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Research Article

Event-Based Sampled-Data Average Consensus

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This study addresses one of the most essential distributed control problems in multiagent systems, called the average consensus issue, using a new event-triggered sampling control perspective. Although the continuous-time sampling for average consensus has provided good results currently, a systematic investigation into the continuous-time agent dynamics with sampled-data control inputs under an event-triggered mechanism is critically lacking. The problem considered in this paper can be formulated into an average consensus problem of hybrid systems. The method considers three types of control schemes, among which periodic sampling is integrant. The first scheme is a classical sampling controller reinvestigated through a lemma. The second scheme realizes aperiodic control update as well as periodic communication, while the third scheme achieves both aspects aperiodically. Corresponding sufficient conditions of the aforementioned three schemes are derived such that the asymptotic stability of systems is ensured by using algebraic graph theory, matrix analysis, and Lyapunov theory. The constraints for the allowed sampling period, event parameter, and maximum eigenvalue of graph Laplacian are explicitly derived. Moreover, the potential Zeno behavior of agents due to the sampling control theory is avoided. Thus, a digitally implementable technique is provided. Finally, some numerical examples are provided to verify the effectiveness of the proposed theoretical analysis.

1. Introduction

Over the past decade, the distributed coordination problems of multiagent systems have attracted immense attention of researchers due to their potential scientific significance and broad engineering application prospects. For instance, multiagent systems will help reveal the working mechanism of brain neurons [1], facilitating the treatment of many types of brain diseases. Multiagent systems are also studied to gain an understanding of the complicated global financial market [2]. Numerous applications of multiagent systems in the engineering field have emerged, such as in formation control [3–5], axis alignment of satellites [6], collective search and rescue operations [7], wireless sensor networks [8], and smart grids [9]. Given these applications, prompt solutions for the distributed coordination of multiagent systems are crucial, and some research has produced good results, mainly with regard to consensus protocols [10–19], consensus tracking [20], containment control [21], distributed average tracking [22], distributed estimates, and optimization [23], among others.

However, in most previous studies, continuous-time sampling, communication, and control update are often assumed for each agent in a distributed consensus protocol. However, continuous sampling and communication are an ideal assumption, so it is more realistic to sample data periodically or aperiodically and exchange information intermittently. To address this contradiction between theory and practice, this study aims to reanalyze one of the most essential problems for multiagent systems, called the average consensus issue, which has core applications in sensor fusion and filters, using a new event-triggered sampling control perspective. Furthermore, consensus theory is the essential principle underlying the collaborative behaviors of bird flocks, fish schools, group of dancers, and any animals or human beings performing similar collective actions. For the sake of simplification of the problem in order to highlight the main concepts of our method, single integrators are chosen as the dynamics of each agent. Nevertheless, the obtained results based on single-order multiagent systems can be potentially extended to more complex agent dynamics. However, the introduction of a communication topology in the system increases the complexity of the control system; therefore, additional constraints need to be further considered with respect to multiagent systems, including constraints of communication delays [24], bandwidth, the quantization effect [25], energy limitation of sensor and actuator nodes, except for motion constraints [26], input saturation [27], and disturbance. This study mainly provides an effective solution for the constraints of communication and energy.

Currently, diverse research exists on periodic event-triggered control or intermittent communication strategies. A continuous sampling method was used to solve the distributed consensus tracking problem for networked Lur'e systems based on an event-triggered mechanism [28]. Further, an effective event-triggered controller that does not need continuous communication and control update was used for the formation tracking problem of unicycle-type robots [29]. However, these studies rely on the assumption of continuous sampling in the theoretical analysis. Recently, Yu et al. [30] proposed a novel idea of intermittent communication between agents, with communication occurring in the first part of each cycle and discontinuing at other times. However, the performance of this strategy still needs to be compared with that of periodic event-triggered control strategy. Moreover, Qin et al. [24] designed a sampled-data controller to analyze the finite-time synchronization of an inertial memristive system. In addition, Liu et al. [31] addressed the sampled-data consensus problem for harmonic oscillators without requiring information exchange over the sampled interval. In addition, Meng and Chen [32] established a framework for the periodic event-triggered consensus problem. However, all these studies adopted communication at each sampling instant. To overcome this shortcoming, Heemels et al. [33] made profound and systematic contributions to the periodic event-triggered control scheme. Meanwhile, the synchronization problem was investigated for a class of heterogeneous master-slave coupling systems consisting of a high-dimensional master system and a low-dimensional slave system by using a periodic event-triggered approach [34]. Further, Wen et al. addressed the global $\mathscr{H}_\infty$ pinning synchronization problem for a class of directed networks with aperiodic sampled-data communication [35]. Liu and Ji [18] considered the consensus problem of time-delayed first-order multiagent systems using the periodic sample and event hybrid control method. Wang et al. [36] studied the cluster formation of nonlinear multiagent systems with a sampled-data event-triggered mechanism. Most recently, Sun et al. [37] studied the bipartite consensus problem of multiagent systems with intermittent interaction under signed directed graph. Qu and Ji [38] introduced a superposition system and studied the problem of how the convergence rate changes by using the inequality of eigenvalues, without destroying the connectivity of the undirected graphs. Furthermore, it is worth mentioning that Ji et al. [39] presented a systematic design and identification process for the complete graphic characterization by taking advantage of controllability destructive nodes. And Ji and Yu [40] also proposed the concept of controllability destructive nodes and designed a method which is proposed to uncover topology structures of QCD nodes for graphs with any size, and a complete graphical characterization is presented for the graphs consisting of five vertices. Because providing an exhaustive list of all existing studies related to the topic considered in this paper is difficult, the authors refer interested readers to the papers mentioned above and references therein.

Based on the observation of the aforementioned studies, most existing work on the average consensus problem is still limited to continuous sampling, communication, and control update; a systematic investigation of an event-triggered sampling control mechanism for the average consensus problem is critically lacking. When sampling control and event-triggered control are well combined, two fundamental problems associated with the closed-loop systems are (i) how to determine the constraint of the sampling period and (ii) how to synthesize a sampled-data controller and event conditions to guarantee the global stability of multiagent systems without loss of asymptotic stability. The difficulties related to the second problem lie in ensuring that the continuous-time Lyapunov function is negative definite with the discrete control input and in deriving the allowed sampling period, the appropriate parameters of event conditions, and the gains of the controller in order to determine an optimal balance between systematical performance and communication cost.

Based on the above discussion, this paper aims to investigate the average consensus problem for single-order multiagent systems under a fixed topology with periodic or aperiodic communication with respect to an intermittent control update mechanism. The main contributions of this study can be summarized as follows: (1) A consensus problem in demonstrational Lemma 1 is solved by a distributed controller based on the fixed periodic feedback of sampling states. The maximum allowed sampling period T is derived. This type of controller relies on periodic sampling, exchange information, and control update rather than on a continuous-time manner that is adopted in most existing theoretical work on consensus. This study provides a digital implementable framework for the consensus of multiagent systems. (2) After recalling the concept of event-triggered control to combine with the sampling control approach in Lemma 1, a distributed controller and an event condition are synchronized to obtain some sufficient conditions that will guarantee the asymptotic stability of closed-loop error systems. The constraints for event parameters, the sampling period, and the maximum eigenvalue of graph Laplacian are derived. In this control strategy, each agent needs to sample every period while communicating with neighbors. However, control update only happens at an event-triggered time instant. Because an event is likely to be triggered after several sampling periods, the total number of control updates is decreased to some extent. (3) Through redesigning the sum of the relative state errors of event conditions while reserving the original distributed controller, we have removed the conservative requirement that needs periodic communication amongst neighboring agents. That is, during the whole evolution of multiagent systems, no periodic communication and control update are requested. Compared to the continuous-time approach, the proposed method greatly reduces communication cost, energy consumption, and mechanical wear. (4) The results can be applied in arbitrary dimensions through the Kronecker product. The underlying methods and concepts of this study can also serve as a useful framework to solve other analog extension issues. (5) The potential Zeno behavior of an event-based controller is avoided by the characteristic of the periodic sampling mechanism, and the lower bound of the interevent interval is equal to the sampling period T.

The rest of this paper is organized as follows. In Section 2, some preliminaries in algebraic graph theory are given, and the average consensus problem is formulated. In Section 3, the main results are derived, including two types of communication control strategies: periodic and aperiodic. In Section 4, the obtained results are briefly extended to switching graphs. In Section 5, two simulation examples are provided to verify the effectiveness of the presented theoretical results. Finally, concluding remarks are presented in Section 6.

1.1. Nomenclature. For a vector $x \in \mathbb{R}^n$, we denote its 1-norm and 2-norm by $\|x\|_1$ and $\|x\|_2 = \sqrt{x^T x}$, respectively. For a symmetric matrix $A \in \mathbb{R}^{n\times n}$, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ represent the maximum and minimum eigenvalues of A, respectively. We thus have other eigenvalues satisfying $\lambda_{\min}(A) \le \lambda_2(A) \le \dots \le \lambda_{\max}(A)$. For an asymmetric matrix $A \in \mathbb{R}^{n\times m}$, we denote the transpose of A by $A^T \in \mathbb{R}^{m\times n}$ and its induced 2-norm by $\|A\|_2 := \sqrt{\lambda_{\max}(A^T A)}$. In addition, I_n denotes the identity matrix and 1_n represents a column vector with n compatible entries equal to 1. Finally, let Inf refer to the infimum of a set and function $\max\{\cdot\}$ return the maximum value of a number series.

2. Preliminaries and Problem Formulation

2.1. Algebraic Graph Theory. We model a group of agents and their interaction relation as a time-invariant undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, where $\mathcal{V} = \{1, 2, \dots, N\}$ is the set of nodes, $\mathrm{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the associated adjacency matrix describing the edge information. The graph also admits matrix representations. An adjacency matrix $\mathcal{A}$ of the graph is a symmetric $N \times N$ matrix encoding the node adjacency relationships, with entries $a_{ij} = 1$, if $(v_j, v_i) \in \varepsilon, i \ne j$, and $a_{ij} = 0$, otherwise. For an undirected graph, the diagonal entries $a_{ij} = a_{ji}$. If there exists an undirected path between any two distinct nodes in the graph, this communication topology is called connected. Another important matrix representation of an undirected graph $\mathcal{G}$ is the Laplacian matrix, defined as $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$ and

$$l_{ij} = \begin{cases} -a_{ij}, & i \ne j, \\ \displaystyle\sum_{k=1,k\ne i}^{n} a_{ik}, & i = j. \end{cases} \tag{1}$$

Let $\mathcal{D} = \operatorname{diag}(d_1, \dots, d_N)$ be the in-degree matrix of the graph, then the Laplacian matrix can also be defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$. Note that the Laplacian matrix of the undirected graph is symmetric and positive semidefinite, with the sum of each row being equal to zero. Moreover, the undirected graph is connected if and only if matrix $\mathcal{L}$ has a simple zero eigenvalue associated with eigenvector 1_N. The second smallest eigenvalue (also known as the Fiedler eigenvalue) of $\mathcal{L}$ is called the algebraic connectivity of the graph, which indicates that the number of links is proportional to the value of this eigenvalue. In addition, N_i is denoted as the set of neighbors of agent i.

2.2. Average Consensus Problem. Consider a multiagent system having single-integrator agent dynamics, which is the most essential model in nature and engineering. Its state can denote an opinion, the solution of a linear equation, animal population, etc. Based on Newtonian mechanics, the state can be position and speed. All the agents are labeled $1, \dots, N$, corresponding to the nodes in the graph. As a typical system, the continuous-time first-order multiagent system can be expressed in the compact form as follows:

$$\dot{x}(t) = u(t), \tag{2}$$

where $x(t) \in \mathbb{R}^{m\times N} = [x_1(t)^T, x_2(t)^T, \dots, x_N(t)^T]^T$ and $x_i(t) \in \mathbb{R}^m = [x_{i1}(t), x_{i2}(t), \dots, x_{im}(t)]^T$ are the state, and control input $u(t)$ is defined by the same dimension.

The control objective in the problem is multivariate, and some of them even cannot be quantitative. Nevertheless, it is most critical to design a distributed event-triggered sampling controller in order to satisfy the following limit:

$$\lim_{t\longrightarrow\infty} \left\| x_i(t) - \frac{1}{N}\sum_{i=1}^{N} x_i(0) \right\|_2 = 0. \tag{3}$$

In addition, our solutions must reduce as much as possible the number of communications and the frequency of control updates, high values of which are detrimental to mechanical components and the limited energy of micro-embedded systems in most cases. In addition, an event detector is configured at each agent to determine how to use the sampled data and control the communication unit.

Assumption 1. The sampling frequency of each agent satisfies the Nyquist–Shannon sampling theorem.

Remark 1. In general, Assumption 1 should be satisfied throughout this paper. Because the frequency of state update for each agent is often low, the allowed sampling period to guarantee the stability of systems can commonly satisfy the requirement of Assumption 1. However, this assumption must consider this fact when the frequency of the state becomes sufficiently high in practice.

2.3. A Simple Case. As a basic preparation to formally present an event-triggered sampling control in the next section, we first provide the readers with a consensus result by adopting a pure sampling control. Thereafter, we will gradually lead the readers deep into the design of a diverse distributed sampled-data controller combined with an event-triggered mechanism, so as to thoroughly improve the control performance step by step.

Lemma 1 (pure sampled-data control). *Consider the continuous-time first-order multiagent system (1) with the following typical sampling version of the continuous-time controller (4) under an undirected connected graph; then, $lim_{t\longrightarrow\infty}\|x_i(t)-x_j(t)\|_2=0$ is achieved as long as the sampling period satisfies $0<T<1/\lambda_{max}(\mathscr{L})$:*

$$u_i(t)=-\sum_{j\in\mathcal{N}_i}\left[x_i(nT)-x_j(nT)\right],\quad t\in[nT,(n+1)T),\quad n=0,1,\ldots,\tag{4}$$

where T denotes the sampling period that is fixed in this paper. The case with time-variant T is of interest.

Proof. The dynamics of follower i for $t\in[nT,nT+T)$ can be given by $\dot{x}_i(t)=u_i(t)=-\sum_{j\in\mathcal{N}_i}[x_i(nT)-x_j(nT)]$; then, we can rewrite the expression in a compact form as follows:

$$\dot{x}(t)=-(\mathscr{L}\otimes I_m)x(nT).\tag{5}$$

Integrating the above formula from nT to t, we can further get $x(t)=x(nT)-(t-nT)(\mathscr{L}\otimes I_m)x(nT)$.

Denote the state error for each agent by $\tilde{x}_i(t)=x_i(t)-\overline{x}$, and rewrite it in the compact form as $\tilde{x}(t)=x(t)-1_N^T\overline{x}$, where $\tilde{x}(t)$ is called the group disagreement vector and $\overline{x}$ is the convergence state, which is the weighted sum of the initial states of agents; thus, $\dot{x}_i(t)=\dot{\tilde{x}}_i(t)$. Next, we transpose the group disagreement vector as follows:

$$\tilde{x}(t)^T=x^T(nT)-(t-nT)x(nT)^T(\mathscr{L}\otimes I_m)-1_N^T\overline{x}.\tag{6}$$

We construct the following Lyapunov function candidate:

$$V(t)=\frac{1}{2}\tilde{x}(t)^T\tilde{x}(t).\tag{7}$$

Taking the time derivative of $V(t)$ along trajectory (5), we obtain

$$\begin{aligned}\dot{V}(t)&=\tilde{x}(t)^T\dot{\tilde{x}}(t)\\&=\left[x^T(nT)-(t-nT)x(nT)^T(\mathscr{L}\otimes I_m)-1_N^T\overline{x}\right]\\&\quad\times\left[-(\mathscr{L}\otimes I_m)x(nT)\right]\\&\leq -x^T(nT)(\mathscr{L}\otimes I_m)x(nT)+T\lambda_{\max}(\mathscr{L}\otimes I_m)\\&\quad\cdot x(nT)^T(\mathscr{L}\otimes I_m)x(nT)\\&=(T\lambda_{\max}-1)x(nT)^T(\mathscr{L}\otimes I_m)x(nT).\end{aligned}\tag{8}$$

Then, $\dot{V}(t)$ is negative definite as long as $T\lambda_{\max}(\mathscr{L})-1<0$. That is, the constraints of the sampling period are given by $T<1/\lambda_{\max}(\mathscr{L})$. Then, the sampled-data control systems are globally stabilizing for systems (1) under the feedback control (4). □

Remark 2. In this case, we solve the consensus problem by using the standard sampled-data control scheme. The closed-loop multiagent system is proven to converge asymptotically fast. Meanwhile, we derive the allowed sampling period $T<1/\lambda_{\max}(\mathscr{L})$. Compared to the continuous-time situation, which is impractical in reality, the scheme proposed in this lemma allows periodic sampling, exchange of information, and control update within the same sampling period T. However, communicating and updating control input at each sampling time instant appear to be slightly strict. That is, how to decrease the communication cost and control update, and thus reduce the energy consumption and mechanical wear, is the purpose of this work.

3. Event-Triggered Communications and Control Update Mechanisms

In this section, we consider two types of event-triggered mechanisms. The first section mainly aims to reduce the continuous communication into periodic information interaction and reduce the control update frequency. By contrast, the second section attempts to drastically decrease the communication cost and control update simultaneously.

3.1. Periodic Communication. The most important feature of the technique in this section is that the communication is periodic, whereas the control update is aperiodic and piecewise constant. We define the sum of the relative state errors for each agent as one part of event conditions:

$$z_i(nT)=\sum_{j\in\mathcal{N}_i}\left[x_i(nT)-x_j(nT)\right].\tag{9}$$

Herein, the event condition of agent i is given as follows:

$$\left\|e_i(nT)\right\|_2^2\leq c_i\left\|z_i(nT)\right\|_2^2,\tag{10}$$

where c_i is a positive scalar that will be determined later, and $e_i(nT) = x_i(t_k^i) - x_i(nT)$. In the study of periodic event-triggered control, nT is just an abbreviation of $t_k^i + lT, l = 0, 1, 2, \ldots$, which denotes the current sampling time instant for agent i. Therefore, event condition (10) leads to

$$e(nT)^T e(nT) \leq c_{\max} z(nT)^T z(nT), \tag{11}$$

where $e(nT) = [e_1(nT)^T, \ldots, e_N(nT)^T]^T$ and $z(nT) = [z_1(nT)^T, \ldots, z_N(nT)^T]^T$, and $c_{\max} = \max\{c_1, \ldots, c_{Nm}\}$.

Based on the consensus literature, let the periodic state tracking error be defined as follows:

$$\varepsilon_i(nT) = x_i(nT) - \varpi, \tag{12}$$

where ϖ is the average value of the sum of the initial states of agents. Substituting (12) into (9) yields

$$\begin{aligned} z_i(nT) &= \sum_{j\in\mathcal{N}_i} \left[x_i(nT) - x_j(nT)\right] \\ &= \sum_{j\in\mathcal{N}_i} \left[\varepsilon_i(nT) - \varepsilon_j(nT)\right]. \end{aligned} \tag{13}$$

Equation (13) can be further rewritten in a stacked form as follows:

$$z(nT) = (\mathcal{L} \otimes I_m)\varepsilon(nT). \tag{14}$$

Furthermore, the quadratic form is given by

$$z(nT)^T z(nT) \leq \lambda_{\max}(\mathcal{L} \otimes I_m)\varepsilon(nT)^T (\mathcal{L} \otimes I_m)\varepsilon(nT). \tag{15}$$

Then, we have

$$\begin{aligned} e(nT)^T (\mathcal{L} \otimes I_m) e(nT) &\leq \lambda_{\max} e(nT)^T e(nT), \\ &\leq c_{\max}\lambda_{\max} z(nT)^T z(nT), \\ &= c_{\max}\lambda_{\max}^2 \varepsilon(nT)^T (\mathcal{L} \otimes I_m)\varepsilon(nT), \end{aligned} \tag{16}$$

where λ_{max} is the abbreviation of $\lambda_{\max}(\mathcal{L} \otimes I_m)$, which will be used in the sequel if no confusion takes place. Note the fact that $\lambda_{\max}((\mathcal{L} \otimes I_m) \otimes I_m) = \lambda_{\max}(\mathcal{L} \otimes I_m)$.

With the above-distributed event condition (10), by using the fixed undirected graph $\mathcal{G}$ to describe the communication topology among agents, the distributed event-triggered sampling controller of agent i $(1 \leq i \leq N)$ for $t \in [t_k^i + lT, t_k^i + lT + T)$ is proposed on the basis of event state feedback:

$$u_i(t) = -\sum_{j\in\mathcal{N}_i} \left[x_i\left(t_k^i\right) - x_j\left(t_{k'(t)}^j\right)\right], \tag{17}$$

where $k\prime(t)$ is the triggering instant being nearest to the present time t for the agent j. Substituting the control input into the agent dynamics, the closed-loop distributed multiagent system is obtained as follows:

$$\begin{aligned} \dot{x}_i(t) &= -\sum_{j\in\mathcal{N}_i} \left[x_i\left(t_k^i\right) - x_j\left(t_{k'(t)}^j\right)\right] \\ &= -\sum_{j\in\mathcal{N}_i} \left[x_i(nT) - x_j(nT)\right] - \sum_{j\in\mathcal{N}_i} \left[e_i(nT) - e_j(nT)\right] \\ &= -\sum_{j\in\mathcal{N}_i} \left[\varepsilon_i(nT) - \varepsilon_j(nT)\right] - \sum_{j\in\mathcal{N}_i} \left[e_i(nT) - e_j(nT)\right]. \end{aligned} \tag{18}$$

Remark 3. In the periodic communication case, each pair of neighboring agents exchanges information with each other at each synchronized sampling time instant, while checking the respective event conditions. When the event condition of an agent is violated, it can be said that its event is triggered. Then, this triggered agent transmits its state to the neighbors, and the state is updated in its controller and its measurement error $e_i(t_k^i + nT)$ is reset to zero. In addition, the zero-order holder is employed to hold the state between two events. It can be observed that the smallest interevent interval is lower bounded by the sampling period, which implies that potential Zeno behavior can be avoided. However, in this section, agents still need to communicate with neighbors periodically, although the control update only occurs at an event-triggered time instant. Relaxing of this conservative requirement directly relates to further reducing communication cost while retaining the originally satisfactory convergence performance. In the next section, a more optimized solution will be investigated.

Theorem 1. *Consider the multiagent system (1) with the distributed sampled-data controller (17) and event condition (10) associated with the sum of relative state errors (9) under the undirected connected graph. Then, control objective (3) is achieved as long as the following constraints are satisfied:*

$$0 < c_{\max} < \frac{0.5 - T - T\lambda_{\max}}{T\lambda_{\max}^3 + T\lambda_{\max}^2 + 0.5\lambda_{\max}^2}, \quad 0 < T < \frac{1}{2 + 2\lambda_{\max}}. \tag{19}$$

Proof. Under the above event-triggered control framework, the dynamical system for agent i can be described by the following form:

$$\dot{x}(t) = -(\mathcal{L} \otimes I_m)e(nT) - (\mathcal{L} \otimes I_m)\varepsilon(nT), \tag{20}$$

where $\varepsilon(t) = [\varepsilon_1(t)^T, \varepsilon_2(t)^T, \ldots, \varepsilon_N(t)^T]^T$. In addition, we define the average value of all states of agents as $\omega(t) = 1/N \sum_{i=1}^N x_i(t)$ and take the time derivative of it as follows:

$$\dot{\omega}(t) = \frac{1}{N}\sum_{i=1}^{N}\dot{x}_i(t) = \frac{1}{N}1_{Nm}^{T}\dot{x}(t) = -\frac{1}{N}1_{Nm}^{T}[(\mathscr{L}\otimes I_m)e(nT) + (\mathscr{L}\otimes I_m)\varepsilon(nT)] = -\frac{1}{N}((\mathscr{L}\otimes I_m)1_{Nm})^{T}[e(nT)+\varepsilon(nT)] \equiv 0, \tag{21}$$

Because $(\mathscr{L}\otimes I_m)1_{Nm} = 0$. Therefore, ϖ is time-invariant. Differentiating $\varepsilon(t)$ with respect to time, we obtain

$$\begin{aligned}\dot{\varepsilon}(t) &= \dot{x}(t) - \dot{\omega}(t)\\ &= -(\mathscr{L}\otimes I_m)e(nT) - (\mathscr{L}\otimes I_m)\varepsilon(nT).\end{aligned} \tag{22}$$

We choose the Lyapunov function candidate as

$$V(t) = \frac{1}{2}\varepsilon(t)^{T}\varepsilon(t). \tag{23}$$

Differentiating (41) with respect to $t \in [nT, nT+T)$, we obtain

$$\begin{aligned}\dot{V}(t) &= \varepsilon(t)^{T}\dot{\varepsilon}(t)\\ &= \varepsilon(t)^{T}[-(\mathscr{L}\otimes I_m)e(nT) - (\mathscr{L}\otimes I_m)\varepsilon(nT)],\end{aligned} \tag{24}$$

because $\varepsilon(t) = -(t-nT)(\mathscr{L}\otimes I_m)e(nT) - (t-nT)(\mathscr{L}\otimes I_m)\ \varepsilon(nT) + \varepsilon(nT)$. Then, its transpose is given by

$$\varepsilon(t)^{T} = -(t-nT)((\mathscr{L}\otimes I_m)e(nT))^{T} - (t-nT)\cdot((\mathscr{L}\otimes I_m)\varepsilon(nT))^{T} + \varepsilon(nT)^{T}. \tag{25}$$

Now, consider the time evolution of the function $V(x(t))$ in (41) along the trajectory generated by (22) for any $t \in [nT, nT+T)$, which is given by

$$\begin{aligned}\dot{V}(t) &= \varepsilon(t)^{T}\dot{\varepsilon}(t)\\ &= \varepsilon(t)^{T}[-(\mathscr{L}\otimes I_m)e(nT) - (\mathscr{L}\otimes I_m)\varepsilon(nT)]\\ &= (t-nT)e(nT)^{T}(\mathscr{L}\otimes I_m)^{2}e(nT) + (t-nT)e(nT)^{T}\\ &\quad\cdot(\mathscr{L}\otimes I_m)^{2}\varepsilon(nT) + (t-nT)\varepsilon(nT)^{T}(\mathscr{L}\otimes I_m)^{2}e(nT)\\ &\quad+(t-nT)\varepsilon(nT)^{T}(\mathscr{L}\otimes I_m)^{2}\varepsilon(nT) - \varepsilon(nT)^{T}\\ &\quad\cdot(\mathscr{L}\otimes I_m)e(nT) - \varepsilon(nT)^{T}(\mathscr{L}\otimes I_m)\varepsilon(nT).\end{aligned} \tag{26}$$

Since $0 \le t - nT < T$, we have

$$\begin{aligned}\dot{V}(t) &\le Te(nT)^{T}(\mathscr{L}\otimes I_m)^{2}e(nT) + T\left|e(nT)^{T}(\mathscr{L}\otimes I_m)^{2}\varepsilon(nT)\right| + T\left|\varepsilon(nT)^{T}(\mathscr{L}\otimes I_m)^{2}e(nT)\right|\\ &\quad+T\varepsilon(nT)^{T}(\mathscr{L}\otimes I_m)^{2}\varepsilon(nT) + \left|\varepsilon(nT)^{T}(\mathscr{L}\otimes I_m)e(nT)\right| - \varepsilon(nT)^{T}(\mathscr{L}\otimes I_m)\varepsilon(nT)\\ &= Te(nT)^{T}(\mathscr{L}\otimes I_m)^{2}e(nT) + (2T+1)\left|e(nT)^{T}(\mathscr{L}\otimes I_m)^{2}\varepsilon(nT)\right| + T\varepsilon(nT)^{T}(\mathscr{L}\otimes I_m)^{2}\varepsilon(nT)\\ &\quad-\varepsilon(nT)^{T}(\mathscr{L}\otimes I_m)\varepsilon(nT)\\ &\le Te(nT)^{T}(\mathscr{L}\otimes I_m)^{2}e(nT) + (2T+1)\left|e(nT)^{T}(\mathscr{L}\otimes I_m)^{2}\varepsilon(nT)\right| + T\lambda_{\max}\varepsilon(nT)^{T}(\mathscr{L}\otimes I_m)\varepsilon(nT)\\ &\quad-\varepsilon(nT)^{T}(\mathscr{L}\otimes I_m)\varepsilon(nT)\\ &= Te(nT)^{T}(\mathscr{L}\otimes I_m)^{2}e(nT) + (2T+1)\left|e(nT)^{T}(\mathscr{L}\otimes I_m)^{2}\varepsilon(nT)\right| + (T\lambda_{\max}-1)\varepsilon(nT)^{T}(\mathscr{L}\otimes I_m)\varepsilon(nT).\end{aligned} \tag{27}$$

By using the inequality

$$\varepsilon^{T}(nT)(\mathscr{L}\otimes I_m)e(nT) \le \frac{1}{2}\varepsilon^{T}(nT)(\mathscr{L}\otimes I_m)\varepsilon(nT) + \frac{1}{2}e^{T}(nT)(\mathscr{L}\otimes I_m)e(nT), \tag{28}$$

inequality (27) can be further derived as follows:

$$\begin{aligned}\dot{V}(t) &\le Te(nT)^{T}(\mathscr{L}\otimes I_m)^{2}e(nT) + (2T+1)\left|\frac{1}{2}\varepsilon^{T}(nT)\right.\\ &\quad\left.\cdot(\mathscr{L}\otimes I_m)\varepsilon(nT) + \frac{1}{2}e^{T}(nT)(\mathscr{L}\otimes I_m)e(nT)\right|\\ &\quad+(T\lambda_{\max}-1)\varepsilon(nT)^{T}(\mathscr{L}\otimes I_m)\varepsilon(nT)\\ &\le Te(nT)^{T}(\mathscr{L}\otimes I_m)^{2}e(nT) + (2T+1)\frac{1}{2}\varepsilon^{T}(nT)(\mathscr{L}\otimes I_m)\varepsilon(nT)\\ &\quad+(2T+1)\frac{1}{2}e^{T}(nT)(\mathscr{L}\otimes I_m)e(nT) + (T\lambda_{\max}-1)\\ &\quad\cdot\varepsilon(nT)^{T}(\mathscr{L}\otimes I_m)\varepsilon(nT).\end{aligned} \tag{29}$$

By invoking inequality (16), calculation continues as follows:

$$\begin{aligned}
&Te(nT)^T(\mathscr{L}\otimes I_m)^2 e(nT) + (2T+1)\frac{1}{2}\varepsilon^T(nT)(\mathscr{L}\otimes I_m)\varepsilon(nT) + (2T+1)\frac{1}{2}e^T(nT)(\mathscr{L}\otimes I_m)e(nT)\\
&\quad + (T\lambda_{\max}-1)\varepsilon(nT)^T(\mathscr{L}\otimes I_m)\varepsilon(nT)\\
&\le Tc_{\max}\lambda_{\max}^3\varepsilon(nT)^T(\mathscr{L}\otimes I_m)\varepsilon(nT) + \left(T+\frac{1}{2}\right)\varepsilon^T(nT)(\mathscr{L}\otimes I_m)\varepsilon(nT)\\
&\quad + \left(T+\frac{1}{2}\right)c_{\max}\lambda_{\max}^2\varepsilon^T(nT)(\mathscr{L}\otimes I_m)\varepsilon(nT) + (T\lambda_{\max}-1)\varepsilon(nT)^T(\mathscr{L}\otimes I_m)\varepsilon(nT)\\
&= \left(Tc_{\max}\lambda_{\max}^3 - \frac{1}{2} + T + \frac{1}{2}c_{\max}\lambda_{\max}^2 + Tc_{\max}\lambda_{\max}^2 + T\lambda_{\max}\right)\varepsilon(nT)^T(\mathscr{L}\otimes I_m)\varepsilon(nT).
\end{aligned} \tag{30}$$

To obtain the sufficient condition that guarantees the Lyapunov function is negative definite, force $Tc_{\max}\lambda_{\max}^3 - 1/2 + T + 1/2c_{\max}\lambda_{\max}^2 + Tc_{\max}\lambda_{\max}^2 + T\lambda_{\max} < 0$; hence, the constraint of the event condition parameter and the allowed sampling period can be derived as follows:

$$0 < c_{\max} < \frac{0.5 - T - T\lambda_{\max}}{T\lambda_{\max}^3 + T\lambda_{\max}^2 + 0.5\lambda_{\max}^2}, \quad 0 < T < \frac{1}{2 + 2\lambda_{\max}}. \tag{31}$$

So far, $\dot{V}(t) < 0$ can be guaranteed, and the designed periodic event-triggered control system converges asymptotically fast; this completes the proof. □

Remark 4. From Theorem 1, it can be observed that the maximum sampling period is less than $1/2 + 2\lambda_{\max}$ in order to guarantee the asymptotic stability of the closed-loop system. It can also be found that the choices of sampling period T and event condition parameter $c_{\max}$ depend on the global information about the communication topology, namely, $\lambda_{\max}$ of the graph Laplacian. The endeavors to eliminate the requirement of global graph information not only in the design stage but also in the implementation stage have been in progress. Li et al. [41] presented a typical work. Furthermore, based on some estimate algorithms such as the result in [42], the upper bound on $\lambda_{\max}$ can be estimated as follows:

$$\lambda_{\max} \le 2(N-1), \tag{32}$$

This can potentially relax the constraint.

Remark 5. In this case, the frequency of the control input update has been decreased dramatically. However, because the communication in each sampling period is required for the event detection, when this value is sufficiently small, the communication cost is still considerably high.

3.2. Aperiodic Communication. In this section, we consider another strategy, namely, aperiodic communication, to check the event condition, rather than the previous periodic information exchange between neighboring agents. The underlying idea is to maintain the structure of the distributed controller, but modify only the sum of relative state errors in the event condition. To this end, variable $z(nT)$ is redesigned as follows:

$$z(nT) = \sum_{j\in\mathcal{N}_i}\left[x_i\left(t_k^i\right) - x_j\left(t_{k'(t)}^j\right)\right], \tag{33}$$

where $k'(t)$ is the triggering instant being nearest to the present time t for the agent j. Then, the following theorem is established.

Theorem 2. *Consider the multiagent system (1) with the distributed sampled-data controller (17) and event condition (10) associated with the sum of relative state errors (33) under the undirected connected graph. Then, control objective (3) is achieved as long as the following constraints are satisfied:*

$$0 < T < \frac{1 - 4\lambda_{\max}^2 c_{\max}}{2 + 2\lambda_{\max}}, \quad 0 < c_{\max} < \frac{1}{4\lambda_{\max}^2}. \tag{34}$$

Proof. The proof follows the same line as controller (17); we rewrite $z(nT)$ in the compact form as

$$z(nT) = (\mathscr{L}\otimes I_m)\varepsilon + (\mathscr{L}\otimes I_m)e. \tag{35}$$

Then, the quadratic form is obtained as follows:

$$\begin{aligned}
z(nT)^T z(nT) &= ((\mathscr{L}\otimes I_m)\varepsilon + (\mathscr{L}\otimes I_m)e)^T((\mathscr{L}\otimes I_m)\varepsilon + (\mathscr{L}\otimes I_m)e)\\
&= \lambda_{\max}\varepsilon^T(\mathscr{L}\otimes I_m)\varepsilon + \lambda_{\max}e^T(\mathscr{L}\otimes I_m)e + 2e^T(\mathscr{L}\otimes I_m)^2\varepsilon.
\end{aligned} \tag{36}$$

Using the inequality

$$\begin{aligned}
\varepsilon^T(nT)(\mathscr{L}\otimes I_m)e(nT) &\le \frac{1}{2}\varepsilon^T(nT)(\mathscr{L}\otimes I_m)\varepsilon(nT)\\
&\quad + \frac{1}{2}e^T(nT)(\mathscr{L}\otimes I_m)e(nT),
\end{aligned} \tag{37}$$

we derive

$$z(nT)^T z(nT) \le 2\lambda_{\max}\varepsilon^T(nT)(\mathscr{L}\otimes I_m)\varepsilon(nT) + 2\lambda_{\max}^2 c_{\max} z(nT)^T z(nT). \quad (38)$$

Finally, $z(nT)^T z(nT)$ can be upper bounded as

$$z(nT)^T z(nT) \le \frac{2\lambda_{\max}\varepsilon^T(nT)(\mathscr{L}\otimes I_m)\varepsilon(nT)}{1-2\lambda_{\max}^2 c_{\max}}, \quad (39)$$

with $0 < c_{\max} < 1/2\lambda_{\max}^2$. Substituting equation (39) into (16), we obtain

$$\begin{aligned} e(nT)^T(\mathscr{L}\otimes I_m)e(nT) &\le \lambda_{\max} e(nT)^T e(nT) \\ &\le c_{\max}\lambda_{\max} z(nT)^T z(nT) \\ &= \frac{2\lambda_{\max}^2 c_{\max}}{1-2\lambda_{\max}^2 c_{\max}}\varepsilon(nT)^T(\mathscr{L}\otimes I_m)\varepsilon(nT). \end{aligned} \quad (40)$$

We choose the same Lyapunov function candidate as in the proof of Theorem 1:

$$V(t) = \frac{1}{2}\varepsilon(t)^T\varepsilon(t). \quad (41)$$

Combining inequality (40) and (30), the $\dot{V}(t)$ can be bounded as follows:

$$\begin{aligned} \dot{V} &\le \lambda_{\max} T e(nT)^T(\mathscr{L}\otimes I_m)e(nT) + \left(T+\frac{1}{2}\right)\varepsilon(nT)^T(\mathscr{L}\otimes I_m) \\ &\quad\cdot \varepsilon(nT) + \left(T+\frac{1}{2}\right)e(nT)^T(\mathscr{L}\otimes I_m)e(nT) \\ &\quad + (T\lambda_{\max}-1)\varepsilon(nT)^T(\mathscr{L}\otimes I_m)\varepsilon(nT) \\ &\le \left[\lambda_{\max} T\frac{2\lambda_{\max}^2 c_{\max}}{1-2\lambda_{\max}^2 c_{\max}} + (T+0.5)\frac{2\lambda_{\max}^2 c_{\max}}{1-2\lambda_{\max}^2 c_{\max}} + T \right. \\ &\quad \left. + T\lambda_{\max} - 0.5\right]\varepsilon(nT)^T(\mathscr{L}\otimes I_m)\varepsilon(nT). \end{aligned} \quad (42)$$

To make the entire formula (42) negative definite while enlarging the available range of the sampling period as much as possible, we can derive $T < 1-4\lambda_{\max}^2 c_{\max}/2+2\lambda_{\max}$. Further, because $1-4\lambda_{\max}^2 c_{\max} > 0$, we get $c_{\max} < 1/4\lambda_{\max}^2$. On the basis of these considerations, we derive the ranges of sampling period T and event parameter $c_{\max}$ as

$$0 < T < \frac{1-4\lambda_{\max}^2 c_{\max}}{2+2\lambda_{\max}}, \quad 0 < c_{\max} < \frac{1}{4\lambda_{\max}^2}. \quad (43)$$

Hereto, $\dot{V}(t) < 0$ is guaranteed, which implies that the designed periodic event-triggered control system converges asymptotically fast; this completes the proof. □

Remark 6. In the scheme of Section 3.2, it is observed that the choice of sampling period T and event parameter $c_{\max}$ is coupled. We first choose event parameter $c_{\max}$ and then determine sampling period T. Note that the maximum eigenvalue of the communication structure plays a critical role in this determination. Intuitively, different graphs will produce diverse communication cost and control updates. How the specific control and communication cost can be guaranteed will be challenging.

Remark 7. The core difference between periodic and aperiodic event-triggered control strategies lies in the fact that the former needs communication of an agent with its neighbors at each sampling time instant in order to judge whether an event condition is violated. Nevertheless, the latter overcomes this shortcoming and realizes communication only at an event-triggered time instant. In general, the aperiodic event-triggered control strategy allows longer communication pause time, which enables other agents to use the saved signal channels; this considerably decreases the communication cost and energy consumption. However, in both strategies, sampling of an agent's own states is performed in a fixed period. Moreover, the control updates of the two strategies happen at a time-triggered instant, which yields a piecewise constant signal. This class of control signals contributes to the reduction of mechanical wear and energy consumption.

4. Extension to Switching Graphs

In practice, formation reconfiguration might occur, changing the topology pattern. Therefore, we will consider multiagent systems under a switching topology in this section. First, the following assumption is adopted.

Assumption 2. The communication topology $\mathscr{G}_l, l \in \mathbb{R}^+$ switches among a finite possible connected undirected graph set $\mathbf{G} = \{\mathscr{G}_1, \mathscr{G}_1, \ldots, \mathscr{G}_l\}$. The switching time instant can be defined randomly as $t_1, t_2, \ldots, t_s$. $\mathscr{G}_{t_s}$ denotes the active topology. In addition, $\lambda_1(\mathbf{G})$ and $\lambda_l(\mathbf{G})$ denote the minimum and maximum eigenvalues for the undirected graph set, respectively.

In the case of switching topologies, the controllers and event conditions can be designed in the similar method as those in Section 3. The same Lyapunov function candidate can be employed to investigate the stability of systems as well as to derive the sufficient condition and constraints for the sampling period and the event parameter.

Then, the following theorem can be given.

Theorem 3. *Consider the multiagent system (1) governed by the distributed controller (17) associated with event condition (10) with respect to the sum of relative state errors (9) or (33). Then, the multiagent systems under switching graphs* $\mathbf{G}$ *reach an average consensus as long as the following sufficient conditions and Assumption 2 are satisfied:*

$$0 < c_{\max} < \frac{0.5-T-T\lambda_l(\mathbf{G})}{T\lambda_l^3(\mathbf{G})+T\lambda_l^2(\mathbf{G})+0.5\lambda_l^2(\mathbf{G})}, \quad 0 < T < \frac{1}{2+2\lambda_l(\mathbf{G})}, \quad (44)$$

or

$$0 < T < \frac{1 - 4\lambda_l^2(\mathbf{G})c_{\max}}{2 + 2\lambda_l(\mathbf{G})}, \quad 0 < c_{\max} < \frac{1}{4\lambda_l^2(\mathbf{G})}. \tag{45}$$

Proof. Following the same line of Theorems 1 and 2, the detailed proof is omitted here because of the limited space. □

5. Numerical Examples

To verify the theoretical results obtained in this paper, three contrastive computer simulations are provided to demonstrate the effectiveness of distributed periodic event-triggered controllers in solving the consensus problem while reducing the communication cost and control update frequency. Consider a scenario where six agents intend to make a rendezvous. Figure 1 shows the corresponding communication topology graph among these agents. Note that the graph is connected.

Based on the communication topology, the adjacency matrix $\mathscr{A}$ is obtained as follows:

$$\mathscr{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}. \tag{46}$$

Thus, the Laplacian matrix is given by

$$\mathscr{L} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & -1 \\ 0 & -1 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}. \tag{47}$$

By calculation, $\lambda_{\max}((\mathscr{L} \otimes I_m)) = \lambda_{\max}(\mathscr{L}) = 4.7321$ and Fiedler eigenvalue $\lambda_2(\mathscr{L}) = 0.5858$. Sampling period limitations $0 < T < 0.2113s$ should be satisfied based on Lemma 1 and $0 < T < 0.0872$ s should be satisfied based on Theorem 1. We choose $T = 0.005$ s. The larger sampling period is chosen in order to intuitively achieve a greater decrease in communication cost and control update. After assuring T, $0 < c_{\max} < 0.0398$ can be computed. In addition, event parameter constraint $0 < c_{\max} < 0.0112$ should be satisfied based on Theorem 2. If we choose $c_{\max} = 0.01$, the sampling period constraint is $0 < T < 0.0091$ s, which satisfies Theorem 1. Based on the above constraints, for facilitating a comparison of the control performance between different cases, the same the sampling period, event parameter, and communication graph were used in the following numerical examples. That is, sampling period $T = 0.005$ s and event parameter $c_{\max} = 0.01$ were chosen.

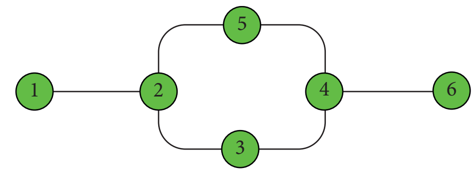

Figure 1: Communication topology.

5.1. Example of Lemma 1. Following Lemma 1, consider a scenario where six agents achieve a rendezvous. Figure 2 shows the evolution of the state response for each agent. It can be seen that the agents reach consensus. Figure 3 shows the data sampling and communication and control update time instants for each agent. According to the simulation setting, the sampling period $T = 0.005$ s; therefore, the sampling and communication and control update times are equal all the time. The times are 120 under the condition $0 \le t \le 0.6$. Figure 4 shows control inputs (4) for agents. A zoomed view of the figure indicates a piecewise (0.005 s) constant function.

5.2. Example of Theorem 1. This case adopts controller (11) and event condition (7) with the sum of relative state errors (6). The initial states are [2 5 9 4 18 7], with an initial average value of 7.5. It can be seen that the agents reach consensus at their initial average. Figure 5 and 6 show the event-triggered time instant from 0 s to 0.6 s. Each agent continues to sample data and communicate once in each cycle, but the control update occurs only at the time instant when an event is triggered. It can be inferred that the number of actuator control updates will dramatically decrease to reach average consensus compared to that in the continuous actuation scheme. Moreover, further reduction in the limited energy in microembedded systems and mechanical wear in brittle components can be obtained. Figure 7 indicates the piecewise constant control input for each agent.

5.3. Example of Theorem 2. This case adopts controller (17) and event condition (10) with the sum of relative state errors (33). The initial states are [2 5 9 4 18 7], with an initial average value of 7.5. The state response presented in Figure 8 indicates that average consensus between agents is achieved. The event-triggered situation of the six agents is shown in Figure 9. Note that in this scenario, each agent samples the data periodically, but does not communicate simultaneously. The communication and control update occur when an event is triggered. This mechanism will tremendously reduce the information exchange and control update. Figure 10 indicates the piecewise constant control inputs.

In addition, some simulation data are reported in Table 1 to explicitly compare different performance indicators of the two types of control strategies. Under the same parameters used in the examples described above, the following can be observed: from 0 s to 0.6 s, the total numbers of communication and control update of pure sampling control (4) are both 120. However, these values for the periodic communication method are 120 and 51. Finally, the corresponding

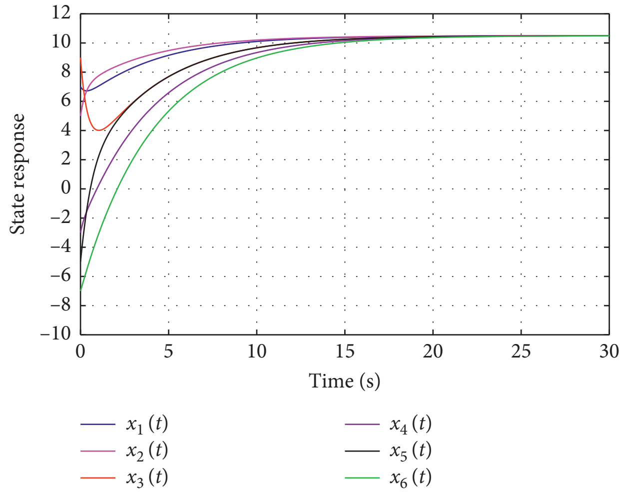

FIGURE 2: Evolution of each agent.

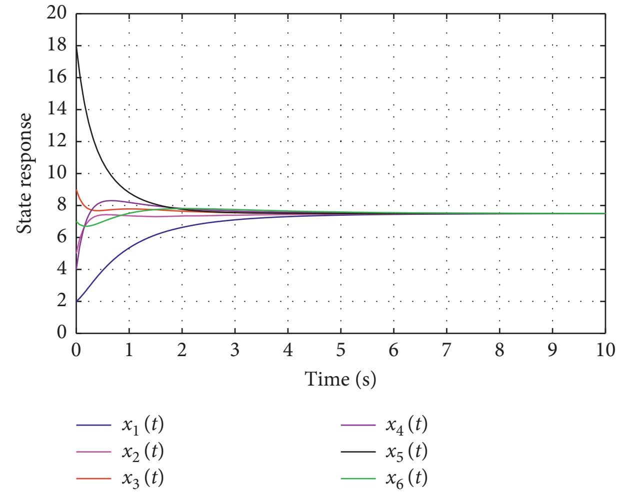

FIGURE 5: Evolution of each agent.

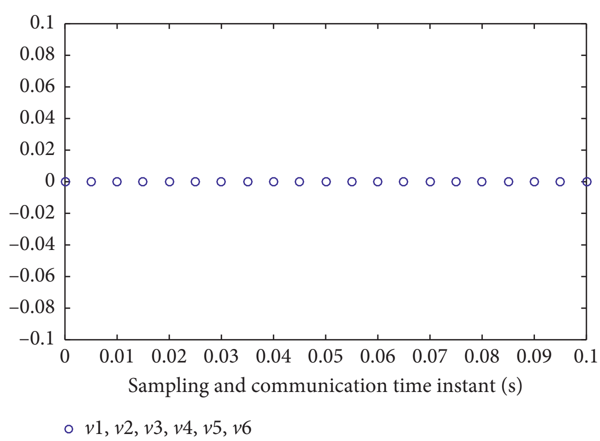

FIGURE 3: Sampling and communication time instant of each agent.

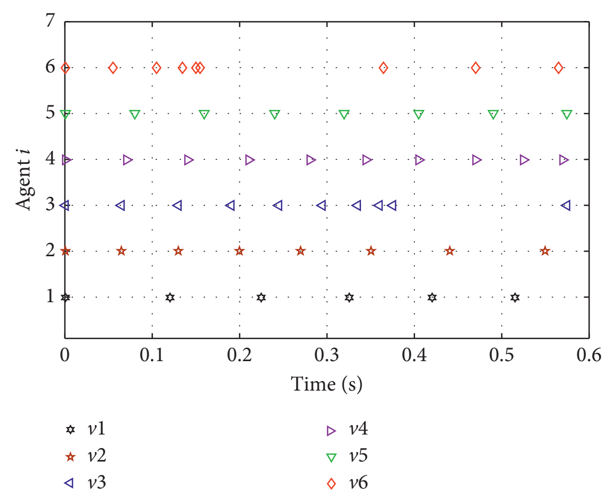

FIGURE 6: Event-triggered time instant of each agent.

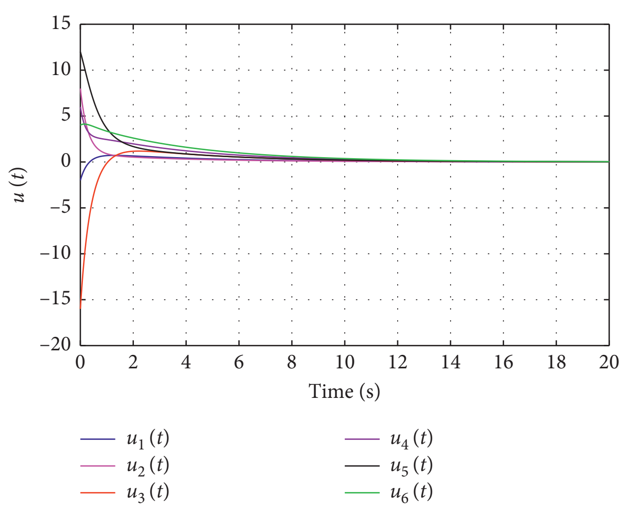

FIGURE 4: Control input of each agent.

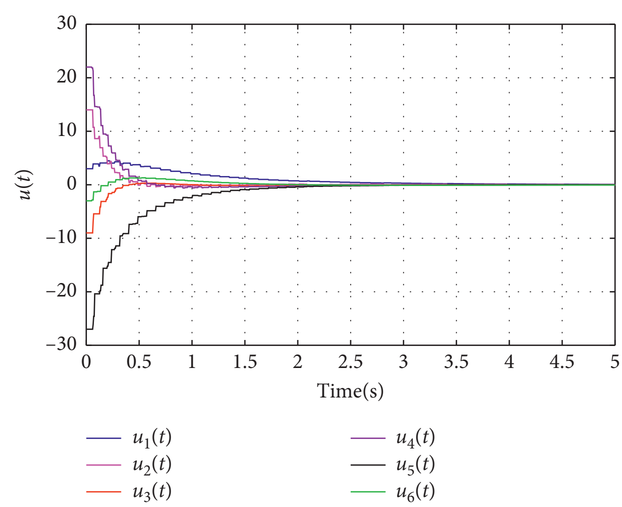

FIGURE 7: Control input of each agent.

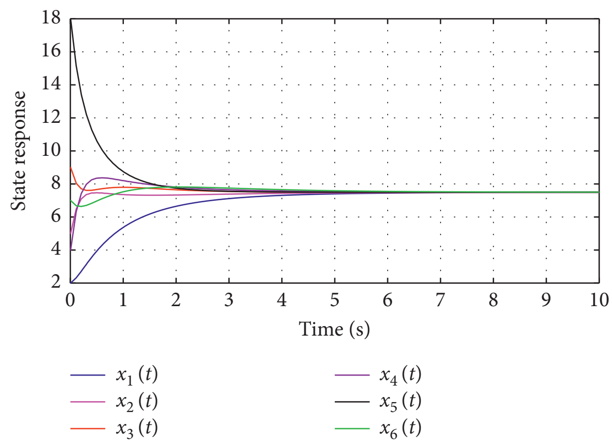

Figure 8: Evolution of each agent.

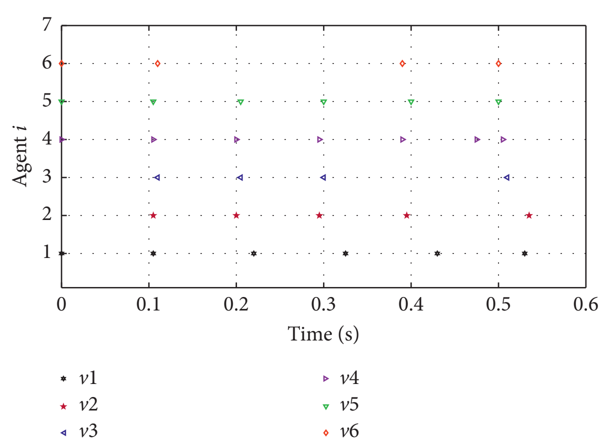

Figure 9: Event-triggered time instant of each agent.

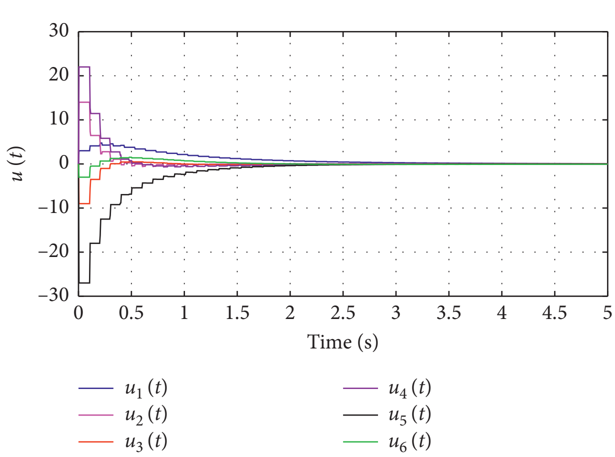

Figure 10: Control input of each agent.

results for the aperiodic communication method are 33. Moreover, for each agent, the number of events in the periodic communication method is more than that in the aperiodic communication method. Note that the events are not triggered when the systems are in steady state.

Table 1: Comparison of Theorems 1 and 2 (0–0.6 s).

Case	1	2
T	0.005	0.005
$c_{\max}$	0.01	0.01
Events of agent 1	6	6
Events of agent 2	8	5
Events of agent 3	10	4
Events of agent 4	10	6
Events of agent 5	8	7
Events of agent 6	9	5
Total number of events triggered	51	33

The results for the switching topologies are analog to fixed graph; those simulations are omitted here due to space limitation.

6. Concluding Remarks

In this study, we proposed an event-triggered sampling control method to drive multiagent systems with fixed and switching topologies contrastive in the sense of average consensus. This method combines the benefits offered by both periodic sampled-data control and event-triggered control. In particular, unlike most existing continuous-time event-triggered control schemes, the underlying idea in this method is to adopt an event-triggering condition that is verified periodically or aperiodically. While periodic and aperiodic communication occurs, control update occurs aperiodically. Corresponding sufficient conditions are derived by ensuring that the continuous-time derivative of the Lyapunov function is negative definite by choosing an appropriate event parameter and sampling period based on graph Laplacian. Moreover, the Zeno behavior for each agent due to the characteristic of periodical sampling is avoided. Our ongoing work is devoted to extending the present control framework to directed graphs and situations with leaders. Other interesting topics for further exploration are proving that the current periodic event-triggered controller will not degenerate to the pure sampling controller and relaxing the fixed sampling period [43].

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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