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*Research Article*

# Antisynchronization of the Hyperchaotic Systems with Uncertainty and Disturbance Using the UDE-Based Control Method

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In this paper, we investigate the antisynchronization problem of a class of hyperchaotic systems with both model uncertainty and external disturbance. Firstly, combining the dynamic feedback control method and the uncertainty and disturbance estimation (UDE)-based control method, we propose a new UDE-based dynamic feedback control method. Secondly, we take the 4D hyperchaotic system as an example and realize the antisynchronization problem of such system. Finally, the effectiveness and correctness of the proposed method is verified by numerical simulation.

## 1. Introduction

Chaotic behavior has been extensively analyzed in many fields, e.g., engineering, medicine, ecology, biology, and economics, even in social science. In 1990, Pecora and Carroll first discovered the phenomenon of chaotic synchronization [1]. Many types of synchronization problems have been proposed successively, including complete synchronization, generalized synchronization, phase synchronization, lag synchronization, and projective synchronization, see References [2–8]. Later, antisynchronization or antiphase synchronization [9] was proposed. So far, many methods have been proposed to realize chaotic antisynchronization, such as active control, adaptive control, linear feedback control, sliding mode control, and time-delay feedback approach, see References [10–22] and the references therein. Among them, the dynamic feedback control method [19] has a wide range of applications because of its simple design and easy implementation. Thus, this dynamic feedback control method is used in this paper.

It should be pointed out that among the abovementioned chaotic and hyperchaotic systems, model uncertainty and external disturbance are not considered. Unfortunately, this is not the case in practice. The UDE-based control method [23] is a good method to deal with the model uncertainty and external disturbance, and it has the following two advantages: one is that the system model or a disturbance model is not known completely; the other is that both structured (or unstructured) uncertainties and external disturbances are robust against. Being an effective robust control strategy, the UDE-based control has found widespread applications in various systems. Naturally, it is of interest to apply the UDE-based control to chaotic and hyperchaotic systems with both model uncertainty and external disturbance. However, to the best of the authors' knowledge, this problem has not been addressed in the existing literature. Therefore, the main goal of this paper is to develop a new UDE-based dynamic feedback control method to realize the antisynchronization problem of the chaotic and hyperchaotic systems.

This paper mainly studies the antisynchronization problem of chaotic and hyperchaotic systems with both model uncertainty and external disturbance. Combining the dynamic dynamic feedback control method and the UDE-based control method, a new UDE-based dynamic feedback control method is proposed. Then, by the obtained new method, the antisynchronization problem of the 4D hyperchaotic system is realized and the effectiveness of the method is verified by the numerical simulation.

## 2. Preliminary

*2.1. Dynamic Feedback Control Method for Antisynchronization.* Consider the following hyperchaotic system:

$$\dot{w} = G(w), \tag{1}$$

where $w \in R^n$ is the state and $G(w) \in R^n$ is a continuous vector function.

Let system (1) be the master system; then, the corresponding slave system with $v$ is given as

$$\dot{v} = G(v) + bu, \tag{2}$$

where $v \in \mathrm{R}^n$ is the state, $G(v) \in R^n$ is a continuous vector function, $b \in R^{n\times s}$ is a constant matrix, and $u \in R^s$ is the designed controller, $s \geq 1$.

Set $E = w + v$; then, the sum system is described as

$$\dot{E} = G(w) + G(v) + bu, \tag{3}$$

where $E \in R^n$ is the state and $b$ and $u$ are given in (2).

*Definition 1.* Consider the sum system (3). If $\lim_{t\longrightarrow\infty} \|E(t)\| = 0$; then, we call the master system (1) and slave system (2) achieve antisynchronization.

*Remark 1.* According to the results in [21], the antisynchronization of system (1) exists only and only if $G(-w) = -G(w)$.

At present, there are many methods for the antisynchronization problem. Among them, the dynamic feedback control method has a wide range of applications because of its simple design and easy implementation. Here is a brief introduction.

According to the existing result in [19–22], we present the following conclusion.

**Lemma 1.** *Consider system (3), where $b = (b_{ij})_{n\times r}$ and $b_{ij} = 0$ or $b_{ij} = 1$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots r$, where $(E(t), b)$ is controllable; then, the dynamic feedback controller is designed as follows:*

$$u = KE, \tag{4}$$

*where $K = k(t)b^T$, $K \in R^{n\times n}$, and the feedback gain $k(t)$ is updated by the following law:*

$$\dot{k}(t) = -\|E(t)\|^2. \tag{5}$$

*2.2. UDE-Based Control Method.* Consider the following controlled systems with model uncertainty and disturbance:

$$\dot{p} = H(p) + \Delta H(p) + bu + d(t), \tag{6}$$

where $p \in R^n$ is the state, $H(p)$ is a continuous vector function, $b \in R^{n\times s}$, $s \geq 1$, $\Delta H(p) \in R^n$ represents the model uncertainty, $d(t) \in R^n$ is the external disturbance vector, $u$ is the controller to be designed, and $(H(x), b)$ is assumed controllable.

The stable linear reference model is presented as follows:

$$\dot{p}_m = A_m p_m + B_m C, \tag{7}$$

where $p_m \in R^n$ is the reference state, $A_m \in R^{n\times n}$ is a Hurwitz constant matrix, $B_m \in R^{n\times s}$ is a vector, and $C \in R^s$ is a command.

According to the existing results in [23], the UDE-based control method is presented as follows.

**Lemma 2** (see [23]). *Consider system (6) and the reference system (7). If the designed filter $g_f(t)$ satisfies the following condition,*

$$\tilde{u}_d = \hat{u}_d - u_d, \tag{8}$$

*where $\hat{u}_d = (\dot{x} - H(p) - bu) * g_f(t)$ and $u_d = \triangle H(x) + d(t)$, then UDE-based controller $u$ is expressed as follows:*

$$u = b^{+}\left\{-H(p) + \ell^{-1}\left[\frac{1}{1 - G_f(s)}\right] * \left(A_m p + B_m C - Kq\right)\right\} - b^{+}\left\{\ell^{-1}\left[\frac{sG_f(s)}{1 - G_f(s)}\right] * p(t)\right\}, \tag{9}$$

*where $q = p - p_m$, $b^{+} = (b^T b)^{-1} b^T$, $\ell^{-1}$ is the inverse Laplace transform operator, $*$ is the convolution operator, and* $G_f(s) = \ell[g_f(t)]$.

*Remark 2.* Since controller in equation (9) cancels $H(p)$ in system (6) directly, thus this controller is too complex to be used in actual chaotic antisynchronization system.

*Remark 3.* According to the existing result in [23], two kinds of filters are introduced. One is the first-order low-pass filter:

$$G_f(s) = \frac{1}{\tau s + 1}, \tag{10}$$

in general, $\tau = 0.001$.

The other is the secondary filter:

$$G_f(s) = \frac{as + b - w_0^2}{s^2 + as + b}, \tag{11}$$

where $w_0 = 4\pi$, $a = 10w_0$, and $b = 100w_0$.

## 3. Problem Formulation

Consider the following hyperchaotic system with both model uncertainty and external disturbance:

$$\dot{x} = f(x) + \Delta f(x) + d(t), \tag{12}$$

where $x \in R^n$ is the state, $f(x) \in R^n$ is a continuous vector function, $\Delta f(x) \in R^n$ denotes system model uncertainty, and $d(t) \in R^n$ stands for the external disturbance.

Let system (12) be the master system; then, the slave system with $y$ is given as

$$\dot{y} = f(y) + Bu, \tag{13}$$

where $x \in R^n$ is the state, $B \in R^{n\times r}$ is a constant matrix, and $u \in R^r$, $r \geq 1$ is the controller to be designed, and it is assumed that $(f(y) + f(x), B)$ is controllable.

Let $e = x + y$, then the sum system is described as follows:

$$\dot{e} = f(x) + f(y) + +\Delta f(x) + d(t) + Bu. \tag{14}$$

The main goal of this paper is to design a controller $u$ to meet the following condition:

$$\lim_{t\longrightarrow\infty} e(t) = \lim_{t\longrightarrow\infty} [y(t) + x(t)] = 0. \tag{15}$$

## 4. Main Results

In this section, we investigate the antisynchronization problem of the hyperchaotic systems with both uncertainty and external disturbance and present the following result.

**Theorem 1** Consider system (14). *If a filter $g_f(t)$ is designed to satisfy the following condition,*

$$\tilde{u}_d = \hat{u}_d - u_d \longrightarrow 0, \tag{16}$$

*where $\hat{u}_d = (\dot{x} - f(x)) * g_f(t)$ and $u_d = \Delta f(x) + d(t)$, then the dynamic feedback UDE-based controller $u$ is designed as follows:*

$$u = u_s + u_{\text{ude}}, \tag{17}$$

*where*

$$u_s = K(t)e = k(t)B^T e, \tag{18}$$

*$k(t)$ is updated by the update law (5), and*

$$u_{\text{ude}} = B^+\left[-u_d * g_f(t)\right] = B^+\left[-(\dot{x} - f(x)) * g_f(t)\right], \tag{19}$$

where $B^+ = (B^T B)^{-1} B^T$ and $*$ stands for the convolution operator.

*Proof.* Substituting controller (17) into system (14), we obtain

$$\begin{aligned}\dot{e} &= f(x) + f(y) + Bu_s + Bu_{\text{ude}} + \Delta f(x) + d(t)\\ &= F(x) + Bu_{\text{ude}} + u_d,\end{aligned} \tag{20}$$

where $F(x) = f(x) + f(y) + Bu_s$ and $u_d = \Delta f(x) + d(t)$.

According to Lemma 1, the system $\dot{e} = F(x)$ is globally asymptotically stable. Noting condition (16), we can obtain

$$Bu_{\text{ude}} = -\hat{u}_d. \tag{21}$$

Thus, system (20) is rewritten as

$$\dot{x} = F(x) + \tilde{u}_d, \tag{22}$$

and this system is globally asymptotically stable, which completes the proof.

## 5. An Illustrative Example with Numerical Simulation

In this section, we take the new 4D hyperchaotic system as an example to apply our theoretical results.

*Example 1.* The new 4D hyperchaotic system with uncertainty and disturbance is given as follows:

$$\dot{x} = f(x) + \Delta f(x) + d(t), \tag{23}$$

where $x \in R^4$, $f(x) \in R^4$ is a continuous vector function, $\Delta f(x) \in R^4$ is the model uncertainty, and $d(t) \in R^4$ is the external disturbance, i.e.,

$$\begin{aligned} f(x) &= \begin{pmatrix} f_1(x)\\ f_2(x)\\ f_3(x)\\ f_4(x)\end{pmatrix} = \begin{pmatrix} 35(x_2 - x_1) + x_2x_3x_4\\ 10(x_1 + x_2) - x_1x_3x_4\\ -x_3 + x_1x_2x_4\\ -10x_4 + x_1x_2x_3\end{pmatrix},\\ \Delta f(x) &= \begin{pmatrix} 0\\ -0.1x_1^2\\ 0\\ 0\end{pmatrix},\\ d(t) &= \begin{pmatrix} 0\\ 0.1\sin(t)\\ 0\\ 0\end{pmatrix}.\end{aligned} \tag{24}$$

Obviously, $f(-x) = -f(x)$. Thus, the antisynchronization problem of the system $\dot{x} = f(x)$ exists.

Let system (23) be the master system, then the corresponding slave system with $y$ is described as

$$\dot{y} = f(y) + Bu, \tag{25}$$

where $y \in R^4$, $u = u_s + u_{\text{ude}}$ is the controller to be designed, and

$$\begin{aligned} f(y) &= \begin{pmatrix} f_1(y)\\ f_2(y)\\ f_3(y)\\ f_4(y)\end{pmatrix} = \begin{pmatrix} 35(y_2 - y_1) + y_2y_3y_4\\ 10(y_1 + y_2) - y_1y_3y_4\\ -y_3 + y_1y_2y_4\\ -10y_4 + y_1y_2y_3\end{pmatrix},\\ B &= \begin{pmatrix} 0 & 0\\ 1 & 0\\ 0 & 1\\ 0 & 0\end{pmatrix}.\end{aligned} \tag{26}$$

Set $e = x + y$, then the sum system is given as follows:

$$\dot{e} = f(x) + f(y) + Bu + \Delta f(x) + d(t), \tag{27}$$

where $e \in R^4$.

The controlled sum system without model uncertainty and external disturbance is presented as

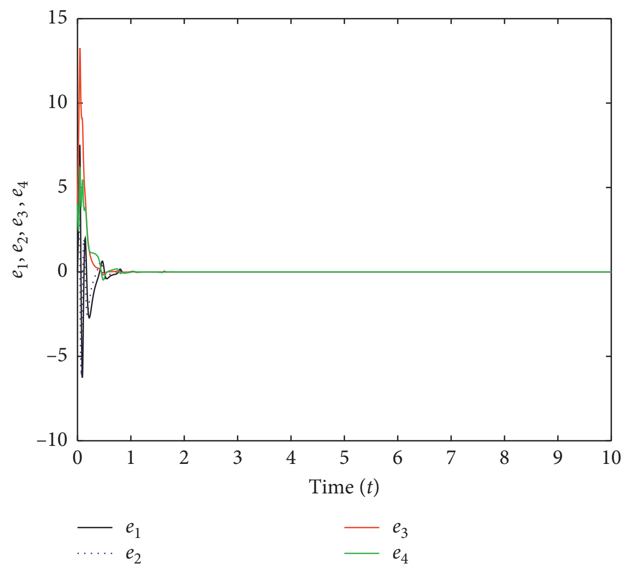


Figure 1: The sum system is asymptotically stable.

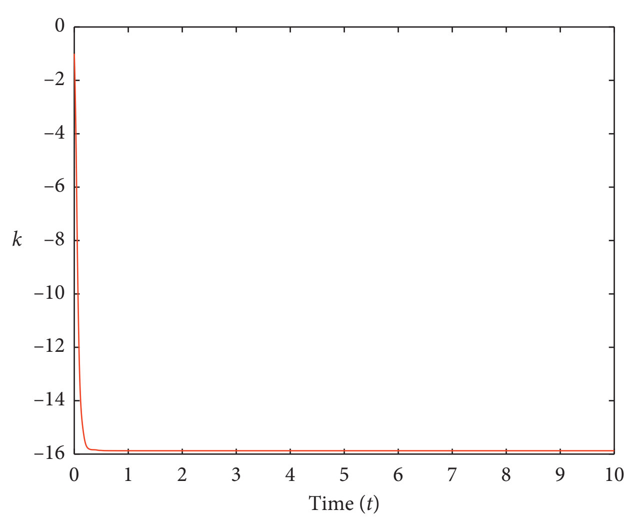


Figure 2: $k(t)$ tends to a constant.

$$\dot{e} = f(x) + f(y) + Bu. \tag{28}$$

Our goal is to design a controller $u = u_s + u_{\text{ude}}$. to stabilize the system (27), i.e., $\lim_{t\longrightarrow\infty}\|e(t)\| = 0$.

The first step is to design controller $u_s$.

For system (28), it is obvious that if $e_2 = 0$ and $e_3 = 0$, we can get that the following two-dimensional system

$$\begin{aligned}\dot{e}_1 &= -35e_1 + x_2x_3e_4,\\ \dot{e}_4 &= -e_4 + x_2x_3e_1,\end{aligned} \tag{29}$$

is globally asymptotically stable.

From Lemma 1, we can design controller $u_s$ as follows:

$$u_s = k(t)B^Te = k(t)\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}e, \tag{30}$$

where $k(t)$ is updated by the update law (5).

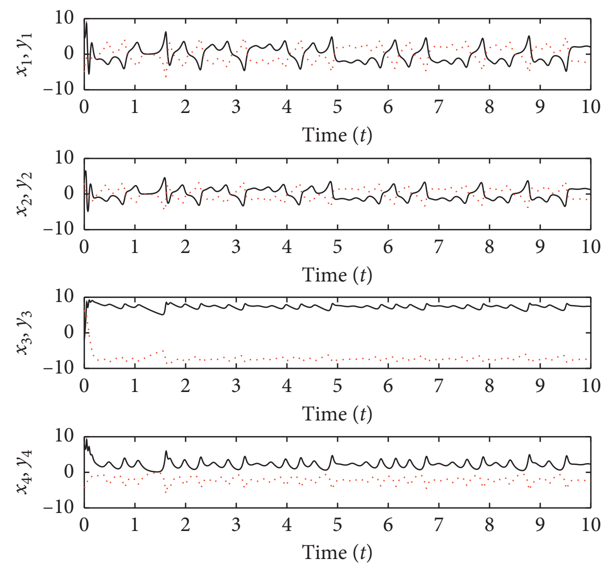


Figure 3: $x_1, x_2, x_3$, and $x_4$ antisynchronizes $y_1, y_2, y_3$, and $y_4$, respectively.

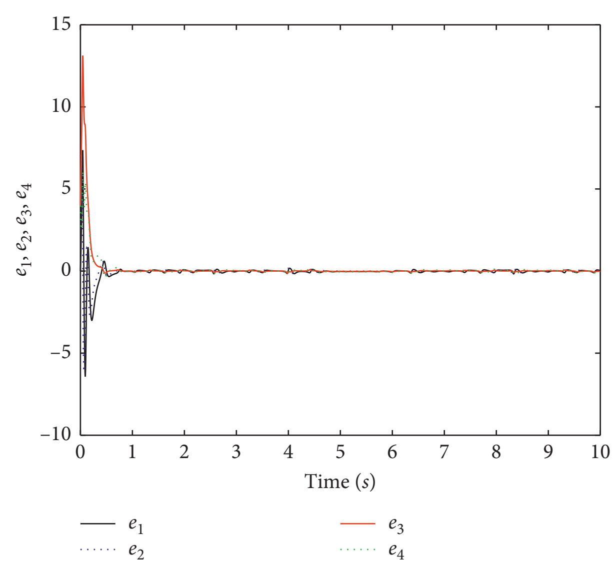


Figure 4: The sum system is also asymptotically stable.

For system (28), the numerical simulation is carried out with the initial conditions: $x(0) = [5, 4, -2, 8]$, $y(0) = [-5, 3, 6, -4]$, $k(0) = -1$. Figure 1 shows that under the abovementioned controller, the sum system is asymptotically stable, i.e., the master-slave system achieves anti-synchronization. Figure 2 shows that the feedback gain converges to an appropriate constant. Figure 3 shows that states of the master system: $x_1, x_2, x_3$, and $x_4$, anti-synchronize, the states of the slave system: $y_1, y_2, y_3$, and $y_4$, respectively.

The second step is to design the UDE controller $u_{\text{ude}}$.

Let $u_d = \Delta f(x) + d(t)$ and $F(x) = f(x) + f(y) + Bu_s$; the system (27) is rewritten as

$$\dot{e} = F(x) + Bu_{\text{ude}} + u_d. \tag{31}$$

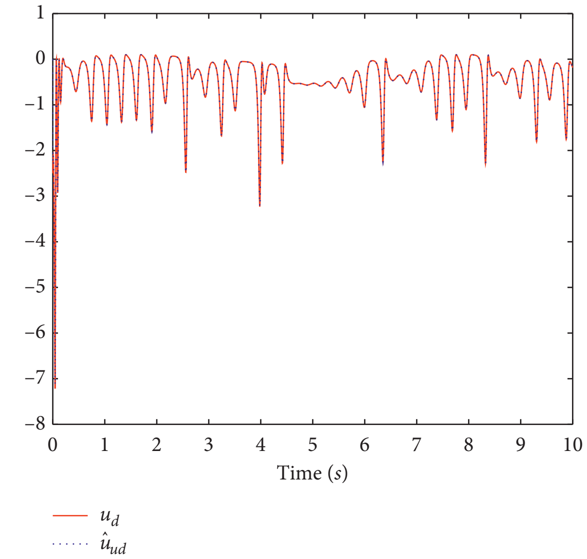


FIGURE 5: $\hat{u}_d$ tends to $u_d$.

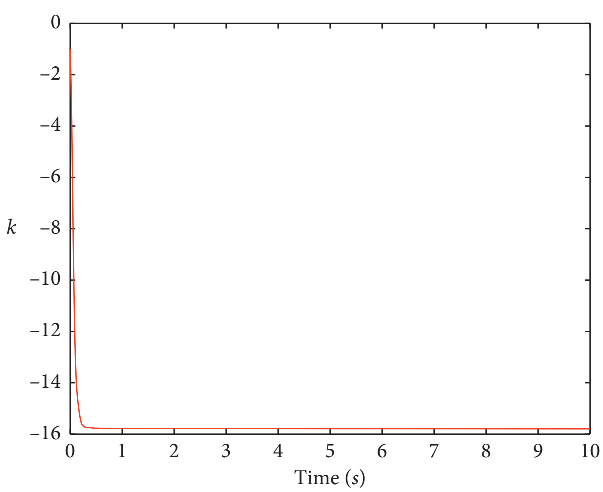


FIGURE 6: $k(t)$ also tends to a constant.

According to Theorem 1, the controller $u_{\text{ude}}$ is designed as

$$u_{\text{ude}} = B^{+}\left(-(\dot{x} - f(x)) * g_f(t)\right), \tag{32}$$

where $B^{+} = (B^T B)^{-1} B^T$ and $*$ stands for the convolution operator.

Thus, $u = u_s + u_{\text{ude}}$ is obtained.

For system (27), the numerical simulation is carried out with the initial conditions: $x(0) = [5, 4, -2, 8]$, $y(0) = [-5, 3, 6, -4]$, and $k(0) = -1$. Figure 3 shows that under the abovementioned controller, the error system is asymptotically stable. Figure 4 shows that $u_d$ and $\hat{u}_d$ after a certain time tend to the same constant. Figure 5 shows that the feedback gain $k(t)$ converges to an appropriate constant. Figure 6 shows that the system achieves antisynchronization. The numerical simulation results show that the new 4D hyperchaotic system achieves antisynchronization under the abovementioned controller. Figure 7 shows that states of the master system: $x_1, x_2, x_3$, and $x_4$ also antisynchronize the states of the slave system: $y_1, y_2, y_3$, and $y_4$, respectively.

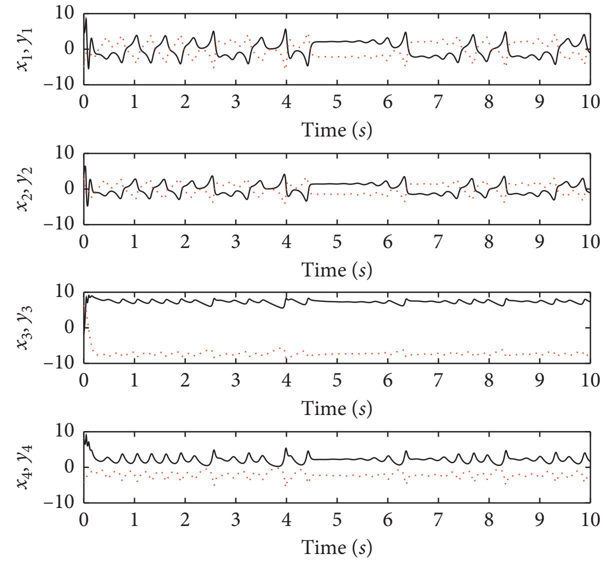


FIGURE 7: $x_1, x_2, x_3$, and $x_4$ also antisynchronizes $y_1, y_2, y_3$, and $y_4$, respectively.

## 6. Conclusion

In this paper, the antisynchronization problem of the hyperchaotic systems has been investigated. A new UDE-based dynamic feedback control method has been proposed, and the antisynchronization of the new 4D hyperchaotic system has been realized by the obtained control method. The correctness and effectiveness of the abovementioned theoretical methods have been verified by numerical simulation.

## Data Availability

All figures are made by Matlab.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of the article.

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## Supplementary Materials

The supplementary file supplied includes MATLAB Programs. (*Supplementary Materials*)

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