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Research Article

A Product Service Supply Chain Network Equilibrium Model considering Capacity Constraints

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With the further development of manufacturing servitization, the supply chain established by enterprises has gradually evolved into a product service supply chain. The introduction of service flow has made supply chain management more complicated. In this paper, we build a product service supply chain network composed of raw material suppliers, service providers, manufacturing integrators, and customers. The equilibrium model for decision-makers at all levels is established by variational inequality. In particular, we emphasize the impact of product and service capacity constraints and changes in the product service integration ratio on network equilibrium. The results show that, while capacity constraints on production tend to stabilize and unify the market price, service-related capacity constraints polarize the customer pay price. That is to say, product capability constraints limit the quality of product service systems, while service capability constraints limit the types of product service systems. Furthermore, the introduction of service flow and integration with products creates a more closely networked relationship between the upper and lower layers of the product service supply chain, and an increase in the service proportion will increase the network equilibrium profit.

1. Introduction

With the rapid development of product technology and changes in customer expectations regarding personalized consumption, competition among manufacturing enterprises has become increasingly fierce, and the integration of products and services has become a new industrial form [1, 2]. The typical characteristics of product service integration are fully demonstrated in the smart phone, shared car, and smart home appliance industries. For example, Apple, Xiaomi, and other mobile phone enterprises, as manufacturing integrators, integrate the parts and components provided by upstream suppliers, such as processors, cameras, and other hardware equipment, with the services provided by service suppliers, such as games, shopping, and other services, to provide customers with smart mobile terminals. Haier Group, also a manufacturing integrator, integrates TV parts provided by upstream suppliers with Tencent video, Iqiyi, and other program platforms to provide customers with smart home appliances [3, 4]. Therefore, in order to meet the needs of customers and achieve high-quality enterprise development, close cooperation and coordinated development among manufacturers, raw material suppliers, and service providers, as well as customers, are needed to achieve a balance of interests between the supply and demand.

Scholars refer to supply chains composed of raw material suppliers, service providers, and manufacturers as product service supply chains [5], and they have analyzed the organizational structure, especially in relation to coordination, of the product service supply chain [6]. However, as far as we know, these studies only consider the coordination of a single supply chain. Although Wang [7] considers competition between product and service supply chains, the study only involves the analysis and proof of two chains. As we all know, the operation of each supply chain is not completely independent, and the cooperation between enterprises is largely affected by their own enterprise decision preferences and other enterprise decision-making behaviors. Therefore, supply chains cross each other and form a huge product service supply chain network [8–10]. Relevant scholars use network equilibrium to study supply chain network coordination, which only involves product flow and price equilibrium [11, 12]. However, the problem of product service supply chain network equilibrium with service flow will be more complex, and the production and service capacity constraints of decision-makers in the network also affect the equilibrium state [13, 14]. Thus, this paper will study the network equilibrium problem of the product service supply chain considering capacity constraints. We propose the following questions:

(1) How does the constraint of capacity affect the network equilibrium of the product service supply chain?

(2) How does the introduction of service flow and the change of its proportion in the product service systems affect network equilibrium?

In order to answer the above questions, we have built a four-tier product service supply chain network with raw material suppliers, service providers, manufacturing integrators, and demand market. The same types of enterprises are non-cooperative competition relations, and the upstream and downstream enterprises are cooperative relations. By means of variational inequality, a network equilibrium model of a product and service supply chain is established, the transaction quantity and transaction price are calculated, and the influence of supply capacity and product and service ratio on network equilibrium is analyzed.

The rest of the paper is organized as follows. A review of the extant literature is presented in Section 2. In Section 3, we establish a network equilibrium model of a product service supply chain with capacity constraints. In Section 4, we use the Lagrange and marginal utility theories to analyze the impact of constraints. In Section 5, we use numerical examples to calculate and analyze the network equilibrium conditions in various states. Section 6 concludes this paper.

2. Literature Review

This paper focuses on the analysis of the network equilibrium problem of a product service supply chain under the limitation of participants' ability. Therefore, we summarize the relevant literature from three aspects, that is, the product service supply chain network, capability constraints of the product service supply chain, and supply chain network equilibrium.

2.1. Product Service Supply Chain. With the development of manufacturing servitization [15, 16], enterprises have begun to place increased emphasis on the use of services to attract and sustain customers, and more and more service elements are pouring into the product supply chain. The simple theory of product supply chain network equilibrium cannot explain the phenomenon of the integration of product and service flows in the network [17]. Johnson and Mena [18] redefined a supply chain that has introduced a service flow as the product service supply chain, and many scholars have also explored supply chain management issues in the context of services [19]. Beuren et al. [20] believed that the introduction of services in the supply chain network can help many manufacturing companies find opportunities, change customer consumption patterns (introducing services to reduce consumption), and integrate products and services as an overall solution to the customer circulation business model, which is more favored by the market [21]. Maull et al. [22] established a process model for the product service supply chain and researched the relationship between products and services. At the same time, a large number of scholars have also studied the product service supply chain focusing on product sales—before or after the sales transaction [23]. Further, the emergence of service flow in the supply chain network will greatly affect decision-making between cooperative enterprises. For example, services bound to products in the supply chain will affect the pricing decisions of retailers [24], and providing a corresponding level of service in a determined supply relationship can change inventory decisions in the supply chain [25]. Finally, service uncertainty in product service systems will affect the cost accounting of the product service enterprise [26].

In addition, due to the different phases of servitization, different types of integration solutions will be generated [27]. In the initial stage of servitization, the services provided by enterprises are mainly to ensure the normal use of product functions [28]; at this time, the integrated solutions are product-oriented. As the customer's customization needs increase, the proportion of services in integrated solutions expands, and the value created by services exceeds products; then, services become the core of integrated solutions [29]. Manufacturers should integrate different levels of services for different products and customers to provide differentiated integrated solutions [30], and customers will choose different integrated solutions according to their feelings about a service when products and services are bundled together [31]. Homburg and Kuehnl [32] think that the innovation and integration of products and services is affected by customer's choices and will cause corporate performance to develop in both positive and negative directions. Tenucci and Supino [33] also mentioned that different types of product service systems can cause different fluctuations in company earnings.

The existing research on product and service integration provides a solid foundation for us to study the integration of product and service flows. On this basis, we explore the balance of product and service as an integrated solution in the supply chain network.

2.2. Capability Constraints of Product Service Supply Chain. When products and services are provided to customers as a whole, the capabilities of participants at all levels greatly affect equilibrium condition changes [34, 35]. The supply capacity of raw materials, the capability of the services suppliers, and efficacy of manufacturing integrators will all have impacts on the decision-making behavior of each participant in the supply chain network [36]. For example, Yang et al. [37] believed that when large-scale manufacturing enterprises provide complete sets of equipment and services, they can transform an industry into cloud manufacturing services due to capacity constraints. Niemann et al. [38] believed that, in the power system, flexible coordination by suppliers and the supply capacity seriously affect the service structure. Peng et al. [39] asserted that suppliers' sustainable and stable supply capacity could reduce the overall cost of the supply chain. Therefore, the ability limitation of suppliers becomes an important problem to be considered in the network equilibrium of product service supply chain. Some scholars have also researched capacity constraints in supply chain networks. Yang et al. [40] proved that, during the balanced evolution of a supply chain network composed of manufacturers and retailers, a limitation in funding capacity will affect investment cooperation between the two. Ahmadi-Javid and Hoseinpour [41] established a supply chain distribution network model with inventory capacity constraints and analyzed the impact of inventory capacity constraints on product pricing decisions. Nagurney [42] studied the problem of product supply chain network equilibrium in the case of production and storage capacity constraints in the supply chain. Later, aiming at the problem of post-disaster reconstruction through humanitarian assistance, a supply chain network equilibrium model considering the constraints of aid agencies' purchasing capacity and transportation capacity was established [43].

The existing literature has studied the capacity of the participants in the supply chain, but most of these studies focus on the decision-making problem in a single supply chain or the equilibrium problem in the product supply chain. Few studies focus on the capacity constraints of the suppliers in the product service supply chain.

2.3. Supply Chain Network Equilibrium. Supply chain network equilibrium studies the distribution of profits among participants at all levels of the network, the goal of which is value co-creation across the entire network under a reasonable distribution of profits. In 2002, Nagurney first proposed the concept of supply chain network equilibrium. She introduced the idea of transportation networks into the study of product flow and determined the network equilibrium conditions using variational inequality.

The current supply chain network has become more cross-cut and complex [44], which is fully reflected in the diversity of equilibrium research. Nagurney and Yu [45] explored the conditions for sustainable management of a fashion supply chain under oligopoly competition and brand differentiation. Ma et al. [46] established a time-based supply chain network equilibrium model, and Hong et al. [47] proved the existence and uniqueness of the equilibrium price and delivery time of products in a competitive manufacturing network. On the basis of Dong et al.'s [48] research on the equilibrium of random demand networks, Liu and Nagurney [49], Daultani et al. [50], and Zhou et al. [51] used real option theory to establish single-period or multi-periods supply chain network equilibrium models that consider uncertain demand, risk aversion, and dynamic loss aversion behavior. Yu [52] used the theory of network equilibrium to study the competition problem of a sustainable supply chain under environmental tax policy. Chan et al. [53] analyzed the supply chain network equilibrium model under multi-attribute behavior.

The above research on supply chain network equilibrium only focuses on the traditional product supply chain network. In contrast, this study investigates the equilibrium state of service flow introduced into the supply chain network and considers the impact of capacity constraints on its equilibrium state.

3. Model Building

In the model, we assume that there are mainly four stakeholders, raw material supplier, service provider, manufacturing integrator, and demand market, consisting of the product service supply chain network, except for the demand market, each of them will face the ability constraints. In this model, there are L raw material suppliers, with a typical supplier denoted by l, N service suppliers, with a typical one denoted by n, M manufacturing integrators, with a typical one denoted by m, and K demand markets, with a typical market denoted by k. Manufacturing integrators, as the core of the network, purchase raw materials and services from raw materials suppliers and service providers, respectively; then they integrate products and service into product service systems and sell them to the demand market. In order to make the model not too complex, we made assumptions like the following. (1) There is a competitive relationship for enterprise with the same type, and they coordinate with other enterprises that are different from them. (2) All stakeholders are free to make transactions with others. (3) The related cost functions are continuously differentiable and convex.

Compared with Nagurney's research before [12, 19], there are some differences with our study. Product transaction volume and the equilibrium conditions for network benefit maximum have been determined in reference [12], while in reference [19], price and quality of internet service in network were the discussion focus. Unlike existing researches, with the process of manufacturing servitization, we construct a model where product flow and service flow are existing in one network and integrated as the whole that would be sold to the demand market. The model is shown clearly in Figure 1. At the same time, with the inspiration of reference [43], we have emphasized on the influence of constraints of all enterprises' ability in the product service supply chain network. The influence of ability limitation of product, service, and integrations are respectively discussed specifically in this study; then, it could be helpful for stakeholders to make decisions in the complex product service supply chain network.

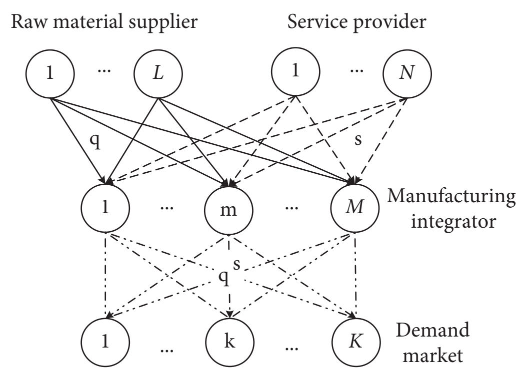

Figure 1: Product service supply chain network equilibrium model.

3.1. Raw Material Suppliers' Profit Maximization Model. Raw material suppliers are responsible for producing raw materials and selling them to manufacturing integrators. We denote $f_l(q_l)$ as the production cost of raw material supplier l, and the vector (q_{lm}) indicates the quantity of material, at unit price (ρ_{lm}), that is delivered to manufacturing integrator m. At the same time, raw material supplier l must also bear a certain amount of transaction costs, which is referred to by $(\widehat{c}_{\mathrm{lm}}(q_{\mathrm{lm}}))$. Therefore, the profit maximization objective function of all raw material suppliers can be expressed as follows:

$$\max U_L = \sum_{l=1}^{L}\left[\sum_{m=1}^{M}\rho_{\mathrm{lm}}q_{\mathrm{lm}} - \sum_{m=1}^{M}\widehat{c}_{\mathrm{lm}}(q_{\mathrm{lm}})\right] - \sum_{l=1}^{L} f_l(q), \quad (1)$$

$$\text{s.t.} \sum_{m=1}^{M} q_{\mathrm{lm}} \le u, \quad (2a)$$

$$q_{\mathrm{lm}} \ge 0. \quad (2b)$$

Formula (2a) indicates that the quantity of raw materials supplied by raw material supplier l to manufacturing integrator m is non-negative, while formula (2b) indicates that, due to the limitation of its own production capacity, the raw material supplier can only provide a limited amount of raw materials.

3.2. Service Providers' Profit Maximization Model. In order to meet the requirements of manufacturing integrators in providing product and service integration solutions to customers, various service providers in the network use their own human, material, and financial resources to deliver various product-related service activities to manufacturing providers, which are carried out for the entire product life cycle. We denote vector s_{nm} as the service that service provider n sells to manufacturing integrator m (service can represent both breadth and depth of service activities), $\pi(s_{nm})$ is the profit for selling a service to the manufacturing integrator, and $f_n(s_n)$ represents the cost of its service activities. Just as with raw material supplier l, we denote $\widehat{c}_{nm}(s_{nm})$ as the transaction costs associated with service delivery. Therefore, the profit maximization objective function of all service providers can be expressed as follows:

$$\max U_N = \sum_{n=1}^{N}\left[\pi(s_{nm}) - \sum_{m=1}^{M}\widehat{c}_{nm}(s_{nm})\right] - \sum_{n=1}^{N} f_n(s_n), \quad (3)$$

$$\text{s.t.} \sum_{n=1}^{N} s_{nm} \le v_n, \quad (4a)$$

$$s_{nm} \ge 0. \quad (4b)$$

Here, formula (4a) indicates that the service supplied by service provider n to manufacturing integrator m is non-negative, while formula (4b) indicates that the amount of service that service provider n can provide is limited.

3.3. Manufacturing Integrators' Profit Maximization Model. Manufacturing integrators integrate purchased raw materials and services to create product service systems. When products and services are integrated, manufacturing integrators can provide different types of product and service system solutions according to customer needs. We believe that, in different types of solutions, the proportion of products and services is in a state of mutual elimination and relative balance. For example, when the logic is product-oriented, the product service system mainly sells the products, and the services are mostly basic and guaranteed services that play an auxiliary role. When the logic is dominated by the service, the product service systems are mainly sold by the service, and the service is mostly promotional or functional and plays a decisive role. However, when the transition from product-oriented logic to service-oriented logic is carried out, the proportion of products and services in the integration scheme gradually changes. Therefore, we regard the product service system as a function of the purchase volume of raw materials (i.e., the production volume of products) and the service purchased [31, 33]. We define vector q^s_{mk} as the integrated solutions delivered by manufacturing integrator m to demand market k; that is $(q^s_{mk} = aq_{lm} + (1-a)s_{nm})$, and a, $(1-a)$, respectively, represent the proportion of products and services in integration solutions.

The unit price for manufacturing integrator m to sell an integrated solution to demand market k is ρ_{mk}. Thus, the total revenue of manufacturing integrator $m\,(m = 1, \ldots, M)$ for selling integration solutions to demand markets is $(\sum_{k=1}^{K}\rho_{mk}q^s_{mk})$, and the cost of integration is $f_m(q^s_m)$. In addition, when manufacturing integrators generate integrated solutions, they need to purchase materials and various services, so the material and service purchase expenses of manufacturing integrator m is $(\sum_{l=1}^{L}\rho_{lm}q_{lm} + \sum_{n=1}^{N}\pi(s_{nm}))$.

In addition to sales revenue and purchase expenses, each manufacturing integrator $m\,(m = 1, \ldots, M)$ also needs to bear a portion of the transaction costs. We, respectively, denote $(\sum_{k=1}^{K} c_{mk}(q^s_{mk}))$, $\sum_{l=1}^{L} c_{lm}(q_{lm})$, and $(\sum_{n=1}^{N} c_{nm}(s_{nm}))$ as the transaction costs related to demand market k, raw material supplier l, and service provider n of manufacturing integrator m. Simultaneously, when manufacturing integrators provide product and service integration solutions to the demand market, they may face the risk of unacceptable services. We define $f_{m\quad d}(q_m^s)$ as the risk cost that integration solutions will not be accepted well, and β indicates the possibility of customer rejection. Therefore, the profit maximization objective function of all manufacturing integrators can be expressed as follows:

$$\max U_M = \sum_{m=1}^{M}\sum_{k=1}^{K}\left[\rho_{mk}q_{mk}^{s} - c_{mk}\left(q_{mk}^{s}\right)\right] \tag{5}$$

$$\text{s.t.} \sum_{k=1}^{K} q_{mk}^{s} \le y_m, \tag{6a}$$

$$\sum_{k=1}^{K} q_{mk}^{s} \le \sum_{m=1}^{M} q_{lm}, \tag{6b}$$

$$\sum_{k=1}^{K} q_{mk}^{s} \le \sum_{l=1}^{L} s_{nm}, \tag{6c}$$

$$q_{mk}^{s} \ge 0. \tag{6d}$$

Formula (6a) indicates that the number of product service system schemes that manufacturing integrators can provide is non-negative. Formulae (6b) and (6c) mean that the quantity of product and service systems will not exceed the quantity of products and services. Formula (6d) indicates that manufacturing integrators cannot infinitely produce product service system schemes, and the total number of product service system schemes delivered to all demand markets is limited.

3.4. Demand Market Utility Maximization Model. Due to the differentiated use requirements, customers in the demand market are no longer satisfied with standardized physical products. Instead, they pursue product service systems that can meet their specific needs. Under the condition that customers pursue their own utility maximization, the demand in market for product service systems is affected by both the price of the integration solutions and the service level provided. We denote ρ_k as the price that customers are willing to pay for the integration solutions, and $(\widehat{c}_{\mathrm{mk}}(q_{\mathrm{mk}}^{s}))$ indicates the participation cost when they make a purchase. When the customers in the demand market are willing to pay the price to cover the cost of participation and the manufacturers' sale price, they can reach a deal. According to the consumer demand function, the customers' decision behavior can be expressed as

$$\begin{aligned}
&\rho_{mk}^{*} + \widehat{c}_{mk}\left(q_{mk}^{s*}\right)\begin{cases} = \rho_k^{*}, & \text{if } q_{mk}^{s} > 0,\\ \ge \rho_k^{*}, & \text{if } q_{mk}^{s} = 0,\end{cases}\\
&d_k\left(\rho_{mk}, s\right)\left\{ = \sum\nolimits_{m=1}^{M} q_{mk}^{s}, \text{if } \rho_k > 0, \le \sum_{m=1}^{M} q_{mk}^{s}, \text{if } \rho_k = 0.\right.
\end{aligned} \tag{7}$$

Furthermore, we describe the constraints faced by all the participants in the product service supply chain network. Some of the constraints are specific to a particular participant, whereas others are common. It is precisely these shared constraints that create the competitive game theory model—one that is defined as a generalized Nash equilibrium.

For manufacturing integrator $(m; m = 1, \ldots, M)$, it can be known from formula (6) that the constraint contains two conditions: the number of product service system schemes transported to the demand market is non-negative and it cannot exceed the upper limit of its supply capacity. We use η_m to indicate a feasible solution that satisfies the two constraints. Then,

$$\eta \equiv \prod_{m=1}^{M} \eta_m. \tag{8}$$

In the same way, φ_l and ϕ_n represent a feasible solution that satisfies the two constrains of raw material suppliers and service suppliers, respectively. Then,

$$\varphi = \prod_{l=1}^{L} \varphi_l, \phi = \prod_{n=1}^{N} \phi_n. \tag{9}$$

We assume that the raw materials and services provided by the raw material and service suppliers, respectively, can meet the needs of the manufacturing integrator to produce product service system schemes. We also assume that the integrated solutions sold by manufacturing integrators also satisfy the needs of customers in the demand market. Therefore, the solution set $(\psi \equiv \eta \cap \varphi \cap \phi)$ is not an empty set.

Definition 1. The generalized Nash equilibrium for manufacturing integrators, raw material suppliers, and service providers. A relief item flow vector, $q^{s*} \in \eta$, sold to the demand market is a generalized Nash equilibrium for each manufacturing integrator $(m; m = 1, \ldots, M)$:

$$U_m\left(q_m^{s*}, \widehat{q}_m^{s*}\right) \ge U_m\left(q^{s}, \widehat{q}^{s*}\right), \quad \forall q_m^{s} \in \eta_m, \tag{10}$$

where $\widehat{q}^{s*} \equiv (q_1^{s*}, \ldots, q_{m-1}^{s*}, q_{m+1}^{s*}, \ldots, q_M^{s*})$.

In the same way, the relief item flow vector, $q^{*} \in \varphi, s^{*} \in \phi$, sold to the manufacturing integrator is also a generalized Nash equilibrium if for each raw material supplier $(l; l = 1, \ldots, L)$ and service provider $(n; n = 1, \ldots, N)$:

$$\begin{aligned}
&U_l\left(q_l^{*}, \widehat{q}_l^{*}\right) \ge U_l\left(q_l, \widehat{q}_l^{*}\right), \quad \forall q_l \in \varphi_l,\\
&U_n\left(s_n^{*}, \widehat{s}_n^{*}\right) \ge U_n\left(s_n, \widehat{s}_n^{*}\right), \quad \forall s_n \in \phi_n,
\end{aligned} \tag{11}$$

where $\widehat{q}_l^{*} \equiv (q_1^{*}, \ldots, q_{l-1}^{*}, q_{l+1}^{*}, \ldots, q_L^{*}), S_n^{*} = (s_1^{*}, \ldots, s_{n-1}^{*}, s_{n+1}^{*}, \ldots, s_N^{*})$.

The above definition means that the interests of every manufacturing integrator, raw material supplier, service provider, and consumer utility purchase are determined not only by their own decisions but also by other decisions in the same industry. Due to the crossover of feasible solutions, the feasible solutions are also mutually influential, making the equilibrium problem of the product service supply chain network a generalized Nash equilibrium problem. We know that, under the imposed assumptions, the feasible sets, $(\eta_m, \varphi_l, \phi_n)$, are convex for each m, l, n. As the generalized Nash equilibrium can be formulated as a variational inequality problem, we use the variational inequality to determine the transaction volume and price of the product and service system, raw material transaction volume, and service level under the equilibrium state of the product service supply chain network.

Definition 2. Variational Equilibrium. The set of vectors $(q^{s*}, \rho^*, q^*, s^*)$ is a variational equilibrium of the above generalized Nash equilibrium problem if the set of $(q^{s*} \in \eta, q^{s*} \in \varphi, q^{s*} \in \phi; q^* \in \varphi; s^* \in \phi)$ is a solution to the following variational inequality:

$$-\sum_{m=1}^{M} \langle \nabla_{q_m^s} U_m(q^{s*}), q^s - q^{s*} \rangle \geq 0, \quad \forall q^s \in \eta, \tag{12}$$

$$-\sum_{l=1}^{L} \langle \nabla_{q_l} U_l(q^*), q - q^* \rangle \geq 0, \quad \forall q \in \varphi, \tag{13}$$

$$-\sum_{n=1}^{N} \langle \nabla_{s_n} U_n(s^*), s - s^* \rangle \geq 0, \quad \forall s \in \phi, \tag{14}$$

$$\begin{aligned} &-\sum_{k=1}^{K} \langle \nabla_{q_m^s} U_k(q^{s*}), q^s - q^{s*} \rangle \\ &\quad -\sum_{k=1}^{K} \langle \nabla_{\rho_k} U_k(\rho_k^*), \rho_k - \rho_k^* \rangle \geq 0, \quad \forall q^s \in \eta. \end{aligned} \tag{15}$$

It is important to note that the variational equilibrium corresponds to the Lagrangian multiplier, with the common constrains being the same for all the manufacturing integrators, raw material suppliers, and service suppliers. It is precisely because they share the same constraints that the product service network is in an equilibrium game competition state.

Expanding variational inequality (12)–(15), we obtain

$$\begin{aligned} &\sum_{m=1}^{M}\sum_{k=1}^{K}\left[\frac{\partial f_m(q_m^{s*})}{q_{mk}^s} + \frac{\partial \beta \quad f_m \quad d(q_m^{s*})}{q_{mk}^s} + \frac{\partial c_{mk}(q_{mk}^{s*})}{q_{mk}^s} + \frac{\partial c_{mk}(q_{mk}^{s*})}{q_{mk}^s} - \rho_k^*\right] \times [q_{mk}^s - q_{mk}^{s*}] \\ &\quad + \sum_{m=1}^{M}\sum_{l=1}^{L}\left[\frac{\partial f_l(q^*)}{\partial q_{lm}} + \frac{\partial \hat{c}_{lm}(q_{lm}^*)}{\partial q_{lm}} + \frac{\partial c_{lm}(q_{lm}^*)}{\partial q_{lm}}\right] \times [q_{lm} - q_{lm}^*] \\ &\quad + \sum_{m=1}^{M}\sum_{n=1}^{N}\left[\frac{\partial f_n(s_n^*)}{\partial s_{nm}} + \frac{\partial \hat{c}_{nm}(s_{nm}^*)}{\partial s_{nm}} + \frac{\partial c_{nm}(s_{nm}^*)}{\partial s_{nm}}\right] \times [s_{nm} - s_{nm}^*] \\ &\quad + \sum_{k=1}^{K}\left[\sum_{m=1}^{M} q_{mk}^{s*} - d_k(\rho_{mk}^*, s_{nm}^*)\right] \times [\rho_k - \rho_k^*] \geq 0. \end{aligned} \tag{16}$$

Now, we put variational inequality (16) into a standard form. We determine $(X^* \in \kappa)$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \kappa, \tag{17}$$

where $\langle \cdot, \cdot \rangle$ is the inner product in N-dimensional Euclidean space, and $(N = \mathrm{MLNK})$ for our model. We define $X \equiv (q, s, q^s, \rho), F(X) = (F_{\mathrm{mn}}, F_{\mathrm{ml}}, F_{\mathrm{mk}}, F_k)$.

Since variational inequality (17) is a continuous function and the feasible set is a convex set, which is limited by the needs and capabilities of the participants at all tiers, our solution is existential and unique.

4. Lagrange Theory and Analysis of the Marginal Utilities

In this section, we study the Lagrangian theory associated with variational inequality (19). Then, we use the Lagrangian multiplier to analyze the marginal utilities and role of each constraint in the model. We also derive alternative variational inequalities to (19), for ease of computation, by setting

$$\begin{aligned}C(q,s,q^{s},\rho) = &\sum_{m=1}^{M}\sum_{k=1}^{K}\left[\frac{\partial f_m(q_m^{s*})}{\partial q_{mk}^{s}} + \frac{\partial \beta \quad f_m \quad _d(q_m^{s*})}{\partial q_{mk}^{s}} + \frac{\partial c_{mk}(q_{mk}^{s*})}{\partial q_{mk}^{s}} + \frac{\partial \widehat{c}_{mk}(q_{mk}^{s*})}{\partial q_{mk}^{s}} - \rho_k^*\right] \times [q_{mk}^{s} - q_{mk}^{s*}] \\ &+ \sum_{m=1}^{M}\sum_{l=1}^{L}\left[\frac{\partial f_l(q_l^*)}{\partial q_{lm}} + \frac{\partial \widehat{c}_{lm}(q_{lm}^*)}{\partial q_{lm}} + \frac{\partial c_{lm}(q_{lm}^*)}{\partial q_{lm}}\right] \times [q_{lm} - q_{lm}^*] \\ &+ \sum_{m=1}^{M}\sum_{n=1}^{N}\left[\frac{\partial f_n(s_n^*)}{\partial s_{nm}} + \frac{\partial \widehat{c}_{nm}(s_{nm}^*)}{\partial s_{nm}} + \frac{\partial c_{nm}(s_{nm}^*)}{\partial s_{nm}}\right] \times [s_{nm} - s_{nm}^*] \\ &+ \sum_{k=1}^{K}\left[\sum_{m=1}^{M} q_{mk}^{s*} - d_k(\rho_{mk}^*, s_{nm}^*)\right] \times [\rho_k - \rho_k^*].\end{aligned} \tag{18}$$

Then, variational inequality (18) could be rewritten as the following minimization problem:

$$\min_{\psi} C(q,s,q^{s},\rho) = C(q^*, s^*, q^{s*}, \rho^*) = 0. \tag{19}$$

According to previous assumptions, all the involved functions in (19) are convex and continuously differentiable. In order to construct the Lagrangian function, we re-represent the constraints with the associated Lagrangian multiplier next to the corresponding constraint:

$$\begin{aligned} g_{mk1} &= -q_{mk}^{s} \le 0, \lambda_{mk1}, \forall m, \forall k, \\ g_{mk2} &= \sum_{k=1}^{K} q_{mk}^{s} - y \le 0, \lambda_{mk2}, \forall m, \\ e_{lm1} &= -q_{lm} \le 0, \varepsilon_{lm1}, \forall l, \forall m, \\ e_{lm2} &= \sum_{m=1}^{M} q_{lm} - u \le 0, \varepsilon_{lm2}, \forall l, \\ b_{nm1} &= -s_{nm} \le 0, \gamma_{nm1}, \forall n, \forall m, \\ b_{nm2} &= \sum_{m=1}^{M} s_{nm} - v \le 0, \gamma_{nm2}, \forall n, \\ \Gamma(q,s,q^{s},\rho) &= (\lambda_{mk1}, \lambda_{mk2}, \varepsilon_{lm1}, \varepsilon_{lm2}, \gamma_{nm1}, \gamma_{nm2})_{m=1,\dots,M, l=1,\dots,L, n=1,\dots,N}. \end{aligned} \tag{20}$$

We now construct the Lagrange function:

$$\begin{aligned}\Phi(q,s,q^{s},\rho,\lambda,\varepsilon,\gamma) = &\sum_{m=1}^{M}\sum_{k=1}^{K}\left[\frac{\partial f_m(q_m^{s*})}{\partial q_{mk}^{s}} + \frac{\partial \beta \quad f_m \quad _d(q_m^{s*})}{\partial q_{mk}^{s}} + \frac{\partial c_{mk}(q_{mk}^{s*})}{\partial q_{mk}^{s}} + \frac{\partial \widehat{c}_{mk}(q_{mk}^{s*})}{\partial q_{mk}^{s}} - \rho_k^*\right] \times [q_{mk}^{s} - q_{mk}^{s*}] \\ &+ \sum_{m=1}^{M}\sum_{l=1}^{L}\left[\frac{\partial f_l(q_l^*)}{\partial q_{lm}} + \frac{\partial \widehat{c}_{lm}(q_{lm}^*)}{\partial q_{lm}} + \frac{\partial c_{lm}(q_{lm}^*)}{\partial q_{lm}}\right] \times [q_{lm} - q_{lm}^*] \\ &+ \sum_{m=1}^{M}\sum_{n=1}^{N}\left[\frac{\partial f_n(s_n^*)}{\partial s_{nm}} + \frac{\partial \widehat{c}_{nm}(s_{nm}^*)}{\partial s_{nm}} + \frac{\partial c_{nm}(s_{nm}^*)}{\partial s_{nm}}\right] \times [s_{nm} - (s_{nm}^*)] \\ &+ \sum_{k=1}^{K}\left[\sum_{m=1}^{M} q_{mk}^{s*} - d_k(\rho_{mk}^*, s_{nm}^*)\right] \times [\rho_k - \rho_k^*] + \sum_{m=1}^{M}\sum_{k=1}^{K}[g_{mk1}\lambda_{mk1} + g_{mk2}\lambda_{mk2}] + \sum_{l=1}^{L}\sum_{m=1}^{M}[e_{lm1}\varepsilon_{lm1} + e_{lm2}\varepsilon_{lm2}] \\ &+ \sum_{n=1}^{N}\sum_{m=1}^{M}[b_{nm1}\gamma_{nm1} + b_{nm2}\gamma_{nm2}], \quad \forall q \in R_+^{LM}, \forall s \in R_+^{NM}, \forall q^{s} \in R_+^{MK}, \forall \rho \in R_+^{MK}, \forall \lambda \in R_+^{MK}, \forall \varepsilon \in R_+^{LM}, \forall \gamma \in R_+^{NM},\end{aligned} \tag{21}$$

where λ is the vector of all (λ_{mk}), ε is the vector of all (ε_{lm}), and γ is the vector of all (γ_{nm}).

Since feasible set ψ is convex and satisfies the Slater condition, if set (q^*, s^*, qs^*, ρ^*) is a minimal solution to problem (20), then there exists $(\lambda^* \in R_+^{MK}, \varepsilon^* \in R_+^{LM}, \gamma^* \in R_+^{NM})$ such that the vector set $(q^*, s^*, qs^*, \rho^*, \lambda^*, \varepsilon^*, \gamma^*)$ is a saddle point of the Lagrange function (21):

$$\begin{aligned}\Phi(q^*, s^*, q^{s*}, \rho^*, \lambda, \varepsilon, \gamma) &\le \Phi(q^*, s^*, q^{s*}, \rho^*, \lambda^*, \varepsilon^*, \gamma^*)\\ &\le \Phi(q, s, q^s, \rho, \lambda^*, \varepsilon^*, \gamma^*),\end{aligned} \tag{22}$$

$$\begin{aligned} g_{mk1}^*\lambda_{mk1}^* &= 0,\\ g_{mk2}^*\lambda_{mk2}^* &= 0,\\ e_{lm1}^*\varepsilon_{lm1}^* &= 0,\\ e_{lm2}^*\varepsilon_{lm2}^* &= 0,\\ b_{nm1}^*\gamma_{nm1}^* &= 0,\\ b_{nm2}^*\gamma_{nm2}^* &= 0.\end{aligned} \tag{23}$$

From (22), we know that set $(q^* \in R_+^{LM}, s^* \in R_+^{NM}, q^{s*} \in R_+^{MK}, \rho^* \in R_+^{MK})$ is the minimal point of function $\Phi(q, s, q^s, \rho, \lambda^*, \varepsilon^*, \gamma^*)$. Therefore, for all $(m = 1, \ldots, M)$, all $(l = 1, \ldots, L)$, and all $(n = 1, \ldots, N)$, we obtain that

$$\begin{aligned}&\sum_{m=1}^{M}\sum_{k=1}^{K}\left[\frac{\partial f_m(q_m^{s*})}{\partial q_{mk}^s} + \frac{\partial \beta \quad f_m \quad {}_d(q_m^{s*})}{\partial q_{mk}^s} + \frac{\partial c_{mk}(q_{mk}^{s*})}{\partial q_{mk}^s} + \frac{\partial \hat{c}_{mk}(q_{mk}^{s*})}{\partial q_{mk}^s} - \rho_k^*\right] - \lambda_{mk1}^* + \lambda_{mk2}^*\\ &+ \sum_{m=1}^{M}\sum_{l=1}^{L}\left[\frac{\partial f_l(q^*)}{\partial q_{lm}} + \frac{\partial \hat{c}_{lm}(q_{lm}^*)}{\partial q_{lm}} + \frac{\partial c_{lm}(q_{lm}^*)}{\partial q_{lm}}\right] - \varepsilon_{lm1}^* + \varepsilon_{lm2}^*\\ &+ \sum_{m=1}^{M}\sum_{n=1}^{N}\left[\frac{\partial f_n(s_n^*)}{\partial s_{nm}} + \frac{\partial \hat{c}_{nm}(s_{nm}^*)}{\partial s_{nm}} + \frac{\partial c_{nm}(s_{nm}^*)}{\partial s_{nm}} - \gamma_{nm1}^* + \gamma_{nm2}^*\right]\\ &+ \sum_{k=1}^{K}\left[\sum_{m=1}^{M} q_{mk}^{s*} - d_k(\rho_{mk}^*, s_{nm}^*)\right] = 0.\end{aligned} \tag{24}$$

Theorem 1. *Alternative Variational Inequality Formulation. Conditions (23) and (24) indicate that the equivalent formula of variational inequality (16) is given by the following formula, and we determine $(q^*, s^*, q^{s*}, \rho^*, \lambda^*, \varepsilon^*, \gamma^*) \in R_+^{MK+LM+NM}$, such that*

$$\begin{aligned}&\sum_{m=1}^{M}\sum_{k=1}^{K}\left[\frac{\partial f_m(q_m^{s*})}{\partial q_{mk}^s} + \frac{\partial \beta \quad f_m \quad {}_d(q_m^{s*})}{\partial q_{mk}^s} + \frac{\partial c_{mk}(q_{mk}^{s*})}{\partial q_{mk}^s} + \frac{\partial \hat{c}_{mk}(q_{mk}^{s*})}{\partial q_{mk}^s} - \rho_k^* - \lambda_{mk1}^* + \lambda_{mk2}^*\right] \times [q_{mk}^s - q_{mk}^{s*}]\\ &+ \sum_{m=1}^{M}\sum_{l=1}^{L}\left[\frac{\partial f_l(q^*)}{\partial q_{lm}} + \frac{\partial \hat{c}_{lm}(q_{lm}^*)}{\partial q_{lm}} + \frac{\partial c_{lm}(q_{lm}^*)}{\partial q_{lm}} - \varepsilon_{lm1}^* + \varepsilon_{lm2}^*\right] \times [q_{lm} - q_{lm}^*]\\ &+ \sum_{m=1}^{M}\sum_{n=1}^{N}\left[\frac{\partial f_n(s_n^*)}{\partial s_{nm}} + \frac{\partial \hat{c}_{nm}(s_{nm}^*)}{\partial s_{nm}} + \frac{\partial c_{nm}(s_{nm}^*)}{\partial s_{nm}} - \gamma_{nm1}^* + \gamma_{nm2}^*\right] \times [s_{nm} - s_{nm}^*]\\ &+ \sum_{k=1}^{K}\left[\sum_{m=1}^{M} q_{mk}^{s*} - d_k(\rho_{mk}^*, s_{nm}^*)\right] \times [\rho_k - \rho_k^*] + \sum_{m=1}^{M}\sum_{k=1}^{K} q_{mk}^{s*} \times [\lambda_{mk1} - \lambda_{mk1}^*]\\ &+ \sum_{m=1}^{M}\left(y - \sum_{k=1}^{K} q_{mk}^{s*}\right) \times [\lambda_{mk2} - \lambda_{mk2}^*] + \sum_{l=1}^{L}\sum_{m=1}^{M} q_{nm}^* \times [\varepsilon_{lm1} - \varepsilon_{lm1}^*] + \sum_{l=1}^{L}\left(u - \sum_{m=1}^{M} q_{lm}^*\right) \times [\varepsilon_{lm2} - \varepsilon_{lm2}^*]\\ &+ \sum_{n=1}^{N}\sum_{m=1}^{M} s_{nm}^* \times [\gamma_{nm1} - \gamma_{nm1}^*] + \sum_{n=1}^{N}\left(v - \sum_{m=1}^{M} s_{nm}^*\right) \times [\gamma_{nm2} - \gamma_{nm2}^*] \ge 0. \quad \forall (q, s, q^s, \rho, \lambda, \varepsilon, \gamma) \in R_+^{MK+LM+NM}.\end{aligned} \tag{25}$$

Now, we provide an explanation for the Lagrange multipliers. We now focus on a situation in which $(q_{lm}^* > 0, s_{nm}^* > 0, q_{mk}^{s*} > 0, \rho_k^* > 0)$. This means the transaction volumes between manufacturing integrator $m$and raw material supplier l, between manufacturing integrator m and service provider n, and between manufacturing integrator m

and demand market k are positive. In the other case, $(q_{lm}^{*} = 0, s_{nm}^{*} = 0, q_{mk}^{s*} = 0, \rho_k^{*} = 0)$. Then, the problem is not interesting. From the first, third, and fifth lines of formula (23), we know that $(\lambda_{mk1}^{*} = 0, \varepsilon_{lm1}^{*} = 0, \gamma_{nm1}^{*} = 0)$.

Now, we consider the case where the constraints are not active; that is,

$$\begin{aligned} &\sum_{k=1}^{K} q_{mk}^{s*} - y < 0, \forall m; \\ &\sum_{m=1}^{M} q_{lm}^{*} - u < 0, \forall l; \\ &\sum_{m=1}^{M} s_{nm}^{*} - v < 0, \forall n, \end{aligned} \tag{26}$$

In this case, all related Lagrangian multipliers are zero, which means $(\lambda_{mk2}^{*} = 0, \forall m, \varepsilon_{lm2}^{*} = 0, \forall l, \gamma_{nm2}^{*} = 0, \forall n)$.

Thus, (24) yields

$$\begin{aligned} &\frac{\partial f_m(q_m^{s*})}{\partial q_{mk}^{s}} + \frac{\partial \beta \quad f_m \quad d(q_m^{s*})}{\partial q_{mk}^{s}} + \frac{\partial c_{mk}(q_{mk}^{s*})}{\partial q_{mk}^{s}} + \frac{\partial \hat{c}_{mk}(q_{mk}^{s*})}{\partial q_{mk}^{s}} \\ &\quad + \frac{\partial f_l(q_l^{*})}{\partial q_{lm}} + \frac{\partial \hat{c}_{lm}(q_{lm}^{*})}{\partial q_{lm}} \\ &\quad + \frac{\partial c_{lm}(q_{lm}^{*})}{\partial q_{lm}} + \frac{\partial f_n(s_n^{*})}{\partial s_{nm}} + \frac{\partial \hat{c}_{nm}(s_{nm}^{*})}{\partial s_{nm}} + \frac{\partial c_{nm}(s_{nm}^{*})}{\partial s_{nm}} = \rho_k^{*} \\ &\sum_{m=1}^{M} q_{mk}^{s*} = d_k(\rho_{mk}^{*}, s_{nm}^{*}). \end{aligned} \tag{27}$$

This means that the sum of the production costs as well as the transaction costs of the manufacturing integrators, raw material suppliers, and service providers is exactly equal to the price paid by customers for the product service systems. The volume of product service systems sold by the manufacturing integrators is equal to the market demand (for the respective (m, l, n, k)).

If constraint (6b) is active for manufacturing integrator$m$ (namely, $(\sum_{k=1}^{K} q_{mk}^{s} = y_m)$), we know from the second line of formula (23) that the associated Lagrange multiplier, λ_{mk2}^{*}, is greater than zero, and formula (28) becomes

$$\begin{aligned} &\frac{\partial f_m(q_m^{s*})}{\partial q_{mk}^{s}} + + \frac{\partial \beta \quad f_m \quad d(q_m^{s*})}{\partial q_{mk}^{s}} \frac{\partial c_{mk}(q_{mk}^{s*})}{\partial q_{mk}^{s}} + \frac{\partial \hat{c}_{mk}(q_{mk}^{s*})}{\partial q_{mk}^{s}} \\ &\quad + \lambda_{mk2}^{*} + \frac{\partial f_l(q_l^{*})}{\partial q_{lm}} + \frac{\partial \hat{c}_{lm}(q_{lm}^{*})}{\partial q_{lm}} \\ &\quad + \frac{\partial c_{lm}(q_{lm}^{*})}{\partial q_{lm}} + \frac{\partial f_n(s_n^{*})}{\partial s_{nm}} + \frac{\partial \hat{c}_{nm}(s_{nm}^{*})}{\partial s_{nm}} + \frac{\partial c_{nm}(s_{nm}^{*})}{\partial s_{nm}} \\ &\quad = \rho_k^{*} \sum_{m=1}^{M} q_{mk}^{s*} = d_k(\rho_{mk}^{*}, s_{nm}^{*}). \end{aligned} \tag{28}$$

This means that the sum of all production and transaction costs for s in a product service supply chain network is less than the price that the customer is willing to pay (for the respective m, l, n, k), which is entirely possible.

Similarly, for raw material supplier l and service provider n, when constraint (2b) or (4b) is active (that is, $(\sum_{m=1}^{M} q_{lm} = u_l)$ or $(\sum_{n=1}^{N} s_{nm} = v_n)$), there will also be cases in the product service supply chain network where the cost of the final delivery of the product service systems is less than the price that the customer is willing to pay.

Through the analysis of the influence of the Lagrangian multiplier and marginal utility in equilibrium, we can conclude that the product service supply chain network can achieve the best equilibrium state under the situation of $(\sum_{k=1}^{K} q_{mk}^{s} = y_m)$, $(\sum_{m=1}^{M} q_{lm} = u_l)$, and $(\sum_{n=1}^{N} s_{nm} = v_n)$. In fact, in this case, the product service systems generated by the manufacturing integrators, raw material suppliers, and service providers for network cooperation can be well accepted by customers, and the costs are all offset.

5. The Algorithm and Numerical Example

5.1. Modified Projection Method. The modified projection algorithm is a new algorithm improved on the bases of projection algorithm. Compared with projection algorithm, this algorithm is more rigorous in logic and more accurate in calculation. The algorithm can be expressed as $(X^{\tau} = P_k (X^{\tau-1} - \alpha F(\overline{X}^{\tau})))$; among $(\overline{X}^{\tau} = P_k (X^{\tau-1} - \alpha F(X^{\tau-1}))), \tau$ is the iteration order and $(P_k X)$ is the vertical mapping of X on K. Please refer to literature [9, 11]. The following are specific steps for examples. Once the functions in the variational inequality are monotone and Lipschitz continuous, the algorithm could be to converge. The calculating method is as follows, where i denotes an iteration counter:

Step 1 (initialization). Set $(X^0 = (q, s, q^s, \rho, \lambda, \varepsilon, \gamma)^0 \in \Omega)$, let $\tau = 1$, $(0 \le \alpha \le (1/L))$, and L is the Lipschitz continuity constant.

Step 2 (computation). Compute $(\overline{X}^{\tau} = (\overline{q}, t\overline{s}n, q\ \overline{q}^{s} h_{\overline{\rho} x}, 7\overline{\lambda}C, ; \overline{\varepsilon}, \overline{\gamma}) \in \Omega)$ by solving the variational inequality subproblem:

$$\left(\overline{X}^{\tau} + \alpha F\left(X^{\tau-1}\right) - X^{\tau-1}\right)^{T} \left(X - \overline{X}^{\tau}\right) \ge 0, \tag{29}$$

where F indicates the function collection of all costs for variable X happened in the product service supply chain.

Step 3 (adaption). Compute $(X^{\tau} \in \Omega)$ by solving the variational inequality subproblem:

$$\left(X^{\tau} + \alpha F\left(\overline{X}^{\tau}\right) - X^{\tau-1}\right)^{T} \left(X - X^{\tau}\right) \ge 0, \tag{30}$$

Step 4 (convergence verification). If $(|q^{\tau} - q^{\tau-1}| \le \mu, |s^{\tau} - s^{\tau-1}| \le \mu, |q^{s\tau} - q^{s\tau-1}| \le \mu, |\rho^{\tau} - \rho^{\tau-1}| \le \mu)$, $(|\lambda^{\tau} - \lambda^{\tau-1}| \le \mu, |\varepsilon^{\tau} - \varepsilon^{\tau-1}| \le \mu, |\gamma^{\tau} - \gamma^{\tau-1}| \le \mu)$, with $\mu > 0$, then stop (which is a prespecified tolerance); else, make $(\tau = \tau + 1)$ and start from Step 1 again. Only F is monotonous and Lipschitz is continuous; the algorithm will converge.

5.2. Numerical Examples. Because the smart-phone industry shows the typical characteristics of product service integration, for this type of manufacturing integrator comes from two smart-phone manufacturers. The two smartphone manufacturers form various types of supply chains with upstream and downstream enterprises and customers, and they cross each other to form a typical product service supply chain network.

Example 1. In Example 1, we first compare the impact on the equilibrium conditions of the supply chain network when various types of enterprises have capacity constraints at the same time. In order to control a single variable, we do not consider the difference of integrated solutions for the time being. We also assume that the product and service in the integrated solutions are in a relatively average state. That is, we assume that $(q^s = 0.5q + (1 - 0.5)s)$.

First of all, we set the procurement cost of the raw material suppliers, service cost of the service provider, integration cost of the manufacturing integrators, and transaction cost function when conducting transactions between the various tiers [24, 42, 43]; these cost functions are shown in Tables 1–4. Among these, we also assume that the risk coefficient of market exclusion for the integration solutions provided by manufacturing integrators is 0.3 [54]. And in this paper, we just need (q, s, q^s, ρ) to confirm the equilibrium statue; thus, to simplify the data, the result of $(\lambda, \varepsilon, \gamma)$ is not given specifically.

First, we calculate the equilibrium conditions of the product service supply chain network in the absence of capacity constraints; all Lagrangian multipliers are set to zero. We set the initial trading volume of raw materials $(q_{11}, q_{12}, q_{21}, q_{22})$ on each link to be 70, and the initial trading volume of services $(s_{11}, s_{12}, s_{21}, s_{22})$ is 5, which is obtained by $(q^s = 0.5q + (1 - 0.5)s)$. Then, the trading volume of all integrated solutions is also a fixed value. A modified projection algorithm is used to solve the equilibrium conditions of the network [11, 42], and the iteration step is set to 0.05. After 692 iterations, we get the equilibrium conditions as

$$\begin{aligned} Q &= \begin{bmatrix} 15.6370 & 12.7884 \\ 11.6890 & 8.8405 \end{bmatrix}, \\ S &= \begin{bmatrix} 15.6097 & 7.3418 \\ 11.7388 & 14.3091 \end{bmatrix}, \\ Q^S &= \begin{bmatrix} 21.7024 & 5.6431 \\ 18.8539 & 2.7946 \end{bmatrix}, \\ \rho &= \begin{bmatrix} 348.1796 & 321.2518 \end{bmatrix}. \end{aligned} \tag{31}$$

The total transaction volume of raw materials and integration solutions, respectively, are

$$\sum_{l=1}^{2} \sum_{m=1}^{2} q_{lm}^{*} = 48.9549, \qquad \sum_{m=1}^{2} \sum_{k=1}^{2} q_{mk}^{s*} = 48.9940. \tag{32}$$

In order to analyze the influence of capacity constraints on the equilibrium conditions of the product service supply chain network, we restrict the transaction value of each link in the network, so that the capacity on the link is less than the traffic value in the case of no constraints. That is, the supply capacity of each supplier is limited. We keep the initial transaction value of raw materials and services unchanged, as well as the initial transaction value of integrated solutions. We set the supply capacity of raw materials to be $(u_{11} = 15, u_{12} = 12, u_{21} = 11, u_{22} = 8)$, the service capacity to be $(v_{11} = 15, v_{12} = 7, v_{21} = 11, v_{22} = 14)$, and integrated solutions to be $(y_{11} = 21, y_{12} = 5, y_{21} = 18, y_{22} = 2)$. At this time, the Lagrange multipliers of the capacity constraints are all 30. Then, we observe what will happen to the equilibrium conditions when all suppliers reach their capacity cap. After 2,255 iterations, we find that the equilibrium conditions at this moment are

$$\begin{aligned} Q &= \begin{bmatrix} 14.9364 & 11.9317 \\ 11.0198 & 8.0200 \end{bmatrix}, \\ S &= \begin{bmatrix} 14.9912 & 6.3440 \\ 10.9915 & 13.6252 \end{bmatrix}, \\ Q^S &= \begin{bmatrix} 20.9857 & 4.9893 \\ 17.9835 & 1.9871 \end{bmatrix}, \\ \rho &= \begin{bmatrix} 347.7898 & 315.5437 \end{bmatrix}. \end{aligned} \tag{33}$$

The total transaction volume of raw materials and integration solutions, respectively, are

$$\sum_{l=1}^{2} \sum_{m=1}^{2} q_{lm}^{*} = 45.9079, \qquad \sum_{m=1}^{2} \sum_{k=1}^{2} q_{mk}^{s*} = 45.9456. \tag{34}$$

By comparing the equilibrium conditions of the two product and service supply chain networks with and without capabilities, we can see that when there is a capacity constraint, the number of raw material transactions integration solutions is reduced, and the service level and price of the integrated solution decline.

Example 2. In Example 1, we demonstrated that capacity constraints will change the equilibrium conditions of the product service supply chain network. Therefore, in Example 2, we will analyze the impact of different supplier capacity changes on the supply chain network equilibrium conditions. In order to facilitate comparison, the same single-variable principle is maintained. When the capacity constraints of a certain type of supplier change, the capabilities of the other two types of suppliers remain unchanged.

First, we will discuss the influence of the change of raw material supplier's capability on the equilibrium condition of the product service supply chain network. Based on the first example in Example 1, we keep the initial transaction volume of raw materials and services as integration solutions unchanged. We still set no capacity restriction between service providers and manufacturing integrators; that is, the Lagrange multiplier of capacity restriction is 0. Because we want to observe the influence of the capability of raw material suppliers, we need to make the capacity of each link of

Table 1: Cost function related to raw material supplier.

Function name	Raw material supplier 1	Raw material supplier 2
Purchase cost of raw material	$(f_1(Q_1) = 2.5q_1^2 + q_1q_2 + 2q_1)$	$(f_2(Q_2) = 3q_2^2 + 1.5q_1q_2 + 2q_2)$
Transaction costs with manufacturing integrator	$(\hat{c}_{11}(q_{11}) = 0.2q_{11}^2 + 1.5q_{11})$ $(\hat{c}_{12}(q_{12}) = 0.2q_{12}^2 + 1.5q_{12})$	$(\hat{c}_{21}(q_{21}) = 0.2q_{21}^2 + 1.5q_{21})$ $(\hat{c}_{22}(q_{22}) = 0.2q_{22}^2 + 1.5q_{22})$

Table 2: Cost function related to service provider.

Function name	Service provider 1	Service provider 2
Service activity costs	$(f_1(S_1) = (2.5s_1^2/2))$	$(f_2(S_2) = (3s_2^2/2))$
Transaction costs with manufacturing integrator	$(\hat{c}_{11}(s_{11}) = 0.2s_{11}^2 + s_{11})$ $(\hat{c}_{12}(s_{12}) = 0.2s_{12}^2 + s_{12})$	$(\hat{c}_{21}(s_{21}) = 0.2s_{21}^2 + s_{21})$ $(\hat{c}_{21}(s_{22}) = 0.2s_{22}^2 + s_{22})$

Table 3: Cost function related to manufacturing integrator.

Function name	Manufacturing integrator 1	Manufacturing integrator 2
Integration cost function	$(f_1(Q_1^S) = 2.5q_1^{s2} + q_1^sq_2^s + 2q_1^s)$	$(f_2(Q_2^S) = 3q_2^{s2} + 1.5q_1^sq_2^s + 2q_2^s)$
Risk cost function	$(\beta f_{1\ d}(Q_1^S) = 0.3(2.5q_1^{s2} + 2q_1^s))$	$(\beta f_{2\ d}(Q_2^S) = 0.3(2.5q_2^{s2} + 2q_2^s))$
Transaction costs with raw material suppliers	$(c_{11}(q_{11}) = 0.2q_{11}^2 + 1.5q_{11})$ $(c_{12}(q_{12}) = 0.2q_{12}^2 + 1.5q_{12})$	$(c_{21}(q_{21}) = 0.2q_{21}^2 + 1.5q_{21})$ $(c_{22}(q_{22}) = 0.2q_{22}^2 + 1.5q_{22})$
Transaction costs with provider suppliers	$(c_{11}(s_{11}) = 0.2s_{11}^2 + s_{11})$ $(c_{12}(s_{12}) = 0.2s_{12}^2 + s_{12})$	$(c_{21}(s_{21}) = 0.2s_{21}^2 + s_{21})$ $(c_{22}(s_{22}) = 0.2s_{22}^2 + s_{22})$

Table 4: Cost function related to demand market.

Function name	Demand market 1	Demand market 2
Transaction costs with manufacturing integrator	$(\hat{c}_{11}(q_{11}^s) = 0.6q_{11}^s + 4)$ $(\hat{c}_{12}(q_{12}^s) = 0.6q_{12}^s + 4)$	$(\hat{c}_{21}(q_{21}^s) = 0.6q_{21}^s + 4)$ $(\hat{c}_{12}(q_{12}^s) = 0.6q_{12}^s + 4)$
Demand for integrated solutions	$(d_1(\rho, s) = 1000 - 2\rho_1 - 1.5\rho_2 + 4s)$	$(d_1(\rho, s) = 1000 - 1.5\rho_1 - 2\rho_2 + 4s)$

raw material less than the maximum value when there is no constraint; that is, we set $(u_{11} = 15, u_{12} = 12, u_{21} = 11, u_{22} = 8)$, and the constraint Lagrange multiplier is still 30. At this point, the equilibrium conditions of the product service supply chain network are as follows:

$$Q = \begin{bmatrix} 14.0960 & 12.0199 \\ 10.7901 & 8.0200 \end{bmatrix},$$
$$S = \begin{bmatrix} 14.0814 & 7.0778 \\ 10.8025 & 12.9798 \end{bmatrix},$$
$$Q^S = \begin{bmatrix} 24.3108 & 0.5725 \\ 20.0573 & 0 \end{bmatrix},$$
$$\rho = \begin{bmatrix} 325.4796 & 335.8327 \end{bmatrix}. \quad (35)$$

The total transaction volume of raw materials and integration solutions, respectively, are

$$\sum_{l=1}^{2}\sum_{m=1}^{2} q_{lm}^{*} = 44.9260, \qquad \sum_{m=1}^{2}\sum_{k=1}^{2} q_{mk}^{s*} = 44.9406. \quad (36)$$

Compared with the result of capability constraint in Example 1, it is found that when there is a supply capacity limitation on the part of the raw material supplier, the raw material transaction volume between that supplier and the manufacturing integrator is significantly reduced, but it has also reached the upper limit of its supply capacity. Because of the limited amount of raw materials, manufacturing integrators cannot make full use of their integration capabilities although themselves are not limited. As a result, the service levels require and integrated solutions they sell also decline. However, there have been two changes in the price paid by customers in the demand market: the higher payment price falls, while the lower one rises.

In the same way, we keep raw material suppliers and manufacturing integrators in a state free from capacity constraints and reduce the service delivery capacity of service providers to explore the impact of their capacity constraints on supply chain network equilibrium. The initial transaction value of the raw material, service, and integration solution does not change, and the capacity of the raw material supplier and manufacturing integrator is not limited, so the Lagrange multiplier of the capacity constraint is 0. We set the service capacity in the link to

Table 5: Changes in supply chain network equilibrium conditions when the ratio of products to services changes in integrated solutions.

α	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
Q	40.3225	42.1706	44.1909	45.8209	45.9079	45.9405	46.0284	46.0317	46.0343
	12.8830	6.0186	9.6691	11.8032	13.4887	6.2698	10.1089	12.3532	14.1264
s	6.5482	10.5785	12.9343	14.9365	6.3825	10.9317	13.5910	14.9912	6.3440
	10.9915	13.6252	14.9952	6.3536	10.9953	13.6304	15.0041	6.3765	11.0037
	13.6424	15.0045	6.3785	11.0039	13.6431	15.0048	6.3799	11.0042	13.6436
Q^s	40.3610	42.2098	44.1836	45.8434	45.9456	45.9707	46.0276	46.0308	46.0332
ρ	336.6512	339.1170	341.5936	346.6419	347.7898	347.7184	347.5847	347.5766	347.5727
	315.5351	316.7610	318.2125	316.4689	315.5437	315.6600	315.8982	315.913	315.9219

($v_{11} = 15$, $v_{12} = 7$, $v_{21} = 11$, $v_{22} = 14$). At this point, we get the equilibrium condition of the product service supply chain network as follows:

$$Q = \begin{bmatrix} 14.9142 & 12.1997 \\ 11.1484 & 8.4339 \end{bmatrix},$$
$$S = \begin{bmatrix} 15.0200 & 6.8710 \\ 11.0199 & 13.7600 \end{bmatrix}, \tag{37}$$
$$Q^S = \begin{bmatrix} 18.2872 & 7.7723 \\ 15.5726 & 5.0577 \end{bmatrix},$$
$$\rho = \begin{bmatrix} 354.6173 & 310.1480 \end{bmatrix}.$$

The total transaction volume of raw materials and integration solutions, respectively, are

$$\sum_{l=1}^{2}\sum_{m=1}^{2} q_{lm}^{*} = 46.6962, \qquad \sum_{m=1}^{2}\sum_{k=1}^{2} q_{mk}^{s*} = 46.6898. \tag{38}$$

Compared with the case of no capacity constraint in Example 1, when the service provider has a capacity constraint, the trading volume of raw materials, integrated solutions, and service under the equilibrium state of the product service supply chain network show a downward trend. Further, the service has reached the upper limit of the capacity of each service provider. In the demand market, the high price is higher, while the low price is lower than before. Due to the limited supply capacity of services, other types of suppliers cannot be fully utilized. The lack of service polarizes the transaction price in demand market.

Finally, we explore the change of supply network equilibrium conditions, under which the integration capability of manufacturing integrators is constrained, and when there is no capacity limitation between raw material suppliers and service providers. Similarly, the initial trading volume of all kinds of values is set to be constant, and the Lagrange multipliers of the capacity constraints of raw material suppliers and service suppliers are all 0. The capacity limit of each integrated solution link is $y_{11} = 21$, $y_{12} = 5$, $y_{21} = 18$, $y_{22} = 2$, and the Lagrange multiplier is 30. The equilibrium conditions of the supply chain network are as follows:

$$Q = \begin{bmatrix} 14.8317 & 11.8280 \\ 11.1289 & 8.1253 \end{bmatrix},$$
$$S = \begin{bmatrix} 15.0261 & 6.3087 \\ 10.9449 & 13.6548 \end{bmatrix}, \tag{39}$$
$$Q^S = \begin{bmatrix} 20.9836 & 4.9891 \\ 17.9801 & 1.9855 \end{bmatrix},$$
$$\rho = \begin{bmatrix} 347.7236 & 315.5714 \end{bmatrix}.$$

The total transaction volume of raw materials and integration solutions, respectively, are

$$\sum_{l=1}^{2}\sum_{m=1}^{2} q_{lm}^{*} = 45.9139, \qquad \sum_{m=1}^{2}\sum_{k=1}^{2} q_{mk}^{s*} = 45.9383. \tag{40}$$

Similarly, compared with the unconstrained case in Example 1, when the manufacturing integrator's capability is constrained, the trading volume of raw materials, trading volume of product service systems, and customer payment price in the service and demand markets all show a downward trend.

It can be seen from the results of Example 2 that when the capacity of each type of enterprise is limited, the transaction value of raw materials, services, and integrated solutions in the link will decrease. However, the capacity constraints of raw material suppliers and manufacturing integrators will cause the transaction price of the demand market to move toward the middle, while the capacity constraints of service suppliers will polarize the transaction price in the demand market. Therefore, we can posit that service can subdivide the types of customers in the demand market more than products; that is, it can provide targeted integrated solutions for customers pursuing different service levels, bringing more added value to the supply chain network.

Example 3. In reality, the equilibrium conditions of the product service supply chain network are not only affected by the ability constraints of various enterprises, the type of product service system scheme is also an important factor. Because product and service integration solutions target customers in different market segments, we need to consider the balanced impact of different integration solutions. Next, we will change the value of the integration ratio coefficient, α, of products and services in the integration solution, and we will explore the changes in equilibrium conditions in different types of product service supply chain networks.

As to the second example in Example 1, we also set the transaction capacity of each link in the product service supply chain network as $(u_{11} = 15, u_{12} = 12, u_{21} = 11, u_{22} = 8)$; $(v_{11} = 15, v_{12} = 7, v_{21} = 11, v_{22} = 14)$; and $(y_{11} = 21, y_{12} = 5, y_{21} = 18, y_{22} = 2)$. We set the Lagrange multiplier as 30. We change the integration ratio of products and services in the product service system scheme and observe the condition changes under a balanced supply chain network. Table 5 shows the changes in each equilibrium condition, when a is 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, and 0.1.

By observing the data in Table 5, we find that, in the presence of capacity limit constraints, the equilibrium conditions of the product service supply chain network show a stable trend as the proportion of products and services changes. With an increase in the service proportion, the transaction volume of raw materials, service transactions, integrated solutions, and customer payment prices in the demand market in the supply chain network are all increasing. Moreover, the increasing trend of each equilibrium condition gradually decreases as the service proportion increases, which tends to be stable. Therefore, we believe that, compared with pure product sales, an integrated solution that combines products and services is more favored by the demand market, and an increase in services leads to the promotion of various transactions in the product service supply chain network. From the increase of the transaction price, we can see that an increase in services can bring higher benefits to enterprises in the network and help the market to maintain stable transactions.

6. Conclusions

In the context of manufacturing servitization, more and more service elements are pouring into the traditional product supply chain network, which makes the coordination of the supply chain network more complex. The equilibrium state of a product supply chain fluctuates due to the introduction of service flow, and all participants in the network need to seek a new equilibrium cooperation state in order to provide customers with satisfactory integrated product and service solutions.

In this paper, we introduce the service flow into the traditional product supply chain and build a four-level product service supply chain network equilibrium model with capacity constraints. Participants maximize their profits and utility through different transaction decisions. This paper solves these problems by determining the equilibrium state of the supply chain network. Specifically, we use Lagrange theory and marginal utility theory to analyze the influence of capacity constraints on the network equilibrium of a product service supply chain. To facilitate the calculation, the benefit maximization model of each layer of participants is expressed as a variational inequality. The cost function of each participant is assumed by numerical examples to determine the equilibrium condition of the product service supply chain network with or without capacity constraints. Further, under supplier capability limitations, we adjust the integration ratio of products and services, analyze changes in network equilibrium conditions, and explore the impact of products and services on the co-creation of value in a supply chain network.

The research shows that the equilibrium conditions of the product service supply chain network are largely affected by the supply capacity of various types of suppliers:

(1) Compared with a situation in which there are no capacity constraints, the existence of capacity constraints reduces the trading volume of raw materials, service trading level, and integrated solutions in the network. Then, the producing limitation of enterprises is not good for enterprises and the whole network.

(2) The raw material ability limitation makes the market price stable and uniform, while the service limitation polarizes the price paid by customers, which proves that production capacity limits the quality of product service systems, while service capacity limits both type and quantity.

(3) As for different kinds of customers, enterprises should corporate with each other to offer various product service integration solutions to meet diversified needs of customers. In this way, not only can they keep and extend customers, but also it is a good measure to solve resources. When it reaches a steady state, both of the enterprises and customers could benefit from the product service supply chain network, and the network value will get improved.

In the future, the research can be expanded from several aspects. This paper only considers the network equilibrium of a product service supply chain with a single service. However, bundling multiple services and products may bring huge profits to enterprises and may also produce a service paradox, so future research on the network equilibrium of a product service supply chain with multiple services will generate new insights.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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