Hindawi
Mathematical Problems in Engineering
Volume 2020, Article ID 2301085, 9 pages
https://doi.org/10.1155/2020/2301085

Research Article

Delay-Dependent H_∞ Control for T-S Fuzzy Systems with Local Nonlinear Models: An LMI Approach

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Received 15 March 2020; Revised 9 April 2020; Accepted 21 April 2020; Published 11 May 2020

Guest Editor: Cuimei Jiang

This paper studies the stabilization design scheme with H_∞ performance for a large class of nonlinear discrete-time systems. The system under study is modeled by Takagi-Sugeno (T-S) model with local nonlinearity and state delay. First, the model is changed into an equivalent fuzzy switching model. And then, according to projection theorem and piecewise Lyapunov function (PLF), two new H_∞ control methods are proposed for fuzzy switched systems, which consider the time delay information of the system. Finally, the relationship among all fuzzy subsystems is considered. Because the results are only expressed by a series of linear matrix inequalities (LMIs), the controller can be directly designed by the linear matrix inequalities toolbox of MATLAB.

1. Introduction

As we all know, the T-S fuzzy method is a kind of common and very effective tool for approaching the discrete-time nonlinear complex system [1]. For instance, in [2, 3], it was shown that coupled chaotic systems are a special class of complex systems, which can be processed by the T-S method [4, 5]. In addition, nonlinear neutral differential equations have numerous applications in engineering and natural sciences [6]. By using the T-S method, Pu et al. [7] studied BP neural network and RBF neural network. Bharathi et al. [8] investigated numerical solutions for sophistication single neutrality differential equations with time delay. As shown in [9], asymptotic suboptimality property of the decentralized methods for the linear-quadratic games is proposed. A game-control method based on the fuzzy linearity quadratic adjuster was presented by Ji et al. [10] for emergency collision avoidance. Therefore, it is very important to study the asymptotic stability and controller design of the T-S fuzzy model [11–13]. However, many papers, such as [11–13], depend on a single common positive-definite symmetric matrix P, which needs to satisfy many LMIs. In reality, such a matrix may not exist, especially for systems with high nonlinearity [14]. Therefore, it is conservative to use the common Lyapunov function method to consider the controller design of the T-S model, and its application scope is limited. In order to increase the feasible region of matrix inequalities, a piecewise Lyapunov function (PLF) is proposed in [15, 16], which studied the filter problem for the T-S model with delay. In references [17–19], the fuzzy Lyapunov function (FLF) method was used to study the controller design approaches for the T-S model.

Recently, the T-S fuzzy system with a local nonlinear model (FSwLNM) has received considerable research (see, for instance, [20–30] and references therein). As shown in [20, 21], T-S FSwLNM requires less fuzzy rules, which can reduce the computational complexity. It can also decrease modeling error compared with conventional T-S fuzzy systems. For a class of T-S FSwLNM, Yang and Wang [22] investigated the problem of fault detection, Klug et al. [23] proposed a convex way to study an output feedback controller, and Chang and Hsu [24] investigated the sliding mode control problem with multiple performance indexes for stochastic nonlinear systems. Nguyen et al. [25] proposed a new method to design a limited controller, which has different fuzzy rules from the system. Zhai et al. [26] observed the problems of network-based fault detection and isolation observer design. Huang and Yang [27] investigated the problem of fault estimation. In [28, 29], the fuzzy polynomial system was studied, and the subsequent part of fuzzy rules was represented by polynomial function. Compared with the general T-S fuzzy system, the fuzzy polynomial system can express the complex nonlinear system more accurately with fewer fuzzy rules.

Based on the above considerations, we will study the delay-dependent controller design method with H_∞ performance index for discrete-time T-S FSwLNM. Adopting PLF and the fuzzy switching model [30], two new design methods of H_∞ controller are derived. These two methods consider the time-delay information of the system, so they are less conservative. By using projection theorem and introducing relaxation matrix variables, there is no product term of Lyapunov matrices and system dynamic matrices in LMIs constraints. Because the derived condition only contains LMIs, the controller gain matrix can be directly designed by the LMIs toolbox of MATLAB.

Notations. This part briefly describes the symbols used in this paper. Symbol $\mathbb{R}^n$ stands for Euclidean space with n dimension. Symbol $\mathbb{R}^{n\times m}$ means the set of real $n\times m$ dimensional matrices. Matrices I and O with appropriate dimensions represent unit matrices and zero matrices. Matrix P is strictly greater than 0, which shows that P is a positive definite symmetric matrix (PDSM). The symbol $\operatorname{diag}\{A_1, A_2, \ldots, A_n\}$ denotes block diagonal matrix. The symbol $\operatorname{sym}\{S\}$ indicates $S+S^T$. Elements of symmetric position of symmetric matrix are represented by *.

2. Preliminaries and Problem Formulation

2.1. Preliminaries. Before giving the main conclusions of this paper, we first present some very important lemmas, which are very important in the process of proving the important conclusions of this paper.

Lemma 1 (see [31]). *Given an $m\times m$-dimensional symmetric real matrix $\mathscr{Z}$ and three matrices $\mathscr{Z}_0$, $\mathscr{X}$, and $\mathscr{Y}$ of proper dimensions, the two sets of inequalities shown as follows are equivalent:*

$$\begin{aligned} &\mathscr{Z} + \operatorname{sym}\left\{\mathscr{X}^T \mathscr{Z}_0 \mathscr{Y}\right\} < 0, \\ &\mathscr{X}_\perp^T \mathscr{Z} \mathscr{X}_\perp < 0, \\ &\mathscr{Y}_\perp^T \mathscr{Z} \mathscr{Y}_\perp < 0, \end{aligned} \tag{1}$$

where two matrices $\mathscr{X}_\perp$ and $\mathscr{Y}_\perp$, whose columns are full rank, satisfy the equalities $\mathscr{X}\mathscr{X}_\perp = 0$ and $\mathscr{Y}\mathscr{Y}_\perp = 0$, respectively.

Lemma 2 (see [32]). *Let ϱ be a given positive integer, $x_h \in \mathbb{R}^n$ be a vector, and $\mathscr{M} \in \mathbb{R}^{n\times n}$ be a semi-PDSM; we have*

$$-\varrho \sum_{h=1}^{\varrho} x_h^T \mathscr{M} x_h \le -\left(\sum_{h=1}^{\varrho} x_h^T\right) \mathscr{M} \left(\sum_{h=1}^{\varrho} x_h\right). \tag{2}$$

Lemma 3 (see [31]). *Given the proper dimension matrices $\mathscr{M}$, $\mathscr{H}$, and $\mathscr{Q}$, the following two sets of inequalities are equivalent:*

$$\begin{aligned} &(1) \begin{bmatrix} \mathscr{Q} & \mathscr{M} \\ * & \mathscr{H} \end{bmatrix} < 0, \\ &(2)\, \mathscr{H} < 0, \mathscr{Q} - \mathscr{M}\mathscr{H}^{-1}\mathscr{M}^T < 0. \end{aligned} \tag{3}$$

In this case, we say that the matrix product $\mathscr{Q} - \mathscr{M}\mathscr{H}^{-1}\mathscr{M}^T$ is Schur complement of matrix $\mathscr{H}$.

2.2. Problem Formulation. In order to improve the approximation effect of the fuzzy system and reduce the number of rules, we consider T-S FSwLNM as follows:

Fuzzy rule m: if ϑ_{1t} is υ_{m1t}, ..., ϑ_{pt} is υ_{mpt}, then

$$\begin{cases} x_{t+1} = A_m x_t + A_{\tau_0 m} x_{t-\tau_0} + G_m \phi_t + G_{\tau_0 m} \phi_{t-\tau_0} + B_{1m}\omega_t + B_{2m} u_t, \\ z_t = C_m x_t + C_{\tau_0 m} x_{t-\tau_0} + G_{zm}\phi_t + G_{z\tau_0 m}\phi_{t-\tau_0} + D_{1m}\omega_t + D_{2m} u_t, \\ x_t = \varsigma_t, \quad -\tau_0 \le t < 0, \end{cases} \tag{4}$$

where $m \in \{1, 2, \ldots, r_0\}$. r_0, υ_{mnt}, and ϑ_{nt} are the rule number, fuzzy sets, and premise variables (usually the state or output of the system), respectively. $x_t \in \mathbb{R}^n$, $z_t \in \mathbb{R}^m$, ς_t, and $u_t \in \mathbb{R}^l$ represent the system state variables, the system output variables to be estimated, the initial conditions, and the controller to be designed, respectively. Positive integer τ_0 represents the constant delay. Disturbance $\omega_t \in l_2[0, \infty)$. $\phi_t \in \mathbb{R}^n$ satisfies

$$\left\|\phi_t\right\|_2 \le \theta \left\|x_t\right\|_2, \quad \theta > 0. \tag{5}$$

Similar to [33], we define open subspace $\Omega_l\ (l = 1, \ldots, k)$ in state space. The symbol Ω_l^c represents the closed subspace and satisfies

$$\Omega_m^c \cap \Omega_n^c = \partial\Omega_i^{\upsilon_0}, \quad m \ne n, m, n = 1, \ldots, \kappa, i \in \{1, \ldots, r_0\}, \tag{6}$$

where $\partial\Omega_m^{\upsilon_0} = \{\vartheta \,|\, h_m(\vartheta) = 1, 0 \le h_m(\vartheta+\delta) < 1, |\delta| \ll 1, \ |\delta| \ne 0\}$. υ_0 is the face indexes set of the polyhedron $\partial\Omega_m = \cup \partial\Omega_m^{\upsilon_0}$. $h_m(\vartheta) = (\omega_m(\vartheta_t)/\sum_{m=1}^{r}\omega_m(\vartheta_t))$, $\omega_m(\vartheta_t) = \prod_{n=1}^{p_0}\mu_{mn}(\vartheta_{nt})$, and $\vartheta_t = [\vartheta_{1t}, \vartheta_{2t}, \ldots, \vartheta_{pt}]$.

Then, we will rewrite system (4) as an equivalent switched fuzzy model according to the idea of [30], as follows:

Global rule n: if $x_t \in \Omega_n$, then there are the following local rules:

if ϑ_{1t} is υ_{nq1t}, ..., ϑ_{pt} is υ_{nqpt}, then

$$\begin{cases} x_{t+1} = A_{nq} x_t + A_{\tau_0 nq} x_{t-\tau_0} + G_{nq}\phi_t + G_{\tau_0 nq}\phi_{t-\tau_0} + B_{1nq}\omega_t + B_{2nq} u_t, \\ z_t = C_{nq} x_t + C_{\tau_0 nq} x_{t-\tau_0} + G_{znq}\phi_t + G_{z\tau_0 nq}\phi_{t-\tau_0} + D_{1nq}\omega_t + D_{2nq} u_t, \\ x_t = \varsigma_t, \quad -\tau_0 \le t < 0, q = 1, \ldots, \lambda_n, n = 1, \ldots, \kappa, \end{cases} \tag{7}$$

where κ is the number of subspaces divided. Ω_n represents the nth subspace. λ_n is the rule number in the nth subspace.

By using the fuzzy method to deal with system (7), we can achieve

$$\begin{cases} x_{t+1} = \sum_{q=1}^{\lambda_n} h_{nq}\left\{A_{nq}x_t + A_{\tau_0 nq}x_{t-\tau_0} + G_{nq}\phi_t + G_{\tau_0 nq}\phi_{t-\tau_0} + B_{1nq}\omega_t + B_{2nq}u_t\right\}, \\ z_t = \sum_{q=1}^{\lambda_n} h_{nq}\left\{C_{nq}x_t + C_{\tau_0 nq}x_{t-\tau_0} + G_{znq}\phi_t + G_{z\tau_0 nq}\phi_{t-\tau_0} + D_{1nq}\omega_t + D_{2nq}u_t\right\}, \\ x_t = \varsigma_t, \quad -\tau_0 \le t < 0, x(t) \in \Omega_n, \end{cases} \tag{8}$$

where $h_{nq} = h_{nq}(\vartheta_t) = (\prod_{l=1}^{p_0} v_{nql}(\vartheta_t)/\sum_{q=1}^{\lambda_n}\prod_{l=1}^{p_0} v_{nql}(\vartheta_t))$.

In each subspace Ω_n, we design independent controllers, that is

Global rule n: if $x_t \in \Omega_n$, then there are the following controllers:

if ϑ_{1t} is v_{nl1t}, ..., ϑ_{pt} is v_{nlpt}, then

$$u_t = F_{anl}x_t + F_{a\tau_0 nl}x_{t-\tau_0} + F_{bnl}\phi_t + F_{b\tau_0 nl}\phi_{t-\tau_0}, \quad l = 1, 2, \ldots, \lambda_n. \tag{9}$$

Using fuzzy reasoning technology, fuzzy controller (9) can be written in the following compact form:

$$u_t = \sum_{l=1}^{\lambda_n} h_{nl}\left\{F_{anl}x_t + F_{a\tau_0 nl}x_{t-\tau_0} + F_{bnl}\phi_t + F_{b\tau_0 nl}\phi_{t-\tau_0}\right\}. \tag{10}$$

Combining (8) with (10), we have

$$\begin{cases} x_{t+1} = \widetilde{A}_{nql}x_t + \widetilde{A}_{\tau_0 nql}x_{t-\tau_0} + \widetilde{B}_{1nq}\omega_t + \widetilde{G}_{nql}\phi_t + \widetilde{G}_{\tau_0 nql}\phi_{t-\tau_0}, \\ z_t = \widetilde{C}_{nql}x_t + \widetilde{C}_{\tau_0 nql}x_{t-\tau_0} + \widetilde{D}_{1nq}\omega_t + \widetilde{G}_{znql}\phi_t + \widetilde{G}_{z\tau_0 nql}\phi_{t-\tau_0}, \end{cases} \tag{11}$$

$$\begin{bmatrix} \widetilde{A}_{nql} & \widetilde{A}_{\tau_0 nql} & \widetilde{B}_{1nq} & \widetilde{G}_{nql} & \widetilde{G}_{\tau_0 nql} \\ \widetilde{C}_{nql} & \widetilde{C}_{\tau_0 nql} & \widetilde{D}_{1nq} & \widetilde{G}_{znql} & \widetilde{G}_{z\tau_0 nql} \end{bmatrix} = \sum_{q=1}^{\lambda_n}\sum_{l=1}^{\lambda_n} h_{nq}h_{nl} \begin{bmatrix} \widehat{A}_{nql} & \widehat{A}_{\tau_0 nql} & B_{1nq} & \widehat{G}_{nql} & \widehat{G}_{\tau_0 nql} \\ \widehat{C}_{nql} & \widehat{C}_{\tau_0 nql} & D_{1nq} & \widehat{G}_{znql} & \widehat{G}_{z\tau_0 nql} \end{bmatrix}, \tag{12}$$

with

$$\begin{aligned} \widehat{A}_{nql} &= A_{nq} + B_{2nq}K_{anl}, \\ \widehat{A}_{\tau_0 nql} &= A_{\tau_0 nq} + B_{2nq}K_{a\tau_0 nl}, \\ \widehat{G}_{nql} &= G_{nq} + B_{2nq}K_{bnl}, \\ \widehat{G}_{\tau_0 nql} &= G_{\tau_0 nk} + B_{2nq}K_{b\tau_0 nl}, \\ \widehat{C}_{nql} &= C_{nq} + D_{2nq}K_{anl}, \\ \widehat{C}_{\tau_0 nql} &= C_{\tau_0 nq} + D_{2nq}K_{a\tau_0 nl}, \\ \widehat{G}_{znql} &= G_{znq} + D_{2nq}K_{bnl}, \\ \widehat{G}_{z\tau_0 nql} &= G_{z\tau_0 nq} + D_{2nq}K_{b\tau_0 nl}. \end{aligned} \tag{13}$$

where $x(t) \in \Omega_j$.

Given the upper bound γ of H_∞ index, where $\gamma > 0$, the purpose is to design independent controllers (10) for system (8) in each subspace Ω_j, so that the following two conditions are met:

(1) When the disturbance $\omega_t = 0$, the equilibrium state $x_t = 0$ of system (11) is asymptotically stable.

(2) When the initial condition $\varsigma_t = 0$, the following norm inequalities hold:

$$\|z_t\|_2 < \gamma\|\omega_t\|_2, \quad \forall\omega_t \ne 0. \tag{14}$$

3. Main Results

The set $\Omega = \{(m, n) \mid x_t \in \Omega_m, x_{t+1} \in \Omega_n\}$ indicates that the system state jumps from subspace Ω_m to subspace Ω_n. Of course, the system state may stay in a certain subspace all the time. Next, we can prove the result in Theorem 1.

Theorem 1. *Given a positive real number γ, the H_∞ control problems of controlled system (11) are solved, if there are symmetric positive definite matrices* $\begin{bmatrix} P_n & W_{1n} \\ * & W_{2n} \end{bmatrix}$, $\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}$, *$R$, the nonsingular matrix F_n and matrices $\mathcal{N}_{ns}(s = 1, \ldots, 7)$, K_{anl}, $K_{a\tau_0 nl}$, K_{bnl}, and $K_{b\tau_0 ql}$, $n = 1, 2, \ldots, \kappa$, $l = 1, 2, \ldots, \lambda_n$, such that*

$$\Pi_{mnqq} < 0, \quad q = 1, 2, \ldots, \lambda_n, (n, m) \in \Omega, \tag{15}$$

$$\Pi_{mnql} + \Pi_{mnlq} < 0, \quad 0 < q < l \le \lambda_n, (n, m) \in \Omega, \tag{16}$$

where

$$\Pi_{mnkl} = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} & \Pi_{16} & \Pi_{17} & \mathcal{N}_{n7} & 0 \\ * & \Phi_1 & \Phi_2 & \Phi_3 & -K_n\mathcal{N}_{n4} & -\mathcal{N}_{n5} & -\mathcal{N}_{n6} & \Phi_4 & \Pi_1^T \\ * & * & \Phi_5 & \Phi_6 & 0 & 0 & 0 & -\mathcal{N}_{n2}^T & \Pi_2^T \\ * & * & * & \Phi_7 & 0 & 0 & 0 & -\mathcal{N}_{n3}^T & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 & -\mathcal{N}_{n4}^T & D_{1nq}^T \\ * & * & * & * & * & -I & 0 & -\mathcal{N}_{j5}^T & \Pi_3^T \\ * & * & * & * & * & * & -I & -\mathcal{N}_{n6} & \Pi_4^T \\ * & * & * & * & * & * & * & \Phi_8 & 0 \\ * & * & * & * & * & * & * & * & -I \end{bmatrix},$$

$$\begin{aligned}
\Pi_{11} &= P_m - \operatorname{sym}\{K_n\}, \\
\Pi_{12} &= W_{1m} + \mathcal{N}_{n1} + A_{nq}K_n + B_{2q}K_{anl}, \\
\Pi_{13} &= \mathcal{N}_{n2} - W_{1m} + A_{\tau_0 nq}K_n + B_{2nq}K_{a\tau_0 nl}, \\
\Pi_{14} &= \mathcal{N}_{n3} + W_{1m}, \\
\Pi_{15} &= K_n\mathcal{N}_{n4} + K_nB_{1nq}, \\
\Pi_{16} &= \mathcal{N}_{n5} + G_{nq}K_n + B_{2nq}K_{bnl}, \\
\Pi_{17} &= \mathcal{N}_{n6} + G_{\tau_0 nq}K_n + B_{2nq}K_{b\tau_0 nl}, \\
\Pi_1^T &= C_{nq}K_n + D_{2nq}K_{anl}, \\
\Pi_2^T &= C_{\tau_0 nq}K_n + D_{2nq}K_{a\tau_0 nl}, \\
\Pi_3^T &= G_{znq}K_n + D_{2nq}K_{bnl}, \\
\Pi_4^T &= G_{z\tau_0 nq}K_n + D_{2nq}K_{b\tau_0 nl}, \\
\Phi_1 &= \theta I + R - \operatorname{sym}\{\mathcal{N}_{n1}\} + W_{2m} + \tau_0^2\mathcal{Q}_{11} - \mathcal{Q}_{22} - P_n, \\
\Phi_2 &= \theta I + \mathcal{Q}_{22} - \mathcal{N}_{n2} - W_{2m}, \\
\Phi_3 &= W_{2m} - \mathcal{N}_{n3} - \mathcal{Q}_{12}^T - W_{1n}, \\
\Phi_4 &= \tau^2\mathcal{Q}_{12} - \mathcal{N}_{n1}^T - N_{n7}, \\
\Phi_5 &= W_{2m} - R - \mathcal{Q}_{22}, \\
\Phi_6 &= \mathcal{Q}_{12}^T - W_{2m}, \\
\Phi_7 &= W_{2m} - \mathcal{Q}_{11} - W_{2n}, \\
\Phi_8 &= \tau_0^2\mathcal{Q}_{22} - \operatorname{sym}\{\mathcal{N}_{n7}\}.
\end{aligned} \tag{17}$$

Moreover, the controllers are given by

$$\begin{aligned}
F_{anl} &= K_{anl}K_n^{-1}, \\
F_{a\tau_0 nl} &= K_{a\tau_0 nl}K_n^{-1}, \\
F_{bnl} &= K_{bnl}K_n^{-1}, \\
F_{b\tau_0 nl} &= K_{b\tau_0 nl}K_n^{-1}.
\end{aligned} \tag{18}$$

Proof. Let $\mathcal{U}_n = K_n^{-1}$ and $\mathcal{U} = \operatorname{diag}\{\mathcal{U}_n, \mathcal{U}_n, \mathcal{U}_n, \mathcal{U}_n, I, \mathcal{U}_n, \mathcal{U}_n, \mathcal{U}_n, I\}$. On left side of inequalities (15) and (16), postmultiplying $\mathcal{U}^T$ and premultiplying $\mathcal{U}$, respectively, one has

$$\Omega_{mnqq} < 0, \quad q = 1, 2, \ldots, \lambda_n, \ (n, m) \in \Omega, \tag{19}$$

$$\Omega_{mnql} + \Omega_{mnlq} < 0, \quad 0 < q < l \leq \lambda_n, \ (n, m) \in \Omega, \tag{20}$$

where

$$\Omega_{mnql}=\begin{bmatrix}\Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} & \Omega_{17} & \overline{\mathcal{N}}_{n7} & 0\\ * & \overline{\Phi}_1 & \overline{\Phi}_2 & \overline{\Phi}_3 & -\mathcal{N}_{n4} & -\overline{\mathcal{N}}_{n5} & -\overline{\mathcal{N}}_{n6} & \overline{\Phi}_4 & \widehat{C}^T_{nql}\\ * & * & \overline{\Phi}_5 & \overline{\Phi}_6 & 0 & 0 & 0 & -\mathcal{N}^T_{n2} & \widehat{C}^T_{\tau nql}\\ * & * & * & \overline{\Phi}_7 & 0 & 0 & 0 & -\overline{\mathcal{N}}^T_{n3} & 0\\ * & * & * & * & -\gamma^2 I & 0 & 0 & -\overline{\mathcal{N}}^T_{n4} & D^T_{1nq}\\ * & * & * & * & * & -\overline{I} & 0 & -\overline{\mathcal{N}}^T_{n5} & \widehat{G}^T_{znql}\\ * & * & * & * & * & * & -\overline{I} & -\overline{\mathcal{N}}_{n6} & \widehat{G}^T_{\tau_0 znql}\\ * & * & * & * & * & * & * & \overline{\Phi}_8 & 0\\ * & * & * & * & * & * & * & * & -I\end{bmatrix} \tag{21}$$

and $\overline{X}$ represents $\mathcal{U}_n^T X \mathcal{U}_n$. $\Omega_{11}=\overline{P}_m-\text{sym}\{\mathcal{U}_n\}$, $\Omega_{12}=\overline{W}_{1m}+\overline{\mathcal{N}}_{n1}+\mathcal{U}_n^T\widehat{A}_{nql}$, $\Omega_{13}=\overline{\mathcal{N}}_{n2}-\overline{W}_{1m}+\mathcal{U}_n^T\widehat{A}_{\tau nql}$, $\Omega_{14}=\overline{\mathcal{N}}_{n3}+\overline{W}_{1m}$, $\Omega_{15}=\mathcal{N}_{n4}+B_{1nq}$, $\Omega_{16}=\overline{\mathcal{N}}_{n5}+\mathcal{U}_n^T\widehat{G}_{nkl}$, and $\Omega_{17}=\overline{\mathcal{N}}_{n6}+\mathcal{U}_n^T\widehat{G}_{\tau_0 nql}$.

Based on (19) and (20), we have

$$\begin{aligned}\Omega &= \sum_{k=1}^{\lambda_n}\sum_{l=1}^{\lambda_n} h_{nk}h_{nl}\Omega_{mnkl}=\sum_{q=1}^{\lambda_n} h_{nk}^2\Omega_{mnqq}\\ &\quad+\sum_{q<l}^{\lambda_n} h_{nq}h_{nl}\left\{\Omega_{mnql}+\Omega_{mnlq}\right\}\\ &= \Omega_0+\text{sym}\left\{\Omega_1^T\Omega_2\right\}<0,\end{aligned} \tag{22}$$

where $\Omega_1=[\mathcal{U}_j,0,0,0,0,0,0,0,0,]$, $\Omega_2=[-I,\widetilde{A}_{nql},\widetilde{A}_{\tau_0 nql},0,\widetilde{B}_{1nq},\widetilde{G}_{nql},\widetilde{G}_{\tau_0 nql},0,0]$, and

$$\Omega_0=\begin{bmatrix}\overline{P}_m & \overline{W}_{1m}+\overline{\mathcal{N}}_{n1} & \overline{\mathcal{N}}_{n2}-\overline{W}_{1m} & \overline{\mathcal{N}}_{n3}+\overline{W}_{1m} & \mathcal{N}_{n4} & \overline{\mathcal{N}}_{n5} & \overline{\mathcal{N}}_{n6} & \overline{\mathcal{N}}_{n7} & 0\\ * & \overline{\Phi}_1 & \overline{\Phi}_2 & \overline{\Phi}_3 & -\mathcal{N}_{n4} & -\overline{\mathcal{N}}_{n5} & -\overline{\mathcal{N}}_{n6} & \overline{\Phi}_4 & \widetilde{C}^T_{nql}\\ * & * & \overline{\Phi}_5 & \overline{\Phi}_6 & 0 & 0 & 0 & -\mathcal{N}^T_{n2} & \widetilde{C}^T_{\tau nql}\\ * & * & * & \overline{\Phi}_7 & 0 & 0 & 0 & -\overline{\mathcal{N}}^T_{n3} & 0\\ * & * & * & * & -\gamma^2 I & 0 & 0 & -\overline{\mathcal{N}}^T_{n4} & \widetilde{D}^T_{1nq}\\ * & * & * & * & * & -\overline{I} & 0 & -\overline{\mathcal{N}}^T_{n5} & \widetilde{G}^T_{znql}\\ * & * & * & * & * & * & -\overline{I} & -\overline{\mathcal{N}}_{n6} & \widetilde{G}^T_{\tau_0 znql}\\ * & * & * & * & * & * & * & \overline{\Phi}_8 & 0\\ * & * & * & * & * & * & * & * & -I\end{bmatrix}. \tag{23}$$

Let $\Omega_{2\perp}=\begin{bmatrix}\widetilde{A}_{nql},\widetilde{A}_{\tau_0 nql},0,\widetilde{B}_{1nq},\widetilde{G}_{nql},\widetilde{G}_{\tau_0 nql},0,0\\ \text{diag}\left\{\underbrace{I,\ldots,I}_{8\text{ blocks}}\right\}\end{bmatrix}$, which satisfies $\Omega_2\Omega_{2\perp}=0$. Moreover, $[\Omega_2^T,\Omega_{2\perp}]$ is a column full-rank matrix. Then, based on Lemma 1 and inequality (22), we obtain

$$\Omega_{2\perp}^T\Omega_0\Omega_{2\perp}=\begin{bmatrix}\Delta_{11}&\Delta_{12}&\Delta_{13}&\Delta_{14}&\Delta_{15}&\Delta_{16}&\Delta_{17}&\widetilde{C}_{nql}^T\\ *&\Delta_{22}&\Delta_{23}&\Delta_{24}&\Delta_{25}&\Delta_{26}&\Delta_{27}&\widetilde{C}_{\tau nql}^T\\ *&*&\Delta_{33}&\Delta_{34}&\Delta_{35}&\Delta_{36}&\Delta_{37}&0\\ *&*&*&\Delta_{44}&\Delta_{45}&\Delta_{46}&\Delta_{47}&\widetilde{D}_{1jq}^T\\ *&*&*&*&\Delta_{55}&\Delta_{56}&\Delta_{57}&\widetilde{G}_{znql}^T\\ *&*&*&*&*&\Delta_{66}&\Delta_{67}&\widetilde{G}_{z\tau_0nql}^T\\ *&*&*&*&*&*&\Delta_{77}&0\\ *&*&*&*&*&*&*&-I\end{bmatrix}<0, \quad (24)$$

where

$$\begin{aligned}
\Delta_{11}&=\widetilde{A}_{nql}^T\overline{P}_i\widetilde{A}_{nql}+H_e\left\{\left(\overline{W}_{1m}+\overline{\mathcal{N}}_{n1}\right)^T\widetilde{A}_{nql}\right\}+\overline{\Phi}_1,\\
\Delta_{12}&=\widetilde{A}_{nql}^T\overline{P}_m\widetilde{A}_{\tau_0nql}+\widetilde{A}_{nql}^T\left(\overline{\mathcal{N}}_{n2}-\overline{W}_{1m}\right)\\
&\quad+\left(\overline{W}_{1m}+\overline{\mathcal{N}}_{n1}\right)^T\widetilde{A}_{\tau_0nql}+\overline{\Phi}_2,\\
\Delta_{13}&=\widetilde{A}_{nql}^T\left(\overline{\mathcal{N}}_{n3}+\overline{W}_{1m}\right)+\overline{\Phi}_3,\\
\Delta_{14}&=\widetilde{A}_{nql}^T\overline{P}_m\widetilde{B}_{1nq}+\left(\overline{W}_{1m}+\overline{\mathcal{N}}_{n1}\right)^T\widetilde{B}_{1nq}+\widetilde{A}_{nql}^T\overline{\mathcal{N}}_{n4}-\overline{\mathcal{N}}_{n4},\\
\Delta_{15}&=\widetilde{A}_{nql}^T\overline{P}_m\widetilde{G}_{nql}+\left(\overline{\mathcal{N}}_{n1}+\overline{W}_{1n}\right)^T\widetilde{G}_{nql}+\widetilde{A}_{nql}^T\overline{\mathcal{N}}_{n5},\\
\Delta_{16}&=\widetilde{A}_{nql}^T\overline{P}_m\widetilde{G}_{\tau_0nql}+\left(\overline{\mathcal{N}}_{n1}+\overline{W}_{1m}\right)^T\widetilde{G}_{\tau_0nql}+\widetilde{A}_{nql}^T\overline{\mathcal{N}}_{n6}-\overline{\mathcal{N}}_{n6},\\
\Delta_{17}&=\widetilde{A}_{nql}^T\overline{\mathcal{N}}_{n7}+\overline{\Phi}_4,\\
\Delta_{22}&=\widetilde{A}_{\tau_0nql}^T\overline{P}_m\widetilde{A}_{\tau_0nql}+H_e\left\{\left(\overline{\mathcal{N}}_{n2}-\overline{W}_{1m}\right)^T\widetilde{A}_{\tau_0nql}\right\}+\overline{\Phi}_5,\\
\Delta_{23}&=\widetilde{A}_{\tau nql}^T\left(\overline{\mathcal{N}}_{n3}+\overline{W}_{1m}\right)+\overline{\Phi}_6,\\
\Delta_{24}&=\widetilde{A}_{\tau_0nql}^T\overline{P}_m\widetilde{B}_{1nq}+\left(\overline{\mathcal{N}}_{n2}-\overline{W}_{1m}\right)^T\widetilde{B}_{1nq}+\widetilde{A}_{\tau_0nql}^T\overline{\mathcal{N}}_{n4},\\
\Delta_{25}&=\widetilde{A}_{\tau_0nql}^T\overline{P}_m\widetilde{G}_{nql}+\left(\overline{\mathcal{N}}_{n2}-\overline{W}_{1m}\right)^T\widetilde{G}_{nql}+\widetilde{A}_{\tau_0nkl}^T\overline{\mathcal{N}}_{n5},\\
\Delta_{26}&=\widetilde{A}_{\tau_0nql}^T\overline{P}_m\widetilde{G}_{\tau_0nql}+\left(\overline{\mathcal{N}}_{n2}-\overline{W}_{1m}\right)^T\widetilde{G}_{\tau_0nql}+\widetilde{A}_{\tau_0nql}^T\overline{\mathcal{N}}_{n6},\\
\Delta_{27}&=\widetilde{A}_{\tau_0nql}^T\overline{\mathcal{N}}_{n7}-\overline{\mathcal{N}}_{n2}^T,\\
\Delta_{33}&=\overline{\Phi}_7,\Delta_{34}=\left(\overline{W}_{1m}+\overline{\mathcal{N}}_{n3}\right)^T\widetilde{B}_{1nq},\\
\Delta_{35}&=\left(\overline{W}_{1m}+\overline{\mathcal{N}}_{n3}\right)^T\widetilde{G}_{nql},\\
\Delta_{36}&=\left(\overline{W}_{1m}+\overline{\mathcal{N}}_{n3}\right)^T\widetilde{G}_{\tau_0nql},\\
\Delta_{37}&=-\overline{\mathcal{N}}_{n3}^T,\\
\Delta_{44}&=\widetilde{B}_{1nq}^T\overline{P}_m\widetilde{B}_{1nq}+H_e\left\{\overline{\mathcal{N}}_{n4}^T\widetilde{B}_{1nq}\right\}-\gamma^2I,\\
\Delta_{45}&=\widetilde{B}_{1nq}^T\overline{P}_m\widetilde{G}_{nql}+\overline{\mathcal{N}}_{n4}^T\widetilde{G}_{nql}+\widetilde{B}_{1nq}\overline{\mathcal{N}}_{n5},\\
\Delta_{46}&=\widetilde{B}_{1nq}^T\overline{P}_m\widetilde{G}_{\tau_0nql}+\overline{\mathcal{N}}_{n4}^T\widetilde{G}_{\tau_0nql}+\widetilde{B}_{1nq}\overline{\mathcal{N}}_{n6},\\
\Delta_{47}&=\widetilde{B}_{1nq}^T\overline{\mathcal{N}}_{n7}-\overline{\mathcal{N}}_{n4}^T,\\
\Delta_{55}&=\widetilde{G}_{nql}^T\overline{P}_m\widetilde{G}_{nql}+H_e\left\{\overline{\mathcal{N}}_{n5}^T\widetilde{G}_{nql}\right\}-\overline{I},\\
\Delta_{57}&=\widetilde{G}_{nql}^T\overline{\mathcal{N}}_{n7}-\overline{\mathcal{N}}_{n5}^T,\\
\Delta_{56}&=\overline{G}_{nql}^TP_m\overline{G}_{\tau_0nql}+\mathcal{N}_{n5}^T\overline{G}_{\tau_0nql}+\overline{G}_{nql}^T\mathcal{N}_{n6},\\
\Delta_{77}&=\overline{\Phi}_8\\
\Delta_{66}&=\widetilde{G}_{\tau_0nql}^T\overline{P}_m\widetilde{G}_{\tau_0nql}+H_e\left\{\overline{\mathcal{N}}_{n6}^T\widetilde{G}_{\tau_0nql}\right\}-\overline{I},\\
\Delta_{67}&=\widetilde{G}_{\tau_0nql}^T\overline{\mathcal{N}}_{n7}-\overline{\mathcal{N}}_{n6}^T.
\end{aligned} \quad (25)$$

Applying the Schur complement described by Lemma 3 to the above inequality, we can obtain

$$\Xi_{mnql}=\begin{bmatrix}\Delta_{11}&\Delta_{12}&\Delta_{13}&\Delta_{14}&\Delta_{15}&\Delta_{16}&\Psi_{17}\\ *&\Delta_{22}&\Delta_{23}&\Delta_{24}&\Delta_{25}&\Delta_{26}&\Delta_{27}\\ *&*&\Delta_{33}&\Delta_{34}&\Delta_{35}&\Delta_{36}&\Delta_{37}\\ *&*&*&\Delta_{44}&\Delta_{45}&\Delta_{46}&\Delta_{47}\\ *&*&*&*&\Delta_{55}&\Delta_{56}&\Delta_{57}\\ *&*&*&*&*&\Delta_{66}&\Delta_{67}\\ *&*&*&*&*&*&\Delta_{77}\end{bmatrix}+\begin{bmatrix}\widetilde{C}_{nql}^T\\ \widetilde{C}_{\tau_0nql}^T\\ 0\\ \widetilde{D}_{1nq}^T\\ \widetilde{G}_{znql}^T\\ \widetilde{G}_{z\tau_0nql}^T\\ 0\end{bmatrix}\begin{bmatrix}\widetilde{C}_{nql}^T\\ \widetilde{C}_{\tau_0nql}^T\\ 0\\ \widetilde{D}_{1nq}^T\\ \widetilde{G}_{znql}^T\\ \widetilde{G}_{z\tau_0nql}^T\\ 0\end{bmatrix}^T<0. \quad (26)$$

Based on (26), we will prove that Theorem 1 is correct. Construct a discrete-time PLF as follows:

$$\begin{aligned}
\mathcal{V}_t&=\mathcal{V}_{1t}+\mathcal{V}_{2t}+\mathcal{V}_{3t},\\
\mathcal{V}_{1t}&=\sum_{l=t-\tau_0}^{t-1}x_l^T\overline{R}x_v,\\
\mathcal{V}_{2t}&=\tau_0\sum_{\theta=-\tau_0}^{-1}\sum_{l=t+\theta}^{t-1}\begin{bmatrix}x_v\\ \pi_l\end{bmatrix}^T\begin{bmatrix}\overline{\mathcal{Q}}_{11}&\overline{\mathcal{Q}}_{12}\\ *&\overline{\mathcal{Q}}_{22}\end{bmatrix}\begin{bmatrix}x_v\\ \pi_l\end{bmatrix},\\
\mathcal{V}_{3t}&=\begin{bmatrix}x_v\\ \sum_{v=t-\tau_0}^{t-1}x_v\end{bmatrix}^T\begin{bmatrix}\overline{P}_n&\overline{W}_{1n}\\ *&\overline{W}_{2n}\end{bmatrix}\begin{bmatrix}x_t\\ \sum_{v=t-\tau_0}^{t-1}x_v\end{bmatrix},
\end{aligned} \quad (27)$$

where $x_t\in\Omega_n$, $n=1,2,\ldots,\kappa$, and $\pi_t=x_{t+1}-x_t$.

Let $\Delta\mathcal{V}_t=\mathcal{V}_{t+1}-\mathcal{V}_t$, we can get

$$\Delta\mathcal{V}_{1t}=x_t^T\overline{R}x_t-x_{t-\tau_0}^T\overline{R}x_{t-\tau_0}, \quad (28)$$

$$\begin{aligned}
\Delta\mathcal{V}_{2t}&=\tau_0^2\begin{bmatrix}x_t\\ \pi_t\end{bmatrix}^T\begin{bmatrix}\overline{\mathcal{Q}}_{11}&\overline{\mathcal{Q}}_{12}\\ *&\overline{\mathcal{Q}}_{22}\end{bmatrix}\begin{bmatrix}x_t\\ \pi_t\end{bmatrix}\\
&\quad-\tau_0\sum_{v=t-\tau_0}^{t-1}\begin{bmatrix}x_v\\ \pi_v\end{bmatrix}^T\begin{bmatrix}\overline{\mathcal{Q}}_{11}&\overline{Q}_{12}\\ *&\overline{\mathcal{Q}}_{22}\end{bmatrix}\begin{bmatrix}x_v\\ \pi_v\end{bmatrix}.
\end{aligned} \quad (29)$$

Using Lemma 2, we can obtain

$$\begin{aligned}
&-\tau_0\sum_{v=t-\tau_0}^{t-1}\begin{bmatrix}x_v\\ \pi_v\end{bmatrix}^T\begin{bmatrix}\overline{\mathcal{Q}}_{11} & \overline{\mathcal{Q}}_{12}\\ * & \overline{\mathcal{Q}}_{22}\end{bmatrix}\begin{bmatrix}x_v\\ \pi_v\end{bmatrix}\\
&\leq-\begin{bmatrix}\sum_{v=t-\tau_0}^{t-1}x_v\\ \sum_{v=t-\tau_0}^{t-1}\pi_v\end{bmatrix}^T\begin{bmatrix}\overline{\mathcal{Q}}_{11} & \overline{\mathcal{Q}}_{12}\\ * & \overline{\mathcal{Q}}_{22}\end{bmatrix}\begin{bmatrix}\sum_{l=t-\tau_0}^{t-1}x_v\\ \sum_{v=t-\tau_0}^{t-1}\pi_v\end{bmatrix}\\
&=\begin{bmatrix}\sum_{v=t-\tau_0}^{t-1}x_v\\ x_t-x_{t-\tau_0}\end{bmatrix}^T\begin{bmatrix}-\overline{\mathcal{Q}}_{11} & -\overline{\mathcal{Q}}_{12}\\ * & -\overline{\mathcal{Q}}_{22}\end{bmatrix}\begin{bmatrix}\sum_{v=t-\tau_0}^{t-1}x_v\\ x_t-x_{t-\tau_0}\end{bmatrix}\\
&=\begin{bmatrix}x_t\\ x_{t-\tau_0}\\ \sum_{v=t-\tau_0}^{t-1}x_v\end{bmatrix}^T\begin{bmatrix}-\overline{\mathcal{Q}}_{22} & \overline{\mathcal{Q}}_{22} & -\overline{\mathcal{Q}}_{12}^T\\ * & -\overline{\mathcal{Q}}_{22} & \overline{\mathcal{Q}}_{12}^T\\ * & * & -\overline{\mathcal{Q}}_{11}\end{bmatrix}\begin{bmatrix}x_t\\ x_{t-\tau_0}\\ \sum_{v=t-\tau_0}^{t-1}x_v\end{bmatrix},
\end{aligned}\tag{30}$$

$$\begin{aligned}
\Delta\mathcal{V}_{3t}&=\begin{bmatrix}x_{t+1}\\ \sum_{v=t+1-\tau_0}^{t}x_v\end{bmatrix}^T\begin{bmatrix}\overline{P}_m & \overline{W}_{1m}\\ * & \overline{W}_{2m}\end{bmatrix}\begin{bmatrix}x_{t+1}\\ \sum_{v=t+1-\tau_0}^{t}x_v\end{bmatrix}\\
&\quad-\begin{bmatrix}x_t\\ \sum_{v=t-\tau_0}^{t-1}x_v\end{bmatrix}^T\begin{bmatrix}\overline{P}_n & \overline{W}_{1n}\\ * & \overline{W}_{2n}\end{bmatrix}\begin{bmatrix}x_t\\ \sum_{v=t-\tau_0}^{t-1}x_v\end{bmatrix}\\
&=\zeta_t^T\left\{\Theta_1\begin{bmatrix}\overline{P}_m & \overline{W}_{1m}\\ * & \overline{W}_{2m}\end{bmatrix}\Theta_1^T-\Theta_2\begin{bmatrix}\overline{P}_n & \overline{W}_{1n}\\ * & \overline{W}_{2n}\end{bmatrix}\Theta_2^T\right\}\zeta_t,
\end{aligned}\tag{31}$$

$$\begin{aligned}
\Theta_1&=\begin{bmatrix}\widetilde{A}_{nql} & \widetilde{A}_{\tau_0nql} & \widetilde{B}_{1nq} & \widetilde{G}_{nql} & \widetilde{G}_{\tau_0nql} & 0 & 0\\ I & -I & I & 0 & 0 & 0 & 0\end{bmatrix}^T,\\
\Theta_2&=\begin{bmatrix}I & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & I & 0 & 0 & 0 & 0\end{bmatrix}^T,\\
\zeta_t^T&=\begin{bmatrix}x_t^T & x_{t-\tau_0}^T & \left(\sum_{v=t-\tau_0}^{t-1}x_l^T\right) & \omega_t^T & \phi_t^T & \phi_{t-\tau_0}^T & \pi_t^T\end{bmatrix}
\end{aligned}\tag{32}$$

where

Observing system (11) and inequality (5), one has

$$\begin{aligned}
\mathcal{N}&=2\xi_t^T\begin{bmatrix}\overline{\mathcal{N}}_{n1} & \overline{\mathcal{N}}_{n2} & \overline{\mathcal{N}}_{n3} & \mathcal{U}_n^T\mathcal{N}_{n4} & \overline{\mathcal{N}}_{n5} & \overline{\mathcal{N}}_{n6} & \overline{\mathcal{N}}_{n7}\end{bmatrix}^T\\
&\quad\times\left[\left(\widetilde{A}_{nql}-I\right)x_t+\widetilde{A}_{\tau_0nql}x_{t-\tau_0}+\widetilde{B}_{1nq}\omega_t\right.\\
&\quad\left.+\widetilde{G}_{nql}\phi_t+\widetilde{G}_{\tau_0nql}\phi_{t-\tau_0}-\pi_t\right]=0,
\end{aligned}\tag{33}$$

$$\theta x_t^T\mathcal{U}_n^T\mathcal{U}_nx_t-\phi_t^T\mathcal{U}_n^T\mathcal{U}_n\phi_t\geq 0,\tag{34}$$

$$\theta x_{t-\tau_0}^T\mathcal{U}_n^T\mathcal{U}_nx_{t-\tau_0}-\phi_{t-\tau_0}^T\mathcal{U}_n^T\mathcal{U}_n\phi_{t-\tau_0}\geq 0.\tag{35}$$

Then, from (28)–(35) and considering system (11), one can obtain

$$\begin{aligned}
&\Delta\mathcal{V}_t+z_t^Tz_t-\gamma^2\omega_t^T\omega_t=\Delta\mathcal{V}_{1t}+\Delta\mathcal{V}_{2t}\\
&\quad+\Delta\mathcal{V}_{3t}+\mathcal{N}+z_t^Tz_t-\gamma^2\omega_t^T\omega_t\\
&\leq\Delta\mathcal{V}_{1t}+\Delta\mathcal{V}_{2t}+\Delta\mathcal{V}_{3t}+\mathcal{N}+z_t^Tz_t-\gamma^2\omega_t^T\omega_t\\
&\quad+\theta x_t^T\mathcal{U}_n^T\mathcal{U}_nx_t-\phi_t^T\mathcal{U}_n^T\mathcal{U}_n\phi_t+\theta x_{t-\tau_0}^T\mathcal{U}_n^T\mathcal{U}_nx_{t-\tau_0}\\
&\quad-\phi_{t-\tau_0}^T\mathcal{U}_n^T\mathcal{U}_n\phi_{t-\tau_0}\\
&\leq\xi_t^T\Xi_{mnql}\xi_t.
\end{aligned}\tag{36}$$

Based on (26), we have

$$\Delta\mathcal{V}_t+z_t^Tz_t-\gamma^2\omega_t^T\omega_t<0.\tag{37}$$

Therefore, when the disturbance is assumed to be zero, the following can be obtained from (28)–(35):

$$\begin{aligned}
&\Delta\mathcal{V}|_{\omega=0}=\left(\Delta\mathcal{V}_1+\Delta\mathcal{V}_2+\Delta\mathcal{V}_3+\mathcal{N}\right)\big|_{\omega_t=0}\\
&\leq\begin{bmatrix}x_t\\ x_{t-\tau_0}\\ \sum_{v=t-\tau_0}^{t-1}x_v\\ \pi_t\end{bmatrix}^T\begin{bmatrix}\Delta_{11} & \Delta_{12} & \Delta_{13} & \Delta_{17}\\ * & \Delta_{22} & \Delta_{23} & \Delta_{27}\\ * & * & \Delta_{33} & \Delta_{37}\\ * & * & * & \Delta_{77}\end{bmatrix}\begin{bmatrix}x_t\\ x_{t-\tau_0}\\ \sum_{v=t-\tau_0}^{t-1}x_v\\ \pi_t\end{bmatrix}.
\end{aligned}\tag{38}$$

Using Schur complement, inequality (26) means that $\Delta V|_{\omega=0}<0$. Therefore, when $\omega(t)=0$, we can easily obtain the asymptotically stability of system (11). Let

$$\mathbb{J}=\sum_{l=0}^{\infty}\left[\left\|z_l\right\|_2^2-\gamma^2\left\|\omega_l\right\|_2^2\right].\tag{39}$$

Under the condition that the initial conditions are zero and inequality (37) holds, we can obtain $\mathbb{J}\leq\sum_{l=0}^{\infty}[-\Delta\mathcal{V}_l]=-\mathcal{V}_\infty+\mathcal{V}_0=-\mathcal{V}_\infty<0$. That is, $\|z\|_2<\gamma\|\omega\|_2$.

In order to increase the feasible range of the results obtained in Theorem 1, the relationship between the subsystems in each subspace is considered. In addition, the introduced relaxation matrix $M_{nql}\,(q<l<\lambda_n)$ may not be symmetric matrices. Consequently, we can give and prove the following improved results. □

Theorem 2. *Designing fuzzy controller (10) for system (8), the controlled system (11) is asymptotically stable and has H_∞property upper bound $\gamma\,(\gamma>0)$, if there exist matrices $\begin{bmatrix}P_n & W_{1n}\\ * & W_{2n}\end{bmatrix}>0$, $\begin{bmatrix}\mathcal{Q}_{11} & \mathcal{Q}_{12}\\ * & \mathcal{Q}_{22}\end{bmatrix}>0$, $R>0$, the nonsingular matrix F_n, matrices $\mathcal{N}_{n\iota}\,(\iota=1,2,\ldots,7)$, K_{anl}, $K_{a\tau_0nl}$, K_{bnl}, $K_{b\tau_0nl}$, and $M_{nlq}=M_{nql}^T$, such that the following are feasible:*

$$\Pi_{mnqq}<M_{nqq},\quad q=1,\ldots,\lambda_n,\ (n,m)\in\Omega,\tag{40}$$

$$\Pi_{mnql}+\Pi_{mnlq}<M_{nql}+M_{nql}^T,\quad 0<k<l\leq\lambda_n,\ (n,m)\in\Omega,\tag{41}$$

$$M_n = \begin{bmatrix} M_{n11} & M_{n12} & \cdots & M_{n1\lambda_n} \\ M_{n21} & M_{n22} & \cdots & M_{n2\lambda_n} \\ \vdots & \vdots & \ddots & \vdots \\ M_{n\lambda_n 1} & M_{n\lambda_n 2} & \cdots & N_{n\lambda_n\lambda_n} \end{bmatrix} < 0, \quad n = 1, 2, \ldots, \kappa. \tag{42}$$

The feedback matrices of the controller can be designed as

$$\begin{aligned} F_{anl} &= K_{anl}K_n^{-1}, \\ F_{a\tau_0 nl} &= K_{a\tau_0 nl}K_n^{-1}, \\ F_{bnl} &= K_{bnl}K_n^{-1}, \\ F_{b\tau_0 nl} &= K_{b\tau_0 nl}K_n^{-1}. \end{aligned} \tag{43}$$

Proof. If inequalities (40)–(42) hold, then

$$\Pi_{mnkl} < \sum_{q=1}^{\lambda_n} h_{nq}^2 M_{nqq} + \sum_{q<l}^{\lambda_n} h_{nq}h_{nl}\left(M_{nql} + M_{nql}^T\right) = \mathbf{H}_n^T M_n \mathbf{H}_n < 0, \tag{44}$$

where $\mathbf{H}_n = \left[\, h_{n1}I \ \ h_{n2}I \ \cdots \ h_{n\lambda_n}I \,\right]^T$.

If inequality (44) has a feasible solution, we can obtain

$$\Delta\mathscr{V}_t + z_t^T z_t - \omega_t^T \omega_t < 0. \tag{45}$$

The next proof process is roughly the same as that of Theorem 2, so it is omitted here. This completes the proof. □

4. Conclusion

By using the fuzzy switched system, PLF, and state movement from one subspace to another, two new H_∞ control schemes with time delay information are derived. The advantages of the proposed method include that the fuzzy posterior contains nonlinear functions, the Lyapunov function is piecewise, and the condition is expressed by linear matrix inequality. In addition, Theorem 2 considers the relationship between fuzzy subsystems. The disadvantage of the proposed method is the introduction of relaxation variables, which increases the computational complexity. However, in order to expand the feasible region of the result and obtain more feasible solutions at the same time, we sometimes need to introduce the relaxation variables. If the conditions have a feasible solution, the controller feedback matrices can be calculated according to the feasible solutions of a set of LMIs. Since fractional-order systems have more dynamic behaviors than integral-order systems [3], we will consider extending the results of this paper to fractional-order T-S fuzzy systems.

Data Availability

The key point of our paper is theoretical derivation. The result simulation can be done with the linear matrix inequality toolbox provided by MATLAB.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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