Research Article

Measurement of Longevity Risk of Life Annuity Based on C-ROSS Framework

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Received 6 June 2020; Accepted 24 July 2020; Published 18 September 2020

Guest Editor: Wenguang Yu

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This paper constructs a model to measure longevity risk and explains the reasons for restricting the supply of annuity products in life insurance companies. According to the Lee–Carter Model and the VaR-based stochastic simulation, it can be found that the risk margin of the first type of longevity risk for ignoring the improvement of mortality rate is about 7%, and the risk margin of the second type of longevity risk for underestimating mortality improvement is about 7%. Therefore, the insurer needs to use cohort life table pricing premium and gradually prepares longevity risk capital during the insurance period.

1. Introduction

With the rapid development of the global economy and the improvement of medical skills, the extension of human life span has become an inevitable trend, which has a significant impact on the stability of life insurance companies. The fact that insurance companies cannot effectively manage longevity risk due to extended life expectancy results in the insufficient motivation to provide lifelong annuity, which limits the supply of annuity. There are two aspects in evaluating the longevity risk of the life annuity products: one is to establish a stochastic mortality model to forecast the improvement of mortality, and the other one is to select an appropriate method to measure longevity risk.

There are lots of studies on the stochastic mortality model. Among all, the Lee–Carter model [1], which is based on the hypothesis that the logarithmic mortality is composed by the independent age and periodic effects, is a pioneering one. Because of its easy form, obvious parameter significance, and easy quantitative calculation, this model is widely used by scholars all over the world. Renshaw and Haberman expand the Lee–Carter model by taking the impact of population cohort on mortality into account to improve the scientificity of the model [2]. Also, due to less exposure of the elderly population and abnormal distribution of death, the

general model cannot fully reflect the mortality curve of the elderly. Cairns et al. propose the CBD model with two factors to solve this problem [3]. Raftery et al. put forward a method based on Bayesian Hierarchical Model (BHM), taking the average life span data from 1950 to 1995 over the world into BHM to forecast life expectancy [4]. According to the cross-check with an actual statistic, Raftery et al. believe that the model could show an accurate prediction interval. At the same time, domestic scholars have explored the application of stochastic mortality model in China, and the results show that the Lee–Carter model is suitable for China [5–7].

As for longevity risk measurement, Olivieri used the present value of the annuity to measure the longevity risk for insurer [8], whereafter Olivieri and Pitacco further studied the solvency capital requirements of annuity insurance under longevity risk based on a stochastic mortality model [9]. In 2010, Borger compared the risk value (VaR) with the long-lived risk metric under standard deterministic mortality volatility and found out that VaR was more reasonable to apply in measurement. Hari et al. point out that longevity risk exists at both the individual and the general levels [10]. The former refers to the fact that individuals spend more money for the desire of longevity than they have accumulated over a lifetime, which can be reduced by using various

pension plans, such as the government's social security pension insurance, the employer's pension for the enterprise, and commercial pension insurance. The overall longevity risk is also called the aggregate longevity risk; that is, the life expectancy exceeds the prediction. In this situation, the budget in the pension plan of the insurance company will exceed the expectation, which will lead to severe debts and cause an economic burden to both government and insurance companies. It is worth noting that we are mainly focusing on the risk of a commercial insurance company. Then, Richards et al. explore the applicability of the stochastic mortality model in different ways, presenting a long-lived risk measurement with a one-year perspective. Antolin proposes a new risk metric that could meet the consistency requirements. Based on Glue VaR, the longevity risk is measured in China's pension system and insurer's survival annuity products.

The structure of this paper is as follows. Section 2 introduces the stochastic mortality model and the method for measuring of longevity risk. Next, Section 3 describes the data features and research scheme. Following this, Section 4 analyses the longevity risk faced by insurers when providing lifelong pension funds. Finally, Section 5 concludes the paper.

2. Longevity Risk Model and Measurement

2.1. Lee-Carter Model. Lee and Carter propose Lee-Carter model, which is superior due to its simple form, obvious practicality of parameters, and convenience for quantitative calculation and is broadly used nowadays. It has become the standard in the US Census Bureau and the United Nations Population Division with good fitting and prediction results. The basic form of the Lee-Carter stochastic mortality model is as follows:

$$\begin{aligned} \ln m_x(t) &= \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t}, \\ \varepsilon_{x,t} &\sim N(0, \sigma^2). \end{aligned} \quad (1)$$

In the above formula, $m_x(t)$ is the central mortality rate of the x -year-old in year t . Generally, it is as the mortality data in the demographic survey; that is,

$$m_x(t) = \frac{D(t, x)}{E(t, x)}, \quad (2)$$

where $D(t, x)$ is the number of deaths of the x -year population during the entire calendar year t . $E(t, x)$ is the average number of people aged x , that is, the number of exposures during the calendar year t . Under the assumption that the deadly force is constant, the age-specific mortality rate, used in the actuarial model frequently, has the following approximate relationship with the central mortality rate:

$$q_x(t) = 1 - \exp[-m_x(t)]. \quad (3)$$

Or, under the assumption that death occurs evenly,

$$q_x(t) = \frac{m_x(t)}{1 + 0.5m_x(t)}. \quad (4)$$

Lee-Carter Model decomposes the population mortality into three parts: the fixed general level of mortality α_x , the refining trend of mortality over time $\beta_x \kappa_t$, and the random fluctuation term. x , which is independent of time, represents the general level of logarithmic mortality at age x . It can take the average of historical data in the time dimension or the value of the last observation year to reflect the difference of mortality at different ages. The model decomposes the trend of mortality over time into the interactive product of age and time. κ_t indicates the relative intensity of overall mortality in each year and gradually decreases with time, reflecting the continuous improvement of mortality over time. β_x shows the sensitivity of the logarithmic mortality rate of the x -year population towards the change in the overall trend, reflecting the inconsistent rate of decline at different ages. The last one is a random error term that reflects the random fluctuation of mortality rate outside the trend.

To ensure the uniqueness of the results, the Lee-Carter model contains two restrictions:

$$\begin{aligned} \sum_x \beta_x &= 1, \\ \sum_t \kappa_t &= 0. \end{aligned} \quad (5)$$

At present, the parameter estimation methods for the Lee-Carter model mainly include singular value decomposition (SVD), ordinary least squares (OLS), weighted least squares (WLS), and Poisson log bilinear model (Poisson log-bilinear). The estimation of α_x is not controversial. Generally, it is

$$\alpha_x = \frac{1}{n} \sum_{t=T_1}^{T_n} \ln m_x(t). \quad (6)$$

Or, to increase the impact weight of the latest observations, the estimate is

$$\alpha_x = \ln m_x(T_n). \quad (7)$$

The main difference among the mentioned methods is how to decompose the trend effect into the age factor β_x and the time factor κ_t . Based on the characteristics of the matrix, singular value decomposition (SVD) extracts the main information of trend effect skillfully. The main estimation method is to singly decompose the matrix $\ln m_x(t) - \alpha_x$, which can be obtained as follows:

$$\text{SVD}[\ln m_x(t) - \alpha_x] = \sum_{i=1}^r \rho_i U_{x,i} V_{i,t}, \quad (8)$$

where r is the rank of the matrix $\ln m_x(t) - \alpha_x$ and $\rho_1, \dots, \rho_2, \dots, \rho_r$ are the singular values of the matrix from large to small. $U_{x,i}$ and $V_{i,t}$ are, respectively, two singular vectors. Since ρ_1 is much larger than the subsequent eigenvalues, most of the information of the matrix $\ln m_x(t) - \alpha_x$ can be extracted only by taking the first term $U_{x,1}$ and $V_{1,t}$ of two singular vectors, and the matrix is as follows:

$$\ln[m_x(t) - \alpha_x] \approx \rho_1 U_{x,1} V_{1,t}. \quad (9)$$

Thus, estimates of β_x and κ_t can be obtained:

$$\begin{aligned}\beta_x &= \frac{U_{x,1}}{\sum_x U_{x,1}}, \\ \kappa_t &= \rho_1 V_{1,t} \sum_x U_{x,1}.\end{aligned}\quad (10)$$

The fitting effect of the singular value decomposition depends on the efficiency of extracting information from the $\ln m_x(t) - \alpha_x$ matrix. It is generally considered that the method can explain more than 90% of the sum of squares of deviations. After obtaining the estimated values of the parameters, it can be found that α_x and β_x are fixed over time, and the variety of mortality over time is mainly reflected by $\{\kappa_t\}$. The prediction value of future mortality can be obtained by extrapolating $\{\kappa_t\}$. It is believed that $\{\kappa_t\}$ is a random walk with drift or ARIMA process. According to the BIC information criterion, $\{\kappa_t\}$ should be the ARIMA (0, 1, 1) Model with drift term.

2.2. Measurement of Longevity Risk

2.2.1. Method of Pressure Trend. Taking the trend risk of mortality prediction into account, the method of pressure trend gives the extreme situation of the long-term decline of mortality. It then measures the difference of the present value paid by insurance companies between the serious cases and the optimal estimation. For instance, $\{\kappa_t\}$ shows the decreasing trend of mortality in the Lee-Carter model. When using not only ARIMA but also drifting random walk model to predict the value of future κ_T , it is a random variable obeying normal distribution. After taking the expected value of κ_T , we can get the mortality in the middle situation, that is, the optimally estimated mortality. If the risk level is given certainly, after taking the $1-\alpha$ quantile of κ_T , we can get the critical value of trend mortality under the risk level. The difference between the pressure trend line and the optimal mortality estimate trend line is the possibility of underestimation of the trend mortality. Under the pressure trend method, measuring formula of longevity risk is defined as

$$\text{LRM} = \left(\frac{\ddot{a}_{x: w-x}^{z=\phi^{-1}(1-\alpha)}}{\ddot{a}_{x: w-x}^{z=0}} \right) \cdot 100\%. \quad (11)$$

Because there is a certain correlation in mortality in different periods, the extreme trend of mortality at each moment in the future will overestimate the risk of longevity to some extent. It can be seen that the method of pressure trend is not based on the VaR, and there is no clear standard for the value of α . Considering that the second quantitative impact study (QIS2) proposes to use the 75% quantile to measure the risk margin, and the European Solvency Regulatory Standard II requires risk margin to be sufficient to ensure that the probability of loss exceeding solvency capital is less than 0.5%, based on which we calculate the margin of longevity risk at 25% and 0.5% risk levels.

2.2.2. Method of Mortality Fluctuation. Method of mortality fluctuation is a standard method required by QIS5 to calculate the marginal risk of longevity. It adjusts for conservative mortality by assuming an immediate downward wave in current and future expected mortality. The formula is defined as follows:

$$q_{x,t}^{\text{shock}} = q_{x,t} \times (1 - f). \quad (12)$$

$q_{x,t}$ is the optimal estimated mortality and f is the mortality adjustment ratio. The formula of the risk margin under this method is

$$\text{LRM} = \left(\frac{\ddot{a}_{x: w-x}^f}{\ddot{a}_{x: w-x}} - 1 \right) \cdot 100\%. \quad (13)$$

According to the requirements of QIS5, f takes 20%, which means that, in the conservative case, a 20% reduction in mortality rate in the future is the best estimate. China's C-ROSS has a distinct demand, that is, for the remaining life insurance period, and different downward ratios are given for each year's mortality. The specific adjustments are as follows.

$$-f = \text{SF}$$

$$= \begin{cases} (1 - 3\%)^t - 1, & 0 < t \leq 5, \\ (1 - 3\%)^5 \times (1 - 2\%)^{t-5} - 1, & 5 < t \leq 10, \\ (1 - 3\%)^5 \times (1 - 2\%)^5 \times (1 - 1\%)^{t-10} - 1, & 10 < t \leq 20, \\ (1 - 3\%)^5 \times (1 - 2\%)^5 \times (1 - 1\%)^{10} - 1, & t > 20. \end{cases} \quad (14)$$

C-ROSS has made minor adjustments to recent mortality rates but major readjustment to long-term rate. Figure 1 shows a comparison of the mortality ratio of the QIS5 and C-ROSS in conservative cases. For QIS5, the mortality ratio f is always 20%, while for C-ROSS, f gradually increases with time, but the increasing trend is slowing down. Within eight years after the evaluation date, the C-ROSS mortality rate is lower than QIS5. After that, the C-ROSS mortality rate becomes higher than QIS5 and remains unchanged at 29.8% after 20 years.

The mortality fluctuation method not only contains the trend risk of mortality in the long-term process but also is a comprehensive measure of the components of longevity risk. Börger points out that the mortality fluctuation method had been underestimating the longevity risk of low age and overestimated the longevity risk of high age longevity [11]. Thus, it is structurally defected. Next, the paper will measure the margin risk of Chinese life annuity products under the QIS5 and C-ROSS standards in the empirical analysis.

2.2.3. Stochastic Simulation Based on VaR and CTE. Widely used in the risk management field, Value at Risk (VaR) embodies the maximum possible loss at a specific level, that is, to ignore the extreme risk at the tail and only consider the maximum loss within a certain probability. Conditional Tail Expectation (CTE) is a conditional expectation in the case of tail extreme risk. It can effectively

FIGURE 1: Comparison of mortality ratios between QIS5 and C-ROSS under conservative cases.

describe the extreme risk of the tail and then make up for the deficiency of VaR to some extent. The EU Solvency II requires that the probability of the insurer's loss exceeding its repayment capital should be less than 0.5%, which is a kind of risk regulation based on VaR. At the same time, the Swiss Solvency Test (SST) takes CTE as the standard method of risk measurement. The general expression formulas for VaR and CTE are as follows:

$$\begin{aligned} \text{VaR}_\alpha &= \min\{x \mid F(x) > \alpha\}, \\ \text{CTE}_\alpha &= \text{VaR}_\alpha + E(X - \text{VaR}_\alpha \mid X > \text{VaR}_\alpha). \end{aligned} \quad (15)$$

The method based on VaR and CTE needs to measure the loss distribution. However, in reality, due to the complexity of the mortality change and the structural characteristics of the annuity products, the loss distribution can only be fitted by random simulation, based on which the values of VaR and CTE are calculated. Plat uses stochastic simulation to measure the longevity risk of lifelong pensions based on population mortality data of the Dutch, but it had some nonsystemic hazard [12]. Here, the method of Plat is revised to be suitable for China, and it is described by taking the Lee-Carter model as an example. The steps are as follows:

- (i) Select a data set with a calendar year from t_L to t_H and age from x_l to x_h , including the number of deaths for each age and the corresponding mid-year population estimates. Then corresponding central mortality rate $m_x(t)$ can be calculated.
- (ii) Fit the data in the first step by the Lee-Carter model and find the corresponding parameters in the model.
- (iii) Perform a random simulation of the future value of $\{\kappa_t\}$ and obtain a random path of $\{\kappa_t\}$. Then the downward trend in future mortality is calculated.
- (iv) Bring the obtained $\{\kappa_t\}$ random path into the normative Lee-Carter model, and add the annual random fluctuation to get a stochastic simulation $\{m_x(t)\}$ of mortality of different ages in each year in the future.

(v) According to the above formula, convert the central mortality rate to the probability of death $\{q_x(t)\}$. Then the simulated life table of a specific cohort can be obtained by taking the death probability value of the corresponding age and year. After that, the lifetime annuity factor $\ddot{a}_{x:w-x}^{(1)}$ of the cohort can be calculated.

(vi) Repeat the above 3 to 5 steps for n times; then we can get n stochastic mortality paths; finally find a set of n annuity factors:

$$S = \{\ddot{a}_{x:w-x}^{(j)}\}, \quad j = 1, 2, \dots, n. \quad (16)$$

(vii) Select the $1-\alpha$ quantile of S as s_{VaR_α} , and calculate the conditional tail expectation as s_{CTE_α} . The corresponding longevity risk metric is

$$\begin{aligned} \text{LRM} &= \left(\frac{s_{\text{VaR}_\alpha}}{\ddot{a}_{x:w-x}} - 1 \right) \cdot 100\%, \\ \text{LRM} &= \left(\frac{s_{\text{CTE}_\alpha}}{\ddot{a}_{x:w-x}} - 1 \right) \cdot 100\%. \end{aligned} \quad (17)$$

According to the request of EU Solvency II, α is 0.5%. So, the number of simulations is generally required to be at least 1000 times. Performing 30,000 simulations in an empirical analysis could improve the accuracy of measuring longevity risk.

2.2.4. VaR-Based Stochastic Simulation from One-Year Perspective. The above methods consider changes in mortality during the entire insurance period. Still, unlike other risks faced by insurers, longevity risks are chronic; that is, the mortality rate reduces slowly during a long period until it is completely exposed. Therefore, in the actual operation, the insurer needs to constantly adjust the mortality model according to the new information of mortality and adjust the reserve level correspondingly, leading to two problems: First, how will the mortality model vary with the new mortality data added in the next year? Second, how will the change of the mortality model lead to the change of reserves? From these two problems, Richards proposes a method based on VaR in a one-year period to calculate the marginal longevity risk. The core idea of this method is to assume that there will be adverse fluctuations in the mortality in the next year so that the mortality model and the predicted mortality level will be changed, and then calculate the future cash payment value of the insurance company under the new situation. This paper constructs a one-year VaR-based stochastic simulation method suitable for China. The stimulation could measure the longevity risk of Chinese commercial pension funds, and the steps are as follows:

- (i) Select a data set with a calendar year from t_L to t_H and age from x_l to x_h , including the number of deaths for each age and the corresponding mid-year population estimate. Then the corresponding central mortality rate $m_x(t)$ can be calculated.

- (ii) Fit the data in the first step using the Lee–Carter model and find the corresponding parameters in the model.
- (iii) Perform a random simulation of κ_{t_H+1} to obtain a value of it.
- (iv) Bring the obtained stochastic simulation value of κ_{t_H+1} into the normative Lee–Carter model, and add the annual random fluctuation to acquire the age-specific mortality rate for the next year; then get a stochastic simulation $\{m_x(t)\}(t_H + 1)$.
- (v) Put $\{m_x(t)\}(t_H + 1)$ into the data set in the first step, refit the new data set with the Lee–Carter model, and predict the optimal estimate of future mortality $\{m_x(t)\}$.
- (vi) According to formula (4), convert the central mortality rate into the probability of death $\{q_x(t)\}$. Then the simulated life table of a specific cohort can be obtained by taking the mortality value of the corresponding age and year. After all, the lifetime annuity factor $\ddot{a}_{x:w-x}^{(1)}$ of the cohort can be calculated.
- (vii) Repeat the above 3 to 5 steps n times, get n stochastic mortality paths, and finally obtain a set of n annuity factors:

$$S = \left\{ \ddot{a}_{x:w-x}^{(j)'} \right\}, \quad j = 1, 2, \dots, n. \quad (18)$$

- (viii) The quantile of $1-\alpha$ of S is selected as $s_{\text{VaR}'_\alpha}$ and the corresponding longevity risk metric is

$$\text{LRM} = \left(\frac{s_{\text{VaR}'_\alpha}}{\ddot{a}_{x:w-x}} - 1 \right) \cdot 100\%. \quad (19)$$

The one-year VaR-based stochastic simulation considers the longevity risk in the short term focusing on the possible adverse changes in mortality in the next year and the long-term effects of such changes. This method, which only considers the risk factors of one year, is to split the long-term longevity risk into each year in advance and help the insurance company shorten the long-term hazard from the perspective of operation. It can also test the sensitivity of mortality model towards new data.

3. Comparison of Longevity Risk Measurement

Lee–Carter model, which directly models mortality, reflects the characteristics of mortality more in line with the characteristics of China's population mortality. Based on the mortality predicted by Lee and Carter, we could compare the different outcomes from different measurements of longevity risk. Since China's population mortality statistics are generally up to 90 or 100 years of age, we have used the Kannisto Model to extrapolate the mortality rate of the elderly over 90 years of age. The Kannisto Model has a good effect on the extrapolation of the mortality rate of the elderly population. The specific form of the Kannisto Model is as follows:

$$m_x = \frac{ae^{bx}}{1 + a(e^{bx} - 1)}. \quad (20)$$

This paper predicted the results of male and female mortality from 60 to 110 years of age using Lee–Carter model and Kannisto model every ten years from 2017 to 2057. It is observed that the projected mortality rate presents a general trend of increasing with age and decreasing with time.

To calculate the longevity risk of life annuity products sold to 60-year-olds in 2017, the benchmark pricing rate is assumed to be 3.5%. The longevity risk is classified into two types. The first type of longevity risk is the longevity risk taken by insurance companies when using the period life table to make a price, regardless of the future mortality decline of the insured cohort. The second type of longevity risk is the longevity risk taken by insurance companies when using the cohort life table to make a price but underestimating the prediction of future cohort mortality. In conclusion, the longevity risk from calculating the period life table belongs to the first type, while that from the cohort life table belongs to the second type.

3.1. Longevity Risk from the Period Life Table. We compared the period life table and the cohort life table of males and females in 2017. At the same time, the confidence interval of trend fluctuation is given under a 99% confidence level of cohort life table based on the pressure trend method. It can be discovered that the period life table only contains the static mortality rate of the current year without considering the decline of the insured mortality rate. The mortality level of the period life table is not only higher than that of the optimal estimation of the cohort life table but also higher than the upper limit of the 99% confidence interval of the cohort life table in the advanced age stage. If the life table is used to make a price for the annuity, the premium level will be lower than the normal level, which leads to the risk of insufficient solvency for the insurance company.

Next, the paper measures the longevity risk faced by insurance companies under the period life table pricing. Table 1 gives the measurement values of longevity risk margin of 60-year-old male and female life annuity under different interest rates. The period life table does not take into account the future mortality decline of the insured cohort. Under the interest rate of 3.5%, the annuity factors of males and females, which are the prices of an annuity product whose annual payment is 1, are 15.5 and 17.1, respectively. But, in fact, the group's future mortality rate should have followed the cohort life table. Under the optimal estimate, the marginal longevity risk is 7.7% for men and 6.7% for women. If the future mortality falls to the lower edge of 99% confidence interval of trend mortality, the extra payments of lifetime annuity products for men and women will reach 17.9% and 13.6% of their premiums. Also, interest rates have significant impacts on the measurement of margin risk. The higher the evaluation interest rate is, the smaller the margin of longevity risk will become and vice versa, which is mainly because the low-interest rate has a less discount effect

TABLE 1: The first type of longevity risk measure for the pension of 60-year-olds.

	Male			Female		
	2.50%	3.50%	4.50%	2.50%	3.50%	4.50%
Annuity factor	17.2	15.5	14.1	19.2	17.1	15.4
LRM (99% high)	-3.5%	-3.3%	-3.1%	-1.2%	-1.2%	-1.2%
LRM (optimal estimate)	9.0%	7.7%	6.6%	7.9%	6.7%	5.7%
LRM (99% low)	20.9%	17.9%	15.4%	15.9%	13.6%	11.6%

on the difference in the payment due to the variety in mortality in future years. In comparison, high-interest rate has a greater impact.

Figure 2 shows the effect from the initial payment age on marginal longevity risk. It can be seen that, for males and females, when initial payment age changes from 60 to 110, the first type of marginal longevity risk increases first and then decreases, with the initial benefit age, reaching the maximum value of 30.4% for men at 93 years of age and 20.4% for women at 92 years of age. For all ages, the first type of risk influences male pension greater than females. The risk margin of insurance policy with low initial payment age is low, which is mainly due to the long insurance period and the greater discount effect of discount rate on the difference of payment value under different mortality rates. However, the risk margin of the insurance policy with high initial payment age decreases mainly because the longevity risk decreases with the shortening of the policy life. In reality, there is almost no annuity product for people over 90 years of age. Therefore, for insurers, the lower the initial payment age of an annuity product is the smaller risk of the first type they will face.

3.2. Longevity Risk from the Cohort Life Table. In reality, insurers can use cohort life table as pricing base to avoid the first type of longevity risk. Still, the second type of longevity risk caused by the uncertainty of future mortality cannot be circumvented. Table 2 shows the risk margin values calculated by different longevity risk measurement methods. It is observed that the risk value level under the pressure trend method significantly affects the risk margin. When α is 0.5%, the marginal longevity risk of male and female life annuities is more than other measures, reaching 9.5% and 6.4%, respectively. When α is 25%, the marginal risk is smaller than the others, which are 2.6% and 1.8%, respectively.

Comparing the results of mortality fluctuation method, the standard of China's C-ROSS longevity risk measurement is more stringent than that of EU QIS5. Specifically, under the intermediate interest rate level, for the life annuity of 60-year-old males, the longevity risk margin is 5.2% under QIS5 standard and 6.6% under C-ROSS standard. For females, the marginal longevity risks measured by QIS5 and C-ROSS are 3.9% and 5.3%, respectively. Similarly, the marginal value of risk measured by C-ROSS is higher than QIS5 at both high-interest and low-interest rates. Therefore, it can be proved that China's second generation of compensation in the declining level of mortality estimate is more conservative.

FIGURE 2: Effect of initial payment age under the first type.

The stochastic simulation based on VaR and CTE accurately defines the conditional expectation of the maximum loss proportion and tail loss faced by insurers under a specific risk level. It is found that the marginal risk of a male life annuity as a standard is 7.9%, which is higher than that of female life annuity by 5.5%. In other words, to prevent an unexpected drop in mortality, the insurer needs to prepare an additional 7.9% of the premium for the male and 5.5% for the female as the venture capital to reduce the risk of being unable to repay due to insufficient capital to 0.5%, meeting the requirements of the EU for the second generation. The standard marginal risk measure under the criterion is greater with 9% for males and 6.1% for females. After comparing stochastic simulation method with the pressure trend method, it can be seen that the latter focuses on the extreme level of the change in the mortality trend, while it ignores the hedging in the risk of mortality fluctuation in different years. Thus, under the same risk level, the pressure trend method overrates the longevity risk, while the marginal longevity risk brought by stochastic simulation has more practical significance.

At the risk level of 0.5%, the marginal longevity risks of male and female lifelong pensions measured by stochastic simulation method in one-year perspective are 3.8% and 3.3%, respectively, which are smaller than those measured by other ways at the same risk level. The main reason is that VaR method based on one-year perspective focuses on the risk situation of the next year rather than the size of longevity risk in the long-term process. It reflects the additional venture capital that the insurer should prepare for the long-term debt changes in response to mortality change next year. Under this method, the longevity risk in the long-term process is gradually reflected in the insurance period through annual calculation.

Also, through comparing the marginal longevity risk of males and females, it can be seen that, same as the first type of longevity risk, the second one faced by men for lifelong pension is also greater than that of women as well, which is mainly because the current mortality level of men is higher than that of women, and the improvement space and possible fluctuation range in the future are also larger, while the current mortality rate of women is already at a lower level. That is to say, the decreasing mortality rate for females

TABLE 2: The second type of longevity risk measurement for the pension of 60-year-olds.

	Male			Female		
	2.5%	3.5%	4.5%	2.5%	3.5%	4.5%
Annuity factor	18.8	16.7	15.0	20.7	18.2	16.2
Pressure trend method (0.5%)	11.0%	9.5%	8.2%	7.5%	6.4%	5.6%
Pressure trend method (25%)	2.9%	2.6%	2.3%	2.1%	1.8%	1.5%
Mortality fluctuation method (QIS5)	6.0%	5.2%	4.6%	4.6%	3.9%	3.4%
Mortality fluctuation method (C-ROSS)	7.8%	6.6%	5.6%	6.3%	5.3%	4.5%
Stochastic simulation (VaR 0.5%)	9.4%	7.9%	6.9%	6.5%	5.5%	4.7%
Stochastic simulation (CTE 0.5%)	10.5%	9.0%	7.6%	7.3%	6.1%	5.3%
One-year stochastic simulation (0.5%)	4.4%	3.8%	3.4%	3.8%	3.3%	2.8%

will bring even less impact on its longevity risk. The effect of interest rates on the second type of longevity risk is the same as that of the first type. For all longevity risk measurement methods, the marginal interest rate and longevity risk are inversely proportional.

Overall, stochastic simulation based on VaR and CTE can better measure the longevity risk in the long run. Therefore, the calculation results have more clear practical significance for insurers [13]. The one-year VaR-based stochastic simulation rule decomposes the long-term longevity risk into each year, which can be used by the insurer to deal with the longevity risk in each policy year of the whole insurance period. The pressure trend method and the mortality fluctuation method are easy and convenient. The pressure trend method, which considers the extreme situation of a trend change, overestimates the longevity risk under a specific risk level. However, the adjustment of mortality fluctuation method to the lower limit of mortality fluctuation is relatively rough. From the calculation results of China's actual data, both QIS5 and C-ROSS measurement standards underestimate the longevity risk.

4. Investment Risk Calculation

Another important risk of a lifelong annuity is investment risk. Here, a simple measurement of investment risk is shown. According to the annually disclosed data of insurance funds' operation, the investment yields of China's insurance funds are 3.39%, 5.04%, 6.30%, 7.56%, and 5.66% from 2012 to 2016, respectively. Here, we calculated the risk of loss for the life annuity when the initial payment age is 60, and the actual investment returns are 3.39%, 3.25%, 3%, 2.75%, and 2.5%. The results are shown in Table 3. It is found that when the actual investment yield is lower than the pricing rate of 3.5%, the insurer will face additional losses. When the practical return of investment falls to 3.39%, the excess loss ratios for male and female life annuities are 1.35% and 1.25%, respectively. As the actual return on investment is lower than the pricing rate, the proportion of additional loss increases rapidly. When the actual investment yield drops to 2.5%, the loss rate of a male life annuity is 13.5%, and the loss rate of female life annuity reaches 12.5%, exceeding the level of longevity risk.

Figure 3 shows the risk of loss for life annuities at different initial payment ages when the actual investment yield is lower than the pricing interest rate of 3.39%. It is found

TABLE 3: Risk of investment loss for the lifelong pension of 60-year-olds.

	Actual return on investment				
	3.39%	3.25%	3.00%	2.75%	2.50%
Male	1.35%	3.11%	6.39%	9.84%	13.48%
Female	1.25%	2.88%	5.92%	9.10%	12.45%

FIGURE 3: Impact of initial payment age on lifetime annuity investment loss risk.

that when the initial payment age is lower than 93, the risk of loss for men is higher than that for women. When it is higher than 93, there is almost no gap between men and women. Overall, the risk of investment loss decreases as the initial payment age increases, which is mainly because the older the initial payment age is, the shorter the insurance period and the lower the investment risk we have.

5. Conclusion

This paper studies the annuity puzzle from the supply perspective, pays attention to the longevity risk faced by insurers, and focuses on the longevity risk measurement. We have established a unified framework of longevity risk measurement, covering population mortality model and longevity risk measurement model. Based on the actual data from China, the differences among diverse mortality models and different longevity risk measurement models were compared, and the longevity risk faced by China's life pension fund was measured.

As for the measurement of longevity risk, different risk measurement methods have their emphasis. The marginal

longevity risk based on the stochastic simulation of VaR and CTE is more realistic for insurers. In comparison, the pressure trend method overestimates the longevity risk, while QIS5 and C-ROSS based mortality volatility underestimates the longevity risk. It can be seen that current regulatory requirements cannot fully reflect the longevity risk. The stochastic simulation based on one-year perspective measures the longevity risk from a short-term perspective and its measured value is relatively small, reflecting the impact of the adverse changes in mortality in the next year. This method can shorten long-term longevity risk, so that insurers can gradually establish longevity risk reserve during the insurance period.

Above all, the life annuity insurance faces longevity risk, and the longevity risk scale could have significant impacts on the insurer's solvency. The inability to accurately assess the longevity risk and manage longevity risks has greatly limited the incentives for insurers to provide lifelong pensions. It is recommended to strengthen the study on longevity risk measurement and use the mortality prediction models that are suitable for China's condition and the risk measurement models that could accurately reflect the longevity risk to help the insurers precisely measure the lifelong longevity risk and prepare for the proper management.

Data Availability

The data used to support the findings of this study are from the Human Mortality Database and the China National Bureau of Statistics.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported by the Beijing Social Science Foundation of China (no. 19YJC042), the Special Foundation for Basic Scientific Research Business Expenses of Universities Owned by the Municipal Government of Beijing of Capital University of Economics and Business, titled "Research on the Measurement of Longevity Risk of Pension System Considering the Change of Death Pattern."

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