Research Article

Fully Distributed Event-Triggered Containment Control of Uncertain Multiagent Systems

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This paper investigates the event-triggered containment control problem of a class of uncertain nonlinear multiagent systems (MASs). By employing the local relative information, we design an adaptive event-triggered containment algorithm. The proposed containment algorithm can cope with the unavailability of global topology information and uncertain dynamics of follower agents. Therefore, the presented containment algorithm is free of global topology information, i.e., the designed algorithm is fully distributed. In addition, it is proved that Zeno behavior will not occur. At last, a numerical example is given to verify our event-triggered containment algorithm.

1. Introduction

During past two decades, cooperative control of MASs has been a hot research field for the reason of broad engineering applications, such as formation control [1], attitude alignment [2], mobile sensor networks [3], multirobot systems [4], and so forth [5, 6]. In practice, MASs are composed of mobile units, such as UAVs and robots, which are equipped with embedded control chips and limited energy supply. To extend the working hours of MASs, researchers developed various cooperative algorithms. Event-triggered cooperative algorithms can effectively reduce communications among neighbor agents and have gained extensive attention.

MASs can be divided into three cases, i.e., leaderless MASs, leader-follower MASs, and multileader MASs. The control objectives of those three kinds of MASs are consensus control, tracking control, and containment control, respectively. Consensus control and tracking control have been widely studied, from linear MASs [1, 7, 8] to nonlinear MASs [9–11]. Event-triggered control technique is also employed to design consensus and tracking controllers for various kinds of MASs [12–16]. Unlike consensus and tracking control, the objective of containment control is that each follower agent enters into the convex hull spanned by

leader agents. In papers [2, 17–19], containment control problems of different kinds of MASs were studied. Event-triggered containment problems are also considered by researchers [20–24]. Miao studied the event-triggered containment control for first/second-order MASs with constant time delay in [20]. Event-triggered containment control of second-order MASs with sampled position data and time-varying input delays are considered in [21, 22]. Rong and her coauthors studied the event-triggered containment control problem for general MIMO linear MASs in [23]. Xu studied the event-triggered containment problem of Euler–Lagrange MASs in [24].

Cooperative algorithms of MASs use the local information to achieve global control tasks. However, most of the existing control algorithms of MASs need the information of Laplacian which is a global information. Control algorithms that do not need any global information are said to be fully distributed. Many researchers devoted to develop the fully distributed algorithms for MASs. Papers [25–28] presented some typical works of fully distributed algorithms. Recently, some researchers studied the fully distributed control problems of MASs via event-triggered approach as well. Zhu and Cheng studied the fully distributed event-triggered control problems for linear MASs in [29, 30]. Li and his

coauthors investigated both event-triggered consensus and tracking problems of second-order nonlinear MASs in [31]. Apart from Euler-Lagrange MASs in [24], event-triggered containment problem of multiple leaders MASs was seldom considered.

Inspired by the above papers, we study the fully distributed event-triggered containment problem of a class of uncertain MASs. Not only the Laplacian of MASs but also the dynamics of follower agents are unavailable to design the containment controllers. To deal with these two unknown data, adaptive techniques are used to design event-triggered containment controllers. The main contributions of this paper are listed as follows: (i) our containment algorithm is fully distributed and does not use any global information of MASs; (ii) our containment algorithm is robust and still works when the dynamics of follower agent is uncertain; and (iii) our presented event-triggered algorithm is free of Zeno behavior.

2. Problem Statement and Preliminaries

Notations. Let $I_N \in \mathbb{R}^{N \times N}$ be the N -dimensional identity matrix, $\mathbf{1}_N \in \mathbb{R}^N$ be a vector with each entry being 1, $\|\cdot\|$ be the Euclidean norm (2-norm), $\otimes$ be the Kronecker product, and $\text{diag}\{a_1, \dots, a_N\}$ be a diagonal matrix. We use $\lambda_{1A} \leq \lambda_{2A} \leq \dots \leq \lambda_{NA}$ to denote N eigenvalues of a symmetric matrix $A \in \mathbb{R}^{N \times N}$. $\|A\|_F = \sqrt{\text{tr}\{A^T A\}}$ denotes the Frobenius norm of matrix A .

2.1. Preliminaries. In this paper, there are $N + M$ agents in the considered MASs. The communication topology among those $N + M$ agents is depicted by a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, where $\mathcal{V} = \{1, \dots, N + M\}$, $\mathcal{E} = \mathcal{V} \times \mathcal{V}$, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{(N+M) \times (N+M)}$ are the agent set, edge set, and adjacency matrix, respectively. $(i, j) \in \mathcal{E}$ denotes a directed edge from agent j to agent i and the associated weighing factor $a_{ij} > 0$; otherwise, $a_{ij} = 0$. If $a_{ij} = a_{ji} > 0$, then there is an undirected edge between agent i and agent j . We assume that there is no self-loop edge, i.e., $a_{ii} = 0, \forall i \in \mathcal{V}$. Edge sequence $(i_2, i_1)(i_3, i_2), \dots, (i_k, i_{k-1})$ denotes a directed path from agent i_1 to agent i_k . $L = [l_{ij}] \in \mathbb{R}^{(N+M) \times (N+M)}$ denotes the Laplacian matrix of graph $\mathcal{G}$, with $l_{ii} = \sum_{j=1}^{N+M} a_{ij}$ and $l_{ij} = -a_{ij}$. Suppose that there are N follower agents and M leader agents in the considered MASs. For simplicity, we assume that agents indexed by $1, \dots, N$ are the followers, and agents indexed by $N + 1, \dots, N + M$ are the leaders. In practice, the desired trajectories of followers are generated by leaders. Hence, we assume that each leader is not influenced by other agents, that is, $a_{ij} = 0, i = N + 1, \dots, N + M$. For brevity, we use $\mathcal{F} = \{1, \dots, N\}$ and $\mathcal{L} = \{N + 1, \dots, N + M\}$ to denote the follower set and leader set, respectively.

For the topology of MASs, we have the following assumption.

Assumption 1. For every follower agent $i \in \mathcal{F}$, there exists at least a leader agent $j \in \mathcal{L}$ that has a path to follower agent i .

Assumption 2. The induced subgraph of follower agents is an undirected graph.

Since each leader agent is not influenced by other agents, the associated Laplacian matrix can be partitioned as follows:

$$L = \begin{bmatrix} L_1 & L_2 \\ 0_{M \times N} & 0_{M \times M} \end{bmatrix}, L_1 \in \mathbb{R}^{N \times N}, L_2 \in \mathbb{R}^{N \times M}. \quad (1)$$

Lemma 1 (see [19]). *If Assumptions 1 and 2 hold, L_1 is positive definite, each entry of $-L_1^{-1}L_2$ is nonnegative and the sum of each row of $-L_1^{-1}L_2$ equals one.*

Definition 1 (see [32]). A set $\mathcal{C} \subseteq \mathbb{R}^n$ is convex if $(1 - \lambda)x + \lambda y \in \mathcal{C}$, for any $x, y \in \mathcal{C}$ and any $\lambda \in [0, 1]$. The convex hull $\text{Co}(X)$ of a finite set of points $X = \{x_1, x_2, \dots, x_q\}$ is the minimal convex set containing all points in X . That is, $\text{Co}(X) = \{\sum_{i=1}^q \lambda_i x_i \mid x_i \in X, \lambda_i \geq 0, \sum_{i=1}^q \lambda_i = 1\}$.

2.2. Problem Statement. In this paper, we study event-triggered containment control problem of uncertain MASs without using any global topology information.

Consider a network system of $N + M$ agents with the following dynamics:

$$\dot{x}_i(t) = Ax_i(t) + B[f_i(x_i(t)) + u_i(t)], \quad i \in \mathcal{F}, \quad (2)$$

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i \in \mathcal{L}, \quad (3)$$

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $x_i(t) \in \mathbb{R}^n$, and $u_i(t) \in \mathbb{R}^m$ are the system matrix, input matrix, state vector, and control input of agent i , respectively; input nonlinearity $f_i(x_i(t))$ is an unknown smooth function.

Based on the fact that $f_i(x_i(t))$ is smooth and Stone-Weierstrass approximation theory [33], the unknown nonlinear function $f_i(x_i(t))$ can be approximated by

$$f_i(x_i(t)) = W_i^T \phi_i(x_i(t)) + \varepsilon_i, \quad \forall x_i(t) \in \Pi, \quad (4)$$

with $\phi_i(\cdot): \mathbb{R}^n \rightarrow \mathbb{R}^s$ being a known basis function, $W_i \in \mathbb{R}^{s \times m}$ being the neural network (NN) weight matrix, $\Pi \subset \mathbb{R}^n$ being a compact set, and $\varepsilon_i \in \mathbb{R}^m$ being the approximation error such that

$$\|\varepsilon_i\|_\infty \leq \varepsilon_M, \quad \forall i \in \mathcal{F}. \quad (5)$$

Suppose that both $\max_{x_i(t) \in \Pi} \{\|\phi_i(x_i(t))\|\}$ and $\|W_i\|_F$ are bounded, i.e., there exist nonnegative numbers $\phi_M, W_M \geq 0$ such that

$$\max_{x_i(t) \in \Pi} \|\phi_i(x_i(t))\| \leq \phi_M, \|W_i\|_F \leq W_M, i \in \mathcal{F}. \quad (6)$$

For the control input of each leader, we assume that $u_i(t)$ satisfies the following assumption, $i \in \mathcal{L}$.

Assumption 3. $u_i(t)$ is bounded by a nonnegative constant τ , that is, $\|u_i(t)\| \leq \tau, \forall i \in \mathcal{L}$.

The control objective of MASs (2) and (3) is to design event-triggered controllers $u_i, i \in \mathcal{F}$ for follower agents such that the states of follower agents enter into the convex hull spanned by leaders' states, that is, $\lim_{t \rightarrow \infty} \text{dist}(x_i(t), \text{Co}(\mathcal{L})) = 0$, where $\text{Co}(\mathcal{L}) = \text{Co}(x_{N+1}(t), \dots, x_{N+M}(t))$.

3. Adaptive Event-Triggered Containment Algorithm

In this section, we will give an adaptive event-triggered algorithm to solve the containment problem of MASs (2) and (3).

Let

$$\begin{aligned} x_f(t) &= [x_1^T(t), \dots, x_N^T(t)]^T, x_l(t) = [x_{N+1}^T(t), \dots, x_{N+M}^T(t)]^T, \\ \gamma(t) &= [\gamma_1^T(t), \dots, \gamma_N^T(t)]^T = -(L_1^{-1}L_2 \otimes I_n)x_l(t). \end{aligned} \quad (7)$$

From Lemma 1, we know that $\gamma_i(t), i \in \mathcal{F}$, is a convex combination of states of leader agents. If follower agent i can track $\gamma_i(t)$, then $x_i(t)$ enters into $Co(\mathcal{L})$, $i \in \mathcal{F}$. We call $\gamma_i(t)$ the target state trajectory of follower agent $i, i \in \mathcal{F}$.

Denote $\delta_i(t) = x_i(t) - \gamma_i(t)$ as the tracking error of follower agent i and $\delta(t) = [\delta_1^T(t), \dots, \delta_N^T(t)]^T$. Then, we have

$$\begin{aligned} \dot{\delta}(t) &= \dot{x}_f(t) + (L_1^{-1}L_2 \otimes I_n)\dot{x}_l(t) \\ &= (I_N \otimes A)x_f(t) + (I_N \otimes B)(F(x_f(t)) + u_f(t)) \\ &\quad + (L_1^{-1}L_2 \otimes I_n)[(I_M \otimes A)x_l(t) + (I_M \otimes B)u_l(t)] \\ &= (I_N \otimes A)\delta(t) + (I_N \otimes B)(F(x_f(t)) + u_f(t)) \\ &\quad + (L_1^{-1}L_2 \otimes B)u_l(t), \end{aligned} \quad (8)$$

where

$$\begin{aligned} F(x_f(t)) &= [f_1^T(x_1(t)), \dots, f_N^T(x_N(t))]^T, \\ u_f(t) &= [u_1^T(t), \dots, u_N^T(t)]^T, \\ u_l(t) &= [u_{N+1}^T(t), \dots, u_{N+M}^T(t)]^T. \end{aligned} \quad (9)$$

Let $\xi_i(t) = \sum_{j=1}^{N+M} a_{ij}(x_i(t) - x_j(t))$ be the local relative state of follower agent $i, i \in \mathcal{F}$. $\xi_i(t_k^i)$ denotes the sample value of $\xi_i(t)$ at sample instant t_k^i , and $e_i(t)$ denotes sample error of $\xi_i(t_k^i)$, i.e.,

$$e_i(t) = \xi_i(t_k^i) - \xi_i(t), t \in [t_k^i, t_{k+1}^i), i \in \mathcal{F}. \quad (10)$$

By using $\xi_i(t_k^i)$, we give the following dynamical containment controller for each follower agent:

$$\begin{aligned} u_i(t) &= -c_i B^T P \xi_i(t_k^i) - \widehat{W}_i^T \phi_i(x_i(t)) - \alpha h(B^T P \xi_i(t_k^i)), \\ \dot{c}_i(t) &= \frac{1}{\eta} \left(\left(1 - \frac{\beta_1}{2} \right) \xi_i^T(t_k^i) P B B^T P \xi_i(t_k^i) - \sigma(c_i(t) - 1) \right), \\ \dot{\widehat{W}}_i(t) &= \phi_i(x_i(t)) \xi_i^T(t_k^i) P B - \rho \widehat{W}_i(t), \quad t \in [t_k^i, t_{k+1}^i), i \in \mathcal{F}, \end{aligned} \quad (11)$$

where $c_i(t)$ is the dynamical gain of $u_i(t)$ with $c_i(0) \geq 1$, $\alpha \geq \varepsilon_M + \tau$ is a positive constant, $\beta_1 \in (0, 2)$ is a constant, $\widehat{W}_i(t)$ is the dynamical estimation of W_i with $\widehat{W}_i(0) = 0$, $\sigma > 0$ and $\rho > 0$ are the turning scalars, and $P > 0$ is the unique solution of the following Riccati equation:

$$PA + A^T P - P B B^T P + I_n = 0, \quad (12)$$

and $h_i(\cdot)$ is a discontinuous function such that, for $z \in \mathbb{R}^m$,

$$h(z) = \begin{cases} \frac{z}{\|z\|}, & \text{if } \|z\| \neq 0, \\ 0, & \text{if } \|z\| = 0. \end{cases} \quad (13)$$

The trigger function for adaptive controller (11) is given as

$$f_i(t) = k_{i1} e_i^T(t) P B B^T P e_i(t) - k_{i2} \xi_i^T(t_k^i) P B B^T P \xi_i(t_k^i) - r, \quad (14)$$

where

$$\begin{aligned} k_{i1} &= \mu(c_i(t) + \phi_M + \alpha) + \frac{(2 - \beta_1)(1 - \beta_4)\beta_5}{2\beta_4}, \quad r > 0 \\ k_{i2} &= \frac{(2 - \beta_1)(1 - \beta_5)}{2}, \quad \mu > 0, \beta_4 \in (0, 1), \beta_5 \in (0, 1). \end{aligned} \quad (15)$$

Remark 1. Usually, the feedback gain of a consensus controller depends on the eigenvalues of the Laplacian matrix, which are the global topology information. Since adaptive gains $c_i(t)$ can be adjusted by $\xi_i(t_k^i)$ automatically, adaptive controller (11) does not rely on any global topology information.

Remark 2. Trigger function (14) is a hybrid trigger function. Unlike trigger functions in other papers, we use a positive constant r to replace the exponential decay term. The exponential decay term only guarantees that there is no Zeno behavior for finite time t . By choosing a relative small r , a satisfactory tracking error $\delta(t)$ and a lower bound of $t_{k+1}^i - t_k^i$ can be guaranteed.

Remark 3. In the design of dynamical controller (11), we use the σ -modification technique. Containment controller (11) and trigger function (14) only guarantee that tracking error $\delta(t)$ is uniformly ultimately bounded. If we do not employ the σ -modification technique, the dynamical gains $c_i(t)$ and estimation parameters $\widehat{W}_i(t)$ may increase unboundedly. To ensure the boundedness of $c_i(t)$ and $\widehat{W}_i(t)$, we use σ -modification technique to adjust the dynamical parameters.

Denote

$$\begin{aligned} \xi(t) &= [\xi_1^T(t), \dots, \xi_N^T(t)]^T, \xi(t_k) = [\xi_1^T(t_k^1), \dots, \xi_N^T(t_k^N)]^T, \\ c(t) &= \text{diag}\{c_1(t), \dots, c_N(t)\}, \varepsilon = [\varepsilon_1^T, \dots, \varepsilon_N^T]^T, \\ \widetilde{W}_i(t) &= W_i - \widehat{W}_i(t), \widetilde{W} = \text{diag}\{\widetilde{W}_1(t), \dots, \widetilde{W}_N(t)\}, \\ \phi(x_f(t)) &= [\phi_1^T(x_1(t)), \dots, \phi_N^T(x_N(t))]^T, \\ H(\xi(t_k)) &= [h^T(B^T P \xi_1(t_k^1)), \dots, h^T(B^T P \xi_N(t_k^N))]^T. \end{aligned} \quad (16)$$

Then, combining (4), (8), and (11), we have, for $t \in [t_k^i, t_{k+1}^i)$,

$$\begin{aligned} \dot{\delta}(t) = & (I_N \otimes A)\delta(t) - (c(t) \otimes BB^T P)\xi(t_k) \\ & + (I_N \otimes B)\left(\tilde{W}^T(t)\phi(x_f(t)) + (I_N \otimes B)\varepsilon\right. \\ & \left. - \alpha(I_N \otimes B)H(\xi(t_k))\right) - (-L_1^{-1}L_2 \otimes B)u_l(t). \end{aligned} \quad (17)$$

Theorem 1. Suppose that Assumptions 1 and 2 hold, containment problem of MASs (2) and (3) will be solved by dynamical controller (11). The tracking errors $\delta_i(t)$ and adaptive parameters $c_i(t)$ and $\tilde{W}_i(t)$ are uniformly ultimately bounded (UUB). Furthermore, controller (11) does not exist Zeno behavior.

Proof. Take Lyapunov function candidate as

$$\begin{aligned} V = & \frac{1}{2}\delta^T(t)(L_1 \otimes P)\delta(t) + \sum_{i=1}^N \frac{\eta_i^2}{2} \tilde{c}_i^2(t) \\ & + \sum_{i=1}^N \frac{1}{2} \text{tr}\left\{\tilde{W}_i^T(t)\tilde{W}_i(t)\right\}, \end{aligned} \quad (18)$$

where $\tilde{c}_i(t) = c_i(t) - \bar{c}$ with $\bar{c}$ being a constant to be determined later.

Along with (11 and 17), we compute $\dot{V}$ as follows:

$$\begin{aligned} \dot{V} = & \delta^T(t)(L_1 \otimes PA)\delta(t) - \delta^T(t)(L_1 c(t) \otimes PBB^T P)\xi(t_k) \\ & + \delta^T(t)(L_1 \otimes PB)\tilde{W}^T(t)\phi(x_f(t)) + \delta^T(t)(L_1 \otimes PB)\varepsilon \\ & - \alpha\delta^T(t)(L_1 \otimes PB)H(\xi(t_k)) + \sum_{i=1}^N \eta_i \tilde{c}_i(t) \dot{\tilde{c}}_i(t) \\ & - \delta^T(t)(-L_1 L_1^{-1} L_2 \otimes PB)u_l(t) + \sum_{i=1}^N \text{tr}\left\{\tilde{W}_i^T(t)\dot{\tilde{W}}_i(t)\right\}. \end{aligned} \quad (19)$$

From the definition of $\xi_i(t)$, one obtains

$$\begin{aligned} \xi(t) = & (L_1 \otimes I_n)x_f(t) + (L_2 \otimes I_n)x_l(t) \\ = & (L_1 \otimes I_n)(x_f(t) - (-L_1^{-1}L_2 \otimes I_n)x_l(t)) \\ = & (L_1 \otimes I_n)\delta(t). \end{aligned} \quad (20)$$

Hence, we get

$$\begin{aligned} \dot{V} = & \delta^T(t)(L_1 \otimes PA)\delta(t) - \xi^T(t)(c(t) \otimes PBB^T P)\xi(t_k) \\ & + \xi^T(t)(I_N \otimes PB)\tilde{W}^T(t)\phi(x_f(t)) + \xi^T(t)(I_N \otimes PB) \\ & \times \varepsilon - \alpha\xi^T(t)(I_N \otimes PB)H(\xi(t_k)) + \sum_{i=1}^N \eta_i \tilde{c}_i(t) \dot{\tilde{c}}_i(t) - \xi^T(t) \\ & \cdot (-L_1^{-1}L_2 \otimes PB)u_l(t) + \sum_{i=1}^N \text{tr}\left\{\tilde{W}_i^T(t)\dot{\tilde{W}}_i(t)\right\} \\ & - \sum_{i=1}^N \alpha \xi_i^T(t)PBh(B^T P \xi_i(t_k^i)) - \sum_{i=1}^N \xi_i^T(t)PBu_{il}(t) \end{aligned}$$

$$\begin{aligned} = & \delta^T(t)(L_1 \otimes PA)\delta(t) - \xi^T(t)(c(t) \otimes PBB^T P)\xi(t_k) \\ & + \sum_{i=1}^N \xi_i^T(t)PB\tilde{W}_i^T(t)\phi_i(x_i(t)) + \sum_{i=1}^N \xi_i^T(t)PB\varepsilon_i \\ & + \sum_{i=1}^N \eta_i \tilde{c}_i(t) \dot{\tilde{c}}_i(t) + \sum_{i=1}^N \text{tr}\left\{\tilde{W}_i^T(t)\dot{\tilde{W}}_i(t)\right\}, \end{aligned} \quad (21)$$

where $u_{il}(t)$ are convex combinations of leaders' inputs with $[u_{1l}^T(t), \dots, u_{Nl}^T(t)]^T = (-L_1^{-1}L_2 \otimes I_m)u_l(t)$. Since the sum of each row of $-L_1^{-1}L_2$ is equal to 1, one has

$$\|u_{il}(t)\| \leq \tau, \quad i \in \mathcal{F}. \quad (22)$$

Denote $e(t) = [e_1^T(t), \dots, e_N^T(t)]^T$. Since $\xi(t) = \xi(t_k) - e(t)$, one gets

$$\begin{aligned} & - \xi^T(t)(c(t) \otimes PBB^T P)\xi(t_k) \\ = & - \xi^T(t_k)(c(t) \otimes PBB^T P)\xi(t_k) \\ & + e^T(t)(c(t) \otimes PBB^T P)\xi(t_k) \\ \leq & - \left(1 - \frac{\beta_1}{2}\right) \xi^T(t_k)(c(t) \otimes PBB^T P)\xi(t_k) \\ & + \sum_{i=1}^N \frac{c_i(t)}{2\beta_1} e_i^T(t)PBB^T P e_i(t), \end{aligned} \quad (23)$$

where $\beta_1 \in (0, 2)$.

For the third term of $\dot{V}$ (21), one has

$$\begin{aligned} & \xi_i^T(t)PB\tilde{W}_i^T(t)\phi_i(x_i(t)) \\ = & \xi_i^T(t_k^i)PB\tilde{W}_i^T(t)\phi_i(x_i(t)) - e_i^T(t)PB\tilde{W}_i^T(t)\phi_i(x_i(t)). \end{aligned} \quad (24)$$

Note that

$$\xi_i^T(t_k^i)PB\tilde{W}_i^T(t)\phi_i(x_i(t)) = \text{tr}\left\{\tilde{W}_i^T(t)\phi_i(x_i(t))\xi_i^T(t_k^i)PB\right\}, \quad (25)$$

$$\begin{aligned} & - e_i^T(t)PB\tilde{W}_i^T(t)\phi_i(x_i(t)) \\ = & - \text{tr}\left\{\tilde{W}_i^T(t)\phi_i(x_i(t))e_i^T(t)PB\right\} \\ \leq & \phi_M \|\tilde{W}_i(t)\|_F \|B^T P e_i(t)\| \\ \leq & \frac{\beta_2 \phi_M}{2} \|\tilde{W}_i(t)\|_F^2 + \frac{\phi_M}{2\beta_2} e_i^T(t)PBB^T P e_i(t), \end{aligned} \quad (26)$$

where $\beta_2 \in (0, \rho/2\phi_M)$.

For the fourth, fifth, and sixth terms, one obtains

$$\begin{aligned}
\xi_i^T(t)PB\varepsilon_i &= \xi_i^T(t_k^i)PB\varepsilon_i - e_i^T(t)PB\varepsilon_i \\
&\leq \varepsilon_M \|B^T P \xi_i(t_k^i)\| + \varepsilon_M \|B^T P e_i(t)\|, \\
&\quad - \alpha \xi_i^T(t)PBh(B^T P \xi_i(t_k^i)) \\
&\leq -\alpha \|B^T P \xi_i(t_k^i)\| + \alpha \|B^T P e_i(t)\|,
\end{aligned} \tag{27}$$

$$-\xi_i^T(t)PBu_{il}(t) \leq \tau \|B^T P \xi_i(t_k^i)\| + \tau \|B^T P e_i(t)\|. \tag{28}$$

Hence, we have

$$\begin{aligned}
&\sum_{i=1}^N (\xi_i^T(t)PB\varepsilon_i - \alpha \xi_i^T(t)PBh(B^T P \xi_i(t_k^i)) - \xi_i^T(t)PBu_{il}(t)) \\
&\leq \sum_{i=1}^N 2\alpha \|B^T P e_i(t)\| \leq \sum_{i=1}^N \left(\alpha \beta_3 + \frac{\alpha}{\beta_3} e_i^T(t)PBB^T P e_i(t) \right),
\end{aligned} \tag{29}$$

where $\beta_3 > 0$ is a real constant. From (11), we have

$$\begin{aligned}
&\sum_{i=1}^N \eta \tilde{c}_i(t) \dot{c}_i(t) \\
&= \left(1 - \frac{\beta_1}{2}\right) \xi^T(t_k) (c(t) \otimes PBB^T P) \xi(t_k) \\
&\quad - \bar{c} \left(1 - \frac{\beta_1}{2}\right) \xi^T(t_k) (I_N \otimes PBB^T P) \xi(t_k) \\
&\quad - \sum_{i=1}^N \sigma \tilde{c}_i(t) (c_i(t) - 1), \\
&\text{tr} \left\{ \tilde{W}_i^T(t) \dot{\tilde{W}}_i(t) \right\} \\
&= -\text{tr} \left\{ \tilde{W}_i^T(t) \phi_i(x_i(t)) \xi_i^T(t_k^i) PB \right\} + \rho \text{tr} \left\{ \tilde{W}_i^T(t) \tilde{W}_i(t) \right\}.
\end{aligned} \tag{30}$$

Substituting (23)–(31) into (21), we obtain

$$\begin{aligned}
\dot{V} &\leq \delta^T(t) (L_1 \otimes PA) \delta(t) + \sum_{i=1}^N \frac{\beta_2 \phi_M}{2} \|\tilde{W}_i(t)\|_F^2 \\
&\quad - \bar{c} \left(1 - \frac{\beta_1}{2}\right) \xi^T(t_k) (I_N \otimes PBB^T P) \xi(t_k) \\
&\quad - \sum_{i=1}^N \sigma \tilde{c}_i(t) (c_i(t) - 1) + \sum_{i=1}^N \rho \text{tr} \left\{ \tilde{W}_i^T(t) \tilde{W}_i(t) \right\} \\
&\quad + \sum_{i=1}^N \left(\frac{c_i(t)}{2\beta_1} + \frac{\phi_M}{2\beta_2} + \frac{\alpha}{\beta_3} \right) e_i^T(t)PBB^T P e_i(t) + N\alpha\beta_3.
\end{aligned} \tag{32}$$

Note that

$$\begin{aligned}
&-\xi^T(t_k) (I_N \otimes PBB^T P) \xi(t_k) \\
&= -\xi^T(t) (I_N \otimes PBB^T P) \xi(t) - 2\xi^T(t) (I_N \otimes PBB^T P) e(t) \\
&\quad - e^T(t) (I_N \otimes PBB^T P) e(t),
\end{aligned} \tag{33}$$

$$\begin{aligned}
&-2\xi^T(t) (I_N \otimes PBB^T P) e(t) \\
&\leq \beta_4 \xi^T(t) (I_N \otimes PBB^T P) \xi(t) + \sum_{i=1}^N \frac{1}{\beta_4} e_i^T(t)PBB^T P e_i(t),
\end{aligned} \tag{34}$$

where $\beta_4 \in (0, 1)$. Since $\xi(t) = (L_1 \otimes I_n) \delta(t)$ and L_1 is positive definite, we have the following inequalities:

$$\begin{aligned}
&\xi^T(t) (I_N \otimes PBB^T P) \xi(t) = \delta^T(t) (L_1^2 \otimes PBB^T P) \delta(t), \\
&\geq \frac{\lambda_{L_1}^2}{\lambda_{L_1}} \delta^T(t) (L_1 \otimes PBB^T P) \delta(t),
\end{aligned} \tag{35}$$

where λ_{L_1} and $\bar{\lambda}_{L_1}$ are the smallest and largest eigenvalues of L_1 , respectively.

Combining (32)–(35), one has

$$\begin{aligned}
\dot{V} &\leq \delta^T(t) (L_1 \otimes PA) \delta(t) + \sum_{i=1}^N \frac{\beta_2 \phi_M}{2} \|\tilde{W}_i(t)\|_F^2 \\
&\quad - \bar{c} \left(1 - \frac{\beta_1}{2}\right) (1 - \beta_4) \beta_5 \frac{\lambda_{L_1}^2}{\lambda_{L_1}} \delta^T(t) (L_1 \otimes PBB^T P) \\
&\quad \times \delta(t) - \sum_{i=1}^N \sigma \tilde{c}_i(t) (c_i(t) - 1) + \sum_{i=1}^N \rho \text{tr} \left\{ \tilde{W}_i^T(t) \tilde{W}_i(t) \right\} \\
&\quad + \sum_{i=1}^N \left(\frac{c_i(t)}{2\beta_1} + \frac{\phi_M}{2\beta_2} + \frac{\alpha}{\beta_3} + \bar{c} \frac{(2 - \beta_1)}{2} \frac{(1 - \beta_4)}{\beta_4} \beta_5 \right) \\
&\quad \times e_i^T(t)PBB^T P e_i(t) - \sum_{i=1}^N \bar{c} \left(1 - \frac{\beta_1}{2}\right) (1 - \beta_5) \\
&\quad \times \xi_i^T(t_k^i)PBB^T P \xi_i(t_k^i) + N\alpha\beta_3,
\end{aligned} \tag{36}$$

where $\beta_5 \in (0, 1)$ is a constant.

Choosing enough large $\bar{c}$ such that

$$\begin{aligned}
\bar{c} &\geq \frac{\bar{\lambda}_{L_1}}{(2 - \beta_1)(1 - \beta_4)\beta_5\lambda_{L_1}^2}, \\
\bar{c} &\geq \frac{1}{2\beta_1\mu}, \bar{c} \geq \frac{1}{2\beta_2\mu}, \bar{c} \geq \frac{1}{\beta_3\mu}.
\end{aligned} \tag{37}$$

Then, Riccati equation (12) and trigger function (14) guarantee that

$$\begin{aligned} \dot{V} \leq & -\frac{1}{2}\delta^T(t)(L_1 \otimes I_n)\delta(t) + \sum_{i=1}^N \frac{\beta_2 \phi_M}{2} \|\tilde{W}_i(t)\|_F^2 + Nr \\ & - \sum_{i=1}^N \sigma \tilde{c}_i(t)(c_i(t) - 1) + \sum_{i=1}^N \rho \text{tr} \left\{ \tilde{W}_i^T(t) \tilde{W}_i(t) \right\} + N\alpha\beta_3. \end{aligned} \quad (38)$$

Since $c_i(t) = \tilde{c}_i(t) + \bar{c}$ and $\tilde{W}_i(t) = W_i - \tilde{W}_i(t)$, one gets

$$\begin{aligned} -\sigma \tilde{c}_i(t)(c_i(t) - 1) &= -\sigma \tilde{c}_i(t)(\tilde{c}_i(t) + \bar{c} - 1) \\ &= -\sigma \tilde{c}_i^2(t) - \sigma \tilde{c}_i(t)(\bar{c} - 1) \\ &\leq -\frac{\sigma}{2} \tilde{c}_i^2(t) + \frac{\sigma}{2}(\bar{c} - 1)^2, \end{aligned} \quad (39)$$

$$\begin{aligned} \text{tr} \left\{ \tilde{W}_i^T(t) \tilde{W}_i(t) \right\} &= \text{tr} \left\{ \tilde{W}_i^T(t) (W_i - \tilde{W}_i(t)) \right\} \\ &\leq W_M \|\tilde{W}_i(t)\|_F - \|\tilde{W}_i(t)\|_F^2 \\ &\leq -\left(1 - \frac{\beta_6 W_M}{2}\right) \|\tilde{W}_i(t)\|_F^2 + \frac{W_M}{2\beta_6}, \end{aligned} \quad (40)$$

where $\beta_6 \in (0, 1/2W_M)$ is a constant.

Combining (38)–(40) yields

$$\begin{aligned} \dot{V} \leq & -\frac{1}{2}\delta^T(t)(L_1 \otimes I_n)\delta(t) - \sum_{i=1}^N \frac{\sigma}{2} \tilde{c}_i^2(t) \\ & - \sum_{i=1}^N \left(\rho \left(1 - \frac{\beta_6 W_M}{2}\right) - \frac{\beta_2 \phi_M}{2} \right) \|\tilde{W}_i(t)\|_F^2 + \varsigma, \end{aligned} \quad (41)$$

where $\varsigma = N\alpha\beta_3 + N\sigma/2(\bar{c} - 1)^2 + NW_M/2\beta_6 + Nr$. Since $\beta_2 \in (0, \rho/2\phi_M)$ and $\beta_6 \in (0, 1/2W_M)$, we get

$$\begin{aligned} \dot{V} \leq & -\frac{1}{2}\delta^T(t)(L_1 \otimes I_n)\delta(t) - \sum_{i=1}^N \frac{\sigma}{2} \tilde{c}_i^2(t) \\ & - \sum_{i=1}^N \frac{\rho}{2} \|\tilde{W}_i(t)\|_F^2 + \varsigma \\ \leq & -\kappa V + \varsigma, \end{aligned} \quad (42)$$

where $\kappa = \min\{1/\bar{\lambda}_p, \sigma/\eta, \rho\}$ with $\bar{\lambda}_p$ being the largest eigenvalue of P .

After simple calculations, we have

$$V(t) \leq \left(V(0) - \frac{\varsigma}{\kappa} \right) e^{-\kappa t} + \frac{\varsigma}{\kappa}, \quad (43)$$

which indicates that tracking error $\delta(t)$, dynamical gains $c_i(t)$, and $\tilde{W}_i(t)$ are UUB.

In the last step, we prove that event-triggered controller (11) does not exist Zeno behavior.

When $t = t_k^i$, sample error $e_i(t)$ will be set to zero. Dini derivative will be used to analyze $e_i(t)$, and we get

$$D^+ \|e_i(t)\| = \frac{d}{dt} \sqrt{e_i^T(t) e_i(t)} \leq \|\dot{e}_i(t)\|. \quad (44)$$

Since $e_i(t) = \xi_i(t_k^i) - \xi_i(t)$, for $t \in [t_k^i, t_{k+1}^i]$, we have

$$\begin{aligned} \dot{e}_i(t) &= -\dot{\xi}_i(t) = -A\xi_i(t) - \sum_{j=1}^N a_{ij}B[f_i(x_i(t)) - f_j(x_j(t))] - \sum_{j=1}^{N+M} a_{ij}(u_i(t) - u_j(t)) \\ &= Ae_i(t) - A\xi_i(t_k^i) - \sum_{j=1}^N a_{ij}B[f_i(x_i(t)) - f_j(x_j(t))] - \sum_{j=1}^{N+M} a_{ij}(u_i(t) - u_j(t)). \end{aligned} \quad (45)$$

Denote

$$\begin{aligned} \varphi_k^i &= \max_{t \in [t_k^i, t_{k+1}^i]} \left\| A\xi_i(t_k^i) + \sum_{j=1}^N a_{ij}B[f_i(x_i(t)) - f_j(x_j(t))] \right. \\ &\quad \left. + \sum_{j=1}^{N+M} a_{ij}(u_i(t) - u_j(t)) \right\|. \end{aligned} \quad (46)$$

And, one gets

$$D^+ \|e_i(t)\| \leq \|A\| \cdot \|e_i(t)\| + \varphi_k^i. \quad (47)$$

Notice that $e_i(t_k^i) = 0$. Hence, for $t \in [t_k^i, t_{k+1}^i]$, $\|e_i(t)\|$ satisfies following inequality:

$$\|e_i(t)\| \leq \frac{1}{\|A\|} \varphi_k^i \left(e^{\|A\|(t-t_k^i)} - 1 \right). \quad (48)$$

Let $\psi_k^i(t) = \sqrt{k_{i2}/k_{i1}} \xi_i^T(t_k^i) P B B^T P \xi_i(t_k^i) + r/k_{i1}$. From trigger function (14), we know that the $(k+1)$ th event of agent i will not be triggered unless $\|B^T P e_i(t)\|$ exceeds $\psi_k^i(t)$, i.e.,

$$\begin{aligned} \|B^T P e_i(t)\| &\leq \psi_k^i(t), \forall t \in [t_k^i, t_{k+1}^i], \\ \lim_{t \rightarrow t_{k+1}^i} \|B^T P e_i(t)\| &= \psi_k^i. \end{aligned} \quad (49)$$

Because $\|B^T P e_i(t)\| \leq \|B^T P\| \cdot \|e_i(t)\|$, one has

$$\lim_{t \rightarrow t_{k+1}^i} \|e_i(t)\| \geq \frac{\psi_k^i(t)}{\|B^T P\|}. \quad (50)$$

By (48) and (50), one obtains the interinterval satisfying

FIGURE 1: $x_{i1}(t)$ trajectories of each agent. FIGURE 2: $x_{i2}(t)$ trajectories of each agent.

$$t_{k+1}^i - t_k^i \geq \frac{1}{\|A\|} \ln \left(\frac{\|A\| \psi_k^i(t)}{\|B^T P\| \phi_k^i} + 1 \right). \quad (51)$$

Due to the existence of positive real $r > 0$, $\psi_k^i(t)$ is greater than zero for any given $t > 0$. That means $t_{k+1}^i - t_k^i$ is strictly positive.

4. Simulations

One numerical example is given in this section to verify the adaptive event-triggered containment controller (11) developed in this paper.

Consider a leader-follower multiagent system with four follower agents and two leader agents. The Laplacian matrix associated with topology is given as follows:

$$L = \begin{bmatrix} 2 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & -1 & 3 & 0 & -1 & 0 \\ -1 & 0 & 0 & 2 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (52)$$

The dynamics of leaders is depicted by the linearized model of the longitudinal dynamics of an aircraft [34], described by (3) with

$$A = \begin{bmatrix} -0.2770 & 1.0000 & -0.0002 \\ -17.1000 & -0.1780 & -12.2000 \\ 0 & 0 & -6.6700 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 6.67 \end{bmatrix}. \quad (53)$$

$x_i(t) = [x_{i1}(t), x_{i2}(t), x_{i3}(t)]^T$ denotes the state of each agent with $x_{i1}(t)$ being the angle of attack, $x_{i2}(t)$ being the pitch rate, and $x_{i3}(t)$ being the elevator angle. From

FIGURE 3: $x_{i3}(t)$ trajectories of each agent.

Stone-Weierstrass approximation theory, the approximation error ε_i can be arbitrarily selected. We set $\phi_i(x_i(t)) = [4x_{i1} \sin(x_{i1}(t)), 2 \cos(x_{i2}(t))]^T$ and $\varepsilon_M = 0.01$. The initial conditions of follower agents are given as $x_1(0) = [1, -1, 0]^T$, $x_2(0) = [2, 0, 1.5]^T$, $x_3(0) = [1.5, 0.5, -1.5]^T$, and $x_4(0) = [-0.9, -0.5, 1.5]^T$. We choose control inputs of leader agents as $u_5(t) = -0.5$ and $u_6(t) = 0.5$ and initial conditions as $x_5(0) = [-1, 0.1, 0.7]^T$ and $x_6(0) = [0.5, 0, -0.8]^T$.

We set the initial values of adaptive gains $c_i(t)$ and $\widehat{W}_i(t)$ as $c_i(0) = 1$ and $\widehat{W}_i(0) = [0, 0]^T$. By solving Riccati equation (12), we get $B^T P = [1.2566, -0.9116, 1.3097]^T$. According to

FIGURE 4: Triggering instants of each follower agent.

FIGURE 5: Adaptive gains $c_i(t)$. FIGURE 6: Norms of $\hat{W}_i(t)$.

controller design (11), we set $\beta_1 = 0.2, \beta_4 = 0.5, \beta_5 = 0.2, \eta = 1, \sigma = 0.2, \rho = 0.1, \mu = 0.2$, and $\alpha = 0.51$.

The simulation results are presented in Figures 1–6. Figures 1 and 3 show the trajectories of follower agents and leader agents. The black lines denote the trajectories of leader agents which indicate that the states of follower agents enter into the convex hull spanned by leaders' states. Figure 4 displays the triggering instants of each agent. Figure 5 displays the adaptive gains $c_i(t)$. Figure 6 presents the norms of estimation coefficients $\hat{W}_i(t)$.

5. Conclusions

In this paper, we have studied the event-triggered containment problem of a class of uncertain MASs. The global topology information is unavailable, and parameters of follower agents are uncertain. Based on adaptive control technique, we have proposed an event-triggered containment controller which does not rely on any global topology information and can estimate the uncertain parameters. Under this containment algorithm, follower agents enter into the convex hull of leader agents. Moreover, the proposed algorithm has no Zeno behavior. In the future, we will concentrate our study on event-triggered consensus problem of MAS with unknown control directions.

Data Availability

The simulation data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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