Research Article

Decentralized Supply Chains under Random Price-Dependent Demand: Noncooperative Equilibria vs. Coordination with Cost-Sharing Contracts

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It is common for a supplier to sell products to multiple retailers. In this paper, we investigate the equilibrium behavior of a decentralized supply chain with multiple retailers facing a random price-dependent demand in the additive form. Here, we consider two kinds of demand functions: the distribution of the demand depends only on the retailer's own retail price (noncompeting retailers) and not only on his own retail price but also on that of the other retailers (competing retailers). We present appropriate wholesale price, buy-back, and lost-sales cost-sharing contracts to coordinate the total supply chain, so that when all the retailers adopt their equilibrium response, the supply chain system coordination is also achieved. Furthermore, the coalition formation among retailers is also analyzed. We find that with buy-back and lost-sales cost-sharing contracts and linear price-dependent demand function, retailers always prefer being in the grand coalition to forming any other coalition.

1. Introduction

In the present global market, it is common that a supplier provides products to several different retailers. On account of different sales channel and logistic efficiency, it might happen that different retailers even sell identical products with different prices, possibly resulting from different logistic and supply chain management costs. A practical case is an international chain of convenience store 7-ELEVEN. For the identical stuffs, for instance, the beverage Coca-Cola and kinds of fruits are more expensive compared to supermarkets PARKnSHOP and Wellcome in Hong Kong. Although with higher retail prices compared to others, the convenience store still attracts customers because we can get the stuff we need at any time. Likewise, for the substitutable products, different retailers may naturally decide distinct retail prices. This phenomenon inevitably causes competitions between different retailers.

The competition of firms becomes increasingly fierce, and the form of competition is not confined within the scope

of prices, and moreover, other important factors, such as service accompanying the products. For example, the SAMSUNG Corporation and its competitor Apple Inc. both offer services for phones to customers, in order to motivate customers' willingness to buy. Fierce competition is not only among retailing companies, but also between seaports. Container ports, the key nodes in the world seaborne trade network that connect the integrated global economy, are actively competing with each other when they serve an overlapping hinterland, such as Los Angeles and Long Beach, along with Seattle, Tacoma, and Vancouver in North America; Antwerp, Rotterdam, Bremen, and Hamburg in West Europe; Singapore and Tanjung Pelepas in Southeast Asia; Hong Kong and Shenzhen in South China [1].

In this paper, we restrict our focus on a distribution system with one supplier and multiple retailers, where a common supplier distributes products to multiple retailers who in turn sell to customers. At the procurement market, the retailers face uncertain price-dependent demand for the products and have different demand functions and

parameters. The supplier decides their individual wholesale prices and the amounts of goods for his products, and simultaneously, the retailers determine retail prices for the products in the procurement market. Here, we make investigations on the noncompeting retailer and competing retailer cases, respectively.

In a decentralized supply chain, all agents primarily aim at optimizing their own individual objectives rather than the chain-wide objective; thus, their self-serving focus may result in a deterioration of the chain-wide performance. Therefore, coordinating supply chains has been a major issue in supply chain management (SCM) research. The importance of effective supply chain coordination is widely recognized since it enables all parties in a supply chain to work together so as to maximize the total profit of the entire supply chain system. To improve the total supply chain's performance, a proper mechanism must be developed to coordinate all channel members so that both the individual supply chain members' objectives and the chain-wide performance can be optimized. Supply chain contracts have been used as an effective means to coordinate the participants in the supply chain to achieve higher system efficiency in a large number of literatures on supply chain coordination. The main and effective contracts are buy-back contract, revenue-sharing contract, wholesale price contract, two-part tariff contract, etc. The use of supply chain mechanism (contracts) during the last decade has witnessed a rapidly increasing interest in supply chain coordination.

In this paper, we utilize game theory to analyze the nonoperative supply chain. We use the Nash equilibria to characterize the supplier's optimal quantity and the retailer's optimal retail price for the simultaneous-move game given the other retailers' decisions under the competing and noncompeting retailer cases. Since the supply chain members make optimal decisions to maximize their own profits, the system-wide expected profit is usually lower than the case when they coordinate their decisions. It is worth noting that the Nash equilibria may not lead to supply chain coordination. Therefore, in order to improve the supply chain performance, we develop appropriate wholesale price, buy-back, and lost-sales cost-sharing contracts to coordinate the total supply chain. That is, we design appropriate parameter settings so that the resulting Nash equilibria will be identical to the globally optimal solution. Under the properly designed contracts, all participants choose their equilibrium solutions, and meanwhile the maximum system-wide profit of the total supply chain is realized. We further explore the impacts of the customer's demand scale and the price-sensitivity coefficient on the optimal amount of goods the supplier provides and the optimal retail price determined by the retailers in the stochastic price-dependent demand. Finally, we investigate the coalition formation of retailers. We reach a conclusion that with wholesale price, buy-back, and lost-sales cost-sharing contracts and linear price-dependent demand function, retailers always prefer being in the grand coalition to forming any other coalition.

The remainder of this paper is organized as follows: Section 2 reviews the literature. In Section 3, we assume that the stochastic price-dependent demand is given in the

noncompeting retailer form and design appropriate wholesale price, buy-back, and lost-sales cost-sharing contracts to coordinate the total supply chain. In Section 4, we suppose that the stochastic price-dependent demand is given in the competing retailer form. The Nash equilibria existence among the retailers is proven, and then we design appropriate parameter settings of wholesale price, buy-back, and lost-sales cost-sharing contracts so that the resulting Nash equilibria is identical to the globally optimal solution. In this section, we further explore the impact of demand scale and price sensitivity coefficient on the optimal amount of goods the supplier provides and the optimal retail price determined by the retailers. Coalition formation among retailers is analyzed in Section 5. Numerical illustrations are presented in Section 6.

2. Literature Review

There is considerable literature devoted to contracts that coordinate a supply chain. We refer readers to Cachon [2] for a summary of supply chain contracts and coordination. As Cachon [2] indicated, supply chain coordination is achieved if and only if all agents in a decentralized supply chain can behave as if they are operating in a centralized supply chain. In this work, we take a stochastic price-sensitive demand function into account. Considering the price-dependent demand, both Wang [3], by using revenue-sharing contracts, and Yin [4], by considering wholesale price contracts, reached the conclusion that all suppliers earn the same profit independent of their individual costs. Moon et al. [5] developed a modified revenue-sharing contract which is proposed to coordinate the centralized distribution system. The analytical study reveals that without coordination among the channel members, the manufacturer always earns maximum profit in decentralized distribution systems. Moon et al. [6] investigated investment decisions in a supply chain of fresh agricultural products. Revenue sharing is coupled with the investment cost-sharing coordinate and a win-win outcome is proposed. Moon et al. [7] considered a two-period supply chain model under demand induced by selling price and investment effort in the presence of strategic inventory. Emmons and Gilbert [8] confirmed that, while a return policy could increase the expected profit of the entire supply chain, return policy alone fails to achieve channel coordination when demand is price dependent. Specifically, when the demand is multiplicative price dependent, Song et al. [9] identified the distribution-free necessary and sufficient conditions for obtaining the optimal return policy which maximizes the expected profit for the manufacturer. Lau et al. [10] considered a contracting problem with asymmetric cost information and price-dependent demand. In this paper, we consider the one supplier and multiple noncompeting and competing retailer model with uncertain price-dependent demand. Chen et al. [11] studied a model with one supplier and multiple noncompeting retailers with deterministic demand. They showed that a single-order quantity discount contract cannot coordinate the action of heterogeneous retailers, and then they proposed a set of transfer payments that

coordinated the whole supply chain. Unlike with stochastic demand, all customer demands were met without back ordering. Viswanathan and Piplani [12] analyzed a supply chain with multiple retailers and quantity discount contracts.

Recently, with game theory getting popular, an increasing number of scholars focus on applications of game theory in the supply chain. Several studies have examined the situation with competing retailers, focusing on the characterization of equilibrium behavior (cf, Bernstein and Federgruen [13] and Van Mieghem and Dada [14]). Bernstein and Federgruen [13] considered the special case of our model with two competing retailers, confining themselves to a characterization of the equilibrium behavior in the retailer game under a given pair of constant wholesale prices. Van Mieghem and Dada [14], as part of a larger study of the value of various types of postponement, analyzed a similar model, except that the retailer's capacities are chosen endogenously by the retailers in a first-stage game and except that the retailers, offering a completely homogeneous product, choose sales quantities rather than retail prices (the retailers thus face Cournot, as opposed to Bertrand, competition). Cachon and Zipkin [15] studied the optimal inventory decisions for a two-stage supply chain with stationary stochastic demand and fixed transportation times. They also compared the supply chain decisions under both competitive and cooperative games and investigated the supplier Stackelberg game against the retailer Stackelberg game. Wang [3] considered joint pricing-production decision problems in supply chains with complementary products for a single period, adopted a multiplicative demand model which is sensitive to sale price, and incorporated the consignment-sales and revenue-sharing contracts into both simultaneous-move and leader-follower games. Granot and Yin [16] investigated competition and cooperation in a multiple-supplier, one-manufacturer supply chain with complementary products. They used the definitions of the Nash equilibrium and farsighted stability to identify stable coalitional structures among suppliers. A Stackelberg game to examine the interactions between the manufacturer and suppliers was also developed. Lau et al. [17] suggested a stochastic and information-asymmetric Stackelberg game framework for a two-echelon supply chain under linear and isoelastic demand curves. Ertek and Griffen [18] explored the impact of power structure on price, sensitivity of market price, and profits in a two-stage supply chain that consists of a single supplier and a single buyer. They derived the pricing scheme when the supplier has dominant bargaining power versus the case where the buyer has a dominant bargaining power. Bernstein and DeCroix [19] considered an assembly supply chain where two components are used to assemble a single final product that is then sold by an assembler to meet the random, price-independent demand. They investigated the equilibrium base-stock levels for the assembler and two component suppliers and described a payment scheme to coordinate the assembly supply chain. In addition, in a recent literature review, Leng and Parlar [20] surveyed a large number of publications that focus on supply chain related to game-theoretic problems with substitutable products. Taleizadeh et al. [21] surveyed a production and inventory problem

under two scenarios in a three-layer supply chain including one distributor, one manufacturer, and one retailer. Nooridaryan et al. [22] examined the optimal pricing, ordering, promised lead time, and supplier-selection policies under three different game-theoretic approaches involving a decentralized approach under an open bid auction strategy, a centralized approach as well as a cooperative approach where demand is uncertain.

The cooperative game theory is relevant to our work since we try to analyze the coalition formation among the retailers. In this paper, unlike the previous works, we consider the coalition formation among the retailers. However, the coalition formation and alliance stability often appear in assembly models in the previous literature. Nagarajan and Sosic [23] examined a supply chain in which n suppliers sold complementary components to a downstream assembler. They, respectively, analyzed the coalition formation stability property between suppliers for the three modes of competition: supplier Stackelberg, vertical Nash, and assembler Stackelberg models. Yin [24] considered complementary suppliers in a price-sensitive assembly system and indicated that whether alliances are formed among suppliers crucially dependent on the form of the demand curve and the pass-through rate.

Different from the existing literature, the novelty of this work is to develop a combination of contracts consisting of appropriate wholesale price, buy-back, and lost-sales cost-sharing contracts to coordinate a distribution system, which will make the decentralized assembly supply chain behave like a centralized one. Furthermore, coalition formation among retailers is analyzed, which is rarely issued in previous supply chain coordination literature.

3. Noncompeting Retailers

We use the following notations throughout the paper:

- (i) p_i : retail price determined by the retailer $i = 1, 2, \dots, N$;
- (ii) $\mathbf{p}$: retail price vector $\mathbf{p} = (p_1, p_2, \dots, p_N)$;
- (iii) q_i : order quantity determined by the supplier for the retailer $i = 1, 2, \dots, N$;
- (iv) b_i : the supplier pays the retailer i 's buy-back price $i = 1, 2, \dots, N$;
- (v) w_i : the supplier charges the retailer $i = 1, 2, \dots, N$'s wholesale price;
- (vi) Π_i : the retailer $i = 1, 2, \dots, N$'s profit; the total supply chain profit when $i = SC$; the supplier's profit when $i = S$;
- (vii) $D_i(\mathbf{p}, \epsilon)$: price-dependent random demand;
- (viii) $d_i(\mathbf{p})$: deterministic component of the random demand of the retailer $i = 1, 2, \dots, N$;
- (ix) ϵ_i : stochastic component of the random demand of the retailer $i = 1, 2, \dots, N$ with cdf $F_i(\cdot)$ and pdf $f_i(\cdot)$ taking values in the range $[A_i, B_i]$, $i = 1, 2, \dots, N$;
- (x) u : underage cost per unit for lost sales;

(xi) ϕ_i : percentage of underage cost u absorbed by the retailer $i = 1, 2, \dots, N$.

To avoid trivial settings, assume $c < b_i < w_i, i = 1, 2, \dots, N$. For noncompeting retailers, the linear price-dependent demand function is denoted by

$$D_i(\mathbf{p}, \varepsilon) = \theta_i - \alpha_i p_i + \varepsilon_i, \quad i = 1, 2, \dots, N, \quad (1)$$

where $\theta_i > 0, \alpha_i > 0$. Here, ε_i is a nonnegative random variable with a finite mean μ that follows a density function $f_i(\cdot)$ and a distribution function $F_i(\cdot)$. We assume that $F_i(\cdot)$ is strictly increasing. Denote the unique inverse function of $F_i(\cdot)$ by $F_i^{-1}(\cdot)$. Since the wholesale price contract is commonly used in supply chains in practice, we use the wholesale price contract as the benchmark against which we compare the wholesale price, buy-back, and lost-sales cost-sharing contracts developed in this paper. With the wholesale price and

buy-back contracts, the supplier charges the retailer i wholesale price $w_i, i = 1, 2, \dots, n$, for per unit product and the retailer i returns the unused products to the supplier at the buy-back price $b_i, i = 1, 2, \dots, n$, per unit purchased product.

The retailer's and the supplier's expected profit functions under the wholesale price contract can be formulated as follows:

$$\begin{aligned} \Pi_i(p, q) &= (p_i - w_i)q_i - (p_i - b_i)[q_i - D_i(\mathbf{p}, \varepsilon)]^+ \\ &\quad - u[D_i(\mathbf{p}, \varepsilon) - q_i]^+, \quad i = 1, 2, \dots, N, \\ \Pi_S &= \sum_{i=1}^N (w_i - c)q_i - b_i[q_i - D_i(\mathbf{p}, \varepsilon)]^+. \end{aligned} \quad (2)$$

Expected profit of the retailer $i = 1, 2, \dots, N$ can now be founded as follows:

$$\begin{aligned} E(\Pi_i(p, q)) &= (p_i - w_i)q_i - (p_i - b_i)E[q_i - D_i(\mathbf{p}, \varepsilon)]^+ - uE[D_i(\mathbf{p}, \varepsilon) - q_i]^+ \\ &= (p_i - w_i)q_i - (p_i - b_i) \left(\int_{A_i}^{q_i - d_i(\mathbf{p})} (q_i - d_i(\mathbf{p}) - x) f(x) dx \right) \\ &\quad - u \left(\int_{q_i - d_i(\mathbf{p})}^{B_i} (d_i(\mathbf{p}) + x - q_i) f(x) dx \right) \\ &= (p_i - w_i)q_i - (p_i - b_i)(q_i - d_i(\mathbf{p}))F(q_i - d_i(\mathbf{p})) \\ &\quad + \int_{A_i}^{q_i - d_i(\mathbf{p})} F_i(x) dx - (q_i - d_i(\mathbf{p}))F(q_i - d_i(\mathbf{p})) \\ &\quad - u \left[(d_i(\mathbf{p}) - q_i)(1 - F(q_i - d_i(\mathbf{p}))) + B_i - (q_i - d_i(\mathbf{p}))F(q_i - d_i(\mathbf{p})) - \int_{q_i - d_i(\mathbf{p})}^{B_i} F(x) dx \right] \\ &= (p_i - w_i)q_i - (p_i - b_i) \int_{A_i}^{q_i - d_i(\mathbf{p})} F_i(x) dx - u \left[-q_i + d_i(\mathbf{p}) + B_i - \int_{q_i - d_i(\mathbf{p})}^{B_i} F(x) dx \right]. \end{aligned} \quad (3)$$

Expected profit of the supplier is also can be formulated as follows:

$$\begin{aligned} E(\Pi_S(p, q)) &= \sum_{i=1}^N (w_i - c)q_i - b_i E[q_i - D_i(\mathbf{p}, \varepsilon)]^+ \\ &= \sum_{i=1}^N (w_i - c)q_i - b_i \int_{A_i}^{q_i - d_i(\mathbf{p})} (q_i - d_i(\mathbf{p}) - x) f(x) dx \\ &= \sum_{i=1}^N (w_i - c)q_i - b_i \int_{A_i}^{q_i - d_i(\mathbf{p})} F_i(x) dx. \end{aligned} \quad (4)$$

With the wholesale price contract, the retailer's $i = 1, 2, \dots, N$ best response retail price p_i^* and the supplier's

production quantity q_i^* are determined as the following nonlinear equations:

$$\begin{cases} q_i^* - \int_{A_i}^{q_i^* - d_i(\mathbf{p}^*)} F(x) dx - \alpha_i(p_i^* - b_i)F(q_i^* - d_i(\mathbf{p}^*)) + \alpha_i u(1 - F(q_i^* - d_i(\mathbf{p}^*))) = 0 \\ q_i^* = F^{-1}\left(\frac{w_i - c}{b_i}\right) + d_i(\mathbf{p}^*). \end{cases} \quad (5)$$

Therefore, with the supplier's optimal production quantity q_i^* and the retailer's optimal retail price

$p_i^*, i = 1, 2, \dots, N$, the retailer's and the supplier's expected profit functions are

$$\begin{aligned} E(\Pi_i(p^*, q^*)) &= (p_i^* - w_i)q_i^* - (p_i^* - b_i) \int_{A_i}^{q_i^* - d_i(\mathbf{p}^*)} F_i(x) dx - u \left[-q_i^* + d_i(\mathbf{p}^*) + B_i - \int_{q_i^* - d_i(\mathbf{p}^*)}^{B_i} F(x) dx \right], \\ E(\Pi_S(p^*, q^*)) &= \sum_{i=1}^N (w_i - c)q_i^* - b_i \int_{A_i}^{q_i^* - d_i(\mathbf{p}^*)} F_i(x) dx, \end{aligned} \quad (6)$$

respectively.

When the amount of goods determined by the supplier for every product is smaller than market demands, it incurs the shortage cost. Therefore, in this paper, we assume that any unsatisfied demand incurs a certain penalty cost. In the lost-sales cost-sharing contract, when shortages arise, the supplier and the retailer share the shortage cost. In the trade

between the supplier and the retailer $i = 1, 2, \dots, n$, we set the cost share percentage of the cost taken by the retailer i as $\phi_i \in [0, 1]$. Therefore, the supplier bears the remaining $1 - \phi_i, i = 1, 2, \dots, n$. Taking the lost-sales cost-sharing contract into account, accordingly, the profits of the retailer and the supplier can be expressed as

$$\begin{aligned} \Pi_i(p, q) &= (p_i - w_i)q_i - (p_i - b_i)[q_i - D_i(\mathbf{p}, \varepsilon)]^+ - \phi_i u[D_i(\mathbf{p}, \varepsilon) - q_i]^+, \quad i = 1, 2, \dots, N, \\ \Pi_S &= \sum_{i=1}^N (w_i - c)q_i - b_i[q_i - D_i(\mathbf{p}, \varepsilon)]^+ - (1 - \phi_i)u[D_i(\mathbf{p}, \varepsilon) - q_i]^+, \end{aligned} \quad (7)$$

respectively. Expected profit of the retailer $i = 1, 2, \dots, N$ can now be expressed as follows:

$$\begin{aligned} E(\Pi_i(p, q)) &= (p_i - w_i)q_i - (p_i - b_i)E[q_i - D_i(\mathbf{p}, \varepsilon)]^+ - \phi_i uE[D_i(\mathbf{p}, \varepsilon) - q_i]^+ \\ &= (p_i - w_i)q_i - (p_i - b_i) \left(\int_{A_i}^{q_i - d_i(\mathbf{p})} (q_i - d_i(\mathbf{p}) - x) f(x) dx \right) \\ &\quad - \phi_i u \left(\int_{q_i - d_i(\mathbf{p})}^{B_i} (d_i(\mathbf{p}) + x - q_i) f(x) dx \right) \\ &= (p_i - w_i)q_i - (p_i - b_i)(q_i - d_i(\mathbf{p}))F(q_i - d_i(\mathbf{p})) \\ &\quad + \int_{A_i}^{q_i - d_i(\mathbf{p})} F_i(x) dx - (q_i - d_i(\mathbf{p}))F(q_i - d_i(\mathbf{p})) \\ &\quad - \phi_i u \left[(d_i(\mathbf{p}) - q_i)(1 - F(q_i - d_i(\mathbf{p}))) + B_i - (q_i - d_i(\mathbf{p}))F(q_i - d_i(\mathbf{p})) - \int_{q_i - d_i(\mathbf{p})}^{B_i} F(x) dx \right] \\ &= (p_i - w_i)q_i - (p_i - b_i) \int_{A_i}^{q_i - d_i(\mathbf{p})} F_i(x) dx \\ &\quad - \phi_i u \left[-q_i + d_i(\mathbf{p}) + B_i - \int_{q_i - d_i(\mathbf{p})}^{B_i} F(x) dx \right]. \end{aligned} \quad (8)$$

Expected profit of the supplier, in turn, can be formulated as follows:

$$\begin{aligned}
 E(\Pi_S(p, q)) &= \sum_{i=1}^N (w_i - c)q_i - b_i E[q_i - D_i(\mathbf{p}, \varepsilon)]^+ - (1 - \phi_i)uE[D_i(\mathbf{p}, \varepsilon) - q_i]^+ \\
 &= \sum_{i=1}^N (w_i - c)q_i - b_i \int_{A_i}^{q_i - d_i(\mathbf{p})} (q_i - d_i(\mathbf{p}) - x)f(x)dx \\
 &\quad - (1 - \phi_i)u \left(\int_{q_i - d_i(\mathbf{p})}^{B_i} (d_i(\mathbf{p}) + x - q_i)f(x)dx \right) \\
 &= \sum_{i=1}^N (w_i - c)q_i - b_i \int_{A_i}^{q_i - d_i(\mathbf{p})} F_i(x)dx - (1 - \phi_i)u \left[-q_i + d_i(\mathbf{p}) + B_i - \int_{q_i - d_i(\mathbf{p})}^{B_i} F(x)dx \right].
 \end{aligned} \tag{9}$$

Likewise, the expected profit of the whole supply chain can be expressed as follows:

$$\Pi^{\text{SC}}(p, q) = \sum_{i=1}^N E(\Pi_i(p, q)) + E(\Pi_S(p, q)) = \sum_{i=1}^N \Pi_i^{\text{SC}}(p, q), \tag{10}$$

where $\Pi_i^{\text{SC}}(p, q) = (p_i - c)q_i - p_i[q_i - D_i(\mathbf{p}, \varepsilon)]^+ - u[D_i(\mathbf{p}, \varepsilon) - q_i]^+$. The expected profit of the whole supply chain can now be founded as follows:

$$\begin{aligned}
 E(\Pi_i^{\text{SC}}(p, q)) &= (p_i - c)q_i - p_i E[q_i - D_i(\mathbf{p}, \varepsilon)]^+ - uE[D_i(\mathbf{p}, \varepsilon) - q_i]^+ \\
 &= (p_i - c)q_i - p_i \left(\int_{A_i}^{q_i - d_i(\mathbf{p})} (q_i - d_i(\mathbf{p}) - x)f(x)dx \right) \\
 &\quad - u \left(\int_{q_i - d_i(\mathbf{p})}^{B_i} (d_i(\mathbf{p}) + x - q_i)f(x)dx \right) \\
 &= (p_i - c)q_i - p_i (q_i - d_i(\mathbf{p}))F(q_i - d_i(\mathbf{p})) + \int_{A_i}^{q_i - d_i(\mathbf{p})} F_i(x)dx \\
 &\quad - (q_i - d_i(\mathbf{p}))F(q_i - d_i(\mathbf{p})) \\
 &\quad - u \left[(d_i(\mathbf{p}) - q_i)(1 - F(q_i - d_i(\mathbf{p}))) + B_i - (q_i - d_i(\mathbf{p}))F(q_i - d_i(\mathbf{p})) - \int_{(q_i - d_i(\mathbf{p}))}^{B_i} F(x)dx \right] \\
 &= (p_i - c)q_i - p_i \int_{A_i}^{q_i - d_i(\mathbf{p})} F_i(x)dx - u \left[-q_i + d_i(\mathbf{p}) + B_i - \int_{(q_i - d_i(\mathbf{p}))}^{B_i} F(x)dx \right].
 \end{aligned} \tag{11}$$

Therefore,

$$\begin{aligned}
 E(\Pi^{\text{SC}}(p, q)) &= \sum_{i=1}^N \left\{ (p_i - c)q_i - p_i \int_{A_i}^{q_i - d_i(\mathbf{p})} F_i(x)dx \right. \\
 &\quad \left. - u \left[-q_i + d_i(\mathbf{p}) + B_i - \int_{(q_i - d_i(\mathbf{p}))}^{B_i} F(x)dx \right] \right\}.
 \end{aligned} \tag{12}$$

We consider the retailer's best response for the retail price and the supplier's response for the production quantity for the simultaneous-move game. We begin by showing that,

for each retailer $i = 1, 2, \dots, N$, the expected profit $E(\Pi_i(p, q))$ is a unimodal function of the retail price p_i . Differentiating $E(\Pi_i(p, q))$ w.r.t. p_i , we have

$$\begin{aligned}
 \frac{\partial E(\Pi_i(p, q))}{\partial p_i} &= q_i - \int_{A_i}^{q_i - d_i(\mathbf{p})} F(x)dx + (p_i - b_i)F(q_i - d_i(\mathbf{p})) \\
 &\quad - \frac{\partial d_i(\mathbf{p})}{\partial p_i} - \phi_i \frac{\partial d_i(\mathbf{p})}{\partial p_i} u + \phi_i u F(q_i - d_i(\mathbf{p})) \frac{\partial d_i(\mathbf{p})}{\partial p_i}.
 \end{aligned} \tag{13}$$

Differentiating $\partial E(\Pi_i(p, q))/\partial p_i$ w.r.t. p_i again, one has

$$\begin{aligned}
\frac{\partial^2 E(\Pi_i(p, q))}{\partial p_i^2} &= F(q_i - d_i(\mathbf{p})) \frac{\partial d_i(\mathbf{p})}{\partial p_i} + F(q_i - d_i(\mathbf{p})) \frac{\partial d_i(\mathbf{p})}{\partial p_i} \\
&\quad + (p_i - b_i) \left[-f(q_i - d_i(\mathbf{p})) \left(\frac{\partial d_i(\mathbf{p})}{\partial p_i} \right)^2 + F(q_i - d_i(\mathbf{p})) \frac{\partial^2 d_i(\mathbf{p})}{\partial p_i^2} \right] \\
&\quad - \phi_i u \frac{\partial^2 d_i(\mathbf{p})}{\partial p_i^2} + \phi_i u \left(-f(q_i - d_i(\mathbf{p})) \left(\frac{\partial d_i(\mathbf{p})}{\partial p_i} \right)^2 + \frac{\partial^2 d_i(\mathbf{p})}{\partial p_i^2} \right) \\
&= -2\alpha_i F(q_i - d_i(\mathbf{p})) - (p_i - b_i) f(q_i - d_i(\mathbf{p})) \alpha_i^2 - \phi_i u \alpha_i^2 f(q_i - d_i(\mathbf{p})) \\
&< 0.
\end{aligned} \tag{14}$$

This implies that every retailer $i = 1, 2, \dots, N$'s expected profit is a unimodal function with respect to the retail price p_i . Furthermore, we derive the supplier's optimal production quantity as follows. Let

$$\begin{aligned}
\frac{\partial E(\Pi_s(p, q))}{\partial q_i} &= (w_i - c) - b_i F(q_i - d_i(\mathbf{p})) \\
&\quad + (1 - \phi_i) u (1 - F(q_i - d_i(\mathbf{p}))) = 0.
\end{aligned} \tag{15}$$

Differentiating $\partial E(\Pi_i(p, q))/\partial q_i$ w.r.t. q_i again, one has

$$\frac{\partial^2 E(\Pi_i(p, q))}{\partial q_i^2} = -(p_i - b_i) f(q_i - d_i(\mathbf{p})) - \phi_i u f(q_i - d_i(\mathbf{p})) < 0. \tag{16}$$

This implies that every retailer $i = 1, 2, \dots, N$'s expected profit function is a concave function with respect to the production quantity q_i . Thus, there exists a unique optimal production quantity $q_i^*, i = 1, 2, \dots, N$. By the simple algebra, we have

$$q_i = F^{-1} \left(\frac{w_i - c + (1 - \phi_i) u}{b_i + (1 - \phi_i) u} \right) + d_i(\mathbf{p}). \tag{17}$$

To summarize, the retailer $i = 1, 2, \dots, N$'s best response retail price p_i^* and the supplier's production quantity q_i^* are determined as the unique solution of the following nonlinear equations:

$$\begin{cases} q_i^* - \int_{A_i}^{q_i^* - d_i(\mathbf{p}^*)} F(x) dx - \alpha_i (p_i^* - b_i) F(q_i^* - d_i(\mathbf{p}^*)) + \alpha_i \phi_i u (1 - F(q_i^* - d_i(\mathbf{p}^*))) = 0, \\ q_i^* = F^{-1} \left(\frac{w_i - c + (1 - \phi_i) u}{b_i + (1 - \phi_i) u} \right) + d_i(\mathbf{p}^*). \end{cases} \tag{18}$$

We first characterize the solution to the centralized system with a central planner who makes all pricing and ordering quantity decisions so as to maximize the channel-wide profits. The system optimal order quantity $q_i^{\text{SC}}, i =$

$1, 2, \dots, N$, and the system optimal retail price $p_i^{\text{SC}}, i = 1, 2, \dots, N$, are obtained from the following nonlinear equations:

$$\begin{cases} q_i^{\text{SC}} - \int_{A_i}^{q_i^{\text{SC}} - d_i(\mathbf{p}^{\text{SC}})} F(x) dx - \alpha_i p_i^{\text{SC}} F(q_i^{\text{SC}} - d_i(\mathbf{p}^{\text{SC}})) + \alpha_i u (1 - F(q_i^{\text{SC}} - d_i(\mathbf{p}^{\text{SC}}))) = 0, \\ q_i^{\text{SC}} = F^{-1} \left(\frac{p_i^{\text{SC}} - c + u}{p_i^{\text{SC}} + u} \right) + d_i(\mathbf{p}^{\text{SC}}). \end{cases} \tag{19}$$

In order to ensure $q_i^* = q_i^{\text{SC}}, i = 1, 2, \dots, N$, and $p_i^* = p_i^{\text{SC}}, i = 1, 2, \dots, N$, equating the stated equations and simplifying,

we need to adjust the parameter values (w_i, ϕ_i, b_i) as follows so that the supply chain coordination can be achieved:

$$\begin{cases} b_i z_i = (1 - z_i)(1 - \phi_i)u, \\ z_i = \frac{w_i - c + (1 - \phi_i)u}{b_i + (1 - \phi_i)u}, \\ z_i = \frac{p_i^{\text{SC}} - c + u}{p_i^{\text{SC}} + u}, \end{cases} \quad (20)$$

for $i = 1, 2, \dots, N$.

By the easy induction and algebra, we get

$$\begin{aligned} \phi_i &= 0, \\ w_i &= c = p_i^{\text{SC}}, \\ b_i &= c. \end{aligned} \quad (21)$$

This shows that supply chain coordination can be achieved by a pair of the properly designed wholesale price, buy-back, and lost-sales cost-sharing contracts with the above parameter setting. However, we learn from the parameter setting that the supplier charges the wholesale prices from all retailers equal to the production costs in order to induce the supply chain coordination and the retailers also set the selling price to be the production costs. Therefore, the

supplier and retailers may all lose the incentive to cooperate for the supply chain coordination. We then show that the centralized solution can be realized in a decentralized system through a proper contract.

Theorem 1. Consider the wholesale price, buy-back, and cost-sharing contracts with

$$w_i = (1 - \phi_i)p_i + \phi_i c, b_i = (1 - \phi_i)p_i, \quad (22)$$

and where $\phi_i \in (0, 1)$. With these contracts, the retailer's profit function is

$$\Pi_i(p, q) = \phi_i \Pi_i^{\text{SC}}(p, q), \quad i = 1, 2, \dots, N. \quad (23)$$

The supplier's profit function is

$$\Pi_S(p, q) = \sum_{i=1}^N (1 - \phi_i) \Pi_i^{\text{SC}}(p, q). \quad (24)$$

Proof. For each retailer $i = 1, 2, \dots, N$, the profits can be expressed as follows according to the above parameter setting:

$$\begin{aligned} \Pi_i(p, q) &= (p_i - w_i)q_i - (p_i - b_i)[q_i - D_i(\mathbf{p}, \varepsilon)]^+ - \phi_i u [D_i(\mathbf{p}, \varepsilon) - q_i]^+ \\ &= (p_i - (1 - \phi_i)p_i + \phi_i c)q_i - (p_i - (1 - \phi_i)p_i)[q_i - D_i(\mathbf{p}, \varepsilon)]^+ - \phi_i u [D_i(\mathbf{p}, \varepsilon) - q_i]^+ \\ &= \phi_i [(p_i - c)q_i - p_i [q_i - D_i(\mathbf{p}, \varepsilon)]^+ - u [D_i(\mathbf{p}, \varepsilon) - q_i]^+] \\ &= \phi_i \Pi_i^{\text{SC}}(p, q). \end{aligned} \quad (25)$$

And hence, the supplier's profit formulation is also shown as follows:

$$\begin{aligned} \Pi_S(p, q) &= \sum_{i=1}^N (w_i - c)q_i - b_i [q_i - D_i(\mathbf{p}, \varepsilon)]^+ - (1 - \phi_i)u [D_i(\mathbf{p}, \varepsilon) - q_i]^+ \\ &= \sum_{i=1}^N ((1 - \phi_i)p_i + \phi_i c - c)q_i - (1 - \phi_i)p_i [q_i - D_i(\mathbf{p}, \varepsilon)]^+ - (1 - \phi_i)u [D_i(\mathbf{p}, \varepsilon) - q_i]^+ \\ &= \sum_{i=1}^N (1 - \phi_i) [(p_i - c)q_i - p_i [q_i - D_i(\mathbf{p}, \varepsilon)]^+ - u [D_i(\mathbf{p}, \varepsilon) - q_i]^+] \\ &= \sum_{i=1}^N (1 - \phi_i) \Pi_i^{\text{SC}}(p, q). \end{aligned} \quad (26)$$

Therefore, $(p_i^{\text{SC}}, q_i^{\text{SC}})$, optimizing $\Pi_i^{\text{SC}}(p, q)$, is also the retailer's and the supplier's optimal retail price and order quantity pair, i.e., those contracts coordinate the total supply chain for the cost division $\phi_i \in (0, 1)$. $\square$

Although there exists the cost division setting $\phi_i \in (0, 1)$ under which the whole supply chain's system-wide optimal can be achieved, the retailers and the supplier may have no

incentive to participate in wholesale price, buy-back, and lost-sales cost-sharing contracts since these contracts do not make them strictly better off than that under the wholesale price contract. Therefore, in order to meet the supplier's and retailers' requirements, we require that the cost division $\phi_i, i = 1, 2, \dots, N$ simultaneously satisfies the following conditions:

$$\begin{aligned} \phi_i &\geq \frac{\Pi_i(p^*, q^*)}{\Pi_i^{\text{SC}}(p^{\text{SC}}, q^{\text{SC}})}, \quad i = 1, 2, \dots, N, \\ \sum_{i=1}^N (1 - \phi_i) \Pi_i^{\text{SC}}(p^{\text{SC}}, q^{\text{SC}}) &\geq \Pi_S(p^*, q^*), \end{aligned} \quad (27)$$

where $\Pi_i(p^*, q^*)$ and $\Pi_S(p^*, q^*)$ denote the optimal retailer i and the supplier's profits under the wholesale price contract, respectively. $\square$

4. Competing Retailers

In this section, we consider the general case of N competing retailers. Here, for the demand faced by any retailer $i = 1, 2, \dots, N$, the cdf of D_i depends on the entire price vector $\mathbf{p}$. Furthermore, because the products offered by the retailers are substitutes, we make the standard assumption that the demand function $d_i(\mathbf{p})$ is differentiable with

Therefore, the demand function can be denoted as

$$D_i(\mathbf{p}, \varepsilon) = \theta_i - \alpha_i p_i + \sum_{j \neq i} \gamma_j p_j + \varepsilon_i, \quad \alpha_i > \gamma_j > 0, \quad \forall j \neq i. \quad (29)$$

Theorem 2. *With the competing retailers, there exists a unique equilibrium of the game in which the supplier offers the buy-back and cost-sharing contract. It is characterized by the following best response retail price functions and the optimal order quantity:*

$$\begin{cases} p_i^* = \frac{F^{-1}(w_i - c + (1 - \phi_i)u/b_i + (1 - \phi_i)u) + \theta_i + \sum_{j \neq i} \gamma_j p_j^* - \int_{A_i}^{F^{-1}(w_i - c + (1 - \phi_i)u/b_i + (1 - \phi_i)u)} F(x) dx + \alpha_i b_i (w_i - c + (1 - \phi_i)u/b_i + (1 - \phi_i)u) + \alpha_i \phi_i u (b_i + c - w_i/b_i + (1 - \phi_i)u)}{\alpha_i (1 + (w_i - c + (1 - \phi_i)u/b_i + (1 - \phi_i)u))}, \\ q_i^* = F^{-1}\left(\frac{w_i - c + (1 - \phi_i)u}{b_i + (1 - \phi_i)u}\right) + \theta_i - \alpha_i p_i^* + \sum_{j \neq i} \gamma_j p_j^*. \end{cases} \quad (30)$$

Proof. As the previous analysis, we have

$$\frac{\partial E(\Pi_i(p, q))}{\partial p_i} = q_i - \int_{A_i}^{q_i - d_i(\mathbf{p})} F(x) dx + (p_i - b_i) F(q_i - d_i(\mathbf{p})) \frac{\partial d_i(\mathbf{p})}{\partial p_i} - \phi_i \frac{\partial d_i(\mathbf{p})}{\partial p_i} u + \phi_i u F(q_i - d_i(\mathbf{p})) \frac{\partial d_i(\mathbf{p})}{\partial p_i}. \quad (31)$$

Differentiating $\partial E(\Pi_i(p, q))/\partial p_i$ w.r.t. p_i again, one has

$$\begin{aligned} \frac{\partial^2 E(\Pi_i(p, q))}{\partial p_i^2} &= F(q_i - d_i(\mathbf{p})) \frac{\partial d_i(\mathbf{p})}{\partial p_i} + F(q_i - d_i(\mathbf{p})) \frac{\partial d_i(\mathbf{p})}{\partial p_i} + (p_i - b_i) \\ &\quad \cdot \left[-f(q_i - d_i(\mathbf{p})) \left(\frac{\partial d_i(\mathbf{p})}{\partial p_i} \right)^2 + F(q_i - d_i(\mathbf{p})) \frac{\partial^2 d_i(\mathbf{p})}{\partial p_i^2} \right] - \phi_i u \frac{\partial^2 d_i(\mathbf{p})}{\partial p_i^2} \\ &\quad + \phi_i u \left(-f(q_i - d_i(\mathbf{p})) \left(\frac{\partial d_i(\mathbf{p})}{\partial p_i} \right)^2 + \frac{\partial^2 d_i(\mathbf{p})}{\partial p_i^2} \right) \\ &= -2\alpha_i F(q_i - d_i(\mathbf{p})) - (p_i - b_i) f(q_i - d_i(\mathbf{p})) \alpha_i^2 - \phi_i u \alpha_i^2 f(q_i - d_i(\mathbf{p})) \\ &< 0, \end{aligned} \quad (32)$$

which verifies that the profit function of the retailer $i = 1, 2, \dots, N$ is concave in the retail price p_i . Thus,

$$\frac{\partial E(\Pi_i(p, q))}{\partial q_i} = (w_i - c) - b_i F(q_i - d_i(\mathbf{p})) + (1 - \phi_i)u(1 - F(q_i - d_i(\mathbf{p}))). \quad (33)$$

Differentiating $\partial E(\Pi_i(p, q))/\partial q_i$ w.r.t. p_i ,

$$\frac{\partial^2 E(\Pi_i(p, q))}{\partial q_i \partial p_i} = -b_i f(q_i - d_i(\mathbf{p}))\alpha_i - (1 - \phi_i)uf(q_i - d_i(\mathbf{p}))\alpha_i. \quad (34)$$

Differentiating $\partial E(\Pi_i(p, q))/\partial q_i$ w.r.t. p_j , one has

$$\frac{\partial^2 E(\Pi_i(p, q))}{\partial q_i \partial p_j} = b_i f(q_i - d_i(\mathbf{p}))\gamma_j + (1 - \phi_i)uf(q_i - d_i(\mathbf{p}))\gamma_j. \quad (35)$$

$$\begin{cases} p_i^* = \frac{F^{-1}(w_i - c + (1 - \phi_i)u/b_i + (1 - \phi_i)u) + \theta_i + \sum_{j \neq i} \gamma_j p_j^* - \int_{A_i}^{F^{-1}(w_i - c + (1 - \phi_i)u/b_i + (1 - \phi_i)u)} F(x) dx + \alpha_i b_i (w_i - c + (1 - \phi_i)u/b_i + (1 - \phi_i)u) + \alpha_i \phi_i u (b_i + c - w_i/b_i + (1 - \phi_i)u)}{\alpha_i (1 + (w_i - c + (1 - \phi_i)u/b_i + (1 - \phi_i)u))}, \\ q_i^* = F^{-1}\left(\frac{w_i - c + (1 - \phi_i)u}{b_i + (1 - \phi_i)u}\right) + \theta_i - \alpha_i p_i^* + \sum_{j \neq i} \gamma_j p_j^*. \end{cases} \quad (37)$$

Next, we investigate the relationship between the optimal price and the optimal order quantity in the meaning of the Nash equilibrium. From the above expressions of the optimal price and the optimal order quantity in the Nash equilibrium, we can easily derive that

$$\begin{aligned} \frac{\partial q_i^*}{\partial p_i^*} &= -\alpha_i < 0, \\ &= \gamma_j > 0, \quad i = 1, 2, \dots, N, j \neq i. \end{aligned} \quad (38)$$

Thus, with the optimal equilibrium price of the retailer i increasing, the optimal equilibrium order quantity of the retailer i decreases. It is easy to understand because with retail price p_i^* increasing, one will receive less demand transferred to the other retailers. So, the retailer i should

Thus, by the implicit function theorem, this yields that

$$\begin{aligned} \frac{\partial p_i}{\partial p_j} &= -\frac{\partial^2 E(\Pi_i(p, q))/\partial q_i \partial p_j}{\partial^2 E(\Pi_i(p, q))/\partial q_i \partial p_i} \\ &= \frac{b_i f(q_i - d_i(\mathbf{p}))\gamma_j + (1 - \phi_i)uf(q_i - d_i(\mathbf{p}))\gamma_j}{b_i f(q_i - d_i(\mathbf{p}))\alpha_i + (1 - \phi_i)uf(q_i - d_i(\mathbf{p}))\alpha_i} = \frac{\gamma_j}{\alpha_i}. \end{aligned} \quad (36)$$

It follows from the reasonable assumptions of $F(\cdot) > 0$, $\alpha_i > \gamma_j > 0$ that $0 < \partial p_i / \partial p_j < 1$. Hence, we have increasing reaction functions with a slope less than 1. This implies that there is a unique equilibrium of the game between the retailers. There also exists a Nash equilibrium between the supplier and retailers. The expression is characterized by the best response retail price and optimal order quantity as follows:

reduce one's order quantity. And, in the other hand, with the optimal equilibrium price of the retailer j increasing, the optimal equilibrium order quantity of the retailer i also increases. With the retail price p_j^* increasing, the retailer i will receive more demand transferred from the retailer j . So, the retailer i should increase one's order quantity.

Like noncompeting retailers, for competing retailers, we have similar nonlinear equations to determine the optimal order quantity of the supplier and optimal retail price of each retailer. The retailer i 's best response retail price p_i^* and order quantity q_i^* , $i = 1, 2, \dots, N$, are determined as the unique solution of the following nonlinear equations in Theorem 2. Similarly, we derive that the optimal order quantity $q_i^{\text{SC}}, i = 1, 2, \dots, N$, and the optimal retail price $p_i^{\text{SC}}, i = 1, 2, \dots, N$, from the following nonlinear equations:

$$\begin{cases} q_i^{\text{SC}} - \int_{A_i}^{q_i^{\text{SC}} - d_i(\mathbf{p}^{\text{SC}})} F(x) dx - \alpha_i p_i^{\text{SC}} F(q_i^{\text{SC}} - d_i(\mathbf{p}^{\text{SC}})) + \alpha_i u (1 - F(q_i^{\text{SC}} - d_i(\mathbf{p}^{\text{SC}}))) = 0, \\ q_i^{\text{SC}} = F^{-1}\left(\frac{p_i^{\text{SC}} - c + u}{p_i^{\text{SC}} + u}\right) + \theta_i - \alpha_i p_i^{\text{SC}} + \sum_{j \neq i} \gamma_j p_j^{\text{SC}}. \end{cases} \quad (39)$$

Theorem 3. Consider the wholesale price, buy-back, and lost-sales cost-sharing contracts with

$$\begin{aligned} w_i &= (1 - \phi_i)p_i + \phi_i c, \\ b_i &= (1 - \phi_i)p_i, \end{aligned} \quad \square \quad (40)$$

and where $\phi_i \in (0, 1)$. With these contracts, the retailer i 's profit function is

$$\Pi_i(p, q) = \phi_i \Pi_i^{\text{SC}}(p, q), \quad i = 1, 2, \dots, N. \quad (41)$$

The supplier's profit function is

$$\Pi_S(p, q) = \sum_{i=1}^N (1 - \phi_i) \Pi_i^{\text{SC}}(p, q). \quad (42)$$

Therefore, $(p_i^{\text{SC}}, q_i^{\text{SC}})$, optimizing $\Pi_i^{\text{SC}}(p, q)$, is also the retailer's and the supplier's optimal retail price and order quantity pair, i.e., those contracts coordinate the total supply chain for arbitrary cost division $\phi_i \in (0, 1)$.

For the demand function with the competing retailers,

$$D_i(\mathbf{p}, \varepsilon) = \theta_i - \alpha_i p_i + \sum_{j \neq i} \gamma_j p_j + \varepsilon_i, \quad \alpha_i > \gamma_j > 0, \quad \forall j \neq i, \quad (43)$$

where the coefficients α_i and $\gamma_j, i \neq j$, are the price sensitivity variables and θ_i presents the customer's demand scale from the retailer i . The larger the number α_i is, the more sensitive attitude the customers have for the retail price. Now, we investigate the relationship between the equilibrium retail price, the equilibrium production quantity, and the parameters θ_i and $\alpha_i, i = 1, 2, \dots, N$, respectively.

As previously discussed, for the retailer $i, i = 1, 2, \dots, N$, we define

$$H_i(p^*, q^*) = q_i^* - \int_{A_i}^{q_i^* - d_i(\mathbf{p}^*)} F(x) dx - \alpha_i (p_i^* - b_i) \cdot F(q_i^* - d_i(\mathbf{p}^*)) + \alpha_i \phi_i u (1 - F(q_i^* - d_i(\mathbf{p}^*))) = 0, \quad (44)$$

where $q_i^* = F^{-1}(w_i - c + (1 - \phi_i)u/b_i + (1 - \phi_i)u) + \theta_i - \alpha_i p_i^* + \sum_{j \neq i} \gamma_j p_j^*, i = 1, 2, \dots, N$. By the implicit function theorem, one has

$$\begin{aligned} \frac{\partial p_i^*}{\partial \theta_i} &= \frac{\partial H_i(p^*, q^*) / \partial \theta_i}{\partial H_i(p^*, q^*) / \partial p_i} = \frac{1}{2\alpha_i} > 0, \quad i = 1, 2, \dots, N, \\ \frac{\partial p_i^*}{\partial \alpha_i} &= \frac{\partial H_i(p^*, q^*) / \partial \alpha_i}{\partial H_i(p^*, q^*) / \partial p_i} = \frac{p_i^* + (p_i^* - b_i)F(q_i^* - d_i(\mathbf{p}^*)) - \phi_i u (1 - F(q_i^* - d_i(\mathbf{p}^*)))}{2\alpha_i} < 0, \quad i = 1, 2, \dots, N. \end{aligned} \quad (45)$$

This is because $p_i^* > u, i = 1, 2, \dots, N$. The equilibrium retail price is increasing with respect to the demand scale and decreasing with respect to the price sensitivity variable. It is easily understood. When the demand scale arises, the retailer $i, i = 1, 2, \dots, N$, can increase the retail price to earn more profits. For the price sensitivity variable, this result tells us that when the customer is risk averse, the retailer should decrease one's retail price. Furthermore, from the expression of $q_i^*, i = 1, 2, \dots, N$,

$$\begin{aligned} \frac{\partial q_i^*}{\partial \theta_i} &= 1, \\ \frac{\partial q_i^*}{\partial \alpha_i} &= -p_i^*, \quad i = 1, 2, \dots, N. \end{aligned} \quad (46)$$

The equilibrium order quantity of the retailer i is linearly increasing with respect to the demand scale and likewise, linearly decreasing with respect to the price sensitivity variable with a slope $-p_i, i = 1, 2, \dots, N$.

In the next section, from the retailers' view, we will investigate whether the coalition formation of the retailers is a better choice for them.

5. Coalition for Retailers

In this section, we analyze the coalition formation of retailers. Before we present the exact conclusion, we first review some concepts from cooperative game theory that are relevant to our work.

Let $N = \{1, 2, \dots, n\}$ be a finite set of players. A subset of $S \subseteq N$ is called a coalition, and $S = N$ is called the grand

coalition. A function $v: 2^N \rightarrow R$, with $v(\emptyset) = 0$, is called a characteristic function. The value $v(S)$ is interpreted as the maximum total profit that coalition S can obtain through cooperation without participation of the players outside the coalition. Assuming that the payoff generated by coalition S can be transferred among the members of S , a pair (N, v) is called a cooperative game with transferable utility. An allocation is a payoff vector $\mathbf{y} = (y_i)_{i \in N} \in R^N$, specifying for each player $i \in N$ his payoff y_i . An allocation $\mathbf{y}$ is called efficient if $\sum_{i \in N} y_i = v(N)$ and individually rational if $y_i \geq v(\{i\})$ for all $i \in N$. Individual rationality implies that every player receives in the grand coalition at least as much as what he could obtain by staying alone. The set of all individually rational and efficient allocations constitutes the imputation set

$$I(v) = \left\{ \mathbf{y} \in R^N \mid \sum_{i \in N} y_i = v(N) \text{ and } y_i \geq v(\{i\}) \text{ for each } i \in N \right\}. \quad (47)$$

A frequently used solution concept in cooperative game theory is the core of the game.

Definition 1. The utility vector $(y_1, y_2, \dots, y_N)$ is in the core of the cooperative game if for all $S \subseteq N, \sum_{i \in S} y_i \geq v(S)$ and $\sum_{i \in N} y_i \geq v(N)$.

Thus, the core consists of all imputations such that no group of retailers has an incentive to leave the grand coalition N and form a smaller coalition because they collectively receive at least as much as they could obtain by themselves.

For $S \subseteq N$, define the vector e^S as follows:

$$e_i^S = \begin{cases} 1, & \text{if } i \in S, \\ 0, & \text{otherwise.} \end{cases} \quad (48)$$

A mapping $\kappa: 2^N/\{\emptyset\} \rightarrow [0, 1]$ is called a balanced map if $\sum_{S \in 2^N/\{\emptyset\}} \kappa(S) e^S = e^N$. Furthermore, a game (N, v) is called balanced if $\sum_{S \in 2^N/\{\emptyset\}} \kappa(S) v(S) \leq v(N)$ for every balanced map $\kappa: 2^N/\{\emptyset\} \rightarrow [0, 1]$.

Theorem 4. *The game (N, v) has a nonempty core.*

Proof. We prove this theorem by showing that the game is balanced. We define for any $S \subseteq N$,

$$v(S) = \max_{p^S \in R_+} (p^S - w^S) q^S - (p^S - b^S) \int_{A_S}^{q^S - d_S(p^S)} F_S(x) dx - \phi^S u \left[-q^S + d_S(p^S) + B_S - \int_{q^S - d_S(p^S)}^{B_S} F_S(x) dx \right], \quad (49)$$

where $q^S = F^{-1}(w^S - c + (1 - \phi_S)u/b^S + (1 - \phi_S)u) + \theta_S - \alpha_S p^S + \sum_{j \neq S} r_j p^j$, which is the Nash equilibrium allocation quantity of products determined by the supplier. In the

coalition S , let q_i^S be an optimal allocation of the following allocation problem:

$$\begin{aligned} & \max_{q_i \in R_+^{|S|}} H_S(p^S, q^S, q_i), \\ & \text{s.t. } \sum_{i \in S} q_i = q^S, \end{aligned} \quad (50)$$

where

$$\begin{aligned} H_S(p^S, q^S, q_i) = & \sum_{i \in S} (p_i^S - w_i) q_i - (p_i^S - b_i) \int_{A_i}^{q_i - d_i(p^S)} F_i(x) dx \\ & - \phi_i u \left[-q_i + d_i(p^S) + B_i - \int_{q_i - d_i(p^S)}^{B_i} F_i(x) dx \right], \end{aligned} \quad (51)$$

and q_S is as previously mentioned. We remark that an optimal allocation exists since $H_S(p^S, q^S, \cdot)$ is continuous, and we can search for an optimal allocation in a nonempty compact set. Without loss of generality, for the coalition S , let κ be a balanced map of N :

$$\begin{aligned} \sum_{S \subseteq N} \kappa(S) v(S) &= \sum_{S \subseteq N} \kappa(S) H_S(p^S, q^S, q_i^S) \\ &= \sum_{S \subseteq N} \kappa(S) \left(\sum_{S \subseteq N | i \in S} (p_i^S - w_i) q_i^S - (p_i^S - b_i) \int_{A_i}^{q_i^S - d_i(p^S)} F_i(x) dx - \phi_i u \left[-q_i^S + d_i(p^S) + B_i - \int_{q_i^S - d_i(p^S)}^{B_i} F_i(x) dx \right] \right) \\ &= \sum_{i \in N} \sum_{S \subseteq N | i \in S} \kappa(S) \left((p_i^S - w_i) q_i^S - (p_i^S - b_i) \int_{A_i}^{q_i^S - d_i(p^S)} F_i(x) dx - \phi_i u \left[-q_i^S + d_i(p^S) + B_i - \int_{q_i^S - d_i(p^S)}^{B_i} F_i(x) dx \right] \right) \\ &\leq \sum_{i \in N} \sum_{S \subseteq N | i \in S} \kappa(S) \left((p_i^S - w_i) \left(F^{-1} \left(\frac{w_i - c + (1 - \phi_i)u}{b_i + (1 - \phi_i)u} \right) + \theta_i - \alpha_i p_i^S + \sum_{j \neq i} \gamma_j p_j \right) \right. \\ &\quad \left. - (p_i^S - b_i) \int_{A_i}^{F^{-1}(w_i - c + (1 - \phi_i)u/b_i + (1 - \phi_i)u)} F_i(x) dx \right. \\ &\quad \left. - \phi_i u \left[-F^{-1} \left(\frac{w_i - c + (1 - \phi_i)u}{b_i + (1 - \phi_i)u} \right) + B_i - \int_{F^{-1}(w_i - c + (1 - \phi_i)u/b_i + (1 - \phi_i)u)}^{B_i} F_i(x) dx \right] \right) \\ &\leq \sum_{i \in N} \left\{ \left(\sum_{S \subseteq N | i \in S} \kappa(S) p_i^S - w_i \right) \left(F^{-1} \left(\frac{w_i - c + (1 - \phi_i)u}{b_i + (1 - \phi_i)u} \right) + \theta_i - \alpha_i \sum_{S \subseteq N | i \in S} \kappa(S) p_i^S + \sum_{j \neq i} \gamma_j p_j \right) \right. \\ &\quad \left. - \left(\sum_{S \subseteq N | i \in S} \kappa(S) p_i^S - b_i \right) \int_{A_i}^{F^{-1}(w_i - c + (1 - \phi_i)u/b_i + (1 - \phi_i)u)} F_i(x) dx \right. \\ &\quad \left. - \phi_i u \left[-F^{-1} \left(\frac{w_i - c + (1 - \phi_i)u}{b_i + (1 - \phi_i)u} \right) + B_i - \int_{F^{-1}(w_i - c + (1 - \phi_i)u/b_i + (1 - \phi_i)u)}^{B_i} F_i(x) dx \right] \right\} \\ &\leq v(N). \end{aligned}$$

TABLE 1: Numerical results of case 1.

	Retail price	Ordering quantity	Profits of retailers	Profit of the supplier	SC profit
Retailer 1	21.9034	112.3009	1099.0442		
Retailer 2	22.9939	107.8335	1223.2364	2892.5609	6686.9744
Retailer 3	24.3269	103.3719	1472.1329		

TABLE 2: Numerical results of case 2.

	Retail price	Ordering quantity	Profits of retailers	Profit of the supplier	SC profit
Retailer 1	21.9032	112.2701	1098.8920		
Retailer 2	22.9937	107.8036	1223.0678	2892.1627	6686.0546
Retailer 3	24.3267	103.3432	1471.9321		

The first inequality holds because $F^{-1}(w_i - c + (1 - \phi_i)u/b_i + (1 - \phi_i)u) + \theta_i - \alpha_i p_i + \sum_{j \neq i} \gamma_j p_j, i = 1, 2, \dots, N$ is the equilibrium optimal solution. The second inequality follows immediately from the concavity property of the function

$$\begin{aligned} & (p_i^S - w_i)q_i^S - (p_i^S - b_i) \int_{A_i}^{q_i^S(p^S)} F_i(x) dx \\ & - \phi_i u \left[-q_i^S + d_i(p^S) + B_i - \int_{q_i^S - d_i(p^S)}^{B_i} F_i(x) dx \right], \end{aligned} \quad (53)$$

with respect to the retail price $p_i^S, i = 1, 2, \dots, N$. The last inequality holds because $\sum_{S \subseteq N | i \in S} \kappa(S) p_i^S$ is a feasible retail price. $\square$

Remark 1. This theorem implies that, with wholesale price, buy-back, and lost-sales cost-sharing contracts and linear price-dependent demand function, retailers always prefer being in the grand coalition to forming any other coalition in the Nash equilibrium sense. In other words, the grand coalition structure among retailers is stable.

6. Numerical Illustrations

6.1. Comparisons of Optimal Decisions between Noncompeting Retailers. In order to verify the efficiency of the obtained results, we take a numerical experiment on practical distribution systems. Assume there are three noncompeting retailers who face price-dependent and uncertain demand, that is,

$$\begin{aligned} D_1 &= 230 - 5.5p_1 + \varepsilon, \\ D_2 &= 220 - 5p_2 + \varepsilon, \\ D_3 &= 210 - 4.5p_3 + \varepsilon. \end{aligned} \quad (54)$$

We set the parameters as follows: $c = 2$, $\phi_1 = 0.5$, $\phi_2 = 0.55$, and $\phi_3 = 0.65$. Here, ε is a nonnegative random variable that follows exponential distribution $\varepsilon \sim \exp(1)$, $A = 0$, $B = 1$.

- Case 1: $b_1 = 14, b_2 = 13, b_3 = 12, u = 10$
- Case 2: $b_1 = 12, b_2 = 11, b_3 = 10, u = 9$
- Case 3: $b_1 = 11, b_2 = 10, b_3 = 9, u = 8$

6.2. Comparisons of Optimal Decisions between Competing Retailers. In this section, we assume there are three competing retailers who face price-dependent and uncertain demand, that is,

$$\begin{aligned} D_1 &= 230 - 5.5p_1 + 2p_2 + 1.5p_3 + \varepsilon, \\ D_2 &= 220 - 5p_2 + 2.5p_1 + 1.5p_3 + \varepsilon, \\ D_3 &= 210 - 4.5p_3 + 2.5p_1 + 1.5p_2 + \varepsilon. \end{aligned} \quad (55)$$

We set the parameters as follows: $c = 2$, $\phi_1 = 0.5$, $\phi_2 = 0.55$, and $\phi_3 = 0.65$. Here, ε is a nonnegative random variable that follows uniform distribution on the interval $[0, 1]$, i.e., $\varepsilon \sim U[0, 1]$, $A = 0$, $B = 1$.

- Case 1: $b_1 = 14, b_2 = 13, b_3 = 12, u = 10$
- Case 2: $b_1 = 12, b_2 = 11, b_3 = 10, u = 9$
- Case 3: $b_1 = 11, b_2 = 10, b_3 = 9, u = 8$

As illustrated in Tables 1–6, we have considered decentralized supply chain distributions with one common supplier providing substitutable products to downstream three retailers who face uncertain price-dependent demand. For the noncompeting and competing retailer cases, we design appropriate wholesale price, buy-back, and cost-sharing contracts to coordinate the total supply chain, so that when all the retailers adopt their equilibrium response, the supply chain system coordination is also achieved. On the other hand, under the competing retailer situation, it has higher retailing prices, ordering quantities, and higher supply chain profit performances compared to the non-competing case.

6.2.1. Managerial Insights. In this paper, we utilize game theory to analyze the nonoperative supply chain. We use the Nash equilibria to characterize the supplier's optimal quantity and the retailer's optimal retail price for the simultaneous-move game given the other retailers' decisions under the competing and noncompeting retailer cases. Through properly designed buy-back and lost-sales cost-sharing contracts, the resulting Nash equilibria will be identical to the globally optimal solution. On the other hand, we investigate the coalition formation of retailers. We reach a conclusion that retailers always prefer being in the grand coalition to forming any other coalition.

TABLE 3: Numerical results of case 3.

	Retail price	Ordering quantity	Profits of retailers	Profit of the supplier	SC profit
Retailer 1	21.9030	112.2383	1098.7408		
Retailer 2	22.9935	107.7729	1222.9003	2891.7670	6685.1406
Retailer 3	24.3265	103.3137	1471.7325		

TABLE 4: Numerical results of case 1.

	Retail price	Ordering quantity	Profits of retailers	Profit of the supplier	SC profit
Retailer 1	34.6735	180.1605	2935.3239		
Retailer 2	38.0910	180.9145	3581.5394	5865.6743	19757.2691
Retailer 3	42.4850	182.6452	4793.5754		

TABLE 5: Numerical results of case 2.

	Retail price	Ordering quantity	Profits of retailers	Profit of the supplier	SC profit
Retailer 1	34.6735	180.1595	2935.3241		
Retailer 2	38.0910	180.9136	3581.5394	5865.6745	19757.2690
Retailer 3	42.4850	182.6444	4793.5752		

TABLE 6: Numerical results of case 3.

	Retail price	Ordering quantity	Profits of retailers	Profit of the supplier	SC profit
Retailer 1	34.6735	180.1585	2935.3242		
Retailer 2	38.0910	180.9127	3581.5394	5865.6746	19757.2688
Retailer 3	42.4850	182.6437	4793.5749		

7. Conclusions and Future Research

In this paper, we have considered a supply chain distribution model with one common supplier providing products to downstream multiple retailers who face uncertain price-dependent demand. For the noncompeting and competing retailer cases, we successfully develop appropriate wholesale price, buy-back, and cost-sharing contracts to coordinate the supply chain, so that when all the retailers adopt their equilibrium response, the supply chain system coordination is also achieved. We also analyze the coalition formation among retailers. Numerical analysis further confirms that the efficiency of the designed contract and the grand coalition structure among retailers are stable.

In the future research, we will discuss the coalition formation with wholesale price, buy-back, and lost-sales cost-sharing contracts under the multiplicative form demand. In another future research direction, nowadays with risk theory getting popular, we will focus on the supply chain coordination with participants' different risk preferences.

Data Availability

Data were used to support this study and are available in the following reference: M. Leng and M. Parlar, "Game-Theoretic Analyses of Decentralized Assembly Supply Chains: Non-Cooperative Equilibria vs. Coordination with Cost-Sharing Contracts," *European Journal of Operational Research*, vol. 204, issue 1, pp. 96–104, 2010. The prior studies

(and datasets) are cited at relevant places within the text as references.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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