Hindawi
Mathematical Problems in Engineering
Volume 2020, Article ID 1672595, 11 pages
https://doi.org/10.1155/2020/1672595

Research Article

Global Robust Stabilization Control for Nonlinear Time-Delay Systems with Dead-Zone Input and Complex Dynamics

Lingrong Xue,[1] Zhen-Guo Liu ,[2] Junjun Chen,[2] and Fujing Xu[2]

[1]*School of Information Management, Shanxi University of Finance and Economics, Taiyuan 030006, China*
[2]*Department of Automation, Shanxi University, Taiyuan 030006, China*

Correspondence should be addressed to Zhen-Guo Liu; lzg819@163.com

Received 28 February 2020; Revised 23 April 2020; Accepted 24 April 2020; Published 26 May 2020

Guest Editor: Rongwei Guo

This article investigates the robust stabilization problem for nonlinear time-delay systems with dead-zone input and complex dynamics. By flexibly using the inequality technique, the backstepping control method, and skillfully introducing a new Lyapunov–Krasovskii functional, we obtain a stable controller without using unmeasurable signals in the dynamic subsystem. The control system is guaranteed to be stable finally. Two simulation examples are given to verify the control strategy.

1. Introduction

Many practical models in engineering are nonlinear systems such as the flexible-joint robot [1], the wheeled inverted pendulum [2], and the autonomous underwater vehicle [3]. In the past few years, scholars have focused on studying nonlinear systems, such as [4–11]. For numerous nonlinear systems, the time-delay phenomenon which may lead to system instability often exists and is inevitable [12]. Nonlinear systems sometimes involve complex dynamics, in which the information of the states is not available. For more results on nonlinear systems with complex dynamics, we refer the reader to [13]. Besides the time delay, nonlinear conditions, and complex dynamics, specific control inputs such as the dead-zone input [14, 15] can also have a significant impact on the system. Considering the above facts, the nonlinear time-delay systems with dead-zone input and complex dynamics are investigated in this paper.

In recent years, some complicated linear systems have been studied. For example, Xu and Zhang [16] considered the stochastic large population system and presented a novel strategy for linear-quadratic games. However, different from linear systems, control problems of nonlinear systems are often difficult since they have more complicated dynamics. Specially, Guo [17] studied nonlinear chaotic systems and raised a physically implementable controller to solve the projective synchronization problem. In engineering practice, lots of nonlinear systems can be approximated by using linear systems at the origin; thus, the theory of linear systems can be applied. However, some systems may not be linearized at the origin or can be linearized but have uncontrollable Jacobian linearization [18, 19]. So, it is necessary to study nonlinear control design methods for those systems. Besides, time-delay problems cannot be ignored for the system control design, since ignoring it may make the system unstable. Many scholars have studied the associated control design for systems with time delay (for example, see the adaptive control problem [20], the stabilization problem [21], and the tracking problem [22]).

In recent years, the control design for systems with complex dynamics has been one of the interesting topics. Particularly, with the choice of a state observer, the adaptive control problem of systems containing complex dynamics was solved in [23]. By utilizing a new Lyapunov function, the adaptive tracking problem was studied in [24] for systems with input saturation and complex dynamics. On the other hand, time delay may bring negative effects to the stability of systems. Therefore, scholars have studied control problems for nonlinear systems with time delay. For systems involving time delay and complex dynamics, in [25], by the neural network method and the Lyapunov–Krasovskii functional, the tracking control design was studied. In [26], a modified strategy of adding a power integrator was applied for stochastic delayed nonlinear systems with complex dynamics. Subsequently, this technique was further applied to systems with uncertainty in [27].

In practice, dead zone may exist in the actuator or the control input of the system. There have been some related reports mainly discussing the neural method and the fuzzy method. Specially, the neural control method [28] and the adaptive fuzzy control method [29] were applied to solve adaptive control problems of nonlinear systems. Recently, the Lyapunov–Krasovskii functional control approach has been used to study the nonlinear tracking control of systems containing dead-zone input and implies that the approach is important for systems with time delay, see [30]. However, this method is not extended to solve the robust stabilization problem for systems involving time delay, dead-zone input, and complex dynamics. Also, few studies in the literature considered the robust control problem for the system.

The difficulty and the contribution of this paper are as follows:

(i) Considering that the system of this paper involves complicated dynamics, time delay, external disturbances, and input dead zone, the robust stabilization control problem of this paper is more challenging. The existing methods are difficult to solve the problem of this paper. Particularly, the homogeneous domination approach [5] has not proposed a strategy to solve the input dead-zone problem, and the tuning functions-based robust control method [7] has not give a solution to the complicated dynamics. The neural control method [28] and the fuzzy method [29] are difficult to give appropriate bounds for the nonlinear time-delay terms. The continuous control methods in [31, 32] did not provide a strategy to deal with the external disturbances. This article will consider more complicated nonlinear systems and present a control strategy to solve the control problem.

(ii) A new robust stabilization strategy is raised. By recursively selecting a Lyapunov–Krasovskii functional and by presenting a modified backstepping control technique, a stable controller without utilizing the unmeasurable states is successfully constructed.

2. Problem Formulation and Preliminaries

Consider the nonlinear system as follows:

$$\begin{cases} \dot{\eta} = A\eta + \phi_0(x_1, x_1(t-\tau)), \\ \dot{x}_i = d_i(t)x_{i+1} + \phi_i(\eta, \eta(t-\tau), \overline{x}_i, \overline{x}_i(t-\tau)), \quad i = 1, \ldots, n-1, \\ \dot{x}_n = d_n(t)u(v) + \phi_n(\eta, \eta(t-\tau), x, x(t-\tau)), \end{cases} \tag{1}$$

where $\eta = [\eta_1, \ldots, \eta_m]^\top \in R^m$ is the unmeasurable dynamics and $x = [x_1, \ldots, x_n]^\top \in \mathscr{R}^n$ is the state vector which is measurable. $\overline{x}_i = [x_1, \ldots, x_i]^\top$. $\eta(t-\tau) = [\eta_1(t-\tau), \ldots, \eta_m(t-\tau)]^\top$ and $\overline{x}_i(t-\tau) = [x_1(t-\tau), \ldots, x_i(t-\tau)]^\top$. τ is the constant time delay. $A \in R^{m\times m}$ is the Hurwitz matrix that satisfies $A^\top P + PA \leq -Q$, where P and Q are positive definite symmetric matrices. The control coefficients $d_j(t) \in \mathscr{R}$, $j = 1, \ldots, n$ satisfy $\underline{d} \leq d_j(t) \leq \overline{d}$ with $\underline{d} \geq 0$ and $\overline{d} \geq 0$. $\phi_k \in \mathscr{R}$, $k = 0, 1, \ldots, n$, are continuous functions. The dead-zone input $u(v)$ is

$$u(v) = \begin{cases} m_r(t)(v - b_r(t)), & v \geq b_r(t), \\ 0, & -b_l(t) < v < b_r(t), \\ m_l(t)(v + b_l(t)), & v \leq -b_l(t), \end{cases} \tag{2}$$

where $m_r(t) > 0, m_l(t) > 0, b_r(t) > 0$, and $b_l(t) > 0$ the are time-varying functions.

To facilitate the problem simply, let $\varpi = (q/p) \geq 0$, where q and p are the even integer and the odd integer, respectively. Therefore, define the constants $\varepsilon_1 = 1$ and $\varepsilon_j = \varepsilon_{j-1} + \varpi$, $j = 2, \ldots, n$. Next, we need the following assumptions.

Assumption 1. The nonlinear terms satisfy the following:

$$\begin{aligned} &\left|\phi_0(x_1, x_1(t-\tau))\right| \leq C_0\left(\left|x_1\right| + \left|x_1(t-\tau)\right|\right), \\ &\left|\phi_i(\eta, \eta(t-\tau), \overline{x}_i, \overline{x}_i(t-\tau))\right| \leq C\left(\|\eta\|^{\varepsilon_{i+1}} + \|\eta(t-\tau)\|^{\varepsilon_{i+1}} + \sum_{j=1}^{i}\left|x_j\right|^{\varepsilon_{i+1}/\varepsilon_j} + \sum_{j=1}^{i}\left|x_j(t-\tau)\right|^{\varepsilon_{i+1}/\varepsilon_j}\right) + M_0(t), \quad i = 1, \ldots, n, \end{aligned} \tag{3}$$

where $C_0 > 0$ and $C > 0$ are constants and $M_0(t)$ is a bounded disturbance.

Assumption 2. There are constants $\underline{m}_r > 0, \underline{m}_l > 0, \overline{b}_r > 0$, and $\overline{b}_l > 0$ satisfying

$$\underline{m}_r \le m_r(t), \quad \underline{m}_l \le m_l(t), \quad b_r(t) \le \overline{b}_r, \text{ and } \quad b_l(t) \le \overline{b}_l. \tag{4}$$

Next, Lemma 1 is provided for control design.

Lemma 1 (see [30]). *For given $m > 0$ and $n > 0$ and functions $a(x, y) > 0$ and $c(x, y) > 0$, there holds*

$$\left|a(x, y)x^m y^n\right| \le c(x, y)|x|^{m+n} + \frac{n}{m+n} \cdot \left(\frac{m}{(m+n)c(x, y)}\right)^{m/n} |a(x, y)|^{m+n/n}|y|^{m+n}. \tag{5}$$

3. Main Results

Theorem 1. *For system (1), suppose that Assumptions 1-2 hold. Then, under the following transformation*

$$\begin{aligned} \zeta_1 &= x_1, \\ \zeta_i &= x_i - \alpha_{i-1}, \\ \alpha_i &= -g_i \zeta_i^{\varepsilon_{i+1}/\varepsilon_i}, \quad i = 2, \ldots, n, \end{aligned} \tag{6}$$

there exists a robust controller:

$$v = \begin{cases} \dfrac{\alpha_n}{\underline{m}_r} + \overline{b}_r, & \alpha_n > 0, \\ 0, & \alpha_n = 0, \\ \dfrac{\alpha_n}{\underline{m}_l} - \overline{b}_l, & \alpha_n < 0. \end{cases} \tag{7}$$

where $\alpha_n = -g_n \zeta_n^{\varepsilon_{n+1}/\varepsilon_n}$. The controller ensures that the considered system is globally stable.

Proof. We divide the proof into several parts.

3.1. Part I: Robust Control Design. We construct the controller by using the modified backstepping technique.

Step 1: defining $\sigma = \varepsilon_n$ and choosing $U_0 = 1/\sigma(\eta^\top P\eta)^\sigma$, we have

$$\begin{aligned} \dot{U}_0 &= (\eta^\top P\eta)^{\sigma-1}\left(\eta^\top P(A\eta + \phi_0) + (A\eta + \phi_0)^\top P\eta\right) \\ &= (\eta^\top P\eta)^{\sigma-1}(\eta^\top(PA + A^\top P)\eta + 2\eta^\top P\phi_0). \end{aligned} \tag{8}$$

Noting that $\lambda_p\|\eta\|^2 \le \eta^\top P\eta \le \lambda_P\|\eta\|^2$ and $\eta^\top(PA + A^\top P)\eta \le -\lambda_q\|\eta\|^2$, where λ_p is the minimal eigenvalue of P and λ_P is the maximal eigenvalues of P. λ_q is the minimal eigenvalue of Q. Then, introduce $\zeta_1 = x_1$ and define $\lambda = \min\{\lambda_p, \lambda_q\}$. From Assumption 1 and Lemma 1, we get

$$\dot{U}_0 \le -\lambda^\sigma\|\eta\|^{2\sigma} + 2(\eta^\top P\eta)^{\sigma-1}\eta^\top P\phi_0 = -\lambda^\sigma\|\eta\|^{2\sigma} + \Delta, \tag{9}$$

$$\begin{aligned} \Delta &= 2(\eta^\top P\eta)^{\sigma-1}\eta^\top P\phi_0 \le 2\lambda_P^{\sigma-1}\|\eta\|^{2\sigma-2}\cdot\|\eta\|\cdot\|P\|\cdot|\phi_0| \\ &\le 2\lambda_P^{\sigma-1}\|P\|\|\eta\|^{2\sigma-1}C_0\left(|\zeta_1| + |\zeta_1(t-\tau)|\right) \\ &\le \frac{1}{n+2}\lambda^\sigma\|\eta\|^{2\sigma} + \gamma_{01}(C_0, \lambda_P)\zeta_1^{\frac{2\sigma}{\varepsilon_1}} + \gamma_{02}(C_0, \lambda_P)e^{-\mu\tau}\zeta_1^{\frac{2\sigma}{\varepsilon_1}}(t-\tau), \end{aligned} \tag{10}$$

where $\gamma_{01}(C_0, \lambda_P)$ and $\gamma_{02}(C_0, \lambda_P)$ are the positive constants depended on C_0 and λ_P. Substituting (10) into (9), it yields

$$\begin{aligned} \dot{U}_0 \le &-\frac{n+1}{n+2}\lambda^\sigma\|\eta\|^{2\sigma} + \gamma_{01}(C_0, \lambda_P)\zeta_1^{2\sigma/\varepsilon_1} \\ &+ \gamma_{02}(C_0, \lambda_P)e^{-\mu\tau}\zeta_1^{2\sigma/\varepsilon_1}(t-\tau). \end{aligned} \tag{11}$$

By Lemma 1 and Assumption 1, there are constants $\gamma_{11} > 0$ and $M_1 > 0$ such that

$$\begin{aligned} \left|x_1^{2\sigma-\varepsilon_2/\varepsilon_1}\phi_1\right| &\le C|x_1|^{2\sigma-\varepsilon_2/\varepsilon_1}\Big(\|\eta\|^{\varepsilon_2} + \|\eta(t-\tau)\|^{\varepsilon_2} + |x_1|^{\varepsilon_2/\varepsilon_1} \\ &\quad + |x_1(t-\tau)|^{\varepsilon_2/\varepsilon_1}\Big) + |M_0||x_1|^{2\sigma-\varepsilon_2/\varepsilon_1} \\ &\le \gamma_{11}\zeta_1^{2\sigma/\varepsilon_1} + \frac{1}{n+2}\lambda^\sigma\|\eta\|^{2\sigma} \\ &\quad + \frac{1}{2(n+2)}e^{-\mu\tau}\lambda^\sigma\|\eta(t-\tau)\|^{2\sigma} \\ &\quad + e^{-\mu\tau}\zeta_1^{2\sigma/\varepsilon_1}(t-\tau) + \frac{M_0^{2\sigma/\varepsilon_2}}{M_1}. \end{aligned} \tag{12}$$

Introduce the transformation $\zeta_2 = x_2 - \alpha_1$, and choose $V_1 = U_0 + U_1 + W_1 + T_1$, where $U_1 = n/2(n+2)\lambda^\sigma \int_{t-\tau}^{t} e^{\mu(s-t)}\|\eta(s)\|^{2\sigma}\mathrm{d}s.$, $W_1 = (\varepsilon_1/2\sigma - \varpi)\zeta_1^{2\sigma-\varpi/\varepsilon_1}$, and $T_1 = (n + \gamma_{02})\int_{t-\tau}^{t} e^{\mu(s-t)}\zeta_1^{2\sigma/\varepsilon_1}(s)\mathrm{d}s$. $\mu > 0$ is a constant. Then, it follows from (11) and (12) that

$$\begin{aligned}\dot{V}_1 &= \dot{U}_0 + \zeta_1^{2\sigma-\varepsilon_2/\varepsilon_1}(d_1 x_2 + \phi_1) + (n+\gamma_{02})\zeta_1^{2\sigma/\varepsilon_1} - (n+\gamma_{02})e^{-\mu\tau}\zeta_1^{2\sigma/\varepsilon_1}(t-\tau) + \frac{n}{2(n+2)}\lambda^\sigma\|\eta\|^{2\sigma} \\ &\quad - \frac{n}{2(n+2)}e^{-\mu\tau}\lambda^\sigma\|\eta(t-\tau)\|^{2\sigma} - \mu(U_1+T_1) \\ &\le -\frac{n}{2(n+2)}\lambda^\sigma\|\eta\|^{2\sigma} + d_1\zeta_2\zeta_1^{2\sigma-\varepsilon_2/\varepsilon_1} + d_1\alpha_1\zeta_1^{2\sigma-\varepsilon_2/\varepsilon_1} + (n+\gamma_{01}+\gamma_{02}+\gamma_{11})\zeta_1^{2\sigma/\varepsilon_1} \\ &\quad + \frac{M_0^{2\sigma/\varepsilon_2}}{M_1} - (n-1)e^{-\mu\tau}\zeta_1^{2\sigma/\varepsilon_1}(t-\tau) - \frac{n-1}{2(n+2)}e^{-\mu\tau}\lambda^\sigma\|\eta(t-\tau)\|^{2\sigma} - \mu(U_1+T_1).\end{aligned} \tag{13}$$

Selecting the virtual control $\alpha_1 = -1/d(2n+\gamma_{01}+\gamma_{02}+\gamma_{11})\zeta_1^{(\varepsilon_2/\varepsilon_1)} \overset{\text{def}}{=} -g_1\zeta_1^{\varepsilon_2/\varepsilon_1}$, and substituting it into (13), it yields that

$$\begin{aligned}\dot{V}_1 &\le -\frac{n}{2(n+2)}\lambda^\sigma\|\eta\|^{2\sigma} - n\zeta_1^{2\sigma/\varepsilon_1} + d_1\zeta_2\zeta_1^{2\sigma-\varepsilon_2/\varepsilon_1} + \frac{M_0^{2\sigma/\varepsilon_2}}{M_1} \\ &\quad - \mu(U_1+T_1) - (n-1)e^{-\mu\tau}\zeta_1^{2\sigma/\varepsilon_1}(t-\tau) \\ &\quad - \frac{n-1}{2(n+2)}e^{-\mu\tau}\lambda^\sigma\|\eta(t-\tau)\|^{2\sigma}.\end{aligned} \tag{14}$$

Step k ($k = 2,\dots,n-1$): for step $k-1$, suppose that there exist transformations (6) and a Lyapunov–Krasovskii functional V_{k-1} such that there exist the following inequality:

$$\begin{aligned}\dot{V}_{k-1} &\le -\frac{n-k+2}{2(n+2)}\lambda^\sigma\|\eta\|^{2\sigma} - (n-k+2)\sum_{j=1}^{k-1}\zeta_j^{2\sigma/\varepsilon_j} + d_{k-1}\zeta_k\zeta_{k-1}^{2\sigma-\varepsilon_k/\varepsilon_{k-1}} \\ &\quad + \sum_{j=1}^{k-1}\frac{M_0^{2\sigma/\varepsilon_{j+1}}}{M_j} - \mu\left(U_1 + \sum_{j=1}^{k-1}T_j\right) - (n-k+1)e^{-\mu\tau}\sum_{j=1}^{k-1}\zeta_j^{2\sigma/\varepsilon_j} \\ &\quad \cdot (t-\tau) - \frac{n-k+1}{2(n+2)}e^{-\mu\tau}\lambda^\sigma\|\eta(t-\tau)\|^{2\sigma}.\end{aligned} \tag{15}$$

Next, we prove it still holds for $i = k$. Choosing

$$\begin{aligned}V_k &= V_{k-1} + W_k + T_k, \\ W_k &= \frac{\varepsilon_k}{2\sigma-\varpi}\zeta_k^{2\sigma-\varpi/\varepsilon_k}, \\ T_k &= (n-i+1)\int_{t-\tau}^{t} e^{\mu(s-t)}\zeta_k^{2\sigma/\varepsilon_k}(s)ds.\end{aligned} \tag{16}$$

introducing $\zeta_{k+1} = x_{k+1} - \alpha_k$, and taking the derivative of V_k, it can be deduced that

$$\begin{aligned}\dot{V}_k &\le -\frac{n-k+2}{2(n+2)}\lambda^\sigma\|\eta\|^{2\sigma} - (n-k+2)\sum_{j=1}^{k-1}\zeta_j^{2\sigma/\varepsilon_j} \\ &\quad + d_{k-1}\zeta_k\zeta_{k-1}^{2\sigma-\varepsilon_k/\varepsilon_{k-1}} + \sum_{j=1}^{k-1}\frac{M_0^{2\sigma/\varepsilon_{j+1}}}{M_j} - \mu\left(U_1 + \sum_{j=1}^{k}T_j\right) \\ &\quad - (n-k+1)e^{-\mu\tau}\sum_{j=1}^{k}\zeta_j^{2\sigma/\varepsilon_j}(t-\tau) \\ &\quad - \frac{n-k+1}{2(n+2)}e^{-\mu\tau}\lambda^\sigma\|\eta(t-\tau)\|^{2\sigma} + (n-k+1)\zeta_k^{2\sigma/\varepsilon_k} \\ &\quad + \zeta_k^{2\sigma-\varepsilon_{k+1}/\varepsilon_k}\left(\phi_k - \sum_{j=1}^{k-1}\frac{\partial\alpha_{k-1}}{\partial x_j}(d_j x_{j+1} + \phi_j)\right) \\ &\quad + d_k\zeta_{k+1}\zeta_k^{2\sigma-\varepsilon_{k+1}/\varepsilon_k} + d_k\alpha_k\zeta_k^{2\sigma-\varepsilon_{k+1}/\varepsilon_k}.\end{aligned} \tag{17}$$

On the basis of $d_{k-1} \le \overline{d}$ and Lemma 1, there is a constant $\gamma_{11} > 0$ such that

$$d_{k-1}\zeta_k\zeta_{k-1}^{2\sigma-\varepsilon_k/\varepsilon_{k-1}} \le \frac{1}{3}\zeta_{k-1}^{2\sigma/\varepsilon_{k-1}} + \gamma_{k1}\zeta_k^{2\sigma/\varepsilon_k}. \tag{18}$$

Similarly to the proof of (12), one obtains

$$\begin{aligned}\left|\zeta_k^{2\sigma-\varepsilon_{k+1}/\varepsilon_k}\phi_k\right| &\le C\left|\zeta_k\right|^{2\sigma-\varepsilon_{k+1}/\varepsilon_k}\Bigg(\|\eta\|^{\varepsilon_{k+1}}+\|\eta(t-\tau)\|^{\varepsilon_{k+1}} \\ &\quad+\sum_{j=1}^{k}\left|\zeta_j-g_{j-1}\zeta_{j-1}^{\varepsilon_j/\varepsilon_{j-1}}\right|^{\varepsilon_{k+1}/\varepsilon_j} \\ &\quad+\sum_{j=1}^{k}\left|\zeta_j(t-\tau)-g_{j-1}\zeta_{j-1}^{\varepsilon_j/\varepsilon_{j-1}}(t-\tau)\right|^{\varepsilon_{k+1}/\varepsilon_j}\Bigg) \\ &\quad+\left|M_0\right|\left|\zeta_k\right|^{2\sigma-\varepsilon_{k+1}/\varepsilon_k} \\ &\le\gamma_{k2}\zeta_k^{2\sigma/\varepsilon_k}+\frac{1}{4(n+2)}\lambda^{\sigma}\|\eta\|^{2\sigma} \\ &\quad+\frac{1}{4(n+2)}e^{-\mu\tau}\lambda^{\sigma}\|\eta(t-\tau)\|^{2\sigma} \\ &\quad+\frac{1}{2}e^{-\mu\tau}\sum_{j=1}^{k}\zeta_j^{2\sigma/\varepsilon_j}(t-\tau)+\frac{M_0^{2\sigma/\varepsilon_{k+1}}}{M_{k1}}+\frac{1}{3}\sum_{j=1}^{k-1}\zeta_j^{2\sigma/\varepsilon_j},\end{aligned} \tag{19}$$

where $\gamma_{k2}>0$ and $M_{k1}>0$ are constants. From Assumption 1, we have

$$\begin{aligned}\left|d_jx_{j+1}+\phi_j\right| &\le d\left(\left|\zeta_{j+1}\right|+g_j\left|\zeta_j\right|^{\epsilon_{j+1}/\epsilon_j}\right)+C\Bigg(\|\eta\|^{\epsilon_{j+1}}+\|\eta(t-\tau)\|^{\epsilon_{j+1}}+\sum_{i=1}^{j}\left|\zeta_i-g_{i-1}\zeta_{i-1}^{\epsilon_i/\epsilon_{i-1}}\right|^{\epsilon_{j+1}/\epsilon_i}+\sum_{i=1}^{j}\left|\zeta_i(t-\tau)-g_{i-1}\zeta_{i-1}^{\epsilon_i/\epsilon_{i-1}}(t-\tau)\right|^{\epsilon_{j+1}/\epsilon_i}\Bigg) \\ &\quad+\left|M_0\right|.\end{aligned} \tag{20}$$

Utilizing (20) and Lemma 1 and noting that $|(\partial\alpha_{k-1}/\partial x_j)| = g_jg_{j+1}\cdots g_{k-1}(\varepsilon_k/\varepsilon_j)|\zeta_j|^{\varpi/\varepsilon_j}|\zeta_{j+1}|^{\varpi/\varepsilon_{j+1}}\cdots|\zeta_{k-1}|^{\varpi/\varepsilon_{k-1}}$, it follows that

$$\begin{aligned}\left|\frac{\partial\alpha_{k-1}}{\partial x_j}\right|\left|d_jx_{j+1}+\phi_j\right| &\le\Bigg(d\left(\left|\zeta_{j+1}\right|+g_j\left|\zeta_j\right|^{\varepsilon_{j+1}/\varepsilon_j}\right)+C\Bigg(\|\eta\|^{\varepsilon_{j+1}}+\|\eta(t-\tau)\|^{\varepsilon_{j+1}}+\sum_{i=1}^{j}\left|\zeta_i-g_{i-1}\zeta_{i-1}^{\varepsilon_i/\varepsilon_{i-1}}\right|^{\varepsilon_{j+1}/\varepsilon_i} \\ &\quad+\sum_{i=1}^{j}\left|\zeta_i(t-\tau)-g_{i-1}\zeta_{i-1}^{\varepsilon_i/\varepsilon_{i-1}}(t-\tau)\right|^{\varepsilon_{j+1}/\varepsilon_i}\Bigg)+\left|M_0\right|\Bigg)g_jg_{j+1}\cdots g_{k-1}\frac{\varepsilon_k}{\varepsilon_j}\left|\zeta_j\right|^{\varpi/\varepsilon_j}\left|\zeta_{j+1}\right|^{\varpi/\varepsilon_{j+1}}\cdots\left|\zeta_{k-1}\right|^{\varpi/\varepsilon_{k-1}} \\ &\le C_{j1}\sum_{i=1}^{j}\left|\zeta_i\right|^{\varepsilon_{j+1}/\varepsilon_i}+C_{j2}\sum_{i=1}^{j}\left|\zeta_i(t-\tau)\right|^{\varepsilon_{j+1}/\varepsilon_i}+C_{j3}\|\eta\|^{\varepsilon_{k+1}}+C_{j4}\|\eta(t-\tau)\|^{\varepsilon_{k+1}} \\ &\quad+\left|M_0\right|g_jg_{j+1}\cdots g_{k-1}\frac{\varepsilon_k}{\varepsilon_j}\left|\zeta_j\right|^{\varpi/\varepsilon_j}\left|\zeta_{j+1}\right|^{\varpi/\varepsilon_{j+1}}\cdots\left|\zeta_{k-1}\right|^{\varpi/\varepsilon_{k-1}},\end{aligned} \tag{21}$$

where $C_{jk} > 0, k = 1, \ldots, 4$ are constants. It can be deduced from (21) and Lemma 1 that

$$\left|-\zeta_k^{2\sigma-\varepsilon_{k+1}/\varepsilon_k} \sum_{j=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial x_j}\left(d_j x_{j+1} + \phi_j\right)\right| \le \gamma_{k3}\zeta_k^{2\sigma/\varepsilon_k} + \frac{1}{4(n+2)}\lambda^\sigma \|\eta\|^{2\sigma} + \frac{1}{4(n+2)} e^{-\mu\tau}\lambda^\sigma \|\eta(t-\tau)\|^{2\sigma} + \frac{1}{2}e^{-\mu\tau}\sum_{j=1}^{k}\zeta_j^{2\sigma/\varepsilon_j}(t-\tau) + \frac{M_0^{2\sigma/\varepsilon_{k+1}}}{M_{k2}} + \frac{1}{3}\sum_{j=1}^{k-1}\zeta_j^{2\sigma/\varepsilon_j}, \tag{22}$$

where $\gamma_{k3} > 0$ and $M_{k2} > 0$ are constants. Choosing M_k such that $1/M_k = (1/M_{k1}) + (1/M_{k2})$ and using (19) and (22), it yields that

$$\zeta_k^{2\sigma-\varepsilon_{k+1}/\varepsilon_k}\left(\phi_k - \sum_{j=1}^{k-1}\frac{\partial \alpha_{k-1}}{\partial x_j}\left(d_j x_{j+1} + \phi_j\right)\right) \le \left(\gamma_{k2} + \gamma_{k3}\right)\zeta_k^{2\sigma/\varepsilon_k} + \frac{1}{2(n+2)}\lambda^\sigma\|\eta\|^{2\sigma} + \frac{1}{2(n+2)}e^{-\mu\tau}\lambda^\sigma|\eta(t-\tau)|^{2\sigma} + e^{-\mu\tau}\sum_{j=1}^{k}\zeta_j^{2\sigma/\varepsilon_j}(t-\tau) + \frac{M_0^{2\sigma/\varepsilon_{k+1}}}{M_k} + \frac{2}{3}\sum_{j=1}^{k-1}\zeta_j^{2\sigma/\varepsilon_j}. \tag{23}$$

Now, we construct the virtual control $\alpha_k = -1/d(2n - 2k + 2 + \gamma_{k1} + \gamma_{k2} + \gamma_{k3})\zeta_k^{(\varepsilon_{k+1}/\varepsilon_k)} \overset{\text{def}}{=} -g_k\zeta_k^{\varepsilon_{k+1}/\varepsilon_k}$. With the help of (17), (18), and (23), it is deduced that

$$\dot{V}_k \le -\frac{n-k+1}{2(n+2)}\lambda^\sigma\|\eta\|^{2\sigma} - (n-k+1)\sum_{j=1}^{k}\zeta_j^{2\sigma/\varepsilon_j} + d_k\zeta_{k+1}\zeta_k^{2\sigma-\varepsilon_{k+1}/\varepsilon_k} - \mu\left(U_1 + \sum_{j=1}^{k}T_j\right) - (n-k)e^{-\mu\tau}\sum_{j=1}^{k}\zeta_j^{2\sigma/\varepsilon_j}(t-\tau) - \frac{n-k}{2(n+2)}e^{-\mu\tau}\lambda^\sigma\|\eta(t-\tau)\|^{2\sigma} + \sum_{j=1}^{k}\frac{M_0^{2\sigma/\varepsilon_{j+1}}}{M_j}. \tag{24}$$

This completes the control design for step $k = 2, \ldots, n-1$.

Step n: in this step, we choose $V_n = V_{n-1} + W_n + T_n, W_n = (\varepsilon_n/2\sigma - \varpi)\zeta_n^{2\sigma-\varpi/\varepsilon_n}$ and $T_n = \int_{t-\tau}^{t} e^{\mu(s-t)}\zeta_n^{2\sigma/\varepsilon_n}(s)\mathrm{d}s$, and using (24), some simple computations lead to

$$\dot{V}_n \le -\frac{2}{2(n+2)}\lambda^\sigma\|\eta\|^{2\sigma} - 2\sum_{j=1}^{n-1}\zeta_j^{2\sigma/\varepsilon_j} + d_{n-1}\zeta_n\zeta_{n-1}^{2\sigma-\varepsilon_n/\varepsilon_{n-1}} + \sum_{j=1}^{n-1}\frac{M_0^{2\sigma/\varepsilon_{j+1}}}{M_j} - \mu\left(U_1 + \sum_{j=1}^{n}T_j\right) - e^{-\mu\tau}\sum_{j=1}^{n}\zeta_j^{2\sigma/\varepsilon_j}(t-\tau) - \frac{1}{2(n+2)}e^{-\mu\tau}\lambda^\sigma\|\eta(t-\tau)\|^{2\sigma} + \zeta_n^{2\sigma/\varepsilon_n} + \zeta_n^{2\sigma-\varepsilon_{n+1}/\varepsilon_n}\left(\phi_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\left(d_j x_{j+1} + \phi_j\right)\right) + d_n u(v)\zeta_n^{2\sigma-\varepsilon_{n+1}/\varepsilon_n}. \tag{25}$$

Similar to (18) and (23), by Lemma 1, we obtain that

$$d_{n-1}\zeta_n\zeta_{n-1}^{2\sigma-\varepsilon_n/\varepsilon_{n-1}} \leq \frac{1}{3}\zeta_{n-1}^{2\sigma/\varepsilon_{n-1}} + \gamma_{n1}\zeta_n^{2\sigma/\varepsilon_n}, \tag{26}$$

$$\begin{aligned}\zeta_n^{2\sigma-\varepsilon_{n+1}/\varepsilon_n}\left(\phi_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\left(d_j x_{j+1} + \phi_j\right)\right) &\leq (\gamma_{n2}+\gamma_{n3})\zeta_n^{2\sigma/\varepsilon_n} + \frac{1}{2(n+2)}\lambda^\sigma\|\eta\|^{2\sigma} \\ &+ \frac{1}{2(n+2)}e^{-\mu\tau}\lambda^\sigma\|\eta(t-\tau)\|^{2\sigma} + e^{-\mu\tau}\sum_{j=1}^{n}\zeta_j^{2\sigma/\varepsilon_j}(t-\tau) + \frac{M_0^{2\sigma/\varepsilon_{n+1}}}{M_n} + \frac{2}{3}\sum_{j=1}^{n-1}\zeta_j^{2\sigma/\varepsilon_j},\end{aligned} \tag{27}$$

where the constants $\gamma_{n1}\geq 0, \gamma_{n2}\geq 0, \gamma_{n3}\geq 0$, and $M_n\geq 0$. Now, we choose the virtual control $\alpha_n = -1/d(2+\gamma_{n1}+\gamma_{n2}+\gamma_{n3})\zeta_n^{(\varepsilon_{n+1}/\varepsilon_n)} \underset{=}{\text{def}} -g_n\zeta_n^{\varepsilon_{n+1}/\varepsilon_n}$, which, and (25)–(27), give that

$$\begin{aligned}\dot{V}_n \leq &-\frac{1}{2(n+2)}\lambda^\sigma\|\eta\|^{2\sigma} - \sum_{j=1}^{n}\zeta_j^{2\sigma/\varepsilon_j} - \mu\left(U_1 + \sum_{j=1}^{n}T_j\right) \\ &+ \sum_{j=1}^{n}\frac{M_0^{2\sigma/\varepsilon_{j+1}}}{M_j} + d_n(u(v)-\alpha_n)\zeta_n^{2\sigma-\varepsilon_{n+1}/\varepsilon_n}.\end{aligned} \tag{28}$$

Finally, we choose the control input v as (7), Then, by using (2), it follows that

$$u(v)-\alpha_n = \begin{cases} m_r\left(\dfrac{\alpha_n}{\underline{m}_r} + \overline{b}_r - b_r\right) - \alpha_n, & \alpha_n > 0, \\ 0, & \alpha_n = 0, \\ m_l\left(\dfrac{\alpha_n}{\underline{m}_l} + \overline{b}_l - b_l\right) - \alpha_n, & \alpha_n < 0, \end{cases} \tag{29}$$

which further renders that $d_n(u(v)-\alpha_n)\zeta_n^{2\sigma-\varepsilon_{n+1}/\varepsilon_n}\leq 0$. Then, using (28), we get

$$\begin{aligned}\dot{V}_n \leq &-\frac{1}{2(n+2)}\lambda^\sigma\|\eta\|^{2\sigma} - \sum_{j=1}^{n}\zeta_j^{2\sigma/\varepsilon_j} \\ &- \mu\left(U_1 + \sum_{j=1}^{n}T_j\right) + \sum_{j=1}^{n}\frac{M_0^{2\sigma/\varepsilon_{j+1}}}{M_j}.\end{aligned} \tag{30}$$

3.2. Part II: Stability Analysis. Since $U_0 = 1/\sigma(\eta^\top P\eta)^\sigma$ and $\lambda_p\|\eta\|^2 \leq \eta^\top P\eta \leq \lambda_P\|\eta\|^2$, we have $\|\eta\|^{2\sigma} \geq (\sigma/\lambda_P^\sigma)U_0$, which leads to

$$-\frac{1}{2(n+2)}\lambda^\sigma\|\eta\|^{2\sigma} \leq -\frac{\sigma}{2(n+2)\lambda_P^\sigma}\lambda^\sigma U_0. \tag{31}$$

Defining $\Delta_j = (2\sigma-\varpi/2\sigma)(\delta_j\varpi/2\sigma)^{\varpi/2\sigma-\varpi}(\varepsilon_j/2\sigma-\varpi)^{2\sigma/2\sigma-\varpi}$ and $\Delta m = \min_{1\leq j\leq n}(1/\Delta_j)$, it follows from Lemma 1 that $W_j = (\varepsilon_j/2\sigma-\varpi)\zeta_j^{2\sigma-\varpi/\varepsilon_j} \leq (1/\delta_j) + \Delta_j\zeta_j^{2\sigma/\varepsilon_j}$, which renders that

$$-\sum_{j=1}^{n}\zeta_j^{2\sigma/\varepsilon_j} \leq -\Delta m\sum_{j=1}^{n}W_j + \sum_{j=1}^{n}\frac{1}{\delta_j\Delta_j}. \tag{32}$$

Substituting (31) and (32) into (28), it yields that

$$\begin{aligned}\dot{V}_n \leq &-\frac{\sigma}{2(n+2)\lambda_P^\sigma}\lambda^\sigma U_0 - \Delta m\sum_{j=1}^{n}W_j + \sum_{j=1}^{n}\frac{1}{\delta_j\Delta_j} \\ &- \mu\left(U_1 + \sum_{j=1}^{n}T_j\right) + \sum_{j=1}^{n}\frac{M_0^{2\sigma/\varepsilon_{j+1}}}{M_j} \leq -\rho_1 V_n + \rho_2,\end{aligned} \tag{33}$$

where $\rho_1 = \min\{(\sigma/2(n+2)\lambda_P^\sigma)\lambda^\sigma, \Delta m, \mu\}$ and $\rho_2 = \sum_{j=1}^{n}(1/\delta_j\Delta_j) + \sum_{j=1}^{n}(M_0^{2\sigma/\varepsilon_{j+1}}/M_j)$. By the definition of U_0, U_1, W_i, and $T_i, i = 1,\ldots,n$, it follows that

$$\begin{aligned}U_0 + U_1 &= \frac{1}{\sigma}(\eta^\top P\eta)^\sigma + \frac{n}{2(n+2)}\lambda^\sigma\int_{t-\tau}^{t}e^{\mu(s-t)}\|\eta(s)\|^{2\sigma}ds \\ &\leq \frac{\lambda_P^\sigma}{\sigma}\|\eta\|^{2\sigma} + \frac{n\tau\lambda^\sigma}{2(n+2)}\sup_{-\tau\leq k\leq 0}e^{\mu k}\|\eta(k+t)\|^{2\sigma} \\ &\leq \left(\frac{\lambda_P^\sigma}{\sigma} + \frac{n\tau\lambda^\sigma}{2(n+2)}\right)\sup_{-\tau\leq k\leq 0}\|\eta(k+t)\|^{2\sigma}, \\ \sum_{i=1}^{n}W_i &= \sum_{i=1}^{n}\frac{\varepsilon_i}{2\sigma-\varpi}\zeta_i^{2\sigma-\varpi/\varepsilon_i} \leq \frac{\varepsilon_n}{2\sigma-\varpi}\sum_{i=1}^{n}\sup_{-\tau\leq k\leq 0}\zeta_i^{2\sigma-\varpi/\varepsilon_i}(k+t), \\ \sum_{i=1}^{n}T_i &= \gamma_{02}\int_{t-\tau}^{t}e^{\mu(s-t)}\zeta_1^{2\sigma/\varepsilon_1}(s)ds + \sum_{i=1}^{n}(n-i+1) \\ &\cdot\int_{t-\tau}^{t}e^{\mu(s-t)}\zeta_i^{2\sigma/\varepsilon_i}(s)ds \leq (n+\gamma_{02})\tau\sum_{i=1}^{n}\sup_{-\tau\leq k\leq 0}\zeta_i^{2\sigma/\varepsilon_i}(k+t).\end{aligned} \tag{34}$$

which lead to

$$V_n = U_0 + U_1 + \sum_{i=1}^{n}W_i + \sum_{i=1}^{n}T_i \leq \varphi_1\left(\sup_{-\tau\leq k\leq 0}\Xi(k+t)\right), \tag{35}$$

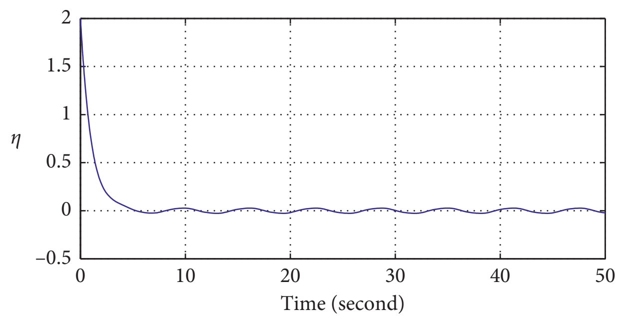

Figure 1: : The trajectory of η.

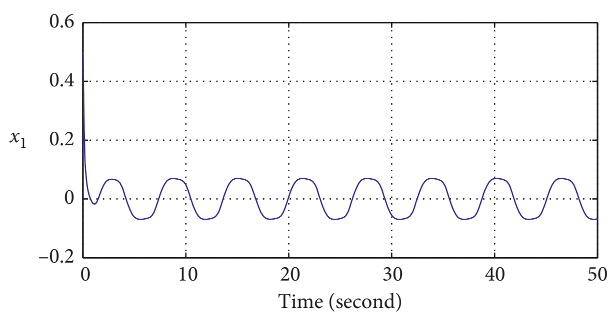

Figure 2: : The trajectory of x_1.

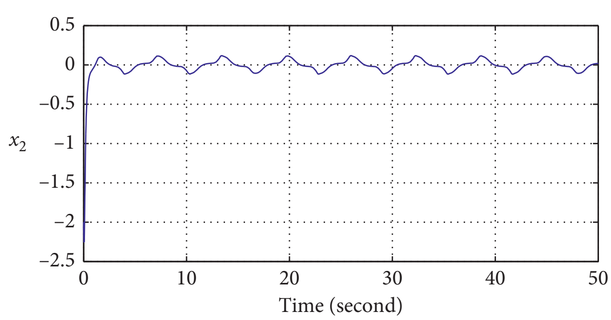

Figure 3: : The trajectory of x_2.

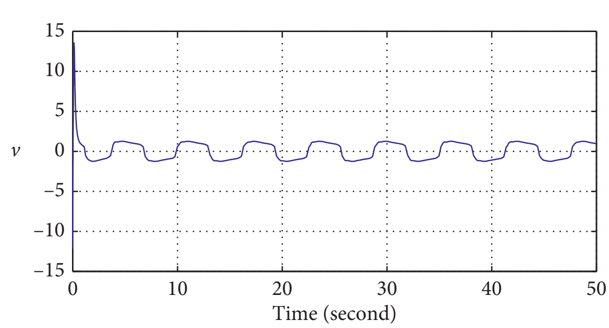

Figure 4: : The trajectory of ν.

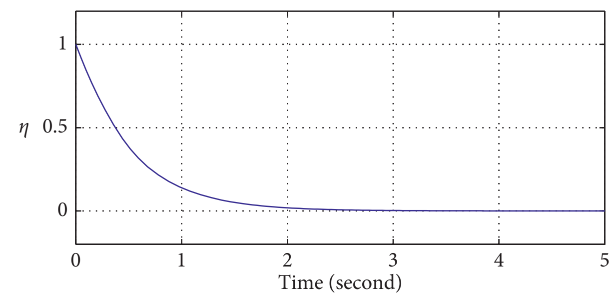

Figure 5: : The trajectory of η.

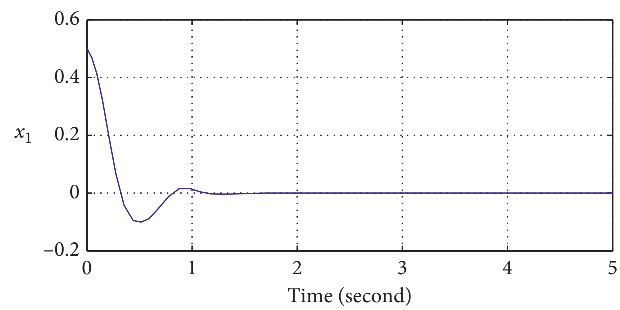

Figure 6: : The trajectory of x_1.

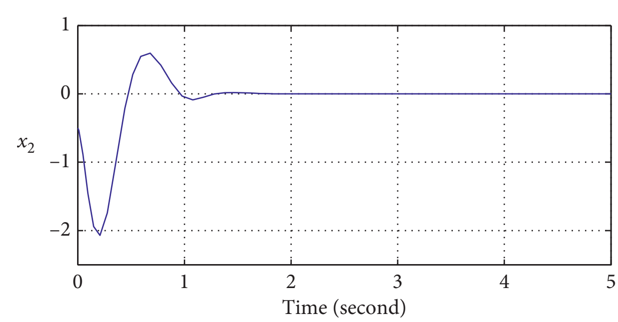

Figure 7: The trajectory of x_2.

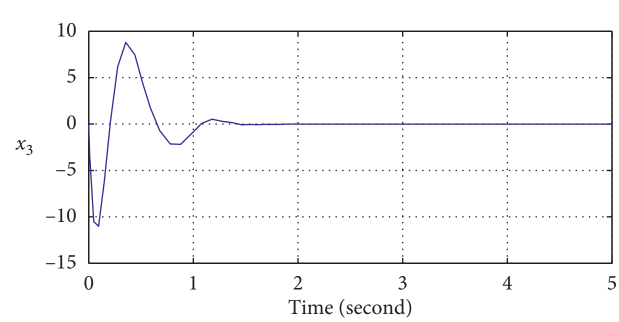

Figure 8: The trajectory of x_3.

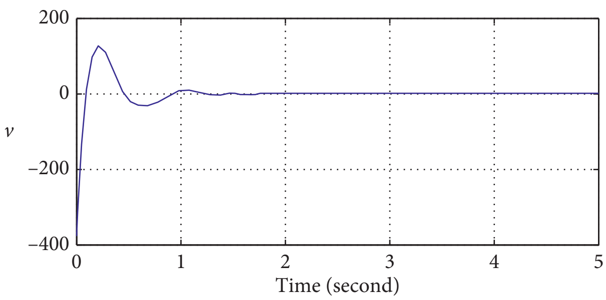

Figure 9: The trajectory of v.

where the function $\varphi_1 \in K_\infty$ and $\Xi(t) = [\eta(t)^\top, \zeta(t)^\top]^\top$. On the other hand, there exists a function $\varphi_2 \in K_\infty$ such that

$$V_n = U_0 + U_1 + \sum_{i=1}^{n} W_i + \sum_{i=1}^{n} T_i \geq \frac{\lambda_p^\sigma}{\sigma}\|\eta\|^{2\sigma} + \sum_{i=1}^{n} \frac{\varepsilon_i}{2\sigma - \varpi}\zeta_i^{2\sigma-\varpi/\varepsilon_i} \geq \varphi_2(\Xi(t)). \quad (36)$$

In view of (28), (35), and (36), and using Lemma 3.2.4 in [33], we obtain

$$\dot{V}_n \leq \left(V_n(0) - \frac{\rho_2}{\rho_1}\right)e^{-\rho_1 t} + \frac{\rho_2}{\rho_1}, \quad (37)$$

which indicates that η and ζ are bounded. From $\zeta_1 = x_1, \zeta_i = x_i - \alpha_{i-1}, \alpha_1 = -g_1\zeta_1^{\varepsilon_2/\varepsilon_1}$, and $\alpha_i = -g_i\zeta_i^{\varepsilon_{i+1}/\varepsilon_i}, i = 2, \ldots, n$, the states of $\eta, x_1, \ldots, x_n$ are bounded, which indicates the system composed of (1) and (7) is globally stable.

4. Simulation Example

Example 1. Consider the following nonlinear time-delay system:

$$\begin{cases} \dot{\eta} = -\eta + 0.5x_1, \\ \dot{x}_1 = x_2 + x_1^{5/3}(t - 0.2), \\ \dot{x}_2 = u(v) + x_2^{7/5} + 0.3\sin(t), \end{cases} \quad (38)$$

where η is the unmeasurable state, x_1 and x_2 are the measurable states, $u(v)$ is the dead-zone input given in (2) with $b_r = 0.4 + 0.1\sin(t), b_l = 0.4 - 0.1\sin(t)$, and v is the input of the system. We see that Assumptions 1-2 are satisfied for system (32) with $\varpi = 2/3, \varepsilon_1 = 1, \varepsilon_2 = 5/3, \varepsilon_3 = 7/3$, and $\overline{b}_r = \overline{b}_l = 0.5$, and $\underline{m}_r = \underline{m}_l = m_r = m_l = 1$. Applying the above control method, we choose $\alpha_2 = -15(x_2 + 9x_1^{5/3})^{7/5}$. Then, the actual controller is constructed as

$$v = \begin{cases} \alpha_2 + \overline{b}_r, & \alpha_2 > 0, \\ 0, & \alpha_2 = 0, \\ \alpha_2 - \overline{b}_l, & \alpha_2 < 0. \end{cases} \quad (39)$$

In the simulation, choose $\eta(0) = 2, x_1(0) = 0.5$, and $x_2(0) = -2$. Figures 1–4 give the trajectories of η, x_1, and x_2 and the control input v. All signals in systems (38) and (39) are bounded. Hence, the validity of the presented control method is verified.

Example 2. Consider the following system:

$$\begin{cases} \dot{\eta} = -2\eta + \dfrac{1}{1+\eta^2}\eta x_1, \\ \dot{x}_1 = x_2 + x_1^{1/3}x_1^{2/3}(t - 0.5), \\ \dot{x}_2 = x_3 - 5\sin(x_1) - x_2 + \sin(x_1(t - 0.5)), \\ \dot{x}_3 = u(v) + 10x_2 - 5x_3, \end{cases} \quad (40)$$

where η is the unmeasurable state, x_1, x_2, x_3 are the system states, v is the control input, and $u(v)$ is the dead-zone input defined in (2) with $b_r = 0.5 + \sin(t)$ and $b_l = 0.5 - \sin(t)$. It can be deduced that $|(x_1\eta/1 + \eta^2)| \leq |x_1|$, $|x_1^{1/3}x_1^{2/3}(t - 0.5) \leq |x_1| + 2/3|x_1(t - 0.5)|$, $| - 5\sin(x_1) - x_2 + \sin(x_1 \quad (t - 0.5))| \leq 5|x_1| + |x_2| + |x_1(t - 0.5)|$, and $|10x_2 - 5x_3| \leq 10|x_2| + 5|x_3|$. In this example, $\varpi = 0, \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 1$, and $\overline{b}_r = \overline{b}_l = 1.5$, and $\underline{m}_r = \underline{m}_l == 1$. Thus, all conditions of Assumptions 1-2 are satisfied. Using the control design method in Section 3, we can design the actual controller as follows:

$$v = \begin{cases} \alpha_3 + \overline{b}_r, & \alpha_3 > 0, \\ 0, & \alpha_3 = 0, \\ \alpha_3 - \overline{b}_l, & \alpha_3 < 0. \end{cases} \quad (41)$$

where $\alpha_3 = -15(x_3 + 10(x_2 + 6x_1))$.

In the simulation, the initial conditions are selected as $\eta(0) = 1, x_1(0) = 0.5, x_2(0) = -0.5$, and $x_3(0) = 0$. Figures 5–9 show the trajectories of η, x_1, x_2, x_3, and the control input v. It can be seen that all signals in systems (34) and (35) are bounded. Hence, the presented control method is effective.

5. Conclusions

The robust stabilization for nonlinear systems with dead-zone input and time delay has been studied. Because the system involves the dead-zone input, time-delay, disturbance, and unmeasurable states, the stabilization control in this work is more challenging. A robust stable controller has been designed via the Lyapunov–Krasovskii functional and the backstepping technique. Another interesting problem is as follows: When the considered system includes uncertainty parameters, and only the system output is measurable, how can we design the adaptive controller via the output feedback control method?

Data Availability

The data used to support the findings are included within this article.

Conflicts of Interest

The authors declare no conflicts of interest in preparing this article.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant no. 61903239), Youth Science and Technology Research Foundation of the ShanXi Science and Technology Department of China (Grant no. 201801D221167), and Scientific and Technological Innovation Programs of Higher Education Institutions in Shanxi (STIP) (Grant no. 2019L0492).

References

[1] W. Sun, S. F. Su, J. Xia, and V. T. Nguyen, "Adaptive fuzzy tracking control of flexible-joint robots with full-state constraints," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 11, pp. 2201–2209, 2018.

[2] W. Sun, S. Su, J. Xia, and Y. Wu, "Adaptive tracking control of wheeled inverted pendulums with periodic disturbances," *IEEE Transactions on Cybernetics*, vol. 50, no. 5, pp. 1867–1876, 2018.

[3] Z. Zhang and Y. Wu, "Adaptive fuzzy tracking control of autonomous underwater vehicles with output constraints," *IEEE Transactions on Fuzzy Systems*, 2020.

[4] C. Qian, "A homogeneous domination approach for global output feedback stabilization of a class of nonlinear systems," in *Proceedings of the American Control Conference*, pp. 4708–4715, IEEE, Portland, OR, USA, June 2005.

[5] J. Polendo and C. Qian, "A generalized homogeneous domination approach for global stabilization of inherently nonlinear systems via output feedback," *International Journal of Robust and Nonlinear Control*, vol. 17, no. 7, pp. 605–629, 2007.

[6] C. C. Chen, "A unified approach to finite-time stabilization of high-order nonlinear systems with and without an output constraint," *International Journal of Robust and Nonlinear Control*, vol. 29, no. 2, pp. 393–407, 2019.

[7] Z. Zhang, J. Lu, and S. Xu, "Tuning functions-based robust adaptive tracking control of a class of nonlinear systems with time delays," *International Journal of Robust and Nonlinear Control*, vol. 22, no. 14, pp. 1631–1646, 2012.

[8] Z.-Y. Sun, C.-H. Zhang, and Z. Wang, "Adaptive disturbance attenuation for generalized high-order uncertain nonlinear systems," *Automatica*, vol. 80, pp. 102–109, 2017.

[9] N. Duan, H. Min, and X. Qin, "Adaptive stabilization of stochastic nonlinear systems disturbed by unknown time delay and covariance noise," *Mathematical Problems in Engineering*, vol. 2017, Article ID 8084529, 9 pages, 2017.

[10] Q. Meng, T. Zhao, C. Qian, Z.-y. Sun, and P. Ge, "Integrated stability control of AFS and DYC for electric vehicle based on non-smooth control," *International Journal of Systems Science*, vol. 49, no. 7, pp. 1518–1528, 2018.

[11] Q. Meng, C. Qian, and R. Liu, "Dual-rate sampled-data stabilization for active suspension system of electric vehicle," *International Journal of Robust and Nonlinear Control*, vol. 28, no. 5, pp. 1610–1623, 2018.

[12] J. K. Hale and S. M. V. Lunel, *Introduction to Functional Differential Equations*, Springer Science & Business Media, Berlin, Germany, 2013.

[13] M. Krstić, P. V. Kokotovic, and I. Kanellakopoulos, *Nonlinear and Adaptive Control Design*, John Wiley & Sons, Inc., Hoboken, NJ, USA, 1995.

[14] X. Zhao, P. Shi, X. Zheng, and L. Zhang, "Adaptive tracking control for switched stochastic nonlinear systems with unknown actuator dead-zone," *Automatica*, vol. 60, pp. 193–200, 2015.

[15] F. Gao, Y. Wu, and Y. Liu, "Finite-time stabilization for a class of switched stochastic nonlinear systems with dead-zone input nonlinearities," *International Journal of Robust and Nonlinear Control*, vol. 28, no. 9, pp. 3239–3257, 2018.

[16] R. Xu and F. Zhang, "ϵ-Nash mean-field games for general linear-quadratic systems with applications," *Automatica*, vol. 114, p. 108835, 2020.

[17] R. Guo, "Projective synchronization of a class of chaotic systems by dynamic feedback control method," *Nonlinear Dynamics*, vol. 90, no. 1, pp. 53–64, 2017.

[18] C. Qian and W. Lin, "A continuous feedback approach to global strong stabilization of nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 46, no. 7, pp. 1061–1079, 2001.

[19] W. Zha, C. Qian, and S. Yan, "Controller design for a class of nontriangular nonlinear systems with input dependent growth rate," *International Journal of Robust and Nonlinear Control*, vol. 29, no. 5, pp. 1325–1338, 2019.

[20] Z.-G. Liu and Y.-Q. Wu, "Universal strategies to explicit adaptive control of nonlinear time-delay systems with different structures," *Automatica*, vol. 89, pp. 151–159, 2018.

[21] F. Gao, Y. Wu, and F. Yuan, "Global output feedback stabilisation of high-order nonlinear systems with multiple time-varying delays," *International Journal of Systems Science*, vol. 47, no. 10, pp. 2382–2392, 2016.

[22] L. Xue, Z. Liu, Z. Sun, and W. Sun, "New results on robust tracking control for a class of high-order nonlinear time-delay systems," *International Journal of Systems Science*, vol. 50, no. 10, pp. 2002–2014, 2019.

[23] M. Cui, "Adaptive output feedback stabilization of random nonlinear systems with unmodeled dynamics driven by colored noise," *Mathematical Problems in Engineering*, vol. 2019, Article ID 8581374, 10 pages, 2019.

[24] Z. Song, P. Li, Z. Wang, X. Huang, and W. Liu, "Adaptive tracking control for switched uncertain nonlinear systems with input saturation and unmodeled dynamics," *IEEE Transactions on Circuits and Systems II: Express Briefs*, 2020.

[25] H. Wang, Y. Zou, P. X. Liu, X. Zhao, J. Bao, and Y. Zhou, "Neural-network-based tracking control for a class of time-delay nonlinear systems with unmodeled dynamics," *Neurocomputing*, 2019.

[26] L. Xue, W. Zhang, and Y. Lin, "Global output tracking control for high-order stochastic nonlinear systems with SISS inverse dynamics and time-varying delays," *Journal of the Franklin Institute*, vol. 353, no. 13, pp. 3249–3270, 2016.

[27] L. Xue, W. Zhang, and X. Xie, "Global practical tracking for stochastic time-delay nonlinear systems with SISS-like inverse dynamics," *Science China Information Sciences*, vol. 60, no. 12, Article ID 122201, 2017.

[28] Z. Wang, J. Yuan, Y. Pan, and D. Che, "Adaptive neural control for high order Markovian jump nonlinear systems with unmodeled dynamics and dead zone inputs," *Neurocomputing*, vol. 247, pp. 62–72, 2017.

[29] S. Yin, P. Shi, and H. Yang, "Adaptive fuzzy control of strict-feedback nonlinear time-delay systems with unmodeled dynamics," *IEEE Transactions on Cybernetics*, vol. 46, no. 8, pp. 1926–1938, 2015.

[30] Z. Liu, L. Xue, W. Sun, and Z. Sun, "Robust output feedback tracking control for a class of high-order time-delay nonlinear systems with input dead-zone and disturbances," *Nonlinear Dynamics*, vol. 97, no. 2, pp. 921–935, 2019.

[31] Z. Sun, Y. Shao, C. Chen, and Q. Meng, "Global output-feedback stabilization for stochastic nonlinear systems: a double-domination approach," *International Journal of Robust and Nonlinear Control*, vol. 28, no. 15, pp. 4635–4646, 2018.

[32] Z.-Y. Sun, Y. Shao, and C.-C. Chen, "Fast finite-time stability and its application in adaptive control of high-order nonlinear system," *Automatica*, vol. 106, pp. 339–348, 2019.

[33] P. A. Ioannou and J. Sun, *Robust Adaptive Control*, Courier Corporation, Chelmsford, MA, USA, 1996.