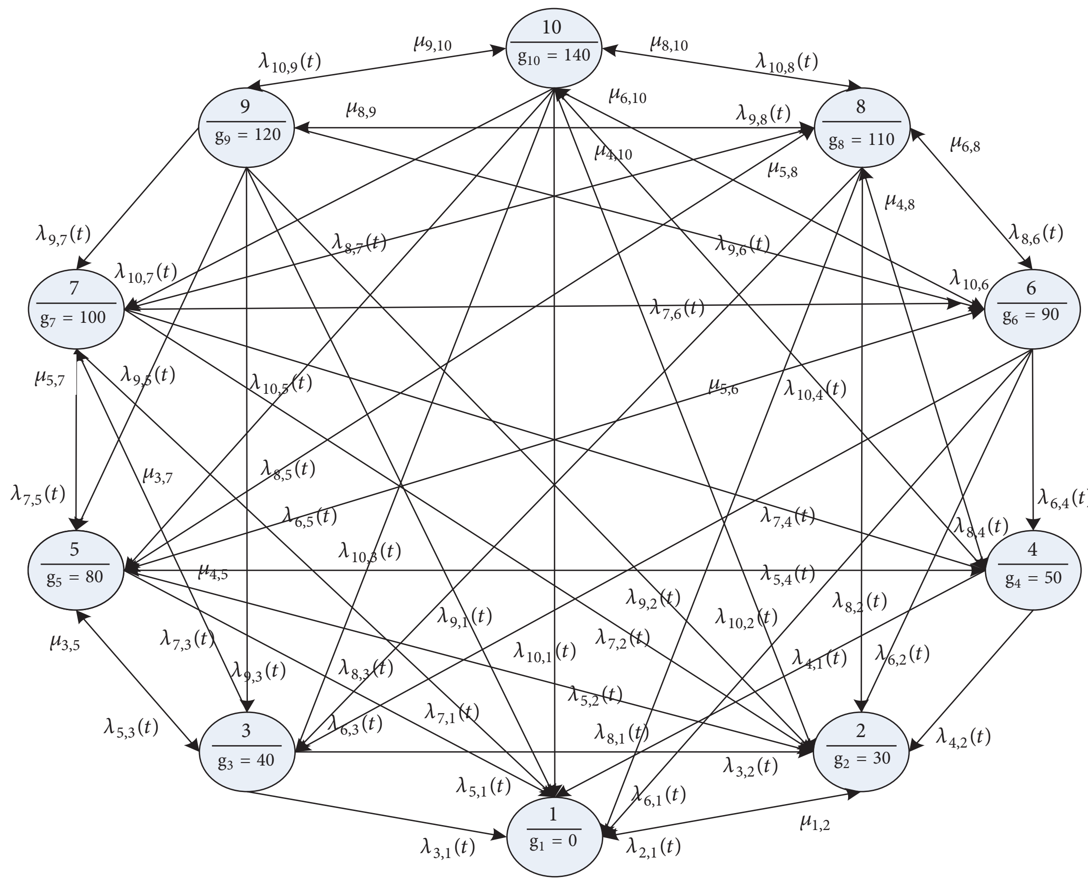

Figure 3: State-transition diagram of reduced MSS.

Table 3: Cost parameters of maintenance activities.

Maintenance activities	Cost		
	Component 1	Component 2	Component 3
Minor service	54	72	120
Major service	72	96	160
Minor repair	90	120	200
Major repair	180	240	400

Table 4: Replacement parameters.

C_r (\$/day)	t_{rep} (day)	C_{rep} (\$)
50	2	200

Table 5: MSS performance and benefits.

State	10	9	8	7	6	5	4	3	2	1
Performance	140	120	110	100	90	80	50	40	30	0
Benefit (\$)	1000	800	700	600	500	400	300	200	100	0

MATLAB programs to execute the mathematical calculations involved in the proposed approach.

According to the previously mentioned parameter settings, this study mimics two PM policies to illustrate the ramifications of the proposed approach. The first PM policy is derived from Huang and Wang [11]. Figures 4–6 show the state-transition diagrams for the three components. Apparently, minor services are implemented in component 1 when this PM policy reaches states 4 and 1; minor repair

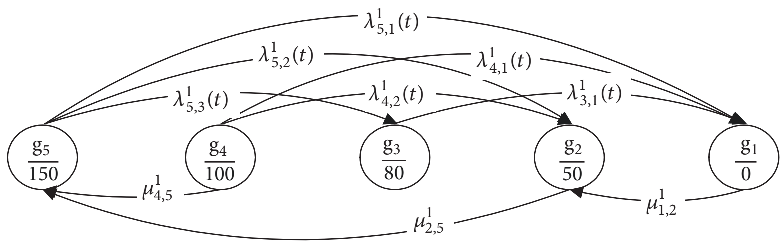

Figure 4: State-transition diagram of component 1 with PM policy 1.

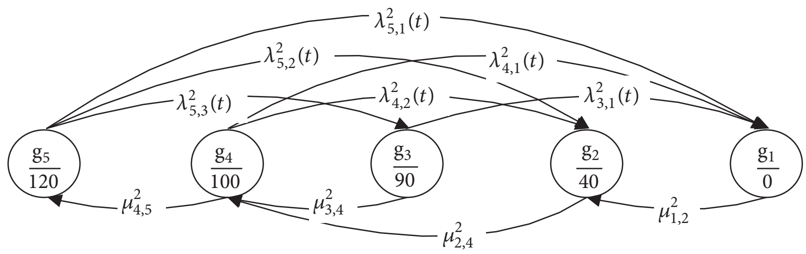

Figure 5: State-transition diagram of component 2 with PM policy 1.

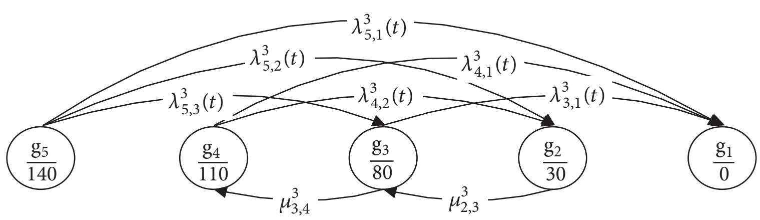

Figure 6: State-transition diagram of component 3 with PM policy 1.

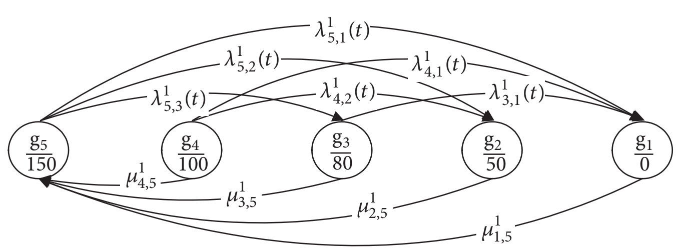

Figure 7: State-transition diagram of component 1 with PM policy 2.

is implemented when this policy reaches state 2. However, no maintenance activity is implemented in state 3. In component 2, minor services are undertaken in states 4, 3, and 1; major service is implemented when the PM policy reaches state 2. For component 3, no maintenance activity is performed until states 3 and 2 are reached, at which points minor services are implemented. Figures 7–9 show the state-transition diagrams of the three components for the second PM policy. This PM policy involves restoring the system to the best state whenever a component falls into a degradation state.

Figure 10 shows a trend diagram of the LREB values with time at the continuous time horizon for PM policy 1 and PM policy 2. The time interval for calculation of LREB values is 1 day, which is obtained by dividing the time of 1000 days into 1000 time intervals. Obviously, for policy 1, at the onset of this system's operation, the LREB values increase with time until the 818th day, reaching a maximum average LREB value of 36,046 with lower bound and upper bound 35,501 and 36,590, respectively, and then falling with time dramatically. The gap between lower bound and upper bound of LREB dwindles as the number of intervals N increases. In this case, the lower bound and upper bond of LREB for the 818th day are 32,724 and 39,368 with N=100, revealing the difference of 16.88%, while given N=1000, the difference considerably reduces to 2.88% which is rather small and can be negligible. We can see a similar trend for PM policy 2, which takes until the 635th day to reach its maximum average LREB value of 34,933 with lower bound and upper bound being 34,485 and 35,380, given

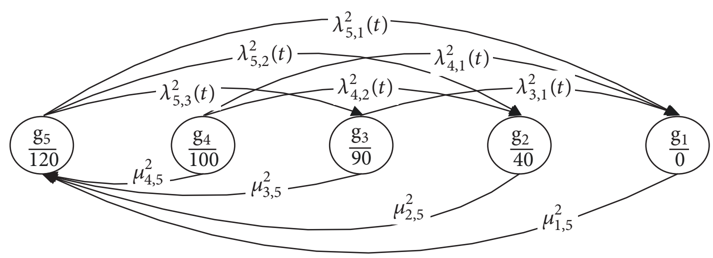

FIGURE 8: State-transition diagram of component 2 with PM policy 2.

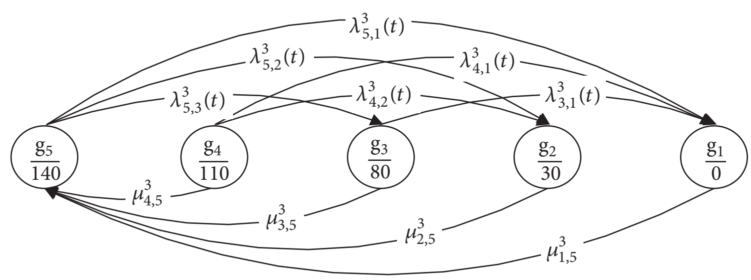

FIGURE 9: State-transition diagram of component 3 with PM policy 2.

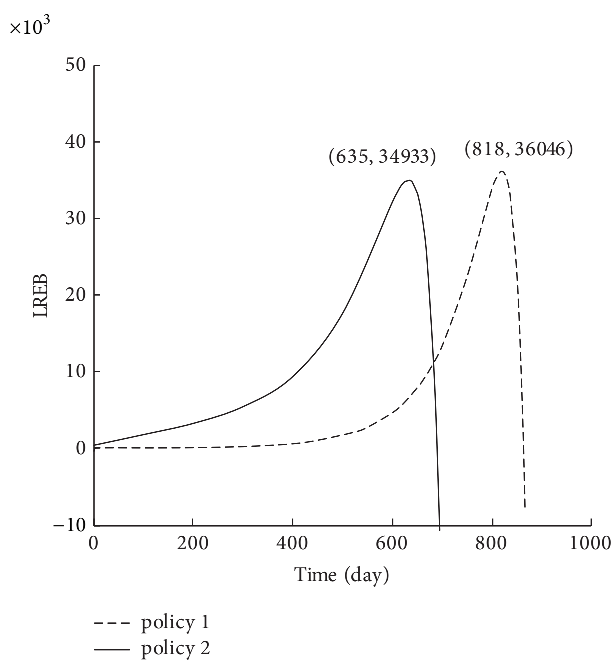

FIGURE 10: LREB trend diagrams through lifetime with two PM policies.

N=1000, respectively. Certainly, Figure 10 reveals that the best time to replace this system is the 818th day and the 635th day for PM policy 1 and PM policy 2, respectively. Both trend diagrams show that more frequent maintenance is necessary for the system to satisfy the performance requirements as the system ages and induces a large maintenance cost. For comparison, PM policy 1 obtains a higher LREB value at a longer replacement time than does PM policy 2. The main reason is that PM policy 1, derived from Huang and Wang [11], is the result of an optimizing procedure, whereas PM policy 2 mainly characterizes implementing the best maintenance alternative that restores the system to the best state after an MSC falls into any degradation state. From the viewpoint of the PM establishment, PM policy 1 outperforms PM policy 2 in maintenance costs and benefits; excessive maintenance may not be necessary for economic consideration. Certainly, the superiority of the PM policy directly affects the results of the proposed time-replacement policy.

To verify the proposed approach further, a sensitivity analysis with multiple increases in the failure rates for component 3 was performed by a given PM policy 1. Figure 11 shows the corresponding LREB trend diagrams. As expected, the time to replace the MSS decreases with the multiple increases in the failure rates. Furthermore, in contrast, we increase the repair rates multiple times for component 3 given PM policy 1. Figure 12 shows the corresponding trend diagrams. Noticeably, the system requires conducting this MSS replacement a short time after starting the operation with multiple additions of repair rates. We select component 3 to conduct sensitivity analysis because of its importance, being connected in series in the MSS. Additionally, a sensitivity analysis with varied system demands was performed. Table 6 summarizes the ramifications of this analysis. For PM policy 1, the optimal replacement time remains the same at the 818^{th} day for all demands, but the corresponding LREB values decrease when demand for system performance increases. For PM policy 2, the result of the sensitivity analysis is similar to PM policy 1: the optimal replacement time is the same at the 632^{nd} day, except for demand 100. The desktop computing and executing the proposed approach involves operation

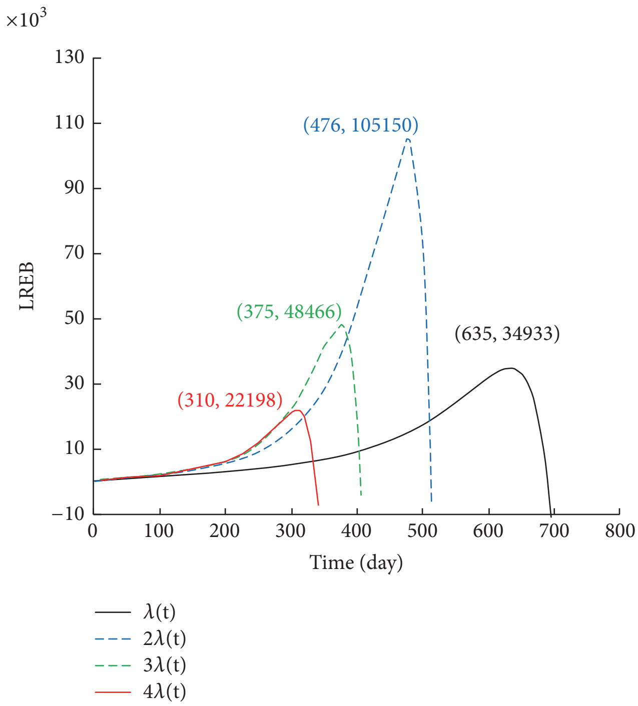

Figure 11: LREB trend diagrams with multiple increases in failure rates for component 3 given PM policy 1.

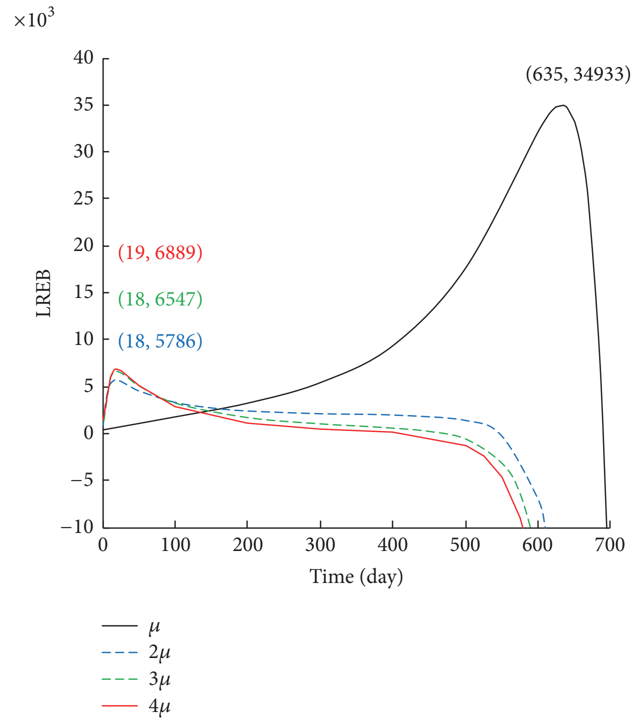

Figure 12: LREB trend diagrams with multiple increases in repair rates for component 3 given PM policy 1.

Table 6: Optimal replacement time T and LREB versus demand w.

PM policy	Demand w	T (days)	LREB ($/day)
1	100	818	36,046
	110	818	35,913
	120	818	35,824
	140	818	35,795
2	100	635	34,933
	110	632	34,678
	120	632	34,490
	140	632	34,283

system of Windows 7 (64 bits), Intel i5 CPU of 2.3 GHz, and 8 GB RAM with R2014a version of MATLAB coded. The time is between 600 and 900 seconds.

6. Conclusions and Discussions

This study aims at the PM model from the component perspective for the MSS with aging MSCs, in which the maintenance alternatives are implemented when the MSCs fall into degradation states, to propose a time-replacement policy. A time-dependent LREB index of system performance was developed on the basis of the continuous time Markov theory. By maximizing the LREB values throughout the system's lifetime, we determined the optimal time to replace this type of MSS economically. The proposed approach provides further insight into the relationship between PM policy setting and long-term system benefits; it also verifies a time-replacement policy. For future study, the ramifications of the current study can be extended to situations that consider the repair difficulties as MSCs age. Namely, the repair rate is a decreasing function of time. Using this method, the analyzed PM models more adequately fulfill the practical requirements. However, the mathematical calculation involved is a difficult challenge in striving to obtain precise results.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the Ministry of Science and Technology of Taiwan (NSC 104-2221-E-606-004 and 107-2221-E-012-001-MY2).

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