Research Article

Computing a Complex Network Hierarchical Structure for Financial Market Networks on the Basis of the Hybrid Heuristic Algorithm

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The recent empirical studies showed that money center networks in interbank markets are more robust and stable. Therefore, the research on layered financial networks is a key part of the systemic risk management. Various methods have been proposed in prior studies to find optimal partitioning of interbank networks into core and periphery subsets. However, these methods that have been adopted with approximation methods, in general, do not guarantee optimal bipartition. In this paper, a genetic simulated annealing algorithm is presented to detect a hierarchical structure in interbank networks as a hybrid heuristic algorithm, while its effects are also analyzed. The optimization of the error score for the core-periphery model is mathematically developed firstly as an improved expression of the optimization function, which incorporates the genetic algorithm into a simulated annealing algorithm to guarantee the optimal bipartition and to jump from a local optimization. The results of this algorithm are finally verified by empirical analysis of interbank networks; and, through the immunity strategy under the risk diffusion model, the significance of core-periphery structure to risk management is verified.

1. Introduction

In recent years, the financial crisis has shown that the threat of systemic risks takes many forms and is highly dynamic. The risk contagion was caused by the bankruptcy of US investment bank Lehman Brothers in September, 2008; starting in December 2009, the Greek sovereign debt crisis within the euro zone spread; COVID-19 may also result in widespread market earthquake. Therefore, the studies on financial systemic risks have received great attention over the past decade.

Traditionally, the interbank markets have been analyzed without taking the network structure into account. Meanwhile, as research continues, a number of studies have realized that the structure of interbank market network has a huge impact on systematic risk. Fabio Caccioli et al. [1] have summarized the works about the core-periphery structure in interbank networks. Ramazan Gençay et al.'s [2] study has

shown that the endogenous core-periphery structure is best suited to alleviate the spread of the financial crisis but focuses systemic risks on the core of banks. The research of Grigorios Asimakopoulos et al. [3] showed that external shocks are easier to deal with pressure on core banks. In any case, core banks outperformed peripheral banks throughout the period.

Besides, empirical analysis has illustrated prominent characteristics of an interbank network, which has a complex structure with multiple centers and the law of degree distribution. Nier et al. [4] investigate how systemic risk is affected by the structure of the financial system and implies that banking systems show a high degree of tiering, which contain many properties, some of which are modular [5–7] and core-periphery structures [8]. Recently, researchers have begun to research interbank networks via multitiered networks. In a groundbreaking study, Ben Craig and Goetz von Peter [9] demonstrated that interbank markets are multitier

instead of single-layer, because transactions between banks were not direct but through the money central bank via intermediator. Daan in't Veld, Iman van Lelyveld [10] figured out the transition matrix between the states of being in the core and in the periphery, comparing the core-periphery model (CPM) for the Dutch interbank market to Germany (Craig and von Peter [9]). This paper discusses interbank network layering through a core-periphery model (CPM), which has maintained wide recognition in interbank network analysis methods for decades.

The continuous developments of CPM have started by Borgatti and Everett [11] who sought to formalize the intuitive notion of a core-periphery structure and design algorithms for detecting this structure. They offered discrete formulation of core-periphery structures as an architectural layering issue and demonstrated that a genetic algorithm (GA) would be able to fit a CPM. They concluded that GA is a convenient method, though perhaps it is not the fastest. Then, Craig and von Peter [9] relied on a greedy algorithm to find an optimal subset that could switch the proportions of core/periphery nodes in order to minimize errors. Furthermore, Brusco [12] developed an alternative methodological proposal with branch-and-bound methods. Boyd et al. [13] compared four algorithms, namely, exhaustive search, differential evolution, simulated annealing, and the Kernighan–Lin (KL) algorithm, to detect core-periphery structures. In this paper, we developed a CPM error score as the objective function for the algorithm to detect layers of an interbank network. Moreover, we have put forward an improved genetic simulated annealing algorithm (GSA) to optimize the error score and test algorithm performance; further details and conclusions are illustrated below. Finally, this paper completes the empirical analysis based on the European banking network. The cascading default mechanism of the Gai–Kapadia model [14] combined with CPM is used to predict risk contagion, and the most effective risk immunity strategy has been given.

2. Methods

2.1. Core-Periphery Model (CPM).
From the perspective of social networks, the CPM is not formally defined. The continuous development of CPM began with Borgatti and Everett [11] who tried to formalize the intuitive concept of the core-periphery structure and the design algorithm used to detect the structure. They provide discrete and continuous representations of the core-periphery structure as an architectural layering problem and demonstrated that a genetic algorithm (GA) would be able to fit a CPM. They concluded that GA is a convenient method, although perhaps it is not the fastest. In 2014, the concept was applied to interbank networks by Craig and von Peter [8]. To capture a structural characteristic of the interbank network, they divided each bank into core or periphery and relied on a greedy algorithm to find an optimal subset that could switch the proportions of core/periphery nodes in order to minimize errors. This was illustrated by a system of interbank credit relations, which helped to summarize a series of definitions about interbank network structures. Veld and

van Lelyveld [10] acknowledged that the formal definition of CPM by Craig and von Peter was more detailed than that of Borgatti and Everett.

After the overview of an ideal core-periphery structure, we define the difference between the core and the peripheral nodes as follows: the core and the peripheral nodes are as follows; the core forms a subgraph of the entire network, where the nodes are closely connected to each other. Peripheral nodes are connected to core nodes but not to other peripheral nodes. Therefore, the definition of the ideal core-periphery structure is outlined in Figure 1.

In general, a core-periphery structure of interbank networks consists of four blocks, as shown in equation (1). Block CC (core-core) specifies how core banks are related to each other. When they all lend each other according to Condition 1, CC must be an all-one matrix, which indicates the presence of interbank linkages. With regard to any banks in the periphery not lending each other, PP must be a square-zero matrix. Core banks lend or borrow from periphery banks at least once. Namely, we make sure that CC = 1 and PP = 0; the CP or PC block must be a non-empty-square matrix in the ideal core-periphery structure. So far, ideal CPM in interbank networks stylizes M as per the following equation:

$$M = \begin{pmatrix} CC & CP \\ PC & PP \end{pmatrix} = \begin{pmatrix} 1 & \text{no empty} \\ \text{no empty} & 0 \end{pmatrix}. \quad (1)$$

2.2. Detecting Core-Periphery Structures in Interbank Networks.
A formal definition of the ideal CPM in interbank networks is already obtained. But the tiering network should be optimized from ideal theory and verified to be practical. In order to evaluate the applicability of the model, we calculated the error score of the definition, namely, minimizing the number of errors for any chosen set of core banks; errors with respect to the ideal CPM can be calculated according to the definition. More specifically, error score e in the CC block is deemed to be the number of core banks that fail to borrow from (lend) other core banks. As an analogy to this definition, total error score e in four blocks can be normalized as a proportion of the total linkages in M:

$$e = \frac{e_{CC} + e_{PP} + e_{CP} + e_{PC}}{\sum_i \sum_j a_{ij}}, \quad (2)$$

where e_{CC}, e_{PP}, e_{CP}, and e_{PC} represent error score e in different blocks of M, respectively. The optimal structure of CP captures minimal total error score e; thus, we can transform the fit of the CPM to the optimization problem. An algorithm was developed from the research of Lip [15] to formulate this problem after the specific definitions for interbank networks and core-periphery tiering issues was made. An interbank network consists of banks and loan relationships that can be described as directed graph $G(V, E)$, banks (vertices) are expressed as V, and loan relationships (edges) are expressed as E. When i and j are vertices of $G(V, E)$, if an edge a_{ij} from i and j exists, it is said to connect i and j, and it denotes loan relationships i and j.
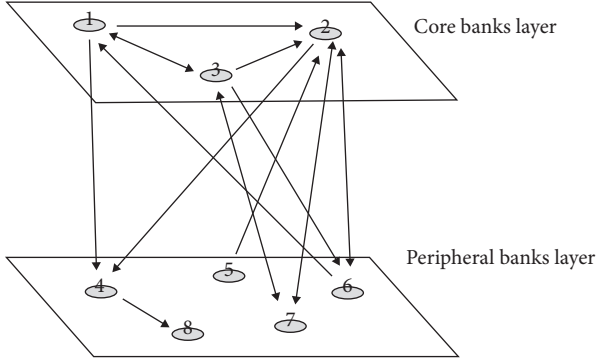
FIGURE 1: Example of an ideal core-periphery structure.

At this point, $a_{ij} = 1$; otherwise, $a_{ij} = 0$. Loan relationships to each other can be represented by an $n \times n$ adjacency matrix $\Re_{n \times n} = [a_{ij}]$, $1 \le i \neq j \le n$, where n is the number of banks in interbank network. The degree of a node i is the number of edges with other nodes and it can be denoted by $k(i) = \sum_{j \in V} a_{ij}$. Let x be the number of banks in s_C; we can write

$$e = \frac{e_{CC} + e_{PP} + e_{CP} + e_{PC}}{\sum_i \sum_j a_{ij}}$$

$$= \frac{\left(x(x-1) + \sum_{i \notin s_C} k(i) - \sum_{i \notin S} l_{s_C}(i) \right)}{\sum_i \sum_j a_{ij}}, \quad (3)$$

subject to $S = \{1, 2, \ldots, n\} = \{s_C, s_P\}$ and $1 \le i \neq j \le n$.

Therefore, the improved error score e is illustrated for interbank networks. Appendix 1 has a detailed mathematical derivation process. Using this result to research CPM is more effective than using the study by Ben and Peter. We then began to detect core-periphery structures by using GSA.

2.3. Using GSA to Detect Core-Periphery Structures.

In this section, we propose an improved genetic algorithm for solving CPM. In Section 1, we reviewed the previous studies on detecting core-periphery structures, which were somewhat flawed on optimal approaches. For example, exhaustive search or greedy algorithm is with slow convergence and low oscillation due to high time complexity. Although the Kernighan–Lin algorithm and branch-and-bound algorithm are much faster than other methods, switching the proportion of core/periphery nodes is not acceptable, where minimal total error score e cannot be obtained. On the other hand, it is easy for GA to quickly reach the optimization value, but it has a shortcoming: it is prone to be trapped in a local optimum; namely, there exists a high probability of premature convergence. Fortunately, the simulated annealing algorithm (SA) is an available tool to jump from local optimizations and provide a feasible scheme for searching for the best global solution [16]. We have to incorporate the GA into an SA because a GA ensures the rapid astringency of the algorithm, and the SA is utilized to seek a better solution that could be chosen. GSA for a core-periphery structure in interbank networks is described as below.

2.3.1. Genetic Algorithms (GA). Genetic algorithm (GA) mimics the principal of natural genetics and is used to solve search and optimization problems [17]. Genetic algorithms have iterative operations in order to calculate better candidate solutions (individuals) [18]. Working procedure of genetic algorithm is described as per the following: after initializing the population randomly or using knowledge, we gain a population of possible solutions to the given problem. The solutions then undergo recombination and mutation and the process is repeated over various generations. Each candidate solution is assigned a fitness value and the fittest will survive. We keep evolving better solutions over generations until the stopping conditions are satisfied.

In the context of CPM, we initialized genotypes in the chromosome by randomly arranging a number of n banks as core-periphery. In this method, encoding a chromosome can be written as

$$\text{ch} = [gt(1), gt(2), \ldots, gt(i), \ldots, gt(n)], 1 \le i \le n, \text{ let } gt(i)$$
$$= \{0 \text{ or } 1\}. \quad (4)$$

The research by Wang [19] suggested that a small population could not be comprehensive but a large one reduces the computation speed.

The design of fitness function is very essential in GA as the performance of the algorithm depends heavily upon the design of the fitness function. In this paper, the fitness function relative to the objective function of detecting core-periphery structures is total error score e. The next step is the selection phase which the fittest individuals will select and it will let them pass their genes to the next generation. Next variation operators are crossover and mutation to keep population diversity. This paper chose a roulette-wheel selection algorithm [20] to create a new population.

In the previous sections, we have introduced the policy of encoding a chromosome. Crossover in GA generates new generation the same as natural mutation on the basis of the encoding rule because genetic representation combines these characteristics by selection operator. Crossover is important because it creates offspring different from its parent. Mutation varies based on the chromosome representation and it updates some genes by adding new features. In fact, a mutation is enough to bring randomness to a genotype with small probability if the specified probability is more than a random number generated on an interval of 0 to 1 by using a uniformly distributed rule. In this paper, a two-point crossover was implemented with crossover probability p_1 and the mutation was executed with specified probability p_2.

2.3.2. Simulated Annealing Algorithm (SA). For a CPM, an SA can avoid falling into a local optimum by accepting a certain probability. The concrete principle of SA was concluded by Kirkpatrick et al. [21]. The SA starts from an initial solution at a high temperature and makes a number of changes

according to annealing schedules. For any two iterations, there are two objective values marked as F_t and F_{t-1}, and the difference between objective values $\Delta F = F_t - F_{t-1}$ is calculated at time t. If $\Delta F \leq 0$, then the new solution is accepted with probability $\rho = 1$. Otherwise, it is accepted with a small probability ρ, $\rho = \exp(-\Delta F/T)$, where T is the annealing temperature at t. As cooling proceeds to the set frozen point, the algorithm terminates. There are many articles in which SA parameter optimization has been deeply researched.

2.3.3. Genetic Simulated Annealing (GSA) Algorithm for CPM.

So far, we discussed both GA and SA; the proposed GSA [22] for CPM is illustrated in Figure 2, and the pseudocode of GSA is described in detail in Appendix 2. Research on the best parameters of GA for CPM is a more complex issue, and a number of previous studies summarized the related work as of today.

2.4. Immunization Strategy of Interbank Market Based on Core-Periphery Networks

2.4.1. Network Structure of Interbank Market.

An important application field of core-periphery structure is to study the risk immunity strategy of interbank market through hierarchical network [23]. A lot of researches have studied network models of default cascades, especially Gai-Kapadia cascade model which has been frequently used as a base model of financial contagion [24]. The important assumptions in the Gai-Kapadia model are as follows:

(1) Average distribution of interbank loans

(2) Interbank lending and borrowing according to an Erdos–Renyi random graph

(3) The risk of external assets being not taken into account

This paper generalized the Gai-Kapadia model in core-periphery structures, where banks lend and borrow from each other in different layers. The Gai-Kapadia model forms an interbank network based on directed random graphs and includes the bilateral loans matrix of claims and obligations and the balance sheet of each bank in the interbank market.

In the Gai-Kapadia model, the interbank bilateral exposure can be represented by an adjacency matrix $\mathfrak{R}_{n \times n}$, where each element of $\mathfrak{R}_{n \times n}$ is the total nominal claims u_{ij} from node i to j. When we calculate the adjacency matrix $\mathfrak{R}_{n \times n}$, we must consider the fact that a bank cannot have exposure to itself. Thus, the gross interbank loans of bank i are given by the matrix's row sum as $k_i = \sum_{i, j \in \mathbb{N}} u_{ij}$. In the same way, the gross interbank borrowing of bank i is given by the matrix's column sum as $b_i = \sum_{j, i \in \mathbb{N}} U_{ji}$.

According to the bilateral loans matrix of claims and obligations, suppose that each bank may have two types of elements in the balance sheet: assets A_i and liabilities L_i. Table 1 lists a stylized balance sheet of a bank that participates in the interbank market. The liabilities L_i side of bank $i \in \mathbb{N}$ consists of interbank borrowings b_i, equity q_i, and deposits d_i. On the other hand, the assets A_i side of bank
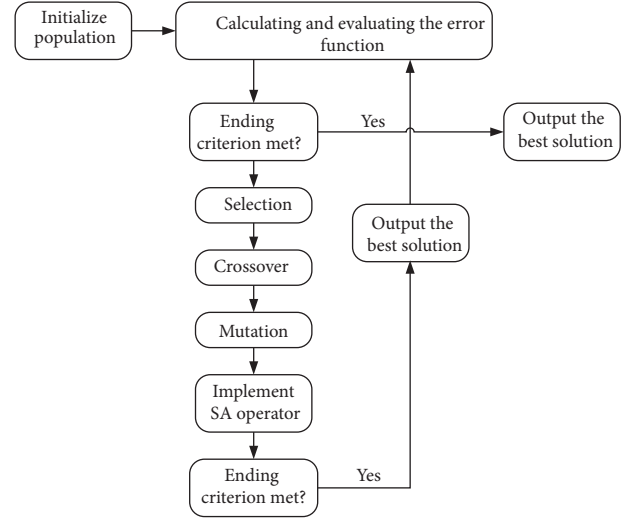

FIGURE 2: Flowchart of GSA.

TABLE 1: The stylized balance sheet of bank $i \in \mathbb{N}$.

Assets A_i	Liabilities L_i
Reserves $r_i = \theta_i k_i$	Equity q_i
External assets f_i	Deposits d_i
Interbank loans o_i	Interbank borrowing b_i
	Shock

$i \in \mathbb{N}$ consists of interbank loans o_i, external assets f_i, and capital reserves r_i. We define the capital reserves ratio as θ, $\theta_i = (r_i/k_i)$; based on the core-periphery structure of the interbank market, capital reserves of core and peripheral banks are represented as θ^C and θ^P. Equation (5) implies that total assets must be equal to total liabilities for each bank $i \in \mathbb{N}$ in the balance sheet:

$$r_i + f_i + o_i = q_i + d_i + b_i. \tag{5}$$

2.4.2. Interbank Market Cascading Default Mechanism.

To simulate the cascading default of the banking system, we trigger a cascading default in interbank network. When the whole banking system is suffering from a common shock [25], the systemic risks will spread via the interbank network, which is observed directly by the bilateral loans matrix. When systemic risks spread to each bank, the risk tolerance of each bank decides on whether the banking system is healthy or not. By analyzing banks' balance sheets, we can assess whether whole banks can absorb further shocks and maintain the system's financial health. If there are one or more banks that have failed, systemic risks will transit to the interbank networks again. Thus, the whole banking system has to share the consequences of the failures.

In this paper, the core-periphery structure of the interbank market is taken into account in interbank market cascading default mechanism. For simplicity, we assume that a common shock attacked the banking system initially, starting the risk contagion mechanism described above. When a bank $x \in \mathbb{N}$ has failed, it implies that its interbank

TABLE 2: Information of banking networks for earlier empirical studies.

Banking network	Sample date	Reference	Number of banks	Network density (%)	Average degree	Maximum degree	Clustering coefficient
Global	End of 2013	[27]	202	1.52	1.54	22	0.624
European	End of 2014	[27]	836	0.77	0.98	27	0.272
German	End of 2008	[26]	2182	0.41	0.54	19	0.095

TABLE 3: Detailed sittings and range of parameters for GSA.

Parameters	Default value	Min	Max	Path
Number of generations		100	1000	50
Size of population		50	500	50
Number of chromosomes	Number of the banks			
Crossover probability		0.1	1	0.1
Mutation probability		0.01	0.1	0.01
Maximum number of generations without improvement		10	50	10
Initial temperature		500	10000	500
Annealing parameter		0.9	0.99	0.01
Maximum iterations of simulated annealing	1000			

borrowing from other banks does not need to be paid back as follows: $b_x = \sum_{j \in \aleph} u_{jx}$ and $u_{ix} = 0, i \in \aleph$. Moreover, there are still interbank loans of bank $x \in \aleph$ to other banks. If the condition is $(u_{xi} - u_{ix}) > 0$, which is represented as $U'_{xi} = (u_{xi} - u_{ix})^+$, it shows that the interbank loans of bank $x \in \aleph$ from bank $i \in \aleph$ is more than the interbank borrowing of bank $i \in \aleph$ from bank $x \in \aleph$. If the condition is $(u_{xi} - u_{ix}) < 0$, which is represented as $U'_{xi} = (u_{xi} - u_{ix})^-$, it shows that the impact of the failure of bank $x \in \aleph$ is not able to be absorbed by the interbank market. Then, we check the balance sheet of bank $i \in \aleph$ to judge whether the

capital reserves r_i supply a sufficient buffer to cover the loss $U'_{xi} = (u_{xi} - u_{ix})^-$. Moreover, according to the balance sheet, it is easy to understand that the capital reserves are $r_i = \theta^C_i k_i$ or $r_i = \theta^P_i k_i$. Meanwhile, external assets f_i keep enough liquidity for an individual bank when the bank $i \in \aleph$ has been affected by the crisis. If bankruptcy is inevitable, the external assets will be forced into a fire sale for extra cash. For simplicity, this paper studies the model without this condition and this step is omitted. Therefore, for the interbank loans vector $U'_{xi}, (i \in \aleph)$, the following condition holds:

$$
u'_{ix} = \begin{cases} u_{xi} - u_{ix}, & \text{case1: if } (u_{xi} - u_{ix})^+ > 0, \\\\ 0, & \text{case2: if } (u_{xi} - u_{ix})^- < 0 \text{ and } \dfrac{-(u_{ix} - u_{xi})^-}{k_i} > \theta^C \text{ or } \theta^P, \\\\ 0, & \text{case3: if } (u_{xi} - u_{ix})^- < 0 \text{ and } \dfrac{-(u_{ix} - u_{xi})^-}{k_i} < \theta^C \text{ or } \theta^P. \end{cases} \tag{6}
$$

3. Results

BANKSCOPE is a banking information database developed by European financial information service provider Bureau van Dijk (BvD) and Fitch Ratings (Fitch), an authoritative banking rating agency. BANKSCOPE provided us with German [26], global [27], and European banking systems [27] as interbank networks samples to verify the performance of our algorithm. Table 2 shows information of banking networks for earlier studies, and the key characteristics of the real network including size and density are fully considered in this paper. We explored detecting the

core-periphery structure of the interbank networks using Monte Carlo simulations.

We ran 100 Monte Carlo simulations of parameters to evaluate the performance of the algorithms, which is good enough to reduce the randomness of the simulation method; Table 3 and Table 4 show detailed parameters for GSA and GA used in these banking systems, respectively. The optimal partition value and mean computation time for each network and each method for the Monte Carlo simulations are shown in Table 5. Subsequent columns illustrate the best result of CPM attribute distribution. Both the number of nodes and the density of the networks impacted the efficiency of the algorithm. Therefore, the proportion of node types and blocks listed here was to facilitate comparative analysis. The best

TABLE 4: Detailed sittings of parameters for GA.

Network	Global	European	German
Number of banks	202	836	2182
Number of generations	500	750	900
Size of population	400	450	450
Crossover probability	0.2	0.2	0.2
Mutation probability	0.8	0.8	0.7
Maximum number of generations without improvement	10	20	20
Initial temperature	8000	6000	6000
Annealing parameter	0.99	0.99	0.99

TABLE 5: Detailed results of simulation experiment.

Algorithm	Network	Number of nodes		Network density (%)				Error scores (%)	Mean of time
		Core	Peripheral	CC	CP	PC	PP		
GSA	Global	16	186	84.3	26.5	16.6	0.99	4.90	23.1 (0.245)
	European	34	802	72.5	13.5	8.9	0.69	6.21	664.2 (111.542)
	German	49	2133	60.7	4.8	12.5	0.35	8.98	3455.3 (121.825)
GA	Global	31	171	85.4	19.5	22.1	1.25	6.92	26.2 (0.561)
	European	52	784	66.2	15.9	13.8	0.57	7.77	622.4 (17.749)
	German	97	2085	61.5	6.8	12.6	0.39	10.75	3352.2 (105.771)
BBA	Global	29	173	92.0	13.6	12.5	1.43	9.9	13.9 (1.456)
	European	110	726	60.2	11.1	7.2	0.53	9.38	233.52 (15.112)
	German	275	1907	564	9.4	11.5	0.17	12.65	1566.6 (335.54)

GSA, genetic simulated annealing algorithm; GA, genetic algorithm; BBA, branch-and-bound algorithm.

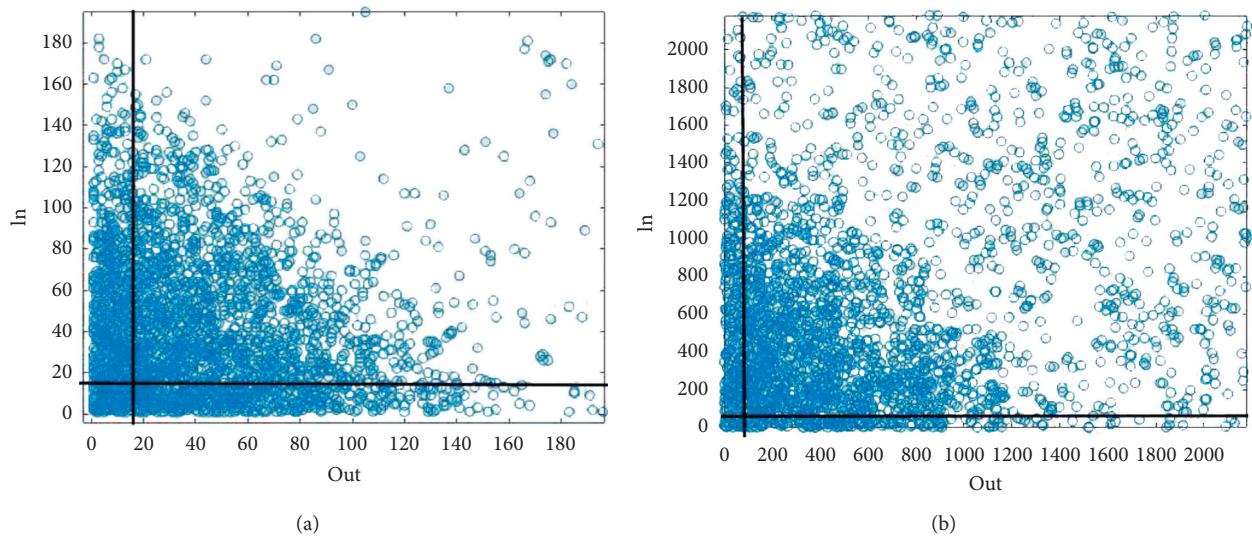(a)   (b)
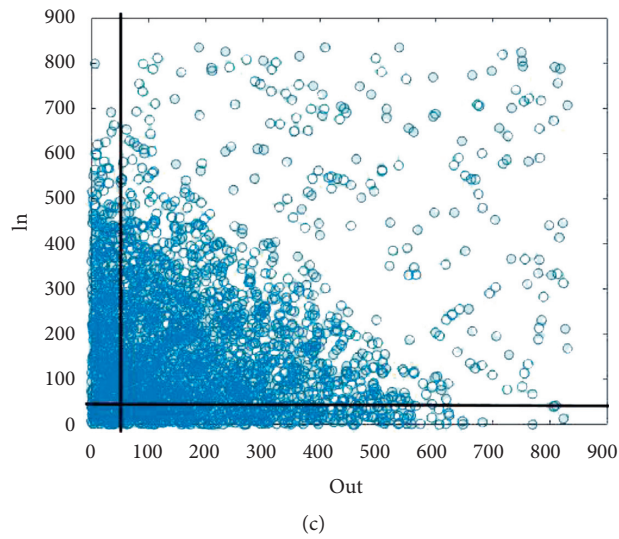
FIGURE 3: Continued.

(c)

FIGURE 3: Simulated adjacency matrix of core-periphery structure: global (a), European (b), and German (c), according to partition by black lines. Block of CC is shown in second quadrant and PP is arranged in fourth quadrant.
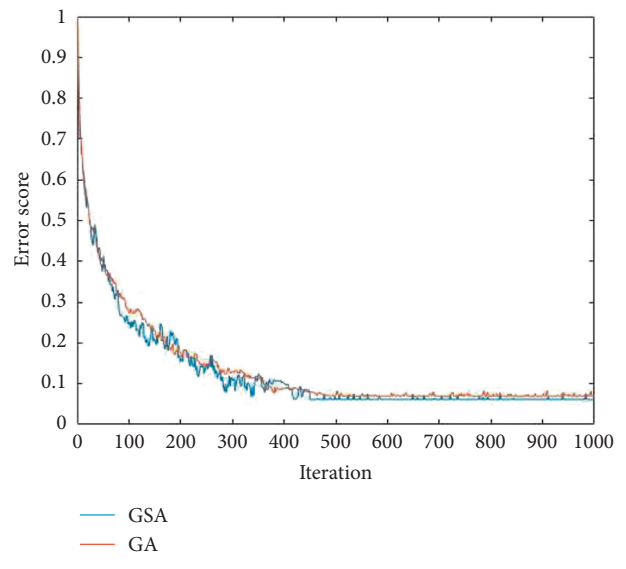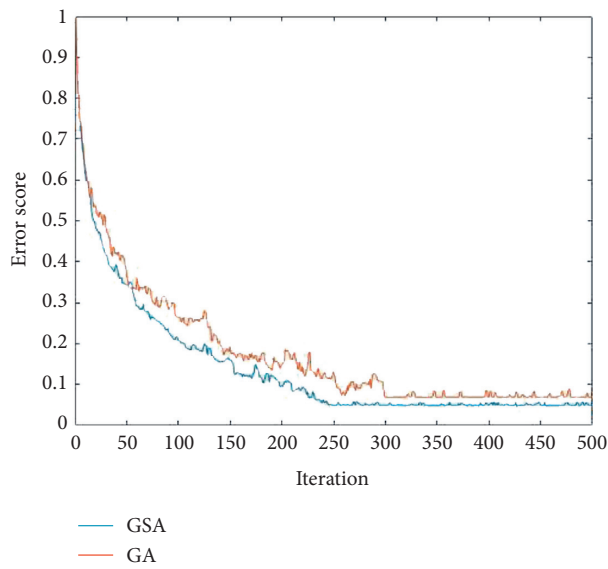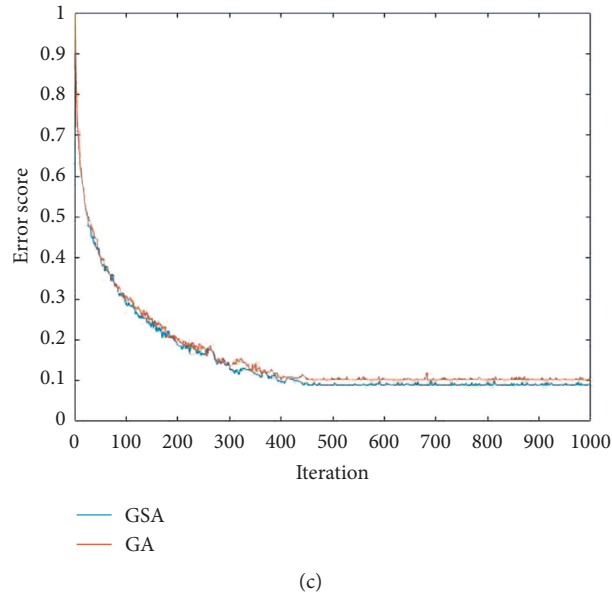

(a)

(b)

FIGURE 4: Continued.

(c)

FIGURE 4: Distribution of error scores for detecting core-periphery structure in global (a), European (b), and German (c) interbank networks.
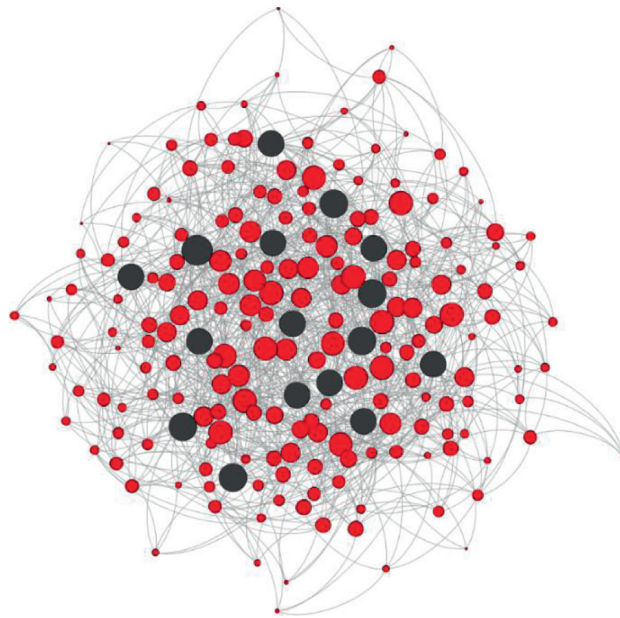

FIGURE 5: Directed global interbank network (202 nodes) based on YIFAN HU layout. Black nodes represent core banks; red nodes represent peripheral banks.

error score and mean of the computation time from fitting CPM to 100 times of the Monte Carlo simulations based on three optimal algorithms are shown in Table 5.

The branch-and-bound algorithm (BBA) computes much faster compared with GA and GSA for discrete CPM. However, it failed to reach best error scores. The contrast between GSA and GA was better because both of them are heuristic methods for discrete CPM without presetting, as the algorithms themselves are able to modify the proportion of core-periphery. Overall, the simulation results

successfully identified the merits and demerits of the three algorithms. BBA is a good method for highly effective and simple processing if calculation speed is preferred to partition results. GSA and GA perform comparably, while GA did not seem to predominate in terms of speed and GSA provided a more feasible scheme for searching for the best global solution than GA did, as is shown in Table 5.

Overall, we analyzed the situation of three algorithms for CPM. Next, we plotted the adjacency matrix of the core-periphery structure (see Figure 3). The four quadrants

Input:
τ: number of generations;
n: size of population;
m: number of chromosomes;
p_1: crossover probability;
p_2: mutation probability;
l: maximum number of generations without improvement;
T: annealing temperature;
σ: annealing parameter.
Output:
Computing a complex network hierarchical structure for financial market networks on the basis of the hybrid heuristic algorithm
Best structure outcome found for CPM.
(1)   Initialize population $\psi(0)$
(2)   Evaluate the error function of each chromosome in $\psi(0)$
(3)   Encode each chromosome in $\psi(0)$ with binary code method.
(4)   While $(t < \ell)$ do
(5)   While (random $R(0, 1) < \Delta \varepsilon(\psi(x))/\sigma T$) do
(6)   For each x in 1 to τ do
(7)   While (random deviate $R(0, 1) < p_1$) do
(8) $ch_1 \leftarrow$ select chromosome at random from $\psi_1(x) \subset \psi(x)$
(9) $ch_2 \leftarrow$ select chromosome at random from $\psi_1(x) \subset \psi(x)$
(10) $ch \leftarrow$ Crossover(ch_1, ch_2).
(11) $\psi(x + 1) \leftarrow$ update$(\psi(x), \psi(x) \cup ch)$
(12) End while
(13) While (random $R(0, 1) < p_2$) do 5
(14) $ch' \leftarrow$ mutate (ch)
(15) $\psi(x + 1) \leftarrow$ update $\psi(x) \cup \{ch'\}$;
(16) End while
(17) Decode and evaluate error function of each chromosome in $\psi(x)$
(18) If $\varepsilon(\psi(x + 1)) > \varepsilon(\psi(x))$
(19) $t \leftarrow t + 1$
(20) End if
(21) $\psi(x + 1) \leftarrow \psi(x)$
(22) $t \leftarrow t + 1$
(23) End while
(24) End while

ALGORITHM 1: GSA proposed to address CPM.

divided by the black solid line represent CC, CP, PC, and PP, respectively. As noted previously, on the basis of the principle of optimal priority, we found that the GSA could be considered as the best candidate to detect a core-periphery model, although computation time is an essential factor. Therefore, a case study of the global banking network was used to show the convergence property and the convergence of the algorithms (see Figures 4 and 5).

4. Conclusion

In this article, we studied the issue of detecting a core-periphery structure in interbank networks with GSA. Specifically, we presented two important improvements to detect a core-periphery structure. First, we mathematically proposed the optimization of the error score for CPM which would give the algorithm access to a simplified error score as an objective function. The second measure was to use an advanced GA to obtain more precise solutions than the original GA. GSA effectiveness is achieved by the prominent benefits of the simulated annealing method. It can find the global minimum of complex functions with a large number of local minimum values and has the ability to use more complex objective functions. Although both GA and GSA algorithms have similar speeds of convergence, as we tested the algorithms in the global, European, and German interbank networks samples, we found that the GSA required fewer iterations than GA did to converge on the smallest error score. In the end, this paper triggers a cascading failure of interbank market with common shock using Gai-Kapadia model and explored an immunization strategy to prevent the spread of risk. In summary, we showed the measure to detect a core-periphery structure in a European interbank network and discussed the significance of the structure to the management of systemic risk contagion.

Appendix

A. Detailed mathematical derivation process of improved error score e

There is a detailed mathematical derivation process of improved error score e. S is bank hierarchy (core or periphery)

and $S\{s_C, s_P\}$; s_C and s_P represent sets of banks assigned to the core or the periphery, respectively. If we only consider banks in block CC which have $x(x-1)$ pairs of links between core banks, then,

$$\sum_{j \in s_C, (i \neq j)} \Theta\{a_{ij} = 0\} + \sum_{j \in s_C, (i \neq j)} \Theta\{a_{ij} = 1\} = x(x-1). \quad (A.1)$$

Here, for the following formulation, an indicator function $\Theta\{\mu\}$ is newly defined: if μ is true, $\Theta\{\mu\} = 1$; otherwise, $\Theta\{\mu\}$ is equal to 0. Furthermore, according to the result from Lip [15], the number of linkages that join bank i to banks in s_c is expressed as $l_{s_C}(i) = \sum_{j \in s_C} a_{ij}$. So, this relationship should be expressed as follows:

$$\sum_{j \in s_C, (i \neq j)} \Theta\{a_{ij} = 1\} = \sum_{i \in S} k(i) - \sum_{i \notin s_C} l_{s_C}(i). \quad (A.2)$$

In addition, there are two important equations, expressed as follows:

$$\sum_{i \notin s_C} k(i) + \sum_{i \in s_C} k(i) = \sum_{i \in S} k(i), \sum_{i \notin s_C} l_{s_C}(i) + \sum_{i \in s_C} l_{s_C}(i) = \sum_{i \in S} l_{s_C}(i). \quad (A.3)$$

Using those definitions and results, total error score e can therefore be written as Algorithm 1.

$$
\begin{aligned}
e(a_{ij}) &= \frac{e_{CC} + e_{PP} + e_{CP} + e_{PC}}{\sum_i \sum_j a_{ij}} \\[2mm]
&= \frac{\left(\sum_{j \in s_C, (i \neq j)} \Theta\{a_{ij} = 0\} + \sum_{j \in s_P, (i \neq j)} \Theta\{a_{ij} = 1\}\right)}{\sum_i \sum_j a_{ij}} \\[2mm]
&= \frac{\left(x(x-1) - \sum_{j \in s_C, (i \neq j)} \Theta\{a_{ij} = 1\} + \sum_{i \notin s_C} k(i) - \sum_{i \notin s_C} l_{s_C}(i)\right)}{\sum_i \sum_j a_{ij}} \\[2mm]
&= \frac{\left(x(x-1) - \sum_{i \in s_C} l_{s_C}(i) + \sum_{i \notin s_C} k(i) - \sum_{i \notin s_C} l_{s_C}(i)\right)}{\sum_i \sum_j a_{ij}} \\[2mm]
&= \frac{\left(x(x-1) + \sum_{i \notin s_C} k(i) - \left(\sum_{i \notin s_C} l_{s_C}(i) + \sum_{i \in s_C} l_{s_C}(i)\right)\right)}{\sum_i \sum_j a_{ij}} \\[2mm]
&= \frac{\left(x(x-1) + \sum_{i \notin s_C} k(i) - \sum_{i \notin S} l_{s_C}(i)\right)}{\sum_i \sum_j a_{ij}},
\end{aligned}
\quad (A.4)
$$

subject to $S = \{1, 2, \ldots, n\} = \{s_C, s_P\}$ and $1 \leq i \neq j \leq n$.

Data Availability

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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