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Research Article

A Novel Disturbance Observer for Multiagent Tracking Control with Matched and Unmatched Uncertainties

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In this paper, multiagent tracking control problem of second-order multiagent systems with unknown leader acceleration, input saturation, and matched and unmatched disturbances is investigated. An auxiliary system is constructed to approximate system position states, and a novel sliding mode disturbance observer is designed to estimate matched and unmatched uncertainties. A sliding mode disturbance observer-based control protocol is proposed by constructing a novel sliding mode manifold based on the sliding mode disturbance observer outputs. In addition, the input saturation and the unknown leader acceleration become a part of lumped uncertainties by using mathematic transformation. The lumped uncertainties estimated by the sliding mode disturbance observer are compensated by the sliding mode disturbance observer-based control protocol. Stability of the second-order multiagent systems is guaranteed via Lyapunov method. Finally, a simulation example is proposed to exhibit advantages and availability of the developed techniques.

1. Introduction

Recently, cooperative control in MASs has received significant attention due to its widespread application in engineering such as unmanned aerial vehicles [1], spacecraft formation [2], and autonomous underwater vehicles [3]. Among these researches, multiagent tracking control, i.e., leader-following control, is extensively scattered in multiagent control, such as adaptive multiagent control [4], backstepping multiagent control [5], intermittent multiagent control [6], and hybrid multiagent control [7]. The multiagent tracking control between leader and followers means that, by partial information, the followers could track a leader reference trajectory and maintain state stability. Significant works of the multiagent tracking control, which deal with different practical conditions, have been conducted in the past decades [8–11]. However, performance of the MASs is usually affected by several uncertainty factors, such as input saturation, unknown leader acceleration, and unknown disturbances of multiagent systems. Therefore, it is an interesting research for designing a control protocol to obtain good performance of MASs with matched and unmatched uncertainties.

A multiagent control problem for second-order MASs in presence of uncertain dynamics and unknown external time-varying disturbances has been investigated in [12]. A robust adaptive neural network controller has been developed for a multiagent tracking control of higher-order nonlinear MASs in which each follower is modeled by a higher-order integrator incorporated with unknown nonlinear dynamics and unknown disturbances [13]. Moreover, methods for a single agent system with matched uncertainties are also applicable to MASs. In [14, 15], a way to deal with matched uncertainties of control systems is that the uncertainties are estimated by an effective observer and compensated by a controller. In [14], an adaptive fuzzy observer design approach has been proposed for control systems with matched uncertainties. An adaptive neural network-based observer has been given in [15] in which the proposed observer could estimate external disturbances online so as to decrease computation burden of control systems with matched uncertainties. Furthermore, results proposed in [16–18] were only focused on the so-called matched uncertainties, which means that the matched uncertainties can only be compensated from input channels. It is well known that unmatched uncertainties are frequently encountered in various engineering systems, such as permanent magnet motor systems and magnetic levitation suspension systems. Due to the fact that the mismatched uncertainties cannot be easily compensated by control inputs, many researchers have devoted their efforts to solve the problem. In [19], an observer-based sliding mode control method has been developed to counteract mismatched disturbances. Reference [20] addressed a finite-time output control problem for control systems with mismatching uncertainties.

On the other hand, input saturation, which may severely deteriorate performance of control systems and even lead to control systems instability, is a common feature for most practical control systems [21]. Furthermore, as the most important nonsmooth nonlinearity, input saturation makes control systems design more complicated. For attitude control system, actuator saturation has been considered in [22]. Discrete-time double-integrator consensus control for MASs with switching topologies and input saturation has also been studied in [23]. In [24], hyperbolic tangent functions have been employed to prevent input saturation, whereas saturation functions have been used in [25]. In [26], a spacecraft finite-time controller with input saturation constraint has been taken into account for the first time, and a global saturated finite-time control scheme has been proposed.

In this paper, a SMDOB based control protocol is proposed for multiagent tracking control of second-order MASs with matched and unmatched uncertainties which are composed of input saturation, unknown leader acceleration, and matched and mismatched disturbance. A novel SMDOB is designed to estimate the matched and unmatched uncertainties of the MASs. An auxiliary system is constructed to approximate the second-order MASs position states. A Lyapunov method is used to show stability of the second-order MASs. An illustrative example is constructed to verify the proposed method developed in this paper. The main contributions of this paper are summarized as follows

(i) A novel SMDOB and an observer-based sliding manifold are proposed to deal with matched and unmatched uncertainties which include input saturation, unknown leader acceleration, and matched and mismatched disturbance in the second-order MASs.

(ii) Compared with literature [2, 27], assumptions of nonlinear functions of all followers in the second-order MASs are developed.

(iii) An auxiliary system is introduced to approximate the second-order MASs position states. A SMDOB based control protocol is proposed based on auxiliary system states.

This paper is organized as follows: Section 2 details the problem formulation and graph theory. Two useful lemmas are also given in this section. The main result of our research is elaborately presented in Section 3. A numerical simulation is shown to verify the effectiveness of the proposed method in Section 4. Finally, the paper is concluded by Section 5.

Notations. Let $\mathscr{R}^n$ denote the n-dimensional Euclidean space; $()^T$ denotes the transpose of matrix or vector; $\otimes$ stands for Kronecker product; I_n represents the $n \times n$ identity matrix. For a given matrix $A \in \mathscr{R}^{m\times n}$, $\|A\| = (Tr(A^TA))^{1/2}$ denotes the Euclidean norms operation; $\lambda(A)$ denotes the eigenvalue of matrix A; $\operatorname{diag}(\cdot)$ is diagonal matrix operation. For vector $x = [x_1, x_2, \ldots, x_m]^T \in \mathscr{R}^m$

$$
\begin{aligned}
&\|x\|_\infty = \max\left(|x_1|, \ldots, |x_m|\right) \\
&|x|^\alpha = \operatorname{diag}\left(|x_1|^\alpha, |x_2|^\alpha, \ldots, |x_m|^\alpha\right), \quad \alpha > 0 \\
&\operatorname{sign}(x) = \left[\operatorname{sign}(x_1), \ldots, \operatorname{sign}(x_m)\right]^T \\
&\int_0^t \operatorname{sign}(x)\, d\tau \\
&\quad = \left[\int_0^t \operatorname{sign}(x_1)\, d\tau, \ldots, \int_0^t \operatorname{sign}(x_m)\, d\tau\right]^T.
\end{aligned}
\tag{1}
$$

2. Problem Description and Preliminary

2.1. Algebraic Graph Theory. For a multiagent system with n connected agents, information can be transmitted between neighboring agents, so it is natural to describe the topology of the information flow by a weighted graph [28]. Let $\mathscr{G} = \{v, \varepsilon, \mathscr{A}\}$ denote a weighted graph, where $v = \{1, \ldots, n\}$ is the node set, $\varepsilon \subset v \times v$ is the edge set, and $\mathscr{A} = [a_{ij}]$ is a weighted adjacency matrix of graph $\mathscr{G}$. Node i denotes the ith agent, and the adjacency element a_{ij} denotes the communication relation between the ith and the jth agent, i.e., $(i,j) \in \varepsilon \Longleftrightarrow a_{ij} > 0$. Meanwhile, nodes i and j are neighbors; it is obtained that $\mathscr{N}_i = \{j \mid (i,j) \in \varepsilon\}$. For any two nodes i and j, the $\mathscr{G}$ is named a connected graph if there exists one path between them. A graph is simple if it has no self-loop or repeated edges. Assume that graph $\mathscr{G}$ is simple and undirected connected in this paper.

Define a Laplacian matrix of a weighted graph $\mathscr{G}$ by $\mathscr{L}$, where diagonal elements and nondiagonal elements of the Laplacian matrix $\mathscr{G}$ satisfy $l_{ii} = \sum_{j=1}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$, respectively.

A graph $\overrightarrow{\mathscr{G}}$ is given, containing n nodes (related to graph $\mathscr{G}$) and a leader labeled 0 with some direct edges from part agents. Let $\mathscr{B} = \operatorname{diag}(b_1, \ldots, b_n)$ be the leader adjacency matrix associated with $\overrightarrow{\mathscr{G}}$, where $b_i > 0$ is a constant if the ith agent has access to the leader; $b_i = 0$, otherwise. For $\overrightarrow{\mathscr{G}}$, if there exists one path from the leader to each follower, then $\overrightarrow{\mathscr{G}}$ is named connected graph. For graph $\overrightarrow{\mathscr{G}}$, the following lemma is given.

Lemma 1 (see [28]). *If $\mathscr{G} = \{v, \varepsilon, \mathscr{A}\}$ is an undirected connected graph, then the Laplacian matrix $\mathscr{L}$ is a symmetric* *matrix and its n real eigenvalues can be arranged in an ascending order*

$$0 = \lambda_1 < \lambda_2 \leq \lambda_3 \leq \cdots \leq \lambda_n \leq 2l_M \quad (2)$$

where $l_M = \max_{1\leqslant i\leqslant n}\{l_{ii}\}$ *and* λ_2 *is called the algebraic connectivity, which is used to analyze the rate of consensus convergence. What is more, the matrix* $\mathscr{L}+\mathscr{B}$ *associated with* $\overrightarrow{\mathscr{G}}$ *is also symmetric and positive definite.*

2.2. Problem Description and Mathematical Preliminaries. A second-order multiagent system (MAS) with an active leader and n followers is considered. A leader dynamic equation labeled by 0 is expressed as follows:

$$\begin{aligned}\dot{x}_0 &= v_0\\ \dot{v}_0 &= u_0\end{aligned} \quad (3)$$

where $x_0, v_0 \in \mathscr{R}^m$ are leader position and velocity vectors, respectively; $u_0 \in \mathscr{R}^m$ is the leader's acceleration. An ith follower's dynamic equation is described as

$$\begin{aligned}\dot{x}_i &= v_i\\ \dot{v}_i &= f_i(x_i, v_i) + u_i + \Delta f_i\end{aligned} \quad (4)$$

where $i = 1,\ldots,n$; $x_i, v_i \in \mathscr{R}^m$ denote the follower's position and velocity states, respectively; $f_i(x_i, v_i) \in \mathscr{R}^m$ are known nonidentical functions; $\Delta f_i \in \mathscr{R}^m$ stand for matched uncertainties which come from the inherent system; $u_i \in \mathscr{R}^m$ are the MAS input vectors. For simplicity, we will ignore declaration expressions (x_i, v_i) and $i = 1,\ldots,n$ in this paper.

To proceed with the design of disturbance observer, the following lemma is given.

Lemma 2 (see [29]). *The following perturbed nonlinear differential equation is shown as follows:*

$$\dot{x} + c_1|x|^{1/2}\operatorname{sign}(x) + c_2\int_0^t \operatorname{sign}(x)\,d\tau = \psi, \quad |\dot{\psi}| \leq c_0 \quad (5)$$

where $x(t) \in \mathscr{R}^m$ *is a solution of (5),* $\psi(t)$ *is unknown bounded disturbance, and* c_0 *is the upper value of the derivative of time-varying disturbance* $|\psi(t)|$*. A solution* $x(t)$ *of (5) and its derivative* $\dot{x}(t)$ *will converge to zeros in finite time* t_f *if parameters satisfy that* $c_1 \geq 1.5\sqrt{c_0}$, $c_2 \geq 1.1c_0$*. In addition, the convergent time* t_f *is decided by*

$$t_f \leq \frac{7.6x(0)}{c_2 - c_0} \quad (6)$$

where $x(0)$ *is the initial value of* $x(t)$*.*

3. Main Result

Our control objective is to design an appropriate control protocol for a second-order MAS with matched and unmatched uncertainties to make followers position and velocity states stability and tracking errors $\|x_i(t) - x_0(t)\|$, $\|v_i(t) - v_0(t)\|$ arbitrary small.

3.1. Control Protocol Design Based on a Novel SMDOB. Here, a SMDOB is designed to estimate the effect of MAS uncertainties. A SMDOB based control protocol is proposed for a MAS to achieve tracking objective. Meanwhile, assume that all the MAS states can be measured. An assumption, which always holds in practical applications, is given as follows.

Assumption 3. A leader acceleration u_0, which satisfies that $\|u_0\| \leqslant \overline{u}_0$ and $\overline{u}_0 > 0$, is an unknown positive real number. In general, the leader position keeps changing throughout the entire motion process and its behaviour is independent of its followers. What is more, for ith follower of a MAS, matched uncertainty Δf_i satisfies that $\|\Delta f_i\| \leqslant \Delta\overline{f}_i$, where $\Delta\overline{f}_i$ is a positive scalar.

Remark 4. Many practical systems can be described as the second-order MAS (4), such as spacecraft attitude dynamics (with some mathematical transformation), satellite orbital control system, and robotic dynamics. Assumption 3 is a common assumption used in most literature works, which means finite available energy of u_0 and bounded change rate of uncertainties Δf_i.

The ith follower velocity takes the following ideal form [30]:

$$v_{di} = -\eta\left(\sum_{j=1}^{n} l_{ij}x_j + b_i(x_i - x_0)\right) + v_0 \quad (7)$$

where l_{ij} is the element of Laplacian matrix $\mathscr{L}$; $\eta > 0$ is the tracking control gain, which is used to adjust the convergence speed. From [30], it is obtained that v_{di} is considered as a reference signal for each follower. Moreover, if the conditions $v_i \longrightarrow v_{di}$ are satisfied, then the control objectives $x_i \longrightarrow x_0$ and $v_i \longrightarrow v_0$ are also obtained. State tracking error variables are introduced.

$$e_{vi} = v_i - v_{di} \quad (8)$$

By invoking the tracking state errors e_{vi} into the second-order MAS (4), a tracking error dynamic equation is obtained as follows:

$$\begin{aligned}\dot{e}_{vi} &= f_i + u_i + \Delta f_i + \eta\left(\sum_{j=1}^{n} l_{ij}v_j + b_i(v_i - v_0)\right) - u_0\\ &= F_i + u_i + \Delta F_i\end{aligned} \quad (9)$$

where $F_i = f_i + \eta(\sum_{j=1}^{n} l_{ij}v_j + b_i(v_i - v_0))$ are known function vectors and $\Delta F_i = \Delta f_i - u_0$ are total uncertainties.

In order to estimate the uncertainties ΔF_i, an auxiliary dynamic equation is introduced as

$$\dot{z}_{vi} = F_i + u_i + \Delta\widehat{F}_i \quad (10)$$

where $z_{vi}, \Delta\widehat{F}_i \in \mathscr{R}^m$ are the estimation of the tracking error e_{vi} (8) and the uncertainties ΔF_i, respectively.

Define the estimation errors $s_{vi} = z_{vi} - e_{vi}$. Considering (9) and (10), one has the following.

$$\begin{aligned} s_{\alpha i} &= \dot{s}_{vi} = \dot{z}_{vi} - \dot{e}_{vi} = F_i + u_i + \Delta\widehat{F}_i - F_i - u_i - \Delta F_i \\ &= \Delta\widehat{F}_i - \Delta F_i \end{aligned} \tag{11}$$

Inspired by [31], uncertainties are viewed as an extended state $\dot{s}_{vi}$ which is estimated by an extended state observer (ESO):

$$\begin{aligned} \xi_{vi} &= s_{vi} - \widehat{s}_{vi} \\ \dot{\widehat{s}}_{vi} &= \widehat{s}_{\alpha i} + \rho_{i1}\left(\left|\xi_{vi}\right|^{\lambda_{i1}} \operatorname{sign}\left(\xi_{vi}\right) + \left|\xi_{vi}\right|^{\theta_{i1}} \operatorname{sign}\left(\xi_{vi}\right)\right) \\ \dot{\widehat{s}}_{\alpha i} &= \rho_{i1}^2\left(\left|\xi_{vi}\right|^{\lambda_{i2}} \operatorname{sign}\left(\xi_{vi}\right) + \left|\xi_{vi}\right|^{\theta_{i2}} \operatorname{sign}\left(\xi_{vi}\right)\right) \end{aligned} \tag{12}$$

where $s_{\alpha i}$ is a derivative of s_{vi}; $\rho_{i1}, \lambda_{i1}, \lambda_{i2}, \theta_{i1}, \theta_{i2}$ are observer parameters. If initial conditions $s_{vi}(0) - \widehat{s}_{vi}(0)$ and $\dot{s}_{vi}(0) - \widehat{s}_{\alpha i}(0)$ are bounded, then $\widehat{s}_{\alpha i}$ can approximate derivative term $\dot{s}_{vi}$ to any arbitrary accuracy.

Remark 5. Stability of the ESO (12) has been obtained by selecting appropriate parameters ρ_{i1} and λ_{i1}. Fundamental selections of parameters can be chosen as $1 < \rho_{i1} < +\infty, 0.5 < \lambda_{i1} < 1, \lambda_{i2} = 2\lambda_{i1} - 1, \theta_{i1} = \lambda_{i1}^{-1}$ and $\theta_{i2} = \lambda_{i1}^{-1} + \lambda_{i1} - 1$. Furthermore, note that ρ_{i1}, λ_{i1} should be large enough to make the estimation errors $\xi_{\alpha i} = \dot{s}_{vi} - \widehat{s}_{\alpha i}$ as small as possible. Thus, via tuning these parameters properly, estimation errors $\xi_{\alpha i}$ are limited to be small enough, which means that $\widehat{s}_{\alpha i}$ converge into a neighborhood of actual states $\dot{s}_{vi}$.

From Lemma 2, a novel SMESO is constructed as follows:

$$\Delta\dot{\widehat{F}}_i = -\widehat{s}_{\alpha i} - \rho_{i2}\left|\overline{s}_i\right|^{1/2} \operatorname{sign}\left(\overline{s}_i\right) - \rho_{i3}\int_0^t \operatorname{sign}\left(\overline{s}_i\right) d\tau \tag{13}$$

where $\overline{s}_i = s_{vi} + \widehat{s}_{\alpha i}$.

By (11), the following is shown.

$$\dot{\overline{s}}_i = \dot{s}_{vi} + \ddot{s}_{vi} - \dot{\xi}_{\alpha i} = \dot{s}_{vi} + \Delta\dot{\widehat{F}}_i - \Delta\dot{F}_i - \dot{\xi}_{\alpha i} \tag{14}$$

Substituting (13) into (14), we obtain the following.

$$\begin{aligned} &\dot{\overline{s}}_i + \rho_{i2}\left|\overline{s}_i\right|^{1/2} \operatorname{sign}\left(\overline{s}_i\right) + \rho_{i3}\int_0^t \operatorname{sign}\left(\overline{s}_i\right) d\tau \\ &\quad = \xi_{\alpha i} - \Delta\dot{F}_i - \dot{\xi}_{\alpha i} \end{aligned} \tag{15}$$

From Lemma 2, if the conditions that a positive constant $\overline{\psi}_{i1}$ such that $\|\dot{\xi}_{\alpha i} - \Delta\ddot{F}_i - \ddot{\xi}_{\alpha i}\|_\infty \leqslant \overline{\psi}_{i1}$ and observer parameters ρ_{i2}, ρ_{i3} such that $\rho_{i2} \geqslant 1.5\sqrt{\overline{\psi}_{i1}}$ and $\rho_{i3} \geqslant 1.1\overline{\psi}_{i1}$ are satisfied, then it is obtained that $\overline{s}_i$ and $\dot{\overline{s}}_i$ will converge to zeros in finite time. From (11), we can also conclude that the SMDOB output $\Delta\widehat{F}_i$ will approach its real value ΔF_i. Meanwhile, it also means that $\|s_{vi}\|$ and $\|\dot{s}_{vi}\|$ would converge to zero in finite time according to (15).

Remark 6. From [29], we can know that the estimation error $\overline{\psi}_{i1}$ can converge to a bounded region, and its derivative $\overline{\psi}_{i1}$ could also converge to zeros.

With $\Delta\widehat{F}_i$, which is estimated by the SMESO (13), a SMESO based control protocol (16) is proposed as follows:

$$u_i = -F_i - \Delta\widehat{F}_i - k_i z_{vi} \tag{16}$$

where $k_i > 0$ is a designed control gain.

The following theorem summarizes the feasibility of the SMESO control protocol (16).

Theorem 7. *Based on the SMESO (13) and the proposed control protocol (16), positions of the second-order MAS (4) with matched uncertainties will track the desired trajectory x_0.*

Proof. To facilitate the stability analysis, the following Lyapunov function is constructed.

$$V = \sum_{i=1}^{n} 0.5 z_{vi}^T z_{vi} \tag{17}$$

With the auxiliary dynamic system (10), the SMESO (13), and the proposed control protocol (16), the time derivative of V is shown as follows.

$$\begin{aligned} \frac{dV}{dt} &= \sum_{i=1}^{n} z_{vi}^T \dot{z}_{vi} = \sum_{i=1}^{n} z_{vi}^T\left(F_i + u_i + \Delta\widehat{F}_i\right) = -\sum_{i=1}^{n} k_i z_{vi}^T z_{vi} \\ &\leqslant 0 \end{aligned} \tag{18}$$

According to (18), it is shown that all system variables are stable, i.e., $z_{vi} \longrightarrow 0$ as $t \longrightarrow \infty$. Since s_{vi} and $\dot{s}_{vi}$ are convergent in finite time, it is obtained that $e_{vi} \longrightarrow 0, v_i \longrightarrow v_{di}$ by considering $s_{vi} = z_{vi} - e_{vi}, e_{vi} = v_i - v_{di}$. Moreover, from (8), the distributed tracking objectives $x_i \longrightarrow x_0, v_i \longrightarrow v_0$ have also been achieved. This concludes the proof. □

Remark 8. For traditional extended disturbance observers, $\dot{s}_{vi}$ involve the term of observer output $\Delta\widehat{F}$ which includes tedious analytic derivatives computation and increases system computation burden. To avoid this, a novel SMDOB is employed to estimate $\dot{s}_{vi}$. However, it should be noted that the estimation errors are unavoidable with the application of SMDOB. On the other hand, it should be also emphasized that the undesired effect of the unknown leader acceleration u_0 is combined with uncertainties Δf_i as compounded uncertainties.

Remark 9. In the SMDOB, derivatives of disturbances are required to be bounded. While taking Assumption 3 and the estimation of the ESO into consideration, it is obtained that the bounded assumption of the time derivative of total lumped disturbance is reasonable.

Remark 10. By the definition of graph Laplacian matrix $\mathscr{L}$, it follows that $l_{ij} \neq 0$ if and only if there is information exchange between ith follower and jth follower. For the diagonal matrix $\mathscr{B}$, $b_i \neq 0$ if and only if there is information exchange between the leader and the ith follower. Therefore, the proposed control protocol can only use the information of its neighbors. Hence, the proposed protocol belongs to the decentralized design fashion with directed communications.

3.2. Control Protocol Design for Unmatched Uncertainties. In practical engineering applications, MASs may suffer from unmatched uncertainties which always affect states directly rather than through the input channels. Compared with traditional matched disturbance observer-based control approaches, one of the intuitive difficulties induced by unmatched uncertainties is that these cannot be compensated in input channels directly. On the other hand, as one of the most important nonsmooth nonlinearity properties, input saturation should be explicitly considered in practical engineering. Here, we consider the multiagent tracking control problem under matched and unmatched uncertainties which include input saturation.

Followers' dynamic equations with input saturation and matched and unmatched uncertainties are considered as

$$\begin{aligned} \dot{x}_i &= v_i + \Delta h_i \\ \dot{v}_i &= f_i(x_i, v_i) + u_i + \Delta f_i \end{aligned} \tag{19}$$

where $\Delta h_i, \Delta f_i \in \mathscr{R}^m$ are the unmatched and matched uncertainties, respectively; $u_i \in \mathscr{R}^m$ denotes unknown nonsymmetric saturation input.

The unknown nonsymmetric saturation input u_{ik} can be expressed as follows:

$$u_{ik} = \begin{cases} u_{\max} & if\ \ v_{rik} \geq u_{\max} \\ v_{rik} & if\ \ u_{\min} < v_{rik} < u_{\max} \\ -u_{\min} & if\ \ v_{rik} \geq u_{\min} \end{cases} \tag{20}$$

where $k = 1, \ldots, m$ and $v_{ri} = [v_{ri1}, \ldots, v_{rim}]^T \in \mathscr{R}^m$ is the designed control input command. Note that $u_{\min} \geq 0$ and $u_{\max} \geq 0$, which are parameters of the unknown nonsymmetric saturation input, denote the upper and lower bounds of the nonsymmetric input saturation.

For further stability analysis, the followers dynamic equations (19) can be transformed as

$$\begin{aligned} \dot{x}_i &= v_i + \Delta h_i \\ \dot{v}_i &= f_i + v_{ri} + d_i \end{aligned} \tag{21}$$

where $d_i = u_i - v_{ri} + \Delta f_i$ are lumped uncertainties. Due to the fact that $u_{\max}$ and $u_{\min}$ are unknown, the compounded uncertainties d_i are also unknown.

The auxiliary dynamic equation is introduced as

$$\dot{z}_{xi} = v_i + \Delta\widehat{h}_i \tag{22}$$

where $\Delta\widehat{h}_i$ are the estimation of the unmatched uncertainties Δh_i.

Considering dynamic equations (21) and dynamic equation (22), an estimation error dynamic is considered as follows.

$$\dot{s}_{xi} = \dot{z}_{xi} - \dot{x}_i = v_i + \Delta\widehat{h}_i - v_i - \Delta h_i = \Delta\widehat{h}_i - \Delta h_i \tag{23}$$

Since $\dot{s}_{xi}$ is unavailable, a ESO is adopted to estimate each element $\dot{s}_{xi}$:

$$\begin{aligned} \dot{\widehat{s}}_{xi} &= \widehat{s}_{vi} + \rho_{i1}\left(\left|\xi_{xi}\right|^{\lambda_{i1}} \operatorname{sign}(\xi_{xi}) + \left|\xi_{xi}\right|^{\theta_{i1}} \operatorname{sign}(\xi_{xi})\right) \\ \dot{\widehat{s}}_{vi} &= \rho_{i1}^2\left(\left|\xi_{xi}\right|^{\lambda_{i2}} \operatorname{sign}(\xi_{xi}) + \left|\xi_{xi}\right|^{\theta_{i2}} \operatorname{sign}(\xi_{xi})\right) \end{aligned} \tag{24}$$

where $\xi_{xi} = \widehat{s}_{xi} - s_{xi}$ is an ESO estimation error. The value of ESO parameters $\lambda_{i1}, \lambda_{i2}, \theta_{i1}$, and θ_{i2} are the same as (12).

According to (24), it is obtained that

$$\dot{s}_{xi} = \widehat{s}_{vi} + \xi_{vi} \tag{25}$$

where $\xi_{vi} \in \mathscr{R}^m$ is the estimation error. Due to the property of the DOB, ξ_{vi} could converge to an arbitrary small range by selecting appropriate parameters.

Defining $\overline{s}_i = s_{xi} + \widehat{s}_{vi}$, a novel SMESO is proposed as follows.

$$\Delta\dot{\widehat{h}}_i = -\widehat{s}_{vi} - \rho_{i2}\left|\overline{s}_i\right|^{1/2} \operatorname{sign}(\overline{s}_i) - \rho_{i3}\int_0^t \operatorname{sign}(\overline{s}_i)\, d\tau \tag{26}$$

With (23) and (26), the following is shown.

$$\dot{\overline{s}}_i = \dot{s}_{xi} + \ddot{s}_{xi} - \dot{\xi}_{vi} = \dot{s}_{xi} + \Delta\dot{\widehat{h}}_i - \Delta\dot{h}_i - \dot{\xi}_{vi} \tag{27}$$

Invoking (25) and substituting (26) into (27), one has the following.

$$\begin{aligned} &\overline{s}_i + \rho_{i2}\left|\overline{s}_i\right|^{1/2} \operatorname{sign}(\overline{s}_i) + \rho_{i3}\int_0^t \operatorname{sign}(\overline{s}_i)\, d\tau \\ &\quad = \xi_{vi} - \Delta\dot{h}_i - \dot{\xi}_{vi} \end{aligned} \tag{28}$$

From Assumption 3 and approximation property of the SMESO, it is shown that $\|\dot{\xi}_{vi} - \Delta\ddot{h}_i - \ddot{\xi}_{vi}\|_\infty \leqslant \overline{\psi}_{i1}$. According to Lemma 2, note that $\|\overline{s}_i\|, \|\dot{\overline{s}}_i\|$ would converge to zeros in finite time if the observer parameters satisfy $\rho_{i2} \geqslant 1.5\sqrt{\overline{\psi}_{i1}}$ and $\rho_{i3} \geqslant 1.1\overline{\psi}_{i1}$. From (23), it follows that the SMESO output $\Delta\widehat{h}_i$ could converge to Δh_i in finite time.

Similar to (7), the followers tracking signal, with input saturation and matched and unmatched uncertainties estimation $\Delta\widehat{h}_i$, yields the following.

$$v_{di} = -\eta\left(\sum_{j=1}^{n} l_{ij}x_j + b_i(x_i - x_0)\right) + v_0 \tag{29}$$

Inspired by [19], a sliding mode manifold equation based on a disturbance observer is defined as follows:

$$\alpha_i = e_{vi} + \Delta\widehat{h}_i \tag{30}$$

where $e_{vi} = v_i - v_{di}$ are reference signal tracking errors.

Thus, a sliding mode manifold equation of the SMESO (26) can be obtained as

$$\begin{aligned}\dot{\alpha}_i &= \dot{e}_{vi} + \Delta\dot{\hat{h}}_i \\ &= f_i + v_{ri} + d_i + \eta\left(\sum_{j=1}^{n} l_{ij}v_j + b_i\left(v_i - v_0\right)\right) \\ &\quad + \eta\left(\sum_{j=1}^{n} l_{ij}\Delta h_j + b_i\Delta h_i\right) - u_0 + \Delta\dot{\hat{h}}_i \\ &= H_i + v_{ri} + \Delta H_i\end{aligned} \tag{31}$$

where $H_i = f_i + \eta(\sum_{j=1}^{n} l_{ij}v_j + b_i(v_i - v_0))$ are known system vector functions and $\Delta H_i = \eta(\sum_{j=1}^{n} l_{ij}\Delta h_j + b_i\Delta h_i) + d_i - u_0 + \Delta\dot{\hat{h}}_i$ are unknown lumped uncertainties.

Similar to (22), in order to estimate the unknown lumped uncertainties ΔH_i, an auxiliary system and a SMESO are proposed as

$$\begin{aligned}\dot{\hat{\alpha}}_i &= H_i + v_{ri} + \Delta\widehat{H}_i \\ \Delta\dot{\widehat{H}}_i &= -\hat{\zeta}_{\alpha i} - \rho_{i5}\left|\overline{\zeta}_i\right|^{1/2}\operatorname{sign}\left(\overline{\zeta}_i\right) - \rho_{i6}\int_0^t \operatorname{sign}\left(\overline{\zeta}_i\right)d\tau\end{aligned} \tag{32}$$

where $\zeta_{xi} = \hat{\alpha}_i - \alpha_i$; note that $\hat{\zeta}_{vi}$ and $\hat{\zeta}_{\alpha i}$ are estimations of $\dot{\zeta}_{xi}$ and $\ddot{\zeta}_{xi}$. $\overline{\zeta}_i = \zeta_{xi} + \hat{\zeta}_{vi}$ are introduced auxiliary variables and $\Delta\widehat{H}_i$ is the estimation of the unknown lumped uncertainties ΔH_i.

Considering Assumption 3 and estimation property of SMDOB, it is shown that $\|\dot{\varepsilon}_{\alpha i} - \dot{\overline{d}}_i - \ddot{\varepsilon}_{\alpha i}\| \leqslant \overline{\psi}_{i2}$ with $\varepsilon_{\alpha i} = \ddot{\zeta}_{xi} - \hat{\zeta}_{\alpha i}$ being the SMDOB approximation error. According to Lemma 2, one has that ζ_{xi} and its derivative $\dot{\zeta}_{xi}$ converge to zeros in finite time if SMDOB parameters satisfy $\rho_{i5} \geqslant 1.5\sqrt{\overline{\psi}_{i2}}$ and $\rho_{i6} \geqslant 1.1\overline{\psi}_{i2}$. Thus, the SMDOB estimation $\Delta\widehat{H}_i$ will converge to ΔH_i in finite time.

With the SMDOB providing the required estimation in (32), the control protocol is proposed as

$$v_{ri} = -H_i - \Delta\widehat{H}_i - k_i\hat{\alpha}_i \tag{33}$$

where $k_i > 0$ is the designed control gain.

The following theorem summarizes the proposed control protocol for the MAS (19) under matched and unmatched uncertainties which include input saturation and unknown leader acceleration.

Theorem 11. *Consider the multiagent tracking control of second-order MAS (19) in presence of matched and unmatched uncertainties which include input saturation and unknown leader acceleration. A finite-time converging performance is obtained according to a SMDOB (32) and a control protocol (33). Then, the MAS (19) is stable and the tracking error will converge to a bounded set as time going.*

Proof. To analyze stability of the MAS (19), we consider the following Lyapunov function candidate.

$$V = 0.5\sum_{i=1}^{n}\hat{\alpha}_i^T\hat{\alpha}_i \tag{34}$$

By the SMDOB (32) and the proposed control protocol (33), the time derivative of V is as follows.

$$\dot{V} = \sum_{i=1}^{n}\hat{\alpha}_i^T\dot{\hat{\alpha}}_i = -\sum_{i=1}^{n}k_i\hat{\alpha}_i^T\hat{\alpha}_i \leqslant 0 \tag{35}$$

According to inequalities (35), we know that all system signals are stable. It implies that $\hat{\alpha}_i \longrightarrow 0$ as $t \longrightarrow \infty$. Since $\zeta_{xi} \longrightarrow 0$, $\Delta\hat{h}_i \longrightarrow \Delta h_i$, and $\Delta\widehat{H}_i \longrightarrow \Delta H_i$ are convergent in finite time, it is obtained that $e_{vi} + \Delta\hat{h}_i \longrightarrow 0$.

Based on above analysis, the following is shown.

$$\dot{x}_i = v_{di} + \Delta h_i - \Delta\hat{h}_i \tag{36}$$

Hence, we can obtain that all followers can track the leader trajectory agreement with bounded error region. This concludes the proof. □

Remark 12. In order to handle unmatched uncertainties, a novel sliding mode manifold is defined based on the SMESO (26). Compared with traditional sliding mode control, integral sliding mode control, and disturbance observer-based sliding mode control in [19], convergent accuracy of the SMESO (26) is dependent on the estimation error $\|\Delta h_i - \Delta\hat{h}_i\|$ rather than $\|\Delta h_i\|$. Since the unmatched uncertainties have been precisely estimated by the SMESO (26), the magnitude of the estimation error $\|\Delta h_i - \Delta\hat{h}_i\|$, which is expected to converge to the neighbor region of zeros, can be kept much smaller than the magnitude of the uncertainties $\|\Delta h_i\|$. It means the tracking trajectory could have the property of chattering reduction as well as excellent dynamic and static performance. The readers can also refer to [32–34] for the same argument.

Remark 13. For (21), the effects of unknown nonsymmetric input saturation are treated as a part of the lumped uncertainties and approximated by using the SMDOB (26). However, the system state has the feasibility of unknown nonsymmetric input saturation, and the bounded state tracking errors are still guaranteed by Lyapunov theory.

Remark 14. The introduced auxiliary system, working together with the SMDOB (26) and the ESO (24), not only improves the control performances, but also reduces real-time computing burden of the MAS. With selecting appropriate parameters, the ultimate convergent sets of state tracking errors $x_i - x_0$ can be tuned. Careful analysis indicates that increasing control gains and disturbances observer parameters could contribute to faster converging speed. Moreover, larger controller gains and observer parameters also lead to larger control power. Therefore, a compromise between control objective and converging speed should be made in practical problem.

Table 1: Parameters selection.

ESO	$\rho_{i1} = 2, \nu_{i1} = 0.6, \nu_{i2} = 0.2, \eta_{i1} = 1.67$ $\eta_{i2} = 1.27, \rho_{i4} = 2, \nu_{i3} = 0.8, \nu_{i4} = 0.6$ $\eta_{i3} = 1.25, \eta_{i4} = 1.05$
SMDOB	$\rho_{i2} = 5, \rho_{i3} = 5, \rho_{i5} = 9, \rho_{i6} = 10, k_i = 2$

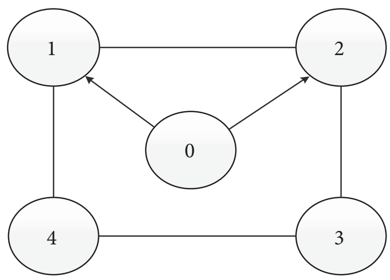

Figure 1: Formation communication topology.

4. Simulation

In this section, a numerical simulation is presented to show the effectiveness of the proposed theorem. The simulation scenario is constructed as follows.

Considering a formation with four followers and a visual leader, the communication graph is shown in Figure 1.

The weighted adjacency matrices $\mathscr{A}$ and $\mathscr{B}$ are defined as

$$\mathscr{A} = \begin{bmatrix} 0 & 0.2 & 0 & 0.3 \\ 0.2 & 0 & 0.3 & 0 \\ 0 & 0.2 & 0 & 0.3 \\ 0.3 & 0 & 0.2 & 0 \end{bmatrix}, \qquad (37)$$

$$\mathscr{B} = \operatorname{diag}(0.3, 0, 0, 0.3).$$

Let us consider the following Van der Pol circuits system [35]:

$$\begin{aligned} \dot{x}_i &= v_i + \Delta h_i \\ v_i &= -2x_i + 3\left(1 - x_i^2\right) v_i + u_i + \Delta d_i \end{aligned} \qquad (38)$$

where the matched and unmatched uncertainties are $\Delta h = [\sin(0.2t), \cos(0.3t), 2\sin(0.1t), 2\cos(0.1t)]'$, $\Delta d = [0.9\sin(t), 1.5\cos(2t), 1.8\sin(3t), 2.1\cos(2t)]'$ and the input saturation parameters are $u_{\min i} = 5, u_{\max i} = 10$.

Parameters of the SMDOB and ESO are listed in Table 1.

In this simulation, initial conditions for all system states are $x(0) = [-0.5649, -0.4979, 0.7858, 0.4064]^T$ and $v(0) = [0.5557, 0.1844, 0.2120, 0.0773]^T$. Moreover, all observer initial conditions are considered as zeros.

Figure 2 is the estimation error of the system uncertainties by using the designed observer. Based on the estimation of system uncertainties, the desired tracking performance and tracking errors for MASs are shown in Figures 3 and 4. Note that Figure 5 is the sliding surface. The evolution

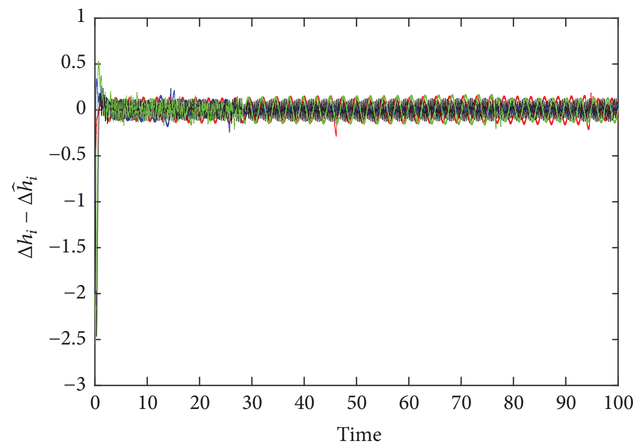

Figure 2: Unmatched uncertainties estimation error.

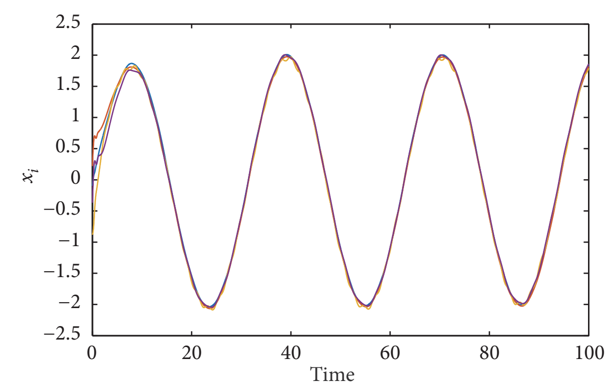

Figure 3: Each agent state.

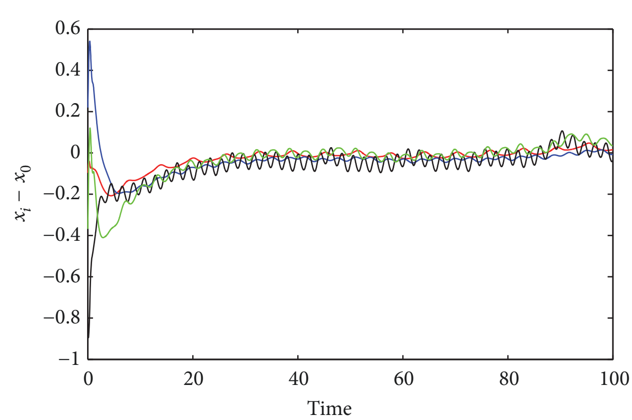

Figure 4: State tracking errors.

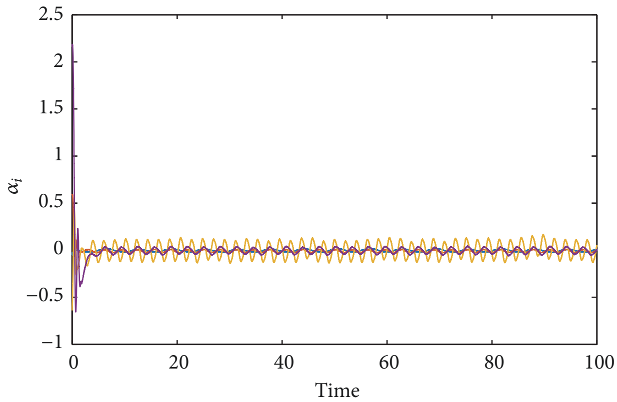

FIGURE 5: Sliding manifolds.

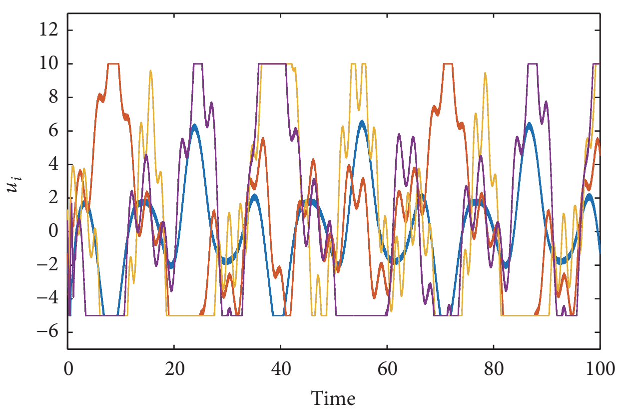

FIGURE 6: Saturation input.

of the saturation control inputs is illustrated in Figure 6. Although, there exist the nonsymmetric input saturation and the time-varying system uncertainties, the system tracking performance is still satisfactory and the tracking errors converge to bounded regions.

From the simulation results, we can obtain that the developed SMDOB control protocol is valid. The proposed protocol can force all agent states following the given desired trajectory; even only a subset of group members has access to the desired signal.

5. Conclusion

In this paper, a multiagent tracking control problem was discussed to make the networked agents achieve tracking objective under matched and unmatched uncertainties, which include input saturation and unknown leader acceleration. A novel SMDOB was proposed to estimate system uncertainties. An auxiliary system was constructed to approximate the follower dynamic. With the aid of designed observer-based sliding manifold, a feedback-type protocol was proposed and the stability conditions were also derived. In contrast to existing results on this aspect [19], the converging accuracies depended on the estimation errors $\|\Delta h_i - \Delta\widehat{h}_i\|$ rather than $\|\Delta h_i\|$. Moreover, the simulations illustrated good performance of the proposed SMDOB based protocol under complicated conditions.

The developed theoretical results can provide new insight into the studies of distributed tracking control for MASs in presence of complicated constraints. This protocol can be applied in some practical systems, such as robotic dynamic, satellite orbital control system.

In the future, there are still some interesting problems on this topic for further study, such as directed communication topology, time delay, and packet dropout.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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