Review Article

Identifying Systemically Important Banks and Firms Based on a Multilayer DebtRank Model

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Received 2 March 2020; Revised 14 July 2020; Accepted 28 July 2020; Published 14 August 2020

Academic Editor: Palle E. Jorgensen

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The stability of the financial system plays a crucial role in the sustainable economic development. Hence, to identify systemically important banks and firms, we take lending relationships with different loan terms and common asset relationships with different investment cycles into consideration to present a multilayer DebtRank model of the bank-firm system. In the light of simulation research, we can obtain the following results. First, the bank-firm system constructed displays a significant core-periphery structure, which exists in the actual financial system. Then, only very few banks and firms show systemically important characteristics, where “important” subjects hold very high net assets and profits, while “fragile” subjects possess negative net assets and serious losses. Furthermore, the bank-firm multilayer DebtRank model presents a great stability to a certain extent. Overall, the multilayer DebtRank model constructed in this paper has certain theoretical reference value for the supervisory authorities to extract the internal characteristics of systemically important banks and firms and identify them effectively.

1. Introduction

The sustainable and healthy development of the economy cannot do without the stability of financial systems [1–3]. The global financial markets have presented multiple complex forms because of the expansion and deepening of financial activities in the process of financial globalization. Since the 2008 financial crisis, regulators have begun to pay more attention to systemically important financial institutions. Once the crisis occurs in systemically important financial institutions, it will cause great damage to the whole financial system. Based on this, in order to ensure the stable and healthy development of the financial system, it is necessary to accurately identify the systemically important financial institutions.

In the aspect of identifying systemically important financial institutions, scholars have given diversified measurement methods from different dimensions such as models and indexes. The CoVaR model is the current mainstream research method [4, 5]. In addition, The Shapley index is also widely used to measure the importance of various banking systems and clarify the important factors affecting the systemic importance of the financial system [6].

Similarly, the SRISK index is used to measure the capital shortage degree of financial institutions, which is an important means to reflect systemically important financial institutions [7]. Some scholars also study the systemic importance of financial systems from the perspective of the marginal expected shortfall (MES) [8] and the systemic expected shortfall (SES) [9].

However, the modern financial system is more and more complex, and the relationship between banks and firms is more and more close, which makes the use of financial network methods to study the systemic importance of financial systems more and more urgent [10]. In the field of the financial network research, scholars have achieved fruitful results. For instance, Martinez-Jaramillo et al. [11] measure systemically important banks from the perspective of the strength centrality, the degree centrality, the betweenness centrality, the closeness centrality, the entropy eigenvector centrality, and the PageRank centrality in financial networks. Nicosia et al. [12] propose the concept of the feedback centrality in complex networks based on the PageRank centrality. Referring to the feedback centrality, Rincón and Villalobos [13] use the authority centrality and the hub centrality to measure the importance of financial systems.

For the study of the systemic importance from the perspective of financial networks, in addition to the above network centrality indicators, the DebtRank centrality has also received attention. Considering that the calculation process of the feedback centrality has multiple feedback effects among network nodes, the DebtRank sets different impact states so that the nodes are not affected by the secondary impact, which can well identify the systemically important financial institutions. Battiston et al. [14] propose the DebtRank model to identify the global systemically important financial institutions from 2008 to 2010 and find that 22 institutions are at the core of the financial system during the financial crisis. The DebtRank model can detect the bank-firm default effect by exerting additional shocks on the bank-firm network and obtain more reliable results. For example, small regional banks will also have a greater impact on the economy by connecting with local important firms [15].

The related research of the DebtRank model mainly focuses on the single risk exposure of financial systems, which is difficult to truly describe the complexity and diversity of the business associations between real financial market subjects. In view of this, considering the lending relationships with different loan terms and the common asset relationships with different investment cycles, the multilayer DebtRank model of the bank-firm system is constructed. Compared with the existing research, the contribution of this paper is to construct a multilayer DebtRank model to identify systemically important banks and firms from the complexity and the diversity of bank-firm business associations. In addition, we analyze the internal characteristics of systemically important banks and firms and verify the robustness of the model. This paper is beneficial to deeply mine the internal relationships between the complex association structure and the systemic importance of financial systems, which has certain theoretical reference value for the supervisory authorities to extract the internal characteristics of systemically important banks and firms and identify them effectively.

The remainder of this paper is organized as follows. Section 2 describes the multilayer DebtRank model of the bank-firm system. In Section 3, we discuss the main simulation results, and the conclusion is in Section 4.

2. The Model

In the artificial financial system, we consider two types of subjects: banks and firms. The entire model is divided into three parts, namely, bank-firm balance sheet construction, bank-firm behavior evolution, and bank-firm systemic importance identification.

2.1. Bank-Firm Balance Sheet Construction

2.1.1. Bank Balance Sheet Construction. We assume that an individual bank's assets include interbank short-term loans (TSBL_z), interbank long-term loans (TLBL_z), firm short-term loans (TSFL_z), firm long-term loans (TLFL_z), short-term investments (TSI_z), long-term investments (TLI_z), and liquid assets (L_z), and that a bank's liabilities are composed of interbank short-term borrowings (TSBB_z),

interbank long-term borrowings (TLBB_z), short-term deposits (TSDP_z), long-term deposits (TLDP_z), and net assets (E_z). At time t , there are N_1 banks. Drawing on the experiences of the research conducted by Georg [16] and Li et al. [17], the bank balance sheet is presented as follows (Tables 1).

The indicators mentioned above can be measured as follows:

$$\begin{aligned}
 \text{TSBL}_{zt} &= \sum_{s=1}^{Q_b^{IL}} \sum_{z' \in \Phi_{t-s+1}} \text{SB}_{t-s+1}^{zz'}, \\
 \text{TLBL}_{zt} &= \sum_{s=1}^{Q_b^{IL}} \sum_{z' \in \Phi_{t-s+1}} \text{LB}_{t-s+1}^{zz'}, \\
 \text{TSFL}_{zt} &= \sum_{s=1}^{Q_b^{IL}} \sum_{j \in \Omega_{t-s+1}} \text{SB}_{t-s+1}^{zj}, \\
 \text{TLFL}_{zt} &= \sum_{s=1}^{Q_b^{IL}} \sum_{j \in \Omega_{t-s+1}} \text{LB}_{t-s+1}^{zj}, \\
 \text{TSI}_{zt} &= \sum_{s=1}^{Q_b^{IL}} \text{SI}_{t-s+1}^z, \\
 \text{TLI}_{zt} &= \sum_{s=1}^{Q_b^{IL}} \text{LI}_{t-s+1}^z, \\
 \text{TSBB}_{zt} &= \sum_{s=1}^{Q_b^{IL}} \sum_{z'' \in \Psi_{t-s+1}} \text{SB}_{t-s+1}^{z''z}, \\
 \text{TLBB}_{zt} &= \sum_{s=1}^{Q_b^{IL}} \sum_{z'' \in \Psi_{t-s+1}} \text{LB}_{t-s+1}^{z''z}, \\
 \text{TSDP}_{zt} &= \sum_{s=1}^{Q_b^{ID}} \text{SDP}_{t-s+1}^z, \\
 \text{TLDP}_{zt} &= \sum_{s=1}^{Q_b^{ID}} \text{LDP}_{t-s+1}^z,
 \end{aligned} \tag{1}$$

where Φ_z is the set of borrowing banks of the bank z ; Ω_z is the set of borrowing firms of the bank z ; Ψ_z is the set of creditor banks of the bank z ; $\text{SB}_t^{zz'} = \theta_b^{sL} B_t^{zz'}$ and $\text{LB}_t^{zz'} = (1 - \theta_b^{sL}) B_t^{zz'}$ denote interbank short-term loans and long-term loans from the bank z of the bank z' , respectively; $\text{SB}_t^{zj} = \theta_f^{sL} B_t^{zj}$ and $\text{LB}_t^{zj} = (1 - \theta_f^{sL}) B_t^{zj}$ represent bank-firm short-term loans and long-term loans from the bank z of the firm j , respectively; $\text{SB}_t^{z''z} = \theta_b^{sL} B_t^{z''z}$ and $\text{LB}_t^{z''z} = (1 - \theta_b^{sL}) B_t^{z''z}$ denote interbank short-term loans and long-term loans from the bank z'' of the bank z , respectively; $\text{SI}_t^z = \theta_b^{sI} I_t^z$ and $\text{LI}_t^z = (1 - \theta_b^{sI}) I_t^z$ represent short-term investments and long-term investments of the bank z , respectively; SDP_t^z and LDP_t^z denote short-term deposits and long-term deposits from depositors of the bank z and satisfy the formulas $\text{SDP}_{zt} = (1 - \gamma_s + 2\chi\gamma_s)\text{SDP}_{zt-1}$ and $\text{LDP}_{zt} = (1 - \gamma_l + 2\chi\gamma_l)\text{LDP}_{zt-1}$, respectively, where $\chi \in [0, 1]$ is a random variable and γ_s and

TABLE 1: Bank balance sheet construction.

Assets	Liabilities
Interbank short-term loans (TSBL _z)	Interbank short-term borrowings (TSBB _z)
Interbank long-term loans (TLBL _z)	Interbank long-term borrowings (TLBB _z)
Firm short-term loans (TSFL _z)	Short-term deposits (TSDP _z)
Firm long-term loans (TLFL _z)	Long-term deposits (TLDP _z)
Short-term investments (TSI _z)	
Long-term investments (TLI _z)	Net assets (E _z)
Liquid assets (L _z)	

γ_l are used to measure the short-term and long-term volatility of deposits; Q_b^{sL} and Q_b^{lL} are the maturity of interbank short-term loans and long-term loans, respectively; Q_f^{sL} and Q_f^{lL} are the maturity of bank-firm short-term loans and long-term loans, respectively; Q_b^{sI} and Q_b^{lI} are the maturity of bank short-term investments and long-term investments, respectively; Q_b^{sD} and Q_b^{lD} are the maturity of short-term deposits and long-term deposits, respectively; and θ_b^{sL} , θ_f^{sL} , and θ_b^{sI} represent the ratio of interbank short-term loans, the ratio of bank-firm short-term loans, and the ratio of bank short-term investments, respectively.

2.1.2. Firm Balance Sheet Construction. We assume that an individual firm's assets include production costs (TCP_j), short-term investments (TSI_j), long-term investments (TLI_j), and liquid assets (L_j) and that a firm's liabilities are composed of sale revenues (TSR_j), bank short-term loans (TSFL_j), bank long-term loans (TLFL_j), and net assets (E_j). At time t , there are N_2 banks. Drawing on the experiences of the research conducted by Ma et al. [18], the firm balance sheet is presented as follows (Table2).

The indicators mentioned above can be measured as follows:

$$\begin{aligned}
TCP_{jt} &= wN_{jt}^{\text{real}}, \\
TSR_j &= \mu_{jt}Y_{jt}^{\text{real}}, \\
TSI_{jt} &= \sum_{s=1}^{Q_f^{sI}} SI_{t-s+1}^j, \\
TSFL_{jt} &= \sum_{s=1}^{Q_f^{sL}} \sum_{z \in \Xi_{t-s+1}} SB_{t-s+1}^{zj}, \\
TLF_{jt} &= \sum_{s=1}^{Q_f^{lI}} LI_{t-s+1}^j, \\
TLFL_{j\bar{F}} &= \sum_{s=1}^{Q_f^{lL}} \sum_{z \in \Xi_{t-s+1}} LB_{t-s+1}^{zj},
\end{aligned} \tag{2}$$

TABLE 2: Firm balance sheet construction.

Assets	Liabilities
Production costs (TCP _j)	Sale revenues (TSR _j)
Short-term investments (TSI _j)	Bank short-term loans (TSFL _j)
Long-term investments (TLI _j)	Bank long-term loans (TLFL _j)
Liquid assets (L _j)	Net assets (E _j)

where w denotes the labor wage for producing the products; N_{jt}^{real} represents the actual labor for producing the products; $SI_t^j = \theta_f^{sI} I_t^j$ and $LI_t^j = (1 - \theta_f^{sI}) I_t^j$ denote short-term investments and long-term investments of the firm j , respectively; μ_{jt} means that the product price per unit output is randomly taken from the interval $(u_{\min}, u_{\max})$; Y_{jt}^{real} represents the actual output for producing the products and satisfies the formula $Y_{jt} = \varphi_1 E_{jt}^{\varphi_2} N_{jt}^{1-\varphi_2}$, where φ_1 , φ_2 , and $1 - \varphi_2$ denote the comprehensive technical level, capital output elasticity coefficient, and labor output elasticity coefficient, respectively; Ξ_z denotes the set of creditor banks of the firm j ; $SB_t^{zj} = \theta_f^{sL} B_t^{zj}$ and $LB_t^{zj} = (1 - \theta_f^{sL}) B_t^{zj}$ represent bank-firm short-term loans and long-term loans from the bank z of the firm j , respectively; Q_f^{sI} and Q_f^{lI} are the maturity of firm short-term investments and long-term investments, respectively; Q_f^{sL} and Q_f^{lL} are the maturity of bank-firm short-term loans and long-term loans, respectively; and θ_f^{sI} is the ratio of firm short-term investments.

2.2. Bank-Firm Behavior Evolution

2.2.1. Bank-Firm Lending. For simplicity, $\hat{L}_{it}$ represents the intraperiod cash of the bank or the firm. At time t , banks and firms with negative liquidity can be classified as potential debt banks and debt firms ($\hat{L}_{i't} < 0$). In addition, banks with sufficient liquidity can be classified as potential creditor banks ($\hat{L}_{i''t} > 0$). Banks and firms with negative liquidity conduct loan applications to banks with sufficient liquidity.

For the debt bank i' or the debt firm i' , each potential borrower randomly selects a certain proportion M^L of potential creditor banks and observes the credit lending rates that they can provide. The short-term lending rate and the long-term lending rate set by the potential creditor banks are shown in equations (3) and (4), respectively [19]:

$$sr_t^{i'i''} = r_0 + \alpha_b^s \left(\frac{L_{i''t}}{\text{TSDP}_{i''t} + \text{TLDP}_{i''t}} \right)^{-\alpha_b^s} + \alpha_b^s \left(\frac{\text{TSBB}_t^{i'i''} + \text{TLBB}_t^{i'i''}}{E_{i't}} \right)^{\alpha_b^s}, \quad (3)$$

$$lr_t^{i'i''} = r_0 + \alpha_b^l \left(\frac{L_{i''t}}{\text{TSDP}_{i''t} + \text{TLDP}_{i''t}} \right)^{-\alpha_b^l} + \alpha_b^l \left(\frac{\text{TSBB}_t^{i'i''} + \text{TLBB}_t^{i'i''}}{E_{i't}} \right)^{\alpha_b^l}, \quad (4)$$

where r_0 represents the risk-free interest rate; α_b^s and α_b^l denote the sensitivity of the short-term lending rate and the long-term lending rate to the bank-firm risk, respectively; $L_{i''t}/\text{TSDP}_{i''t} + \text{TLDP}_{i''t}$ represents the current ratio of creditor bank i'' ; and $\text{TSBB}_t^{i'i''} + \text{TLBB}_t^{i'i''}/E_{i't}$ denotes the debt leverage ratio of the debt bank i' or the debt firm i' .

The potential debt banks and debt firms borrow funds from the potential creditor banks based on the optimal partner selection mechanism. If they cannot obtain the sufficient liquidity from the first potential creditor bank, they contact other banks for the remaining funds until their total demand for liquidity is satisfied or all loanable funds are exhausted. As a potential creditor bank, the amount that can be used for fund lending is $\hat{L}_{i''t} - \pi(\text{TSDP}_{i''t} + \text{TLDP}_{i''t})$, where π denotes the deposit reserve ratio. If the creditor bank i'' has enough liquidity, it satisfies all its potential borrowers. Otherwise, it allocates its surplus liquidity in sequence according to the rank of the potential borrowers' net assets from high to low until all its loanable funds are exhausted.

2.2.2. Bank-Firm Investments. At time t , if the bank z and the firm j have residual liquidity, they will carry out investment activities. The investment amount satisfies the following formulas, respectively:

$$\begin{aligned} I_{zt} &= \max(0, \min(\text{Net } P_{zt}, \hat{L}_{zt} - \pi(\text{TSDP}_{z''t} + \text{TLDP}_{z''t}))), \\ I_{jt} &= \max(0, \min(\text{Net } P_{jt}, \hat{L}_{jt})), \end{aligned} \quad (5)$$

where $\text{Net } P_{zt}$ denotes the net profit of the bank z and $\text{Net } P_{zt} = E_{zt} - E_{zt-1}$; $\text{Net } P_{jt}$ represents the net profit of the firm j and $\text{Net } P_{jt} = E_{jt} - E_{jt-1}$.

Suppose there are N_3 types of risky assets; the return rate of the k^{th} risky asset is x_k , which is abbreviated as $x = (x_1, \dots, x_{N_3})^T$. The return rate of risky assets is affected by common risk factors and their idiosyncratic risk factors. Therefore, the return rate of the k^{th} risky asset satisfies $x_{kt} = \beta_k x_{kt}^c + x_{kt}^g$, where the return rate of the common risk factor x_{kt}^c and the return rate of the idiosyncratic risk factor x_{kt}^g obey the normal distributions $x_{kt}^c \sim N(0, \sigma_c^2)$ and $x_{kt}^g \sim N(0, \sigma_g^2)$, respectively. β_k represents the degree and the direction of the k^{th} risky asset affected by the common risk factor and is randomly taken from the interval $[-1, 1]$.

Banks and firms randomly choose a certain proportion of risk assets for equal investments. For simplicity, the selected risky assets are recorded as $i = 1, 2, \dots, N_4$, and the corresponding return rates are abbreviated as

$x' = (x'_1, \dots, x'_{N_4})^T$. Then, the short-term returns and the long-term returns of investments by banks and firms are measured as follows:

$$\begin{aligned} sp_t^i &= \left(\prod_{s=1}^{Q^s} \left(1 + \left(\frac{\sum_{k=1}^{N_4} x'_{kt}}{N_4} \right) \right) \right) - 1, \\ lp_t^i &= \left(\prod_{s=1}^{Q^l} \left(1 + \left(\frac{\sum_{k=1}^{N_4} x'_{kt}}{N_4} \right) \right) \right) - 1. \end{aligned} \quad (6)$$

2.3. Bank-Firm Systemic Importance Identification. In this paper, the systemic importance of the bank-firm system refers to the impact of losses suffered by banks and firms on the whole bank-firm system through lending correlations and common asset correlations. Considering the lending relationships with different loan terms and the common asset relationships with different investment cycles, we use the multilayer pressure diffusion to evaluate the multilayer DebtRank of banks and firms and then identify the systematically important banks and firms.

Drawing on the experience of research conducted by Bardoscia et al. [20], the pressure level of banks $hb(t)$ and the pressure level of firms $hf(t)$ are given at time t , respectively. The pressure levels satisfy $hb(t) \in [0, 1]$ and $hf(t) \in [0, 1]$, respectively. The default probability of the bank under the pressure level $hb(t)$ is $pb(t)$ and satisfies $pb(t) = hb(t)e^{\alpha[nb(t)-1]}$, where α represents the default amplification factor. Similarly, the default probability of the firm under the pressure level $hf(t)$ is $pf(t)$ and satisfies $pf(t) = hf(t)e^{\alpha[hf(t)-1]}$.

Considering that if there is a loop in the network, the pressure will circulate between nodes. In order to avoid the nodes participating in the pressure diffusion process repeatedly, drawing on the experiences of the research conducted by Battiston et al. [14] and Poledna et al. [21], the nodes in the bank-firm system are given three pressure states, pressured (D), unpressured (U), and inactive (I) at time t . For simplicity, $s_z^b(t)$ and $s_j^f(t)$ denote the pressure states of the bank z and the firm j at time t , respectively. The whole process of the pressure diffusion is as follows. When the node in the unpressured state (U) is impacted by the external shock, it will be transformed into the pressured state (D), which will infect other nodes in the next round of the pressure diffusion and transform itself into the inactive state (I). When all nodes are in the states of unpressured (U) or inactive (I), the whole process of the pressure diffusion ends. The stress states of banks and firms are calculated as follows:

$$s_z^b(t) = \begin{cases} D, & \text{if } hb_z(t) > 0 \text{ and } s_z^b(t-1) \neq I, \\ U, & \text{if } hb_z(t) = 0 \text{ and } s_z^b(t-1) \neq I, \\ I, & \text{otherwise,} \end{cases} \quad (7)$$

$$s_j^f(t) = \begin{cases} D, & \text{if } hf_j(t) > 0 \text{ and } s_j^f(t-1) \neq I, \\ U, & \text{if } hf_j(t) = 0 \text{ and } s_j^f(t-1) \neq I, \\ I, & \text{otherwise,} \end{cases}$$

where $\forall z \in B$ and $\forall j \in F$. B denotes the set of all banks and F denotes the set of all firms.

At the initial time $t = 0$, we assume that S is the set of impacted nodes of banks and firms. The pressure levels and pressure states of banks and firms are set as follows:

$$\begin{aligned} hb_z(0) &= \begin{cases} 1, & z \in S, \\ 0, & z \notin S, \end{cases} \\ hf_j(0) &= \begin{cases} 1, & j \in S, \\ 0, & j \notin S, \end{cases} \\ s_z^b(0) &= \begin{cases} D, & z \in S, \\ U, & z \notin S, \end{cases} \\ s_j^f(0) &= \begin{cases} D, & j \in S, \\ U, & j \notin S. \end{cases} \end{aligned} \quad (8)$$

In order to further describe the bank-firm pressure level in different network layers, we set the number of layers of the complex network as m and $\{m_1, m_2, m_3, m_4\} \in m$, where m_1, m_2, m_3 , and m_4 , respectively, represent the short-term common asset network, the short-term lending network, the long-term common asset network, and the long-term lending network between banks and firms. In addition, we set X^1, X^2, X^3 , and X^4 as the short-term common asset scale matrix, the short-term lending scale matrix, the long-term common asset size matrix, and the long-term lending scale matrix between banks and firms, respectively.

At time t , the pressure level of the bank z in the different network layers is expressed as the sum of the pressure level in the previous state and the pressure level caused by all firm nodes in the present state. The calculation formula is as follows:

$$hb_z^m(t) = \min \left(1, hb_z^m(t-1) + \sum_{j \in F, s_j^f(t-1)=D} W_{zj}^m \Delta p f_j^m(t-1) \right), \quad (9)$$

where $W_{zj}^{m_1}, W_{zj}^{m_2}, W_{zj}^{m_3}$, and $W_{zj}^{m_4}$ represent the short-term common asset sensitivity coefficient, the short-term lending sensitivity coefficient, the long-term common asset sensitivity coefficient, and the long-term lending sensitivity coefficient of the bank z to the firm j , respectively. They satisfy the calculation formulas as follows:

$$\begin{aligned} W_{zj}^{m_1} &= \frac{X_{zj}^1}{\sum_{j \in F} X_{zj}^1}, \\ W_{zj}^{m_2} &= \frac{X_{zj}^2}{\sum_{j \in F} X_{zj}^2}, \\ W_{zj}^{m_3} &= \frac{X_{zj}^3}{\sum_{j \in F} X_{zj}^3}, \\ W_{zj}^{m_4} &= \frac{X_{zj}^4}{\sum_{j \in F} X_{zj}^4}. \end{aligned} \quad (10)$$

In addition, $\Delta p f_j^m(t-1)$ denotes the difference of the default probability received by the bank z from the pressured nodes and satisfies $\Delta p f_j^m(t-1) = p f_j^m(t-1) - p f_j^m(t-2)$.

Similarly, at time t , the pressure level of the firm j in the different network layers is expressed as the sum of the pressure level in the previous state and the pressure level caused by all bank nodes in the present state. The calculation formula is as follows:

$$hf_j^m(t) = \min \left(1, hf_j^m(t-1) + \sum_{z \in B, s_z^b(t-1)=D} W_{jz} \Delta p b_z^m(t-1) \right), \quad (11)$$

where $W_{jz}^{m_1}, W_{jz}^{m_2}, W_{jz}^{m_3}$, and $W_{jz}^{m_4}$ represent the short-term common asset sensitivity coefficient, the short-term lending sensitivity coefficient, the long-term common asset sensitivity coefficient, and the long-term lending sensitivity coefficient of the firm j to the bank z , respectively. They satisfy the calculation formulas as follows:

$$\begin{aligned} W_{jz}^{m_1} &= \frac{X_{jz}^1}{\sum_{z \in B} X_{jz}^1}, \\ W_{jz}^{m_2} &= \frac{X_{jz}^2}{\sum_{z \in B} X_{jz}^2}, \\ W_{jz}^{m_3} &= \frac{X_{jz}^3}{\sum_{z \in B} X_{jz}^3}, \\ W_{jz}^{m_4} &= \frac{X_{jz}^4}{\sum_{z \in B} X_{jz}^4}. \end{aligned} \quad (12)$$

In addition, $\Delta p b_z^m(t-1)$ denotes the difference of the default probability received by the firm j from the pressured nodes and satisfies $\Delta p b_z^m(t-1) = p b_z^m(t-1) - p b_z^m(t-2)$.

Assume that the pressure diffusion stops at $t = T$. The DebtRank of the impacted node $i (i \in S)$ in the different network layers is expressed as the difference between the total pressure level of the four network layers at time $t = T$ and the pressure level of the impacted node $i (i \in S)$ at the initial time $t = 0$. The corresponding calculation formula is as follows:

$$DR_i^m = \frac{\sum_{z \in B} hb_z^m(t) v_z^m + \sum_{j \in F} hf_j^m(t) v_j^m}{2} - hb_i^m(0) v_i^m, \quad (13)$$

where $v_z^m = \{v_z^{m_1}, v_z^{m_2}, v_z^{m_3}, v_z^{m_4}\}$ represent the short-term investment weight, the short-term lending weight, the long-term investment weight, and the long-term lending weight held by the bank node z in all banks at time t , respectively. $v_j^m = \{v_j^{m_1}, v_j^{m_2}, v_j^{m_3}, v_j^{m_4}\}$ represent the short-term investment weight, the short-term lending weight, the long-term investment weight, and the long-term lending weight held by the firm node j in all firms at time t , respectively. In addition, v_z^m and v_j^m can be calculated as follows:

$$\begin{aligned} v_z^{m_1} &= \frac{\sum_{j \in F} X_{zj}^1}{\sum_{z \in B} \sum_{j \in F} X_{zj}^1}, \\ v_z^{m_2} &= \frac{\sum_{j \in F} X_{zj}^2}{\sum_{z \in B} \sum_{j \in F} X_{zj}^2}, \\ v_z^{m_3} &= \frac{\sum_{j \in F} X_{zj}^3}{\sum_{z \in B} \sum_{j \in F} X_{zj}^3}, \\ v_z^{m_4} &= \frac{\sum_{j \in F} X_{zj}^4}{\sum_{z \in B} \sum_{j \in F} X_{zj}^4}, \\ v_j^{m_1} &= \frac{\sum_{z \in B} X_{zj}^1}{\sum_{j \in F} \sum_{z \in B} X_{zj}^1}, \\ v_j^{m_2} &= \frac{\sum_{z \in B} X_{zj}^2}{\sum_{j \in F} \sum_{z \in B} X_{zj}^2}, \\ v_j^{m_3} &= \frac{\sum_{z \in B} X_{zj}^3}{\sum_{j \in F} \sum_{z \in B} X_{zj}^3}, \\ v_j^{m_4} &= \frac{\sum_{z \in B} X_{zj}^4}{\sum_{j \in F} \sum_{z \in B} X_{zj}^4}. \end{aligned} \quad (14)$$

Based on the above analysis, the multilayer DebtRank of the impacted node i in the bank-firm multilayer network can be expressed as follows:

$$DR_i = \frac{DR_i^{m_1} + DR_i^{m_2} + DR_i^{m_3} + DR_i^{m_4}}{4}. \quad (15)$$

3. Simulations

3.1. Model Parameters. Drawing on the experiences of research led by Li et al. [17] and Ma et al. [18], the simulation parameter values are set in this section. The simulation is performed with 50 banks and 100 firms over a time span of 200. At the initial time of the simulation, 50 banks are divided into two categories. The first category includes 10 banks with $E_{z0} = 50$, $SDP_{z0} = 15$, $LDP_{z0} = 35$, and $L_{z0} = 100$. The second category has 40 banks with $E_{z0} = 10$,

$SDP_{z0} = 3$, $LDP_{z0} = 7$, and $L_{z0} = 20$. In addition, 100 firms are divided into two categories. The first category includes 20 firms with $E_{j0} = 5$ and $L_{j0} = 5$. The second category has 80 firms with $E_{j0} = 1$ and $L_{j0} = 1$. Table 3 summarizes the other benchmark parameters of the model.

3.2. Network Structures of Bank-Firm System. The bank-firm system constructed in this paper contains four network layers, including the short-term common asset network, the short-term lending network, the long-term common asset network, and the long-term lending network. The lending network describes the lending relationship between banks and firms, and the common asset network represents the common holding relationship of external assets between banks and firms. Figure 1 illustrates the complex network structure diagram of the bank-firm system at $t = 200$. As is shown, the network is represented by nodes and edges, with the left nodes representing banks and the right nodes representing firms. The connected sides indicate the common asset relationships and the lending relationships between banks and firms. As can be seen from Figure 1, the bank-firm network is closely connected, which shows that the financial business between banks and firms is closely related. The characteristics of bank-firm network structures need to be further explored.

The node degree is a simple and important concept to describe the characteristics of bank-firm network structures. The larger the node degree, the more active the node in the bank-firm network. Drawing on the experience of research conducted by Boccaletti et al. [22], we use the multilayer network node degree to analyze the bank-firm network with four network layers. Figure 2 illustrates the node degree distributions of the bank-firm multilayer networks at different times ($t = 80, t = 120, t = 160$, and $t = 200$), where the horizontal and vertical coordinates represent the bank-firm number and the multilayer network node degree, respectively. Numbers 1–50 are banks and numbers 51–150 are firms.

It can be seen from Figure 2 that the operation results of the bank-firm system at different times are similar. In other words, most banks possess larger node degrees and are at the core of the bank-firm network while a few banks and all firms have smaller node degrees and are at the periphery of the bank-firm network. This indicates that the bank-firm system constructed in this paper has always shown a significant core-periphery structure at different times, which is also verified in the empirical study of Bargigli et al. [23]. This also means that most banks in the whole bank-firm system are highly active while a few banks and all firms have relatively low activities.

In addition, Figure 3 illustrates the complementary cumulative distribution function (CCDF) of total assets of the bank-firm system at different times ($t = 80, t = 120, t = 160$ and $t = 200$). Through the fitting analysis, we can find that the distribution of total assets of the bank-firm system in Figures 3(a) and 3(d) can be described as lognormal distributions with Pareto tails, where the tail exponents are 6.6040, 8.4225, 4.4721, and 6.7831, respectively. Similar conclusions have appeared in the empirical study of Chinese

TABLE 3: Benchmark parameters of the model.

Parameter	Description	Value
Q_b^{sL}	Maturity of interbank short-term loans	1
Q_f^{sL}	Maturity of bank-firm short-term loans	1
Q_b^{sI}	Maturity of bank short-term investments	1
Q_b^{sD}	Maturity of short-term deposits	1
Q_f^{sI}	Maturity of firm short-term investments	1
θ_b^{sL}	Ratio of interbank short-term loans	0.5
θ_b^{sI}	Ratio of bank short-term investments	0.5
γ_s	Short-term volatility of deposits	0.3
w	Labor wage for producing products	2
r_0	Risk-free interest rate	0.025
α_b^s	Sensitivity of the short-term lending rate	0.1
φ_1	Firm comprehensive technical level	1.2
σ_c^2	Stock common risk factor variance	0.06
M^I	Selection ratio of risk assets	0.01
α	Default amplification factor	0.1
Q_b^{lL}	Maturity of interbank long-term loans	10
Q_f^{lL}	Maturity of bank-firm long-term loans	10
Q_b^{lI}	Maturity of bank long-term investments	10
Q_b^{lD}	Maturity of long-term deposits	10
Q_f^{lI}	Maturity of firm long-term investments	10
θ_f^{sL}	Ratio of bank-firm short-term loans	0.5
θ_f^{sI}	Ratio of firm short-term investments	0.5
γ_l	Long-term volatility of deposits	0.5
μ	Product price per unit output	(0.5, 2.5)
π	Deposit reserve ratio	0.2
α_b^l	Sensitivity of the long-term lending rate	0.2
φ_2	Firm capital output elasticity coefficient	0.8
σ_g^2	Stock idiosyncratic risk factor variance	0.03
M^L	Selection ratio of creditor banks	0.3

FIGURE 1: Complex network at $t = 200$.

financial market by Xu et al. [24]. This suggests that most banks and firms have large-scale assets while only a few banks and all firms possess a small amount of wealth.

3.3. Systemic Importance of Bank-Firm System. The above research shows that the bank-firm system constructed in this paper shows the core-periphery structure, and most banks

are active at the core of the network, while a few banks and firms are relatively less active at the periphery of the network. It is also very important to identify the systemically important banks and firms. Based on this, we use the multilayer DebtRank to analyze the problem. In order to explore whether there are differences in the systemic importance between banks and firms at different times during the operation of the bank-firm system, Figure 4 calculates the

FIGURE 2: Node degree distributions of bank-firm networks at different times. (a) $t = 80$. (b) $t = 120$. (c) $t = 160$. (d) $t = 200$.

bank-firm multilayer DebtRank at different times ($t = 80$, $t = 120$, $t = 160$, and $t = 200$), where the horizontal and vertical coordinates represent the bank-firm number and the multilayer DebtRank, respectively. Numbers 1–50 are banks and numbers 51–150 are firms.

Figure 4 presents the operation results of the bank-firm system at different times are similar. In other words, most nodes of banks and firms have a multilayer DebtRank between 0 and 0.3, showing weak systemic importance. However, only very few nodes of banks and firms (the bank of No. 7 and the firm of No. 137) have a multilayer DebtRank that exceeds 0.5, showing strong systemic importance. This indicates that only very small numbers of banks and firms show the systemic importance in the bank-firm system composed of 50 banks and 100 firms. Therefore, we should strengthen the monitoring of the financial situations and the operation conditions of these systemically important banks and firms and identify their “importance” and “vulnerability” significantly. This also reflects the problem of “too connected to fail” in the real bank-firm system to prevent the financial systemic risk effectively.

The aforementioned research results show that only very small numbers of banks and firms show the systemic importance in the bank-firm system while the internal characteristics of systemically important banks and firms need to be further explored. Based on this, considering that the financial situations and the operation conditions of banks and firms may have significant impacts on the systemic importance of the bank-firm system [21], we select two indicators of net assets and profits for in-depth analysis.

In the actual operation process of the bank-firm system, due to the differences in asset scales and operation conditions between banks and firms, there may be large

differences in the net assets between banks and firms. Whether this will affect the systemic importance of banks and firms remains to be further studied. Based on this, Figure 5 presents the distribution of the net assets and their multilayer DebtRank of the bank-firm system at $t = 200$. The abscissa represents the net assets of banks and firms, and the ordinate represents the multilayer DebtRank of banks and firms. In Figure 5, all banks are represented by blue diamonds, and all firms are represented by red squares.

It can be seen from Figure 5 that the net assets of most banks and firms are in the interval (0, 200), and the net assets of most banks are higher than those of the firms. The multilayer DebtRank of most banks and firms are between 0 and 0.3, while only the bank of No. 7 and the firm of No. 137 show a strong systemic importance. The bank of No. 7 owns the highest net asset and the highest multilayer DebtRank, which shows an obvious “importance.” However, the firm of No. 137 has the negative net asset and its multilayer DebtRank remains a high level, showing a significant “vulnerability.” It can be concluded that for the systemically important banks and firms, “important” subjects possess high net assets, while “vulnerable” subjects hold negative net assets.

Banks and firms in good operating conditions may gradually evolve into systemically important banks and firms through the accumulation of profits while poorly managed banks and firms may gradually become systemically vulnerable banks and firms due to frequent losses. Whether this will affect the systemic importance of banks and firms remains to be further studied. Based on this, Figure 6 presents the distribution of the profits and their multilayer DebtRank of the bank-firm system at $t = 200$. The abscissa represents the profits of banks and firms, and the ordinate represents the multilayer DebtRank of banks and firms. In Figure 6, all

FIGURE 3: Complementary cumulative distribution function of assets at different times. (a) $t = 80$. (b) $t = 120$. (c) $t = 160$. (d) $t = 200$.

banks are represented by blue circles, and all firms are represented by red triangles.

It can be seen from Figure 6 that the profits of most banks and firms are in the interval $(-2, 5)$, and the multilayer DebtRank of most banks and firms are between 0 and 0.3. We can see that only the bank of No. 7 and the firm of No. 137 show a strong systemic importance. The bank of No. 7 owns the highest profit and the highest multilayer DebtRank, which shows an obvious “importance.” However, the firm of No. 137 has the negative profit and its multilayer DebtRank remains a high level, showing a significant “vulnerability.” This indicates that, for the systemically important banks and firms, “important” subjects hold high profits while “vulnerable” subjects have serious losses.

3.4. Robustness Analysis. In order to ensure the robustness of the simulation results, it is necessary to further verify the robustness of the bank-firm multilayer DebtRank.

Considering that the parameter α represents the bank-firm default amplification factor, it depicts the correlation between the default probability of banks and firms and the pressure level, which plays an important role in the pressure diffusion of the bank-firm system. Particularly, when $\alpha = 0$, the default probability of banks and firms is linearly related to the pressure level. When $\alpha > 0$, there is a nonlinear correlation between the default probability of banks and firms and the pressure level. In order to further verify the robustness of the bank-firm evolution process, Figure 7 presents the bank-firm multilayer DebtRank with different α values ($\alpha = 0$, $\alpha = 0.1$, and $\alpha = 0.3$) at different times ($t = 80$, $t = 120$, $t = 160$, and $t = 200$).

It can be seen from Figure 7(a) that the multilayer DebtRank of banks and firms are similar under different α values ($\alpha = 0$, $\alpha = 0.1$, and $\alpha = 0.3$) at $t = 80$. For the systemically important bank of No. 7, the multilayer DebtRank is 0.593, 0.590, and 0.585 under $\alpha = 0$, $\alpha = 0.1$, and $\alpha = 0.3$, respectively. For the systemically vulnerability firm of No.

FIGURE 4: Bank-firm multilayer DebtRank at different times. (a) $t = 80$. (b) $t = 120$. (c) $t = 160$. (d) $t = 200$.

FIGURE 5: The distribution of the net assets and their multilayer DebtRank.

FIGURE 6: The distribution of the profits and their multilayer DebtRank.

137, the multilayer DebtRank is 0.604, 0.602, and 0.596 in the three cases, respectively. For all nodes of banks and firms, with the gradual increase of α , the multilayer DebtRank decreases slightly. Under different α values, the distribution

of the bank-firm multilayer DebtRank is approximately the same. This shows that the increase of the bank-firm default amplification factor can only reduce the multilayer DebtRank very limitedly, but it can maintain a good robustness to

FIGURE 7: Bank-firm multilayer DebtRank with different α values at different times. (a) $t = 80$. (b) $t = 120$. (c) $t = 160$. (d) $t = 200$.

the overall systemic importance of banks and firms. In addition, it can be seen from Figures 7(a)–7(d) that the overall systemic importance of the bank-firm system keeps a very good stability at different times ($t = 80$, $t = 120$, $t = 160$, and $t = 200$). In other words, the multilayer DebtRank constructed in this paper shows a good robustness to a certain extent.

4. Conclusions

The multilayer DebtRank is beneficial to deeply mine the internal relationships between the complex association structure and the systemic importance of financial systems. Considering the lending relationships with different loan terms and the common asset relationships with different investment cycles, the multilayer DebtRank model of the bank-firm system is constructed. The entire model is divided into three parts, namely, bank-firm balance sheet construction, bank-firm behavior evolution, and bank-firm systemic importance identification. This paper focuses on the identification of systemically important banks and firms, the internal characteristics of systemically important banks and firms, and the robustness test of the model.

The simulation results are as follows. Firstly, the bank-firm system constructed shows a significant core-periphery structure at different times, which means that most banks in

the whole bank-firm system are highly active, while a few banks and all firms have relatively low activities. Secondly, we find that the distributions of total assets of the bank-firm system are lognormal distributions with Pareto tails, which suggests that most of the banks and firms have large-scale assets, while only a few banks and all firms possess a small amount of wealth. Thirdly, we discover that only very small numbers of banks and firms display the systemic importance in the bank-firm system, where “important” subjects hold very high net assets and profits, while “fragile” subjects possess negative net assets and serious losses. This suggests that we should pay more attention to the systemically important banks and firms that are “too connected to fail.” Finally, we verify that the overall systemic importance of the bank-firm system keeps a very good stability at different times.

It is urgent for the government departments to better understand the number and scale distribution of the current Chinese banks and firms, balance the development mode of related business, reasonably optimize the loan mechanism, and establish a multidirectional risk monitoring system. This study reveals the microbasis of complex business associations between banks and firms, which has certain theoretical reference value for the regulatory authorities to extract the internal characteristics of systemically important banks and firms and prevent financial systemic risks.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors' Contributions

Qianting Ma and Kun Yang contributed equally to this work. They are cofirst authors.

Acknowledgments

The authors are grateful to the financial support from the Postgraduate Research and Practice Innovation Program of Jiangsu Province (KYCX20_0165) and the Scientific Research Foundation of the Graduate School of Southeast University (YBPY1971).

References

- [1] S. Rönqvist and P. Sarlin, "Bank distress in the news: describing events through deep learning," *Neurocomputing*, vol. 264, pp. 57–70, 2017.
- [2] H. Dastkhan and N. S. Gharneh, "Simulation of contagion in the stock markets using cross-shareholding networks: a case from an emerging market," *Computational Economics*, vol. 53, no. 3, pp. 1071–1101, 2019.
- [3] P. Giudici, P. Sarlin, and A. Spelta, "The interconnected nature of financial systems: direct and common exposures," *Journal of Banking and Finance*, vol. 112, no. 3, p. 2, 2020.
- [4] T. Adrian and M. K. Brunnermeier, "CoVar: a method for macroprudential regulation," *Federal Reserve Bank of New York Staff Report*, vol. 106, no. 7, pp. 1705–1741, 2009.
- [5] G. Chen, L. Zhong, and Y. Zhang, "Systemic linkages in the Chinese banking system: the asymmetric CoVaR approach," *Xitong Gongcheng Lilun yu Shijian*, vol. 37, no. 1, pp. 61–79, 2017.
- [6] N. Tarashev, C. Borio, and K. Tsatsaronis, *Attributing Systemic Risk to Individual Institutions*, Bank for International Settlements, Basel, Switzerland, 2010.
- [7] G.-J. Wang, Z.-Q. Jiang, M. Lin, C. Xie, and H. E. Stanley, "Interconnectedness and systemic risk of China's financial institutions," *Emerging Markets Review*, vol. 35, pp. 1–18, 2018.
- [8] I. Talasli, "Systemic risk analysis of Turkish financial institutions with systemic expected shortfall," *Central Bank Review*, vol. 13, pp. 25–40, 2013.
- [9] J. Idier, G. Lamé, and J.-S. Mésonnier, "How useful is the marginal expected shortfall for the measurement of systemic exposure? A practical assessment," *Journal of Banking & Finance*, vol. 47, pp. 134–146, 2014.
- [10] H. Dastkhan, "Network-based early warning system to predict financial crisis," *International Journal of Finance & Economics*, 2019.
- [11] S. Martinez-Jaramillo, B. Alexandrova-Kabadjova, B. Bravo-Benitez, and J. P. Solórzano-Margain, "An empirical study of the Mexican banking system's network and its implications for systemic risk," *Journal of Economic Dynamics and Control*, vol. 40, pp. 242–265, 2014.
- [12] V. Nicosia, R. Criado, M. Romance et al., "Controlling centrality in complex networks," *Scientific Reports*, vol. 2, p. 218, 2012.
- [13] C. E. L. Rincón and J. P. Villalobos, *Authority Centrality and Hub Centrality as Metrics of Systemic Importance of Financial Market Infrastructures* (No. 754), Banco de la Republica de Colombia, Bogotá, Colombia, 2013.
- [14] S. Battiston, M. Puliga, R. Kaushik, P. Tasca, and G. Caldarelli, "Debrank: too central to fail? Financial networks, the FED and systemic risk," *Scientific Reports*, vol. 2, no. 1, pp. 1–6, 2012.
- [15] F. Abergel, H. Aoyama, B. K. Chakrabarti, A. Chakraborti, and A. Ghosh, *Econophysics of Agent-Based Models*, Springer, Berlin, Germany, 2013.
- [16] C.-P. Georg, "The effect of the interbank network structure on contagion and common shocks," *Journal of Banking & Finance*, vol. 37, no. 7, pp. 2216–2228, 2013.
- [17] S. Li, X. Sui, and Q. Ma, "An endogenous network model of banking systems," *Complexity*, vol. 21, no. 1, pp. 512–520, 2016.
- [18] Q. Ma, J. He, and S. Li, "Endogenous network of firms and systemic risk," *Physica A: Statistical Mechanics and Its Applications*, vol. 492, pp. 2273–2280, 2018.
- [19] M. Zhang, J. He, and S. Li, "Interbank lending, network structure and default risk contagion," *Physica A: Statistical Mechanics and Its Applications*, vol. 493, pp. 203–209, 2018.
- [20] M. Bardoscia, F. Caccioli, J. I. Perotti et al., "Distress propagation in complex networks: the case of non-linear debt-rank," *PLoS One*, vol. 11, no. 10, Article ID e0163825, 2016.
- [21] S. Poledna, A. Hinteregger, and S. Thurner, "Identifying systemically important companies by using the credit network of an entire nation," *Entropy*, vol. 20, no. 10, p. 792, 2018.
- [22] S. Boccaletti, G. Bianconi, R. Criado et al., "The structure and dynamics of multilayer networks," *Physics Reports*, vol. 544, no. 1, pp. 1–122, 2014.
- [23] L. Bargigli, G. Di Iasio, L. Infante et al., "Interbank markets and multiplex networks: centrality measures and statistical null models," *Understanding Complex Systems*, Springer, Cham, Switzerland, pp. 179–194, 2016.
- [24] Y. Xu, Y. Wang, X. Tao, and L. Ližbetinová, "Evidence of Chinese income dynamics and its effects on income scaling law," *Physica A: Statistical Mechanics and Its Applications*, vol. 487, pp. 143–152, 2017.