Research Article

Multiconsensus of Nonlinear Multiagent Systems with Intermittent Communication

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Compared with single consensus, the multiconsensus of multiagent systems with nonlinear dynamics can reflect some real-world cases. This paper proposes a novel distributed law based only on intermittent relative information to achieve the multiconsensus. By constructing an appropriate Lyapunov function, sufficient conditions on control parameters are derived to undertake the reliability of closed-loop dynamics. Ultimately, the availability of results is completely validated by these numerical examples.

1. Introduction

Multiagent systems have attracted much attention in the field as computer science, vehicle systems, unmanned aerial vehicles, or formation flight of spacecraft since 2009. In most of studies on consensus problem, researchers always adopt the same method that makes all agents finally reach the same value in systems [1–5]. Due to various parameters as sudden changes on environment or cooperative tasks in the reality, the purpose of our research also becomes multiple.

In [6], the consensus problem from asynchronous group of the discrete-time heterogeneous multiagent system under dynamic-change interaction topology is discussed and studied briefly. For different agents, two asynchronous consensus protocols are given in this paper. Based on fixed and switching topology, the multiconsensus of first-order multiagent system is discussed. And we assumed that interactions could reach balance between two subnetworks in [6, 7]. The relationship between multiconsensus ability and the underlying digraph topology is discussed in [8]. For fixed communication networks, the author proposes a consensus protocol which can be applied to two different second-order multiagent systems under the same assumption [9]. By using pinning control method in [10–12], we can obtain some criteria on multiconsensus of networks without assuming the balance of network

topology. For multiconsensus problem from discrete-time multiagent systems with stochastic and fixed topologies, the author performs a professional study in [13]. In [14], the author performs the research on multiconsensus control of switching-directed interaction and fixed topology in common linear multiagent systems, based on pinning control techniques, matrix analysis theory, and Lyapunov stability theory. The cluster consensus of multiagent dynamical systems with impulsive effects and coupling delays was investigated in [15], where interactions among agents were uncertain.

In other words, above conclusion on the consensus of multiagent system with nonlinear dynamics is mostly based on common assumption that information is transmitted continuously between all agents, which means that each single agent shares the information with its neighbors without any communication constraints. However, this is not the fact in reality. For instance, all agents can only get the information from their neighbors during certain disconnected time intervals, as a reason of communication restrictions. In [16–23], intermittent control has attracted more attention. For distributed consensus problem from intermittent control of a time-invariant undirected communication topology in the linear multiagent system, the author designs a type of distributed-observer protocols. In [24], the consensus problem on periodical-intermittent

control of second-order agent networks, which is based on matrix theory, Lyapunov control method, and algebraic graph theory, is discussed. Based on the above discussion and research, this paper mainly studies on some characteristics of the second-order multiagent system, such as multiconsensus. In the same subnet, nonlinear dynamics of all agents are the same, while all agents in different subnet have different dynamics. Multiconsensus indicates that all agents in every subgroup can be consistent. Between different groups, there is no consistent value. The research performed in this paper can be summarized as three points: firstly, the multiconsensus of the multiagent system with nonlinear dynamics is studied. Secondly, a novel multiconsensus law, which is devised through intermittent and relative state information, is more general than other second-order multiconsensus protocol. And, in such a protocol, all agents always need to communicate with their neighbors.

In Section 2 of this paper, the research model has been designed. In Section 3, we perform the study and discussion on the multiconsensus problem of second-order multiagent systems with nonlinear dynamics. In Section 4, two numerical examples are given to prove the effectiveness on the designed protocol. And the conclusion is summarized in Section 5.

2. Preliminary

2.1. Algebraic Graph Theory. In general, the communication topology between agents in a multiagent system is described by a directed graph. Let $G = (V, \varepsilon, A)$ be a system communication topology diagram consisting of N nodes, the vertex set $V = \{v_1, v_2, \dots, v_n\}$ is nonempty finite, the edge set $\varepsilon \subseteq V \times V$, and a nonsymmetric $A = (a_{ij})_{n \times n}$ is nonnegative weighted adjacency matrix. $A = (a_{ij})_{n \times n}$ is defined as $a_{ij} \neq 0$ if $\varepsilon_{ij} \in \varepsilon$ and $a_{ij} = 0$ otherwise. There are no self-loops, i.e., $a_{ii} = 0$. The set of neighbors of agent i is denoted by $N_i = \{v_j \mid \varepsilon_{ij} \in \varepsilon\}$. A directed path is a sequence of distinct vertices $1, 2, \dots, r$ such that $(v_i, v_j) \in \varepsilon, i = 1, 2, \dots, r - 1$.

2.2. Problem Description. Consider a multiagent system with n agents, $\nu = \{1, 2, \dots, N\}$. Suppose the multiagent system composed with p subgroup, ν_p is a set of p subgroup. Note that the corresponding subtopology graph of each subgroup is $\mathcal{G}_p$, and the topology diagram of the whole system is G . The corresponding numbering sets for each subgroup are $\nu_1 = \{1, \dots, l_1\}, \nu_2 = \{l_1 + 1, \dots, l_1 + l_2\}, \nu_p = \{l_1 + l_2 + \dots + l_{p-1} + 1, \dots, N\}$, and $l_1 + l_2 + \dots + l_{p-1} + l_p = N$. $\nu_i \neq \emptyset, \cup_{i=1}^p \nu_i = \nu$, and $\nu_i \cap \nu_j \neq \emptyset$ for $i \neq j$. For $i \in \nu$, let $\bar{i}$ denote the subscript of the subset to which the integer i belongs. It is assumed in what follows that each agent knows which cluster it belongs to.

The dynamics of systems is described as follows:

$$\ddot{q}_i(t) = f_i(q_i, \dot{q}_i, t) + u_i(t), \quad (1)$$

where $q_i(t) \in R^n, \dot{q}_i(t) \in R^n$, and $u_i(t) \in R^n$ are the position, velocity, and control input of agent i , respectively. The function $f_i(q_i, \dot{q}_i, t) \in R^n$, describing the intrinsic dynamics of agent i , is continuously differentiable.

In this work, the leader in each group is described by

$$\ddot{q}_i(t) = f_i(q_i, \dot{q}_i, t). \quad (2)$$

Definition 1. The multiconsensus control for second-order multiagent systems is said to be achieved if

$$\begin{aligned} \lim_{t \rightarrow \infty} \|q_i(t) - q_i^*(t)\| &= 0, \\ \lim_{t \rightarrow \infty} \|\dot{q}_i(t) - \dot{q}_i^*(t)\| &= 0. \end{aligned} \quad (3)$$

Firstly, some basic assumptions and lemmas are given as follows:

Lemma 1. There is a constant $\omega \in R$, and P, Q, M, N are matrices with suitable dimensions. Then, the Kronecker product has the following properties:

$$\begin{aligned} (1) (P \otimes Q)^T &= P^T \otimes Q^T, \\ (2) (\omega P) \otimes Q &= P \otimes (\omega Q), \\ (3) (P + Q) \otimes M &= P \otimes M + Q \otimes M, \\ (4) (P \otimes Q)(M \otimes N) &= (PM \otimes QN). \end{aligned} \quad (4)$$

Lemma 2. The linear matrix inequality

$$\begin{bmatrix} A(s) & B(s) \\ B^T(s) & C(s) \end{bmatrix} > 0, \quad (5)$$

where $A(s) = A^T(s), C(s) = C^T(s)$, and $B(s)$ depend affinely on s , which is equivalent to $C(s) > 0, A(s) - B(s)C^{-1}(s)B^T(s) > 0$.

Lemma 3. Suppose that $S \in R^{n \times n}$ is positive definite and $D \in R^{n \times n}$ is symmetric. Then, for $\forall x \in R^n$, the following inequality holds:

$$\lambda_{\min}(S^{-1}D)x^T Sx \leq x^T D x \leq \lambda_{\max}(S^{-1}D)x^T Sx. \quad (6)$$

Assumption 1

- (1) $\sum_{j=N+1}^{N+M} a_{ij} = 0$ for all $i \in l_1, \sum_{j=1}^N a_{ij} = 0$ for all $i \in l_2$
- (2) The subgraph G_1 and G_2 have a directed spanning tree, respectively

Assumption 2. There exist nonnegative constants p and q such that nonlinear function satisfies the following equality:

$$\|f_i(x_i, v_i, t) - f_i(y_i, z_i, t)\| \leq p_i \|x_i - y_i\|^2 + q_i \|v_i - z_i\|^2, \quad (7)$$

where $x, y, v, z \in R^n, \forall t \geq 0$.

3. Main Results

In this section, the multiconsensus of multiagent system is analyzed.

The distributed feedback controller of agent i is designed as

$$\begin{cases} u_i(t) = \alpha \sum_{j=1}^N a_{ij} [(q_j - q_i) + (\dot{q}_j - \dot{q}_i)] \\ \quad - \alpha d_i [(q_i - q_i) + (\dot{q}_i - \dot{q}_i)], \\ \quad t \in [kH, kH + \delta), \\ u_i(t) = 0, \quad t \in [kH + \delta, (k+1)H), \end{cases} \quad (8)$$

where $H > 0$ is the control period and $\delta > 0$ is called the control time width.

Let $\tilde{q}_i(t) = q_i(t) - q_i(t)$ and $\tilde{\dot{q}}_i(t) = \dot{q}_i(t) - \dot{q}_i(t)$ are the measurement error of position and velocity of the i th agent.

Note that if $j \in V(G_k)$, then $\bar{j} = k$ and $\bar{q}_j(t) = q_j(t) - q_k(t)$. We then observe

$$\begin{aligned} & \sum_{j=1}^N a_{ij} (q_j(t) - q_i(t)) \\ &= - \sum_{k=1}^p \sum_{j \in V(G_k)} l_{ij} q_j(t) \\ &= - \sum_{k=1}^p \sum_{j \in V(G_k)} l_{ij} (q_j(t) - q_k(t) + q_k(t)) \\ &= - \sum_{j=1}^N l_{ij} \tilde{x}_j(t) - \sum_{k=1}^p \left(\sum_{j \in V(G_k)} l_{ij} \right) q_k(t) \sum_{j=1}^N a_{ij} (\tilde{x}_j(t) - \tilde{q}_i(t)). \end{aligned} \quad (9)$$

Obviously, $\sum_{j=1}^N a_{ij} ((\dot{q})_j(t) - \dot{q}_i(t)) = \sum_{j=1}^N a_{ij} (\tilde{\dot{q}}_j(t) - \tilde{\dot{q}}_i(t))$, which in turn together (1) and (2) yields the conclusion

$$\begin{cases} \tilde{\ddot{q}}_i(t) = f_i(q_i, \dot{q}_i, t) - f_i(q_i, \dot{q}_i, t), \\ \quad - \alpha \sum_{j \in V} a_{ij} (\tilde{q}_j(t) - \tilde{q}_i(t) + \tilde{\dot{q}}_j(t) - \tilde{\dot{q}}_i(t)), \\ \quad - \alpha d_i (\tilde{q}_i(t) + \tilde{\dot{q}}_i(t)), \quad t \in [kH, kH + \delta), \\ \tilde{\ddot{q}}_i(t) = f_i(q_i, \dot{q}_i, t) - f_i(q_i, \dot{q}_i, t), \\ \quad t \in [kH + \delta, (k+1)T). \end{cases} \quad (10)$$

Define $\tilde{q}(t) = (\tilde{q}_1^T(t), \tilde{q}_2^T(t), \dots, \tilde{q}_N^T(t))^T$ and $\tilde{\dot{q}}(t) = (\tilde{\dot{q}}_1^T(t), \tilde{\dot{q}}_2^T(t), \dots, \tilde{\dot{q}}_N^T(t))^T$.

Then, system (10) can be rewritten as

$$\begin{cases} \tilde{\ddot{q}}(t) = F_i(q(t), \dot{q}(t), t) - \tilde{F}_i(\tilde{q}(t), \tilde{\dot{q}}(t), t), \\ \quad - \alpha(L + D) \otimes I_n (\tilde{q}(t) + \tilde{\dot{q}}(t)), \\ \quad t \in [kH, kH + \delta), \\ \tilde{\ddot{q}}_i(t) = F_i(q(t), \dot{q}(t), t) - \tilde{F}_i(q_i(t), \dot{q}_i(t), t), \\ \quad t \in [kH + \delta, (k+1)T). \end{cases} \quad (11)$$

where $F(q(t), \dot{q}(t), t) = (f_1^H(q_1(t), \dot{q}_1(t), t), \dots, f_N^H(q_N(t), \dot{q}_N(t), t))^H$, $\tilde{F}(\tilde{q}(t), \tilde{\dot{q}}(t), t) = (f_1^H(q_1(t), \dot{q}_1(t), t), \dots, f_N^H(q_N(t), \dot{q}_N(t), t))^H$, $D = \text{diag}\{d_1, \dots, d_N\}$.

Theorem 1. Under Assumption 1 and Assumption 2, system (1) can reach the multiconsensus if the parameters meet the following conditions:

$$\rho I_N - \frac{\alpha}{2} \bar{L} - \alpha D < 0, \quad (12)$$

$$\delta > \frac{\gamma}{\gamma + \eta} T, \quad (13)$$

where $\rho = \max\{p, q + 1\}$, $p = \max\{p_i\}$, $q = \max\{q_i\}$, $\gamma = 2$

$$\begin{aligned} & \lambda_{\max}(\Omega^{-1}M), \eta = (\lambda_{\min}(Q)/\lambda_{\max}(\Omega)), \Omega = \begin{bmatrix} \alpha \bar{L} + 2\alpha D & I_N \\ I_N & I_N \end{bmatrix}, \\ & Q = \begin{bmatrix} \alpha(\bar{L}/2) + \alpha D - pI_N & 0_N \\ 0_N & \alpha(\bar{L}/2) + \alpha D - (q+1)I_N \end{bmatrix}, \quad M = \\ & \begin{bmatrix} pI_N & (\alpha/2)\bar{L} + \alpha D \\ (\alpha/2)\bar{L} + \alpha D & (q+1)I_N \end{bmatrix}, \text{ and } \bar{L} = L + L^T. \end{aligned}$$

Proof. It follows from (12) that

$$\begin{aligned} & \alpha \frac{\bar{L}}{2} + \alpha D - \rho I_N > 0, \\ & \begin{bmatrix} \alpha \bar{L} + 2\alpha D & I_N \\ I_N & I_N \end{bmatrix} > 0. \end{aligned} \quad (14)$$

Define the Lyapunov function for system (12):

$$V(t) = \frac{1}{2} \tilde{y}^T(t) (\Omega \otimes I_n) \tilde{y}(t), \quad (15)$$

where $\Omega = \begin{bmatrix} \alpha \bar{L} + 2\alpha D & I_N \\ I_N & I_N \end{bmatrix}$ and $\tilde{y}(t) = (\tilde{q}^T(t), \tilde{\dot{q}}^T(t))^T$.

In view of (8), we know that Lyapunov function (15) satisfies $V(t) \geq 0$ and $V(t) = 0$ if and only if $\tilde{q}(t) = \tilde{\dot{q}}(t) = 0$.

$$V(t) = \frac{1}{2} \tilde{q}^T [(\alpha \bar{L} + 2\alpha D) \otimes I_n] \tilde{q} + \tilde{q}^T \tilde{\dot{q}} + \frac{1}{2} \tilde{\dot{q}}^T \tilde{\dot{q}}. \quad (16)$$

For $t \in [kH, t_0 + kH + \delta)$, the time derivative of $V(t)$ along the trajectories of system (11) gives

$$\begin{aligned} \dot{V}(t) &= \tilde{q}^T [(\alpha \bar{L} + 2\alpha D) \otimes I_n] \dot{\tilde{q}} + \tilde{\dot{q}}^T \dot{\tilde{q}} + \tilde{q}^T \dot{\tilde{\dot{q}}} + \tilde{\dot{q}}^T \dot{\tilde{\dot{q}}} \\ &= -\tilde{q}^T [(\alpha L + \alpha D) \otimes I_n] \tilde{q} + \tilde{\dot{q}}^T [(I_N - \alpha L - \alpha D) \otimes I_n] \tilde{\dot{q}} \\ &\quad + (\tilde{q} + \tilde{\dot{q}}) (F_i(q(t), \dot{q}(t), t) - \tilde{F}_i(\tilde{q}(t), \tilde{\dot{q}}(t), t)). \end{aligned} \quad (17)$$

Then, by (A1), we get

$$\begin{aligned}
(\tilde{q}^T + \tilde{q}^T)(F_i(q(t), \dot{q}(t), t) - \tilde{F}_i(\bar{q}(t), \bar{q}(t), t)) &= \sum_{i=1}^N (\tilde{q}_i(t) + \tilde{q}_i(t))^T (f_i(q_i, \dot{q}_i, t) - f_i(\bar{q}_i, \dot{\bar{q}}_i, t)) \\
&= \sum_{i=1}^N (p_i \|\tilde{q}_i(t)\|^2 + q_i \|\tilde{q}_i(t)\|^2) \leq \tilde{q}^T (pI_N \otimes I_n) \tilde{q} + \tilde{q}^T (qI_N \otimes I_n) \tilde{q}.
\end{aligned} \tag{18}$$

It follows from (17) and (18) that

$$\begin{aligned}
\dot{V}(t) &\leq -\tilde{q}^T \left[\left(\alpha \frac{\bar{L}}{2} + \alpha D \right) \otimes I_n \right] \tilde{q} + \tilde{q}^T (qI_N \otimes I_n) \tilde{q} \tilde{q}^T \left[\left(I_N - \alpha \frac{\bar{L}}{2} - \alpha D \right) \otimes I_n \right] \tilde{q} + \tilde{q}^T (pI_N \otimes I_n) \tilde{q} \\
&= \tilde{q}^T \left[\left(-\alpha \frac{\bar{L}}{2} - \alpha D + pI_N \right) \otimes I_n \right] \tilde{q} + \tilde{q}^T \left[\left(-\alpha \frac{\bar{L}}{2} - \alpha D + (q+1)I_N \right) \otimes I_n \right] \tilde{q} = -\tilde{y}^T [Q \otimes I_n] \tilde{y},
\end{aligned} \tag{19}$$

where $Q = \begin{pmatrix} \alpha(\bar{L}/2) + \alpha D - pI_N & 0_N \\ 0_N & \alpha(\bar{L}/2) + \alpha D - (q+1)I_N \end{pmatrix}$.

In view of condition (12), we can get $Q > 0$. It is known that Lyapunov function satisfies that

$$\lambda_{\min}(\Omega) \|\tilde{y}(t)\|^2 \leq V(t) \leq \lambda_{\max}(\Omega) \|\tilde{y}(t)\|^2. \tag{20}$$

Thus, one has

$$\dot{V}(t) \leq -\frac{\lambda_{\min}(Q)}{\lambda_{\max}(\Omega)} V(t) = -\eta V(t), \tag{21}$$

where $\eta = (\lambda_{\min}(Q)/\lambda_{\max}(\Omega))$.

For $t \in [kH + \delta, (k+1)T)$, taking the time derivative of $V(t)$ along the trajectories of system (11)

$$\dot{V}(t) = \tilde{q}^T [(\alpha \bar{L} + 2\alpha D) \otimes I_n] \dot{\tilde{q}} + \tilde{q}^T \dot{H} \tilde{q} + \tilde{q}^T \tilde{q} + \tilde{q}^T \tilde{q} \leq \tilde{y}^T \left(\begin{pmatrix} pI_N & \frac{\alpha}{2} \bar{L} + \alpha D \\ \frac{\alpha}{2} \bar{L} + \alpha D & (q+1)I_N \end{pmatrix} \otimes I_n \right) \tilde{y} \tag{22}$$

$$\tilde{y} \leq \tilde{y}^T [M \otimes I_n] \tilde{y} \leq 2\lambda_{\max}(\Omega^{-1}M) V(t) = \gamma V(t),$$

where $M = \begin{bmatrix} pI_N & (\alpha/2)\bar{L} + \alpha D \\ (\alpha/2)\bar{L} + \alpha D & (q+1)I_N \end{bmatrix}$ and $\gamma = 2\lambda_{\max}(\Omega^{-1}M)$.

Then, by the above differential inequality (21) and (22), we have the following results:

- (1) For $0 < t < \delta$, $V(t) \leq V(0)e^{-\eta t}$ and $V(\delta) \leq V(0)e^{-\eta \delta}$
- (2) For $\delta \leq t < H$, $V(t) \leq V(\delta)e^{(\gamma(t-\delta))} \leq V(0)e^{(\gamma(t-\delta)-\eta\delta)}$ and $V(H) \leq V(0)e^{(\gamma(H-\delta)-\eta\delta)}$
- (3) $H \leq t < H + \delta$, $V(t) \leq V(H)e^{(-\eta(t-H))} \leq V(0)e^{(\gamma(H-\delta)-\eta\delta-\eta(t-H))}$, and $V(H + \delta) \leq V(0)e^{(\gamma(H-\delta)-2\eta\delta)}$
- (4) $H + \delta \leq t < 2H$, $V(t) \leq V(H + \delta)e^{(\gamma(t-H-\delta))} \leq V(0)e^{(\gamma(H-\delta)+\gamma(t-H-\delta)-2\eta\delta)}$ and $V(2H) \leq V(0)e^{(\gamma(H-\delta)+\gamma(H-\delta)-2\eta\delta)} = V(0)e^{(2\gamma(H-\delta)-2\eta\delta)}$

Thus, one has

For $kH \leq t \leq kH + \delta$,

$$\begin{aligned}
V(t) &\leq V(kH)\exp(-\eta(t-kH)) \\
&\leq V(0)\exp(k\gamma(H-\delta) - k\eta\delta - \eta(t-kH)) \\
&\leq V(0)\exp\left(-\frac{\eta\delta - \gamma(H-\delta)}{H}(t-\delta)\right)\exp\left(\frac{H-\delta}{H}\gamma\delta\right) \\
&\leq V(0)\exp\left(-\frac{\eta\delta - \gamma(H-\delta)}{H}(t-\delta)\right)\exp(\gamma(H-\delta)).
\end{aligned} \tag{23}$$

For $kH + \delta \leq t < (k+1)H$,

$$\begin{aligned}
V(t) &\leq V(kH + \delta)e^{\gamma(t-kH-\delta)} \\
&\leq V(0)\exp(k+1)\gamma((H-\delta-k+1)\eta\delta) \\
&\leq V(0)\exp\left(-\frac{\eta\delta - \gamma(H-\delta)}{H}(t-\delta)\right)\exp(\gamma(H-\delta)).
\end{aligned} \tag{24}$$

If $\delta > (\gamma/\gamma + \eta)H$, then defining $\xi = (\eta\delta - (H - \delta)\gamma/H) > 0$. Let $K = V(0)\exp((\eta\delta - \gamma(H - \delta)/H)\delta)\exp(\gamma(H - \delta))$, and from the above analysis, we can draw the following conclusions:

$$V(t) \leq Ke^{-(\eta\delta - \gamma(H - \delta)/H)t}. \quad (25)$$

Therefore, the multiconsensus can be achieved. The proof is completed.

Remark 1. Under the condition of Theorem 1, the multiconsensus of system (1) can be achieved globally exponentially with the presented law.

Remark 2. When each agent has the identical nonlinear function, all the agents have the same virtual leader. The dynamics of the virtual leader can be described as

$$\ddot{q}_r(t) = f(q_r, \dot{q}_r, t). \quad (26)$$

Under the intermittent control, system (1) can reach consensus.

Theorem 2. Under Assumption 1 and Assumption 2, system (1) can reach the multiconsensus if the parameter meets the following conditions:

$$\begin{cases} \ddot{q}_i(t) = -\alpha \sum_{j \in V} a_{ij}(\ddot{q}_j(t) - \ddot{q}_i(t) + \ddot{q}_j(t) - \ddot{q}_i(t)) - \alpha d_i(\ddot{q}_i(t) + \ddot{q}_i(t)), & t \in [kH, kH + \delta), \\ \ddot{q}_i(t) = 0, & t \in [kH + \delta, (k+1)H). \end{cases} \quad (29)$$

where $i = 1, 2, \dots, N$. System (29) can be rewritten as

$$\begin{cases} \ddot{\tilde{q}}(t) = -((\alpha(L + D) \otimes I_n)(\ddot{\tilde{q}}(t) + n\ddot{\tilde{q}}(t))), & t \in [kH, kH + \delta), \\ \ddot{\tilde{q}}(t) = 0, & t \in (kH + \delta, (k+1)H). \end{cases} \quad (30)$$

The proof of this part is similar to that of Theorem 1, which is omitted here.

4. Numerical Simulations

In this part, the effectiveness of presented theoretical results has been proven by numerical examples. The multiconsensus of second-order agent systems has been studied. Actually, seven agents can form a second-order system which is split into two clusters $\nu_1 = \{1, 2, 3\}$ and $\nu_2 = \{4, 5, 6, 7\}$. If $i \in \nu_1$, then $\hat{i} = 1$; and if $i \in \nu_2$, then $\hat{i} = 2$.

The Laplacian matrix is given as follows:

$$\begin{aligned} (1) \quad & I_N - \frac{\alpha}{2}\bar{L} - \alpha D < 0, \\ (2) \quad & \delta > \frac{\gamma}{\gamma + \eta}T, \end{aligned} \quad (27)$$

where $\rho = \max\{\rho_1, \rho_2 + 1\}$, $\gamma = 2\lambda_{\max}(\Omega^{-1}M)$, $\eta = (\lambda_{\min}(Q)/\lambda_{\max}(\Omega))$, $\bar{L} = L + L^T$, $\Omega = \begin{bmatrix} \alpha\bar{L} + 2\alpha D & I_N \\ I_N & I_N \end{bmatrix}$, $Q = \begin{bmatrix} \alpha(\bar{L}/2) + \alpha D & 0_N \\ 0_N & \alpha(\bar{L}/2) + \alpha D - I_N \end{bmatrix}$, and $M = \begin{bmatrix} 0 & (\alpha/2)\bar{L} + \alpha D \\ (\alpha/2)\bar{L} + \alpha D & I_N \end{bmatrix}$.

When $f(q_i, \dot{q}_i, t) = 0$, the dynamics of the i th agent can be described as follows:

$$\ddot{q}_i(t) = u_i(t). \quad (28)$$

Then, (28) shows that the velocity of virtual leader is constant. Under protocol (8), let $\tilde{q}_i(t) = q_i(t) - q_i^*(t)$, $\ddot{\tilde{q}}_i(t) = \ddot{q}_i(t) - \ddot{q}_i^*(t)$, then the closed-loop system (28) becomes:

$$L = \begin{bmatrix} 3 & -3 & 0 & 1 & 0 & 0 & -1 \\ -2 & 3 & -1 & -1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 2 & 0 & 0 & -2 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ -1 & 1 & 0 & -2 & 0 & -1 & 3 \end{bmatrix}, \quad (31)$$

$$D = \text{diag}\{3, 0, 0, 3, 0, 0, 0\}.$$

FIGURE 1: The trajectories of the agents.

FIGURE 2: The error trajectories of the agents.

FIGURE 3: The trajectories of the agents.

FIGURE 4: The trajectories of the agents.

FIGURE 5: The error trajectories of the agents.

Case 1. Set $f_1(x_i, v_i, t) = -x_i$ and $f_2(x_i, v_i, t) = -3v_i$. By Assumption 2, $p_1 = 1, q_1 = 0$, and $p_2 = 0, q_2 = 3$. Then, $\rho = 4$. In view of condition (12), we can choose $\alpha = 15$. By condition (13), we choose $\delta = 0.08$. The initial position and velocity of the virtual leader are given as follows:

$$\begin{aligned} (q_1(0), \dot{q}_1(0)) &= (0, 1), \\ (q_2(0), \dot{q}_2(0)) &= (1, 4). \end{aligned} \quad (32)$$

Figure 1 shows the states and velocity of the virtual leaders and followers. It is easy to see that the multiconsensus of systems (1) can be received. The error tracks of the agents are depicted in Figure 2.

When $f_1(x_i, v_i, t) = f_2(x_i, v_i, t) = -x_i$, as shown in Figure 3, the consensus problem of system (1) is indeed solved.

Case 2. Let $f_1(x_i, v_i, t) = 0$ and $f_2(x_i, v_i, t) = 0$. The initial velocity of the virtual leader are given as follows: $v_1(0) = 2$ and $v_2(0) = 5$. The multiconsensus of the

systems can be achieved. As a special case, when the initial velocity of the virtual leader is same, the system reaches consensus. The simulation result is shown in Figures 4 and 5, which show that the multiconsensus can be achieved.

5. Conclusion

In this paper, we have studied on multiconsensus of second-order multiagent systems with nonlinear dynamics. For the realization of multiconsensus, a distributed protocol based on intermittent relative information has been proposed. Some of sufficient conditions have been given to ensure that the states of all agents could reach more consistent values. In this paper, two simulation examples are given and applied to the effectiveness of theoretical results.

Data Availability

All data included in this study are available upon request by contact with the corresponding author.

Conflicts of Interest

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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