Research Article

Flexure Performance of Externally Bonded CFRP Plates-Strengthened Reinforced Concrete Members

Xin Yuan ¹, Chaoyu Zhu ¹, Wei Zheng,² Jiangbei Hu,³ and Baijian Tang¹

¹School of Civil Engineering, Suzhou University of Science and Technology, Suzhou 215011, China

²Suzhou Zhongheng Access Bridge Co., Ltd., Suzhou 215011, China

³Henan Branch Company, China Southwest Architectural Design and Research Institute Group., Ltd, Zhengzhou 450000, China

Correspondence should be addressed to Xin Yuan; yuanxin@mail.usts.edu.cn

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This paper investigates the flexural behavior of CFRP plate-strengthened concrete structures. Specimens of the CFRP plate-reinforced beam were designed and tested by the four-point flexural test. The load-deflection relationship, failure modes, and crack propagation were analyzed. The results showed that the postcracking stiffness and bearing capacity of the test beams can be improved by the additional anchoring measures for CFRP strengthening. The relationship between flexural moment and curvature was analyzed by introducing a MATLAB program. The calculation model between curvature, flexural moment, and stiffness was derived for the CFRP plate-strengthened structure. The recommended calculation model was applied in the analysis of deflection, and the theoretical values were compared with the test results.

1. Introduction

The carbon fiber-reinforced polymer (CFRP) has advantages of lightweight and high strength [1]. Considering its durability and easy construction compared with other traditional strengthened materials, CFRP has been applied in strengthening and repair of damaged reinforcement concrete structures [2, 3]. The advantages of CFRP-strengthened structure have been studied in static and dynamic methods [4, 5]. Several series of double shear specimen have been tested and analyzed to study the interfacial behavior between CFRP and concrete. The test and analysis results show that the crack growth rate increases with the increase in the stress level, and a calculation model was proposed to describe the change law between the crack growth rate and the stress level [6]. Numerical and theoretical investigations were conducted on a CFRP-strengthened cracked steel plate by considering the stress intensity factors, and numerical results and theoretical results were compared to validate the proposed expressions [7]. In addition, the bond behavior of FRP-concrete interface under sustained loading was studied for reliability analysis, and the CFRP-concrete fracture

energy of the bond behavior was directly obtained by the measured local bond-slip curves. Numerical tests were also performed on prestressed FRP-bonded beams based on a nonlinear model to gain the short-term behavior of prestressed CFRP-concrete beams [8].

In recent years, the CFRP reinforcement method is also increasingly used for strengthening the concrete bridges [9–11]. The numerical analysis of 3D nonlinear finite element method was carried out on the girder bridge with CFRP-strengthened RC beams, and the distributions of load deflection and the concrete-CFRP plate had been investigated [9]. These results show that the new Truss-link interface elements can be effectively adopted for modelling the FRP-strengthened RC structural component. A new type of bridge deck composed of innovative pultruded fiber-reinforced polymer composite sandwich panels was proposed [10]. Based on the first-order shear deformation theory, the failure mode, flexural capacity, and other aspects of bridge decks under different working conditions were researched with four-point flexural tests.

CFRP can effectively extend the ductility and bearing capacity of the concrete structures [11–13]. Seismic behavior

of gravity railway bridge pier strengthened with CFRP was studied [14]. It was found that the load-carrying capacity of the bridge pier can be enhanced by the CFRP wrapping method, and this strengthening method can be applied to improve the vulnerability of gravity bridge pier at the pier-footing region. CFRP-strengthened RC beams were experimentally studied and analyzed under sustaining load [15]. Their research results indicated that the load magnitude during CFRP reinforcement process is an important indicator of the ultimate strength of reinforced concrete beams, while the ultimate strength of CFRP-strengthened concrete beams is basically the same even with small initial load. Experimental and analytical investigations were also carried out on the CFRP shear and flexural strengthened concrete beams [16, 17]. The experimental data indicated that greater thicknesses of CFRP did not increase the shear capacity of specimens with no unanchored method, compared with both control and unanchored CFRP-strengthened beams. It can be concluded that the CFRP anchorage method did lead to moderate increases in shear capacity.

The main failure mode of CFRP-reinforced members is generally due to the stiffness characteristic concentration near the interface of CFRP and concrete. Therefore, accurate prediction of postcracking stiffness is important for preventing the occurrence of debonding failure [18–21]. The test results show that CFRP-strengthened concrete structure can improve the strength and stiffness performance of the structure, and the lateral offset of CFRP from the longitudinal centerline has little effect on the flexural behavior of the concrete-CFRP interface [22]. When the crack width is small, the crack has a certain enhancement effect on the bonding behavior of CFRP-strengthened concrete structure in the initial stage. The crack development at this stage has little effect on the load-deflection curve of CFRP-reinforced concrete structures. When the crack width is large, the bond slippage phenomenon will be increased with the development of concrete cracking. A hybrid system composed of CFRP and reinforced steel was proposed to improve the strength and ductility of CFRP-reinforced concrete structures [23]. The results show that the hybrid reinforcement ratio between CFRP and steel will affect the balance between strength and ductility in flexural design which will affect the flexural performance of hybrid beam, while better strength and ductility of the hybrid beam can be obtained with reasonable mixing and strengthening ratio between FRP and steel. Yuan et al. [24, 25] investigated the crack and mechanical behaviors of CFRP plate-reinforced bridge roofs under high temperature with different anchoring measures. It is shown that the average crack spacing is more effectively reduced by the additional anchoring measures placed at the midspan and the support position.

This paper investigates the flexural behavior of CFRP plate-strengthened concrete structure. CFRP plate-strengthened concrete test beams were designed and constructed according to the structural analysis of the box girder. Four-point flexural tests were carried out on the specimens. The crack propagation, flexural moment-curvature relationship, deflection, and failure modes were investigated. The stiffness variation and the debonding

mechanism of the test beams were comparatively studied. A theoretical calculation model was derived for the CFRP plate-strengthened structure by introducing MATLAB program. An accurate calculation of the relationship between flexural moment and curvature of the CFRP plate-strengthened concrete flexural members was proposed as well. The theoretical model was applied in the deflection analysis of flexural beam. The relationship between flexural moment and stiffness can better predict the deflection of CFRP-strengthened concrete structure by comparing the theoretical calculation results with experimental results.

2. Bridge Diseases and Reinforcement Scheme

Under the coupling effect of vehicle load and a hot humid environment in South Asia, cracks appeared in the box girder roof of the Bangabandhu bridge. The cracks can be divided into several types according to the crack location, cracking reason, and direction. The maximum crack width can reach 6.4 mm. Some representative crack shapes and their locations are presented in Figure 1.

Carbon fiber-reinforced polymer (CFRP) is a lightweight composite material with ultrahigh tensile performance. It is also often used to strengthen structures in civil engineering. In this project, CFRP plates were also adopted to strengthen the roof of box girder, and its main working position was in the tensile side of the negative moment zone. CFRP plates were horizontally pasted on the bridge roof to delay the development of longitudinal cracks in the bridge. The reinforcement scheme of bridge roof using the CFRP plate is shown in Figure 2.

3. Specimen Design and Test Setup

3.1. Specimen Design. Based on the load distribution on the Bangabandhu bridge, the negative moment zone in the range of 2000 mm on both sides of the box girder web was selected to study. Force analysis result of the box girder is shown in Figure 3. The CFRP plate-strengthened specimens were designed according to the principle of action equivalence. The specimen is a concrete slab beam with a rectangular cross section. The length of the specimen is 4.00 m, and the cross-sectional size is 0.65 m in width and 0.28 m in height. The specimen adopted in this study is shown in Figure 3.

According to the structural form of the selected segment of box girder roof, the upper surface of the concrete slab beam specimen was taken as the tension side and the lower surface was taken as the compression side. The concrete adopted in this test was grade C40, and the thickness of concrete cover is 34 mm. The grade of rebar was HRB 400, seven tensile steel bars and seven compressed steel bars were arranged in the specimens, and the reinforcement ratio was 0.87%. Three pieces of CFRP plates were pasted on the upper surface of the specimen. The CFRP plate was 3 400 mm in length and 100 mm in width, and its thickness was 1.4 mm. The interval of each CFRP plate was 250 mm. In order to simulate the high-temperature action of asphalt paving construction, the asphalt layer was also paved on the surface

FIGURE 1: Locations and shapes of different types of cracks.

FIGURE 2: Bridge roof reinforcement scheme.

of the specimen after the CFRP plates were cured. The specimen with asphalt layer is presented in Figure 3.

One control specimen and three CFRP-strengthened specimens were prepared in this test. The classification of the specimens is listed in Table 1.

The control specimen FDBL is a concrete slab beam without strengthened measure and additional anchorage measure. The specimens TM-1 to TM-3 were strengthened with CFRP plates as shown in Figure 4(a). Strain gauges were also arranged on the CFRP plate, and the arrangement

scheme of strain gauges is presented in Figure 4(a). Considering the asphalt layer paved on the surface of the specimen, part of the asphalt layer was removed before placing the strain gauges. The CFRP plate-strengthened specimens with asphalt layer partly removed is presented in Figure 4(a). Additional anchorage measure by horizontally pasting a steel plate on the CFRP plates was adopted to delay the debonding failure occurring in the CFRP-strengthened specimens. The form of the additional anchorage measure is shown in Figure 4(b).

FIGURE 3: Force analysis of the box girder and specimen design.

TABLE 1: Summary of test specimens.

Specimen number	Reinforcement measure	Additional anchoring measure		Purpose
FDBL	No reinforcement	No additional anchorage		Control specimen
TM-1	CFRP plates	Steel plate transverse anchorage	End position	FRP-strengthened specimen
TM-2			End position and midspan position	
TM-3			End position and bearing position	

3.2. Specimen Preparation and Test Setup

3.2.1. Specimen Preparation

(1) *Concrete Pouring Process.* After the reinforced skeleton frame of the specimens was tied, the strain gauges were attached to the upper and lower sides of steel bars at the midspan of the specimen. The pouring process is shown in Figure 5.

(2) *CFRP Strengthening Process.* Firstly, the surface of the CFRP plate is roughened to remove the surface demould wax.

Secondly, the deteriorated layer on the concrete surface was removed with a wire-wheel angle grinder. The surface is cleaned to make it neat and solid. The cement is used to level the surface of the specimen. Before pasting the CFRP plate, the ink line should be prepared to ensure correct positions of the CFRP plate. The surface preparation of the specimen is shown in Figure 6.

Finally, after the CFRP plate is pasted, it is cured by bolting with wood strips. The specific method is as follows: two bolts are placed on both sides of the CFRP plate at intervals of 50 cm, and a vertical long wooden strip is first pressed on the surface of the CFRP plate. Then, the surface of

the vertical long wooden strip is pressed with a horizontal short wooden strip. They are pressed evenly together and are fixed on the surface of the carbon plate. The wooden strips are removed after 3 to 7 days of curing.

(3) *Asphalt Paving Construction.* Considering that the roof of the box girder was paved with asphalt layer after strengthening with CFRP plates, it is necessary to simulate the high temperature conditions of the real construction process. After curing the CFRP plates, an epoxy mortar insulation layer was set according to the pavement scheme (Figure 2). After curing is completed, the asphalt is paved and rolled with heavy equipment. The asphalt paving process is shown in Figure 7.

3.2.2. *Material Performance Test.* Performance of materials was tested according to the corresponding standard code, and the tested results of material performance are shown in Table 2. Six pieces of concrete blocks ($150 \times 150 \times 150$ mm) were tested, and compressive strength and elastic modulus were calculated by the test results. Six steel bars (length = 400 mm, diameter = 28 mm), six CFRP plates (width = 200 mm, thickness = 1.4 mm), and six adhesive

FIGURE 4: CFRP plate-strengthened specimens with additional anchorage measure. (a) Position of additional anchorages and strain gauges. (b) Additional anchorage measure.

specimens were tested as well, and their material performances were also obtained.

3.2.3. Test Setup. According to the structure form of the box girder roof and traffic distribution, two supports were arranged near the midspan position of the specimen. Two 30 t hydraulic jacks were adopted in this test, and they were

arranged on both ends of the specimen to simulate the traffic loads on the box girder. The shear span of specimen is 1400 mm. The test setup is presented in Figure 3. A preload of 5 kN was applied firstly, and then, an increasing load of 20 kN per stage was loaded on the specimen. The loading pattern of the specimen is shown in Figure 8. Loading was stopped when any of the phenomena of concrete crushing, yield of steel bar, or debonding failure of CFRP plate were observed.

FIGURE 5: (a) Assembling the reinforcement of specimens and pasting strain gauge on the steel bar. (b) Concrete pouring of specimens.

FIGURE 6: Main process of strengthening. (a) Surface preparation. (b) Positioning for pasting CFRP plates.

FIGURE 7: Asphalt paving construction process. (a) Asphalt paving construction. (b) Rolling with heavy equipment.

4. Experimental Results and Analysis

4.1. Load-Displacement Relationship. Load-displacement relationship of the control specimen and the strengthened specimens were compared. Different relationships of specimens in this test are shown in Figure 9.

It can be seen from Figure 9 that the curves of strengthened specimens were basically same. Therefore, it can be concluded that additional anchoring measures at different positions have little effect on the bearing capacity of the CFRP-strengthened specimens. In addition, the curves of control specimen and strengthened specimen were similar in

the initial stage. During this stage, the loading level was small and each specimen was in the elastic stage, and the tensile force in the specimen was mostly shared by the concrete and steel bars.

For the control specimen, the concrete in the tensile zone was out of work after the first crack occurred in the pure flexural section, and the tensile force was taken away by the steel bar. While for the CFRP-strengthened specimens, the tensile force was shared by the CFRP plates and steel bars after the concrete was out of work. Due to the application of CFRP plates, the raising rate of neutral axis was delayed.

TABLE 2: Mechanical performance of tested materials.

Material		Tested material performance			
Concrete		Compressive strength (MPa)		Elastic modulus (GPa)	
$\bar{x}$		29.8		33.7	
σ		4.82		4.96	
Steel bar		Yield strength (MPa)		Ultimate strength (MPa)	
$\bar{x}$		408		546	
σ		13.46		18.94	
CFRP plate		Yield strength (MPa)		Elastic modulus (GPa)	
$\bar{x}$		408		210	
σ		13.46		28.5	
Adhesive	Elongation (%)	Flexural strength (MPa)	Tensile strength (MPa)	Compressive strength (MPa)	Elastic modulus (GPa)
$\bar{x}$	1.55	55	25.8	74.6	2.57
σ	0.062	5.19	4.37	5.46	0.089

$\bar{x}$ is the average value of the test results, and σ is the standard deviation.

FIGURE 8: Test setup and load pattern (unit: mm).

FIGURE 9: Load-displacement relationship.

4.2. Specimen Failure Modes. Different failure modes were observed in the constant moment zone of specimens. Debonding failure of the surface concrete and CFRP plate, concrete crushing, and other failure modes can be observed in the CFRP-strengthened specimens as shown in Figure 10.

It can be seen from Figure 10 that CFRP-concrete interfacial debonding failure of the CFRP-strengthened specimens was mainly caused by the flexural shear crack. The debonding failure mainly occurs in the thin layer of concrete below the adhesive layer. During the test, cracks began to appear in the constant moment zone and then gradually extended toward both sides. The CFRP plate at the end position debonded off firstly. Longitudinal slip of CFRP plates occurred at the end of specimen. The CFRP plate in the constant moment zone debonded off from the concrete surface simultaneously.

4.3. Crack Propagation Morphology. After the loading procedure has completed, cracks of specimen were observed. It was found that the crack distribution is mainly concentrated on both sides of the support, including the constant moment zone and at the range of 400 mm on both sides of the bearing. Thus, the cracking in this range was presented. The crack development patterns of specimen are presented in Figure 11. The main flexural crack perpendicular to the concrete tensile side occurred in the initial stage of crack formation. The root crack near the steel bar position appeared at the moment when the specimen approached failure.

As can be seen from Figure 5, the cracking space and height of the specimen were larger than those of the specimen strengthened by the CFRP plate, because the control specimen was not strengthened by the CFRP plate. The bonding shear stress between the interface of CFRP plate and concrete can effectively control the expansion of cracks in the concrete tensile zone. The statistics of crack distribution in the constant moment zone are shown in Table 3.

For the ratio of the maximum crack spacing to the average crack spacing and the ratio of the maximum crack spacing to the minimum crack spacing, it can be seen from the Table 3 that ratio of the control specimen is much smaller than that of the strengthened specimen, while the maximum cracking height of control specimen is higher than that of CFRP plate-strengthened specimens. It can be concluded from Table 3 that the CFRP plate-strengthening method can effectively delay the cracking process of concrete structure.

5. Theoretical Investigation for Relationship of Stiffness and Curvature

The CFRP plate-reinforced concrete structure was taken as the research object. The following basic assumptions were used in this study. Firstly, the specimen follows the plane section assumption, i.e., the section deformation should be a plane. Secondly, the CFRP plate is assumed as a linear elastic material. The stress-strain relationship of the steel bar is considered to be linear before it reaches the yield strength, and when it reaches the yielding strength, its stress is

considered as a constant. However, for the concrete material, the concrete is considered to be cracked when it reached its tensile strength, and the concrete stress at the cracked place dropped to zero.

5.1. Theoretical Calculation Model. Section strain and stress distribution of the specimen are presented in Figure 12. The cross section of the CFRP plate-strengthened concrete specimen can be divided into finite strips. The stress in each strip is assumed to be evenly distributed, and the stress value is equal to the stress value at the center of its strip section.

The strain distribution characteristics in Figure 12 can be calculated with the following formula:

$$\phi = \frac{\varepsilon_c}{x} = \frac{\varepsilon'_s}{x - a'_s} = \frac{\varepsilon_s}{h_0 - x} = \frac{\varepsilon_f}{h_f - x} = \frac{\varepsilon_{ci}}{x_{ci}} = \frac{\varepsilon_{ti}}{x_{ti}}. \quad (1)$$

The following equation can be obtained from Figure 7 by the force balance relationship:

$$N_t + \sigma_s A_s + \sigma_f A_f = N_c + \sigma'_s A'_s. \quad (2)$$

The following equation can also be obtained from Figure 7 by the moment balance relationship:

$$\begin{aligned} M = & \sigma_s A_s (h_0 - x) + \sigma_f A_f (h_f - x) \\ & + \sum_{i=1}^n \sigma_{ti} b \left(i - \frac{1}{2} \right) \left(\frac{h - x}{n} \right)^2 + \sigma'_s A'_s (x - a'_s) \\ & + \sum_{i=1}^n \sigma_{ci} b \left(i - \frac{1}{2} \right) \left(\frac{x}{n} \right)^2, \end{aligned} \quad (3)$$

where n is the number of strips divided in the concrete section. σ_{ti} is the concrete tensile stress at the center of the i^{th} layer of the concrete tension zone, and σ_{ci} is the concrete compressive stress at the center of the i^{th} layer of the concrete compression zone. b is the sample section width.

The flexural moment-curvature relationship of the CFRP plate-strengthened concrete structure was realized by the MATLAB programming method as recommended in the previous study [24]. The specific calculation steps are shown as follows:

- (1) An initial value is given to the compressive strain ε_c of the concrete compression side.
- (2) For each recalculation, the strain at the edge of the concrete compression area is taken as $\varepsilon_c = \varepsilon_c + \Delta\varepsilon_c$, in which the $\Delta\varepsilon_c$ is 0.000005.
- (3) Assuming that the depth of the concrete compression area is x .
- (4) According to the plane section assumption, the strain value corresponding to the center position of each strip, the strain of steel bar, and the strain of the CFRP plate can be obtained from equation (1).
- (5) The corresponding stress value can be calculated by the obtained strain value, and each force on the section can be checked to see whether it meets the force balance equation.

FIGURE 10: Types of debonding failure in the constant moment zone. (a) Specimen FDBL. (b) Specimen TM-1. (c) Specimen TM-2. (d) Specimen TM-3.

FIGURE 11: Crack distribution of specimen (unit: mm) (the number near the crack shows the cracking load level, unit: kN).

(6) If the assumed depth of the concrete compression area x cannot satisfy the equilibrium equation after t cycles calculation, the dichotomy method can be introduced to correct the depth of the concrete compression area. The calculation of steps (4) and

(5) is repeated until the equilibrium equation can be satisfied.

(7) If the equilibrium equation can be satisfied, the depth of the concrete compression area x can be obtained at this time. In addition, the compressive strain of the

TABLE 3: Crack distribution statistics of constant moment zone of specimens.

Specimen number	Crack spacing (mm)			$l_{\max}/l_m$	$l_{\max}/l_{\min}$	Crack height l_h (mm)
	$l_{\max}$	$l_{\min}$	l_m			
FDBL	183.6	137.7	168.2	1.09	1.33	260.1
T-1	174.6	95.23	127.0	1.37	1.83	235.2
T-2	141.5	73.2	105.2	1.35	1.93	234.8
T-3	144.8	65.5	115.0	1.26	2.21	245.8

$l_{\max}$ is the maximum of crack spacing, $l_{\min}$ is the minimum value of crack spacing, l_m is the average value of crack spacing, and l_h is the maximum value of crack height.

FIGURE 12: Distribution of specimen strain and stress.

compression area ε_c and the section flexural moment M can be obtained as well.

- (8) The abovementioned seven calculation steps are repeated until the strain of the concrete compression area reaches its ultimate compressive strain value.

The corresponding calculation process is shown in Figure 13.

5.2. Relationship between Curvature and Moment. Taking the specimens FDBL and TM-1 as examples, the flexural moment-strain relationship of the two specimens is shown in Figure 14.

According to the relationship between the strain and flexural moment obtained from the experimental test (Figure 14), the relationship between section curvature and flexural moment can be calculated by the MATLAB program presented in Figure 13. The comparison of theoretical results and experimental results is shown in Figure 15.

It can be seen from Figure 15 that the calculated cracking moment is much smaller than the test results, and the experimental result and the calculated value of the ultimate flexural moment are not much different. This is mainly because the effect of the asphalt layer on the tension side was not considered when calculating the cracking moment. In addition, because the debonding failure between FRP and concrete was not considered in the MATLAB program, the calculation result of the ultimate flexural moment of the CFRP-reinforced test piece (such as specimen TM-1) is larger than the test result.

In addition, the flexural main crack in the pure flexural section of the specimen will expand with the increasing load, and the CFRP plate will debond from the concrete surface. After the debonding failure of the CFRP plate, loads will no longer be added on the specimen. This is the reason for the short curve length of the test result after the yield of steel bar as shown in Figure 15.

FIGURE 13: Moment-curvature relationship calculation process.

5.3. Relationship between Moment and Stiffness. The average curvature of the section can be calculated based on the flat section assumption, and the formula is shown as follows:

FIGURE 14: Strain change process of steel bar and concrete. (a) Specimen FDBL. (b) Specimen TM-1.

FIGURE 15: Comparison of theoretical results of flexural moment- curvature with experimental results. (a) Specimen FDBL. (b) Specimen TM-1.

$$\phi = \frac{1}{\gamma_m} = \frac{M}{B_s} = \frac{\epsilon_{sm} + \epsilon'_{cm}}{h_0}, \quad (4)$$

where γ_m is the average curvature radius of the section, B_s is the section stiffness under short-term load, ϵ_{sm} is the average strain of the tensile steel bar, and ϵ'_{cm} is the average strain of the concrete at the edge of the compression area.

The section flexural moment can be obtained according to the calculation model shown in Figure 16, and the formula is shown in the following equation:

$$M = \epsilon_s E_s A_s \eta_s h_0 + \epsilon_f E_f A_f \eta_f h_f = \epsilon_s E_s A_s \eta_s h_0 \left(1 + \frac{\epsilon_f E_f A_f \eta_f h_f}{\epsilon_s E_s A_s \eta_s h_0} \right), \quad (5)$$

where η_s and η_f are the internal lever arm coefficient of the steel bar and the CFRP plate, respectively.

Thus, the average strain of the tensile steel bar can be obtained as follows:

$$\epsilon_{sm} = \psi_s \epsilon_s = \psi_s \frac{M}{E_s A_s \eta_s h_0 \left(1 + \frac{\epsilon_f E_f A_f \eta_f h_f}{\epsilon_s E_s A_s \eta_s h_0} \right)}. \quad (6)$$

FIGURE 16: Sectional bending moment calculation model diagram.

It can be seen from Figure 13 that the concrete stress in the compression zone adopts a curve distribution. Therefore, a coefficient ω is introduced when calculating the concrete stress at the edge of the compression zone σ'_c . Thus, the resultant force of the concrete in the compression zone can be calculated by the following formula:

$$C = \omega \sigma'_c (\gamma'_f + \xi) b h_0, \quad (7)$$

where γ'_f is the ratio of the compressed flange area to the effective area of the web in the T-beam section and ξ is the height coefficient of the compression zone at the cracking section.

Considering the force of the concrete in the compression zone and the force in the CFRP plate to the combined point of the tensile reinforcement, the flexural moment can be calculated as follows:

$$\begin{aligned} M &= \omega \sigma'_c (\gamma'_f + \xi) b h_0 \eta_s h_0 + \varepsilon_f E_f A_f a_f \\ &= \varepsilon'_c E_c b h_0^2 \left[\eta_s v_c \omega (\gamma'_f + \xi) + \frac{\varepsilon_f E_f A_f a_f}{\varepsilon'_c E_c b h_0^2} \right], \end{aligned} \quad (8)$$

where v_c is the elastic characteristic coefficient of concrete.

By introducing the nonuniform coefficient of concrete strain in compression zone ψ'_c , the mean value of the concrete strain can be obtained as follows:

$$\begin{aligned} \varepsilon'_{cm} &= \psi'_c \varepsilon'_c = \psi'_c \frac{M}{E_c b h_0^2 \left[\eta_s v_c \omega (\gamma'_f + \xi) + \frac{\varepsilon_f E_f A_f a_f}{\varepsilon'_c E_c b h_0^2} \right]} \\ &= \frac{M}{E_c b h_0^2 \left(\eta_s v_c \omega (\gamma'_f + \xi) + \frac{(\varepsilon_f E_f A_f a_f / \varepsilon'_c E_c b h_0^2)}{\psi'_c} \right)}. \end{aligned} \quad (9)$$

Assuming that ζ is the comprehensive coefficient of the average strain of the concrete in the compression zone, then ζ can be expressed as follows:

$$\zeta = \frac{\eta_s v_c \omega (\gamma'_f + \xi) + (\varepsilon_f E_f A_f a_f / \varepsilon'_c E_c b h_0^2)}{\psi'_c}. \quad (10)$$

Thus, equation (9) can be converted into the following formula:

$$\varepsilon'_{cm} = \frac{M}{\zeta b h_0^2 E_c}. \quad (11)$$

The comprehensive coefficient of the average strain of the concrete in the compression zone ζ can be calculated by MATLAB, and the calculation results are presented in Figure 17.

It can be seen from Figure 17 that the comprehensive coefficient of the average strain ζ decreases with the increase of the section flexural moment. It decreases rapidly after the occurrence of the crack, while it is basically unchanged before the yield of steel bar. Therefore, the calculation of the comprehensive coefficient can ignore the effect of loads before the steel bar yields. The value at the yield point of the steel bar is taken as the numerical fit point of the comprehensive coefficient in this study.

By fitting the test results in this test, the relationship between $\alpha_{Es} \rho_s / \zeta$ and $\alpha_{Es} \rho_s$ can be obtained as shown in Figure 18, and the corresponding formula is shown as follows:

$$\frac{\alpha_{Es} \rho_s}{\zeta} = 9.70078 \alpha_{Es} \rho_s - 0.04895. \quad (12)$$

Thus, the following formula can be derived by the above analysis:

$$\begin{aligned} \frac{M}{B_s} &= \frac{1}{h_0} \left(\psi_s \frac{M}{E_s A_s \eta_s h_0 (1 + (\varepsilon_f E_f A_f \eta_f h_f / \varepsilon_s E_s A_s \eta_s h_0))} + \frac{M}{\zeta b h_0^2 E_c} \right) \\ &= \frac{M}{E_s A_s h_0^2 / ((\psi_s / \eta_s (1 + (\varepsilon_f E_f A_f \eta_f h_f / \varepsilon_s E_s A_s \eta_s h_0))) + (\alpha_{Es} \rho_s / \zeta))}, \end{aligned} \quad (13)$$

where $\alpha_{Es} = E_s / E_c$, $\rho_s = A_s / b h_0$, $(\varepsilon_f / \varepsilon_s) = 1.9 (h / h_0) - 1$, $\eta_s = 0.87$, and $\eta_f h_f = 0.87 h_0 + a_f$. ψ_s is the nonuniform coefficient of steel bar.

Therefore, the short-term stiffness before the yield of steel bar can be expressed as follows:

$$B_s = \frac{E_s A_s h_0^2}{(\psi_s / \eta_s (1 + (\varepsilon_f E_f A_f \eta_f h_f / \varepsilon_s E_s A_s \eta_s h_0))) + 9.70078 \alpha_{Es} \rho_s - 0.04895}. \quad (14)$$

However, the contribution of the longitude steel bar to the section stiffness is small after the yield of the steel bar, and the CFRP plates are still in the state of linear elasticity at this time.

Therefore, the strain of CFRP plate should be adopted when calculating the section curvature and stiffness.

The average curvature of the section after the yield of steel bar can be expressed as follows:

FIGURE 17: Relationship between M/My and ζ under different conditions. (a) With different ρ_f values. (b) With different α_{Ef} values.

FIGURE 18: Diagram of relationship between $\alpha_{Es}\rho_s/\zeta$ and $\alpha_{Es}\rho_s$.

$$\phi = \frac{1}{\gamma_m} = \frac{M}{B_s} = \frac{\varepsilon_{fm} + \varepsilon'_{cm}}{h_f}. \quad (15)$$

And the section flexural moment can be expressed as follows:

$$\begin{aligned} M &= f_y E_s A_s \eta_s h_0 + \varepsilon_f E_f A_f \eta_f h_f \\ &= \varepsilon_f E_f A_f \eta_f h_f \left(1 + \frac{f_y E_s A_s \eta_s h_0}{\varepsilon_f E_f A_f \eta_f h_f} \right). \end{aligned} \quad (16)$$

The average strain of the CFRP plate can be presented as follows:

$$\begin{aligned} \varepsilon_{fm} &= \psi_f \varepsilon_f = \psi_f \frac{M}{E_f A_f \eta_f h_f \left(1 + (f_y E_s A_s \eta_s h_0 / \varepsilon_f E_f A_f \eta_f h_f) \right)} \\ \varepsilon_f &= \frac{M - f_y E_s A_s \eta_s h_0}{E_f A_f \eta_f h_f}. \end{aligned} \quad (17)$$

The section flexural moment can be expressed by using the following equation:

$$\begin{aligned} M &= \omega \sigma'_c (\gamma'_f + \xi) b h_0 \eta_f h_f + f_y E_s A_s a_f \\ &= \varepsilon'_c E_c b h_0 h_f \left[\eta_s v_c \omega (\gamma'_f + \xi) + \frac{f_y E_s A_s a_f}{\varepsilon'_c E_c b h_0 h_f} \right]. \end{aligned} \quad (18)$$

The concrete strain in compression zone can be obtained as follows:

$$\begin{aligned} \varepsilon'_{cm} &= \psi'_c \varepsilon'_c = \psi'_c \frac{M}{E_c b h_0 h_f \left[\eta_s v_c \omega (\gamma'_f + \xi) + (f_y E_s A_s a_f / \varepsilon'_c E_c b h_0 h_f) \right]} \\ &= \frac{M}{E_c b h_0 h_f \left(\eta_s v_c \omega (\gamma'_f + \xi) + (f_y E_s A_s a_f / \varepsilon'_c E_c b h_0 h_f) / \psi'_c \right)}. \end{aligned} \quad (19)$$

Applying the same method, the short-term stiffness formula after the steel bar yields can be derived as follows:

$$B_s = \frac{E_s A_s h_0 h_f}{\left(\psi_f / \left(\eta_f \left(1 + (f_y E_s A_s \eta_s h_0 / \varepsilon_f E_f A_f \eta_f h_f) \right) \right) + (\alpha_{Es} \rho_f / \zeta') \right)}. \quad (20)$$

FIGURE 19: Comparison of theoretical and experimental values of deflection.

5.4. Application in Deflection Analysis. The maximum deflection of the specimen under concentrated load can be obtained according to formula (20):

$$f = \frac{M(3l^2 - 4a^2)}{24B}, \quad (21)$$

where l is the span of beam, a is the distance between the loading point and the support, and B is the section stiffness which can be obtained by using the equations (14) and (20). The comparison of deflection between the theoretical value calculated, simulation results in the previous study [25], and experimental values is shown in Figure 19.

It can be seen from Figure 19 that the theoretical calculation result and the finite element simulation result of the deflection-flexural moment agree well with the experimental results. The relationship between stiffness and curvature derived by the MATLAB program recommended in this paper can better predict the deflection analysis of CFRP plate-strengthened concrete structures.

6. Conclusion

This paper investigates the flexure performance of externally bonded CFRP plates-strengthened reinforced concrete members. Specimens of the CFRP plate-strengthened beam were designed and tested. The crack propagation, moment-curvature relationship, and debonding failure modes were analyzed. The relationship of stiffness-curvature was theoretically investigated by adopting a MATLAB program. The following conclusions can be drawn:

- (1) The CFRP plate has a good inhibitory effect on the cracking expansion. Different positions of additional anchorage measure did little effect on the post-cracking stiffness.
- (2) The yield flexural moment and the ultimate flexural moment value of the CFRP plate-strengthened test

specimen are larger than those of the no-strengthened specimen.

- (3) The analysis of relationship between flexural moment and curvature for the CFRP plate-reinforced concrete flexural members can be realized by the MATLAB programming method adopted in this paper. The deflection of specimens can be predicted by applying the relationship of moment-curvature concluded in this paper.

Data Availability

The figure and table data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors' Contributions

X. Yuan and J. B. Hu were responsible for conceptualization. X. Yuan, J. B. Hu, and C. Y. Zhu provided the methodology. X. Yuan, C. Y. Zhu, J. B. Hu, and W. Zheng were involved in investigation. X. Yuan, B. J. Tang, and C. Y. Zhu wrote the original draft. X. Yuan contributed to writing, reviewing, and editing. X. Yuan and C. Y. Zhu carried out funding acquisition. X. Yuan and C. Y. Zhu provided the resources. X. Yuan and W. Zheng supervised the study.

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