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Research Article

Analysis of Rotary Vibration of Rigid Friction Pipe Pile in Unsaturated Soil

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Based on the mixture theory and previous work, the governing equation of the rotary vibration of rigid friction pipe pile in unsaturated soil is established. The analytical solution of this equation can be used to analyze the displacements and the complex stiffness of rotary vibration. The results show that the contribution to stiffness is as follows: solid < liquid < gas; and the contribution to rotational impedance is as follows: solid > liquid > gas. In addition, when the fluid permeability coefficient decreases, the stiffness decreases and the rotational impedance increases, but the influence is not obvious (especially the gas permeability coefficient). Four different kinds of degradation problems are also presented. Relevant conclusions can provide reference for engineering application.

1. Introduction

The seismic response analysis of pile foundation is an important problem in foundation engineering, and it is widely used in many engineering fields, such as bridge engineering or hospital construction. There is much research about the analysis of situation whether pile foundation is under horizontal or vertical dynamic loading. But, superstructure will also rotate under the action of earthquake or some sort of periodic dynamic force (for example, some mechanical or medical equipment with periodic natural vibration), and that will have a great influence on pile foundation which is under abutments, slender transmission towers, or special mechanical equipment. For end bearing pile, since the bottom of the pile is fixed, the rotation of the superstructure will cause torsional vibrations of the pile; for rigid friction piles such as concrete pipe pile, since the modulus of soil is far less than the modulus of the pile (the difference is about 3 orders of magnitude), the rotation of the superstructure will cause rotary vibrations of the whole pile.

For static torsion problem of a single pile, the solution has been found in principle since SaintVenant problem in elastic mechanics had been solved [1]. There is also much valuable research even when considering the effect of soil surrounding pile. By comparison, the vibration problem of pile foundation under dynamic torsion load is hard to get analytical solution like static problem because of its complexity, but there is also some valuable research work. Novak and Sachs [2] applied the plane strain soil model to study the vibration characteristics of embedded rigid foundation under dynamic torsional loading and derived an approximate analytical solution for torsional vibrations of footings partially embedded in a semi-infinite medium or a stratum. Wang et al. [3] developed an analytical solution to investigate the torsional vibration of an end bearing pile embedded in a homogeneous pyroclastic medium and subjected to a time-harmonic torsional loading. Zhang [4] derived dynamic governing equations of transversely isotropic saturated soil in cylindrical coordinates on the basis of the dynamic equations of saturated soil and the stress-strain relationships of transversely isotropic medium. In experimental study, Fattah et al. [5–7] analyzed the dynamic response of pile foundation in dry and saturated sandy soil excited by two opposite rotary machines and found that in dry soil the pile tip load decreased for all (L/d) ratios and operating frequencies. Whereas in saturated soil, the pile tip load increased for small (L/d) ratios and low operating frequencies. Fattah et al. [8] also studied load sharing and behavior of single pile embedded in unsaturated swelling soil, the results showed that the ultimate skin resistance increased to about 49% when the initial degree of saturation decreased from 90% to 70%. In addition, Chikr et al. [9–11] used shear deformation theory to make lots of analysis of the plates on the elastic foundation, and some other scholars [12, 13] used Hamiltonian principle or virtual displacement principle to reduce the unknown quantities needed in the calculation of plates.

It can be seen from previous research work that many scholars have done a lot of work deeply in this field. But, on the one hand, these researches mainly simplified the soil around the pile into single-phase soil or saturated soil, and there were few researches on unsaturated soil. On the other hand, the previous researches mainly focus on the dynamic torsion of end bearing piles, and the dynamic rotation of friction piles is seldom studied. The rotary vibration of rigid friction pipe pile in an unsaturated medium is studied in this article, and this problem is proposed by the author of this article first. This dynamic problem involves a pile-soil coupling system. Since solid pile is the special case when the inner radius of pipe pile tends to be 0, the rigid friction pipe pile analyzed in this article can be regarded as the extension of traditional solid pile. Considering the rapid development of prestressed concrete pipe piles in recent years (prestressed concrete pipe pile industry scale in China reached 73.058 billion Yuan in 2018, with a total length of about 336 million meters) and seismic research is an important subject in pile foundation engineering, we hope that this article can provide reference for the engineering application of prestressed concrete pipe piles.

The concept of rotary vibration of rigid friction pile is given first. In engineering, single pile foundations can be adopted in some high-voltage transmission towers, wind turbines (including onshore and offshore wind turbines), bridge piers, or some mechanical equipment with periodic dynamic response. When the superstructure is subjected to wind load, wave load or the equipment itself generates periodic dynamic response during operation and the dynamic couple with rotational effect may be generated on the pile top, so as to make the pile itself produce rotational dynamic response. For the end bearing pile, the pile bottom is firmly supported and the torsion may happen to the pile. But, for the rigid friction pipe pile, its bearing capacity is mainly derived from the friction on the side of the pile. As the constraint at the bottom of the pile is weaker than that at the end bearing pile and the stiffness of the pile itself is larger, it is assumed that the phase of rotation of the upper and lower ends of the rigid friction pipe pile can be approximately regarded as the same when torsion occurs (see Figure 1, where u_θ is the amplitude of

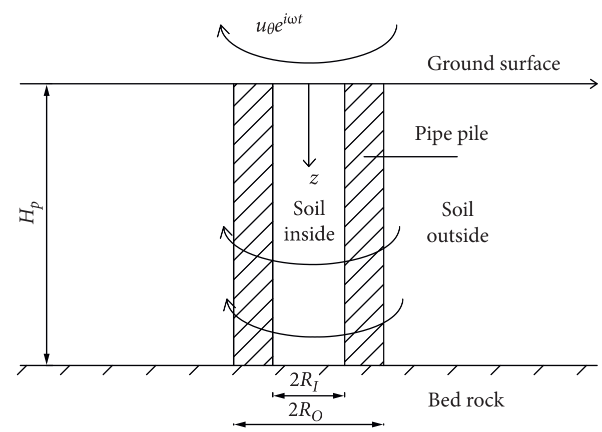

Figure 1: Schematic diagram of rotary vibration of rigid friction pipe pile.

rotational vibration, e is the base of the natural logarithm, $i^2 = -1$, ω is the circular frequency, t is the time, H_p is the pile length, and R_I/R_O is the internal/external radius of the pipe pile, respectively). The plane strain condition is approximately satisfied beyond a certain distance from both the ends, it can be regarded as a rotating vibration around the symmetry axis of the pile, and the problem can be regarded as a plane problem.

In addition, for the friction pile group pile foundation, when the ground vibration causes the upper structure rotation, the instantaneous center of rotation of the pile will have similar dynamic response. Because of the irregularity of the rotation, there may be several rotational centers successively in the whole vibration process; this means the rotation may happen to more than one pile. When the superstructure is irregular, the effect will be intensified. Therefore, for rigid friction piles, this problem may occur on either single pile foundation or group pile foundation. On this basis, it is assumed that the physical quantities (such as velocity and rotation angle) are all small in the vibration process, and the pile-soil interface does not separate. This is the rotary vibration model of rigid friction pile. Since there are many symbols in this article, we have included a symbol table in the Table 1 for the convenience of readers.

2. Basic Governing Equations for Dynamic Problems of Unsaturated Medium

This section will refer to the method of Vardoulakis and Beskos [14] to establish the basic governing equations of unsaturated soil elastodynamics and then establish and solve the governing equations of rotary vibration of rigid friction pipe pile. Solids, liquids, and gases are represented, respectively, by the subscript s, l, and g. The relative densities of solid, liquid, and gas are defined as follows:

Table 1: Symbols table.

Symbol	Meaning
b^l, b^g	Unit liquid and gas penetration force, equations (18) and (19)
G	Shear modulus
H	The effective pile length participating in the rotary vibration
$H_1^{(1)}$	The first-order Hankel function
I_p	Moment of inertia of pile, equation (59)
I_1, I_2	The first and second orders of the first Bessel function of imaginary argument
J_1, J_2	The first- and second-order Bessel function of real arguments
K_1, K_2	The first and second orders of the second Bessel function of imaginary argument
K	Intrinsic permeability of solids
$k_M = k_{M1} + ik_{M2}$	The rotary complex stiffness of the pile body, equations (60) and (61)
M	Peak of the couple
M_p	The simple harmonic circumferential couple which the pile top sustains, equation (56)
M_s	The circumferential couple of the pile body under the coaction of soil inside and outside the pile, equation (57)
M	m represents solid, liquid, or gas, respectively, when $m = s$, l, or g
N	Porosity
p^l, p^g	Pressure of liquid and gas
Q_i^m	Volume discharge, equations (5)~(7)
$\overline{Q}_i^l$, $\overline{Q}_i^g$	Relative specific discharge, equations (8) and (9)
q	Equation (29)
q_i^m	Mass discharge, equation (4)
R_I, R_O	Internal and external radii of the pipe pile
S	Saturation
u_i	Displacement of the solid component
u_θ	Amplitude of rotational vibration
v_i^m	Velocity
w_θ, v_θ	Circumferential displacement of the liquid and gas phase components relative to the solid phase component
β	Volume compressibility of soil
β_s	Compressibility of the solid phase component material
γ	Equation (17)
δ_{ij}	Kronecker symbol
ε_{ij}	Strain of solid
η	Equation (27)
θ_p	The maximum rotation angle of the pile body
λ	Lamé constants of dry soil
μ	Lamé constants of dry soil
ν_l, ν_g	Dynamic viscosity (absolute viscosity) of liquids or gases
ρ	Density of unsaturated soil, equation (25)
ρ_m	Mass density
ρ_p	Density of pile
$\overline{\rho}_m$	Relative density, equations (1) and~(3)
σ_{ij}	Total stress, equation (10)
σ_{ij}^m	Stress corresponding to component m, equations (11) and (13)
τ	Shear stress, equation (41)
τ_{ij}	σ_{ij}^s, equation (11)
τ_{ij}'	Effective stress, equations (15) and (16)
ω	Circular frequency

$$\overline{\rho}_s = (1-n)\rho_s, \tag{1}$$

$$\overline{\rho}_l = Sn\rho_l, \tag{2}$$

$$\overline{\rho}_g = (1-S)n\rho_g, \tag{3}$$

where ρ_m ($m = s$, l, and g) represents the mass density of the corresponding component, n is the porosity, and S is the saturation. Besides, the relationship between mass discharge q_i^m and volume discharge Q_i^m is

$$q_i^m = \rho_m Q_i^m, \tag{4}$$

and the relationship between volume discharge Q_i^m and the velocity of the components in the medium v_i^m is

$$Q_i^s = (1-n)v_i^s, \tag{5}$$

$$Q_i^l = Snv_i^l, \tag{6}$$

$$Q_i^g = (1-S)nv_i^g. \tag{7}$$

According to [14], introduce $\overline{Q}_i^l$ and $\overline{Q}_i^g$ as relative specific discharge:

$$\overline{Q}_i^l = Sn\left(v_i^l - v_i^s\right), \tag{8}$$

$$\overline{Q}_i^g = (1 - S)n\left(v_i^g - v_i^s\right). \tag{9}$$

For the convenience, the superscript s can be omitted for the displacement component of the solid. Based on the above definitions and mixture theory [15], the total stress can be defined as follows:

$$\sigma_{ij} = \sigma_{ij}^s + \sigma_{ij}^l + \sigma_{ij}^g, \tag{10}$$

where s, l, and g represent the solid, liquid, and gas, respectively, and the solid stress which is the first term on right side, namely, "intergranular stress", is transmitted by the contact of the particles in the earth framework. It can be also represented as τ_{ij}:

$$\sigma_{ij}^s = \tau_{ij}. \tag{11}$$

Assuming that the liquid and gas are ideal fluids, the stress has the following forms:

$$\sigma_{ij}^l = -Sp^l\delta_{ij}, \tag{12}$$

$$\sigma_{ij}^g = -(1 - S)p^g\delta_{ij}. \tag{13}$$

In the above equation, δ_{ij} is the Kronecker symbol, p^l and p^g are the pressure of liquid and gas, respectively, and there is a minus sign in front since the fluid can only be compressed. By rewriting (10) in the incremental form, we can get

$$\Delta\sigma_{ij} = \Delta\tau_{ij} - S\Delta p^l\delta_{ij} - (1 - S)\Delta p^g\delta_{ij}. \tag{14}$$

According to the Verruijt's and Luger's view [16] which is based on the effective stress, the relationship between particles stress $\Delta\tau_{ij}$ and effective stress $\Delta\tau'_{ij}$ is as follows:

$$\Delta\tau_{ij} = \Delta\tau'_{ij} + \gamma\Delta p^l\delta_{ij}, \tag{15}$$

where $\Delta\tau'_{ij}$ depends only on the deformation of solid components. According to the linear elasticity theory, there is

$$\Delta\tau'_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij}, \tag{16}$$

where λ and μ are Lamé constants of dry soil and ε_{ij} is the strain tensor of solid. γ can be defined as follows:

$$\gamma = \frac{\beta_s}{\beta} \ll 1, \tag{17}$$

where β_s is the compressibility of the solid phase component material and β is the volume compressibility of soil. Besides, unit liquid and gas penetration forces b^l and b^g are defined as follows:

$$b^l = \frac{\nu_l}{k}, \tag{18}$$

$$b^g = \frac{\nu_g}{k}, \tag{19}$$

where ν_l and ν_g,respectively, correspond to the dynamic viscosities of liquids and gases, also called absolute viscosities, k is the intrinsic permeability of solids. Based on equation (14) and the nearly saturation hypothesis, reference [14] established the dynamic governing equation of unsaturated soil:

$$(\lambda + \mu)\frac{\partial^2 u_k}{\partial x_k \partial x_i} + \mu\frac{\partial^2 u_i}{\partial x_k \partial x_k} + \gamma\frac{\partial\Delta p^l}{\partial x_i} = -b^l\overline{Q}_i^l - b^g\overline{Q}_i^g + \overline{\rho}_s\ddot{u}_i, \tag{20}$$

$$-S\frac{\partial\Delta p^l}{\partial x_i} = b^l\overline{Q}_i^l + \rho_l\dot{\overline{Q}}_i^l + \overline{\rho}_l\ddot{u}_i, \tag{21}$$

$$-(1 - S)\frac{\partial\Delta p^g}{\partial x_i} = b^g\overline{Q}_i^g + \rho_g\dot{\overline{Q}}_i^g + \overline{\rho}_g\ddot{u}_i, \tag{22}$$

where u_i is the displacement of the solid component.

3. Equations and Solutions of Rotary Vibration of Rigid Friction Pipe Pile

3.1. Governing Equation. Firstly, rewrite the displacement of liquid and gas relative to solid as w_i and v_i [17]. Suppose that the pile top sustains a simple harmonic circumferential couple $M_p = Me^{i\omega t}$ (M is the peak of the couple). Considering the symmetry of the model and assuming that all the movements are simple harmonic, the circumferential displacement of the solid phase component has the form $u_\theta(r, t) = u_\theta e^{i\omega t}$ and the circumferential displacement of the liquid and gas phase components relative to the solid phase component is $w_\theta(r, t) = w_\theta e^{i\omega t}$ and $v_\theta(r, t) = v_\theta e^{i\omega t}$ (u_θ, w_θ, and v_θ are the amplitude of rotational vibration), and the rest of the components of the displacement are zero. Equations (20)~(22) can be simplified as follows:

$$\left.\begin{aligned} &\mu\left(\nabla^2 - \frac{1}{r^2}\right)u_\theta = -\rho\omega^2 u_\theta - \rho_l\omega^2 w_\theta - \rho_g\omega^2 v_\theta \\ &i\omega b^l w_\theta - \rho_l\omega^2 w_\theta - \overline{\rho}_l\omega^2 u_\theta = 0 \\ &i\omega b^g v_\theta - \rho_g\omega^2 v_\theta - \overline{\rho}_g\omega^2 u_\theta = 0 \end{aligned}\right\}, \tag{23}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial}{\partial r}. \tag{24}$$

It is the Laplace operator of a central symmetric plane problem; ρ is the density of unsaturated soil:

$$\rho = \overline{\rho}_s + \overline{\rho}_l + \overline{\rho}_g. \tag{25}$$

Substitute the latter two equations of (23) into the first equation and rewrite μ as the shear modulus G:

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial}{\partial r} - \frac{1}{r^2}\right)u_\theta = \frac{\eta}{G}u_\theta, \tag{26}$$

where

$$\eta = -\omega^2\left(\rho + \frac{\bar{\rho}_l\omega}{ig/(k_l-\omega)} + \frac{\bar{\rho}_g\omega}{ig/\left(k_g-\omega\right)}\right). \tag{27}$$

Equation (26) is the basic governing equation of the rotary vibration of rigid friction pile in unsaturated soil to be studied in this article.

3.2. Solution of Rotary Vibration Displacement of Rigid Friction Pipe Pile. Rearrange equation (26) as follows:

$$\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial u_\theta}{\partial r} - \left(\frac{1}{r^2} + q^2\right)u_\theta = 0, \tag{28}$$

where

$$q^2 = \frac{\eta}{G}. \tag{29}$$

Equation (28) is the first-order Bessel equation of imaginary argument, the solution of which is

$$u_\theta = C_a I_1(qr) + C_b K_1(qr), \tag{30}$$

where I_1 and K_1 are the first order of the first and second Bessel functions of imaginary argument, respectively, and C_a and C_b are arbitrary constants. I_1 and K_1 have the following properties (according to chapter 7 of reference [18]):

$$\left.\begin{array}{l} I_1(z)|_{z=0} = 0 \\ I_1(z)|_{z=\infty} = \infty \end{array}\right\}, \tag{31}$$

$$\left.\begin{array}{l} K_1(z)|_{z=0} = \infty \\ K_1(z)|_{z=\infty} = 0 \end{array}\right\}, \tag{32}$$

where z is a complex number whose real part is nonzero. In the rotary problem, the pile modulus (dozens GPa) is much larger than that of soil around the pile (dozens MPa), so the pile can be considered as rigid body. For the pipe pile, assume the inside is also filled with soil and the maximum rotation angle of the pile body is θ_p, then the boundary conditions can be taken as follows:

$$u_{I\theta}|_{r=R_I} = R_I\theta_p, \tag{33}$$

$$u_{O\theta}|_{r=R_O} = R_O\theta_p, \tag{34}$$

where the subscript I represents inside and O represents outside. Naturally, the following boundary conditions shall be provided at the origin and at infinity, respectively:

$$u_{I\theta}|_{r=0} < \infty, \tag{35}$$

$$u_{O\theta}|_{r=\infty} < \infty. \tag{36}$$

For in-pile soil, arbitrary constants in (30) can be determined by using equations (31), (33), and (35):

$$\begin{aligned} C_a &= \frac{R_I\theta_p}{I_1(qR_I)}, \\ C_b &= 0. \end{aligned} \tag{37}$$

Similarly, for out-pile soil, arbitrary constants in (30) can be determined by using equations (32), (34), and (36):

$$\begin{aligned} C_a &= 0, \\ C_b &= \frac{R_O\theta_p}{K_1(qR_O)}. \end{aligned} \tag{38}$$

Therefore, for soil inside and outside the pile, the displacement without factor $e^{i\omega t}$ is as follows, respectively:

$$u_{I\theta} = \frac{R_I\theta_p}{I_1(qR_I)} I_1(qr), \tag{39}$$

$$u_{O\theta} = \frac{R_O\theta_p}{K_1(qR_O)} K_1(qr). \tag{40}$$

3.3. Calculation of Complex Stiffness of Rotary Vibration of Rigid Friction Pipe Pile. In order to calculate the dynamic stiffness of pile, the stress is first calculated. The shear stress is as follows:

$$\tau = G\left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right)e^{i\omega t}. \tag{41}$$

For soil inside and outside the pile, there are

$$\tau_I = G\frac{R_I\theta_p}{I_1(qR_I)}\left(\frac{\partial I_1(qr)}{\partial r} - \frac{I_1(qr)}{r}\right)e^{i\omega t}, \tag{42}$$

$$\tau_O = G\frac{R_O\theta_p}{K_1(qR_O)}\left(\frac{\partial K_1(qr)}{\partial r} - \frac{K_1(qr)}{r}\right)e^{i\omega t}. \tag{43}$$

In order to calculate the above two equations, the properties of the first and the second-order Bessel functions of imaginary argument need to be considered. According to chapter 7 of reference [18],

$$I_1(z) = \begin{cases} e^{-(\pi/2)i} J_1\left(ze^{(i\pi/2)}\right), & \left(-\pi < \arg z < \dfrac{\pi}{2}\right), \\ e^{(3/2)\pi i} J_1\left(ze^{-(3/2)\pi i}\right), & \left(\dfrac{\pi}{2} < \arg z < \pi\right), \end{cases} \tag{44}$$

$$K_1(z) = \frac{\pi i}{2} e^{(\pi/2)i} H_1^{(1)}\left(ze^{(\pi/2)i}\right), \tag{45}$$

where J_1 is the first-order Bessel function of real arguments and $H_1^{(1)}$ is the first order of the first Hankel function. To simplify the above two equations, when q is calculated according to equations (29) and (27), the real part of q can be nonnegative. In this way, the amplitude angle of the independent variable can be taken between $-\pi$ and $\pi/2$, so that equation (44) can be taken with only the upper half branch. In addition, according to Euler's formula in complex analysis,

$$e^{i\theta} = \cos\theta + i\sin\theta. \tag{46}$$

If $\theta = \pi$, the equations (44) and (45) can be reduced to

$$I_1(z) = i^{-1}J_1(iz), \tag{47}$$

$$K_1(z) = \frac{\pi}{2}i^2 H_1^{(1)}(iz). \tag{48}$$

According to chapter 7 of reference [18], the following equations can be established:

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(z^{-1}J_1\right) = -z^{-1}J_2, \tag{49}$$

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(z^{-1}H_1^{(1)}\right) = -z^{-1}H_1^{(1)}. \tag{50}$$

After substituting equations (47) and (48) into (49) and (50), respectively, we can get

$$\frac{\partial I_1(z)}{\partial z} - \frac{I_1(z)}{z} = -I_2(z), \tag{51}$$

$$\frac{\partial K_1(z)}{\partial z} - \frac{K_1(z)}{z} = -K_2(z). \tag{52}$$

Substitute the above two equations into equations (42) and (43), respectively, and we can get

$$\tau_I = -Gq\frac{R_I\theta_p}{I_1(qR_I)}I_2(qr)e^{i\omega t}, \tag{53}$$

$$\tau_O = -Gq\frac{R_O\theta_p}{K_1(qR_O)}K_2(qr)e^{i\omega t}. \tag{54}$$

Considering that the pile body rotates under the coaction of external load on the pile top and soil around the pile, the dynamic balance equation is as follows:

$$M_p + M_s = I_p\frac{\mathrm{d}^2\left(\theta_p e^{i\omega t}\right)}{\mathrm{d}t^2} = -\omega^2 I_p\theta_p e^{i\omega t}, \tag{55}$$

where M_p is the simple harmonic circumferential couple which the pile top sustains:

$$M_p = Me^{i\omega t}, \tag{56}$$

where M is the peak of the couple of forces and M_s is the circumferential couple of the pile body under the coaction of soil inside and outside the pile:

$$M_s = H\cdot 2\pi R_O\cdot \tau_O|_{r=R_O}\cdot R_O - H\cdot 2\pi R_I\cdot \tau_I|_{r=R_I}\cdot R_I, \tag{57}$$

where H is the effective pile length participating in the rotary vibration. By substituting equations (53) and (54) into (57), it can be obtained that

$$M_s = -2\pi GqH\theta_p e^{i\omega t}\left[R_O^3\frac{K_2(qR_O)}{K_1(qR_O)} - R_I^3\frac{I_2(qR_I)}{I_1(qR_I)}\right]. \tag{58}$$

In addition, I_p is the moment of inertia of pile:

$$I_p = \pi\rho_p H\frac{R_O^4 - R_I^4}{2}, \tag{59}$$

where ρ_p is the density of the pile.

After substituting equations (56), (58), and (59) into (55), it can be simplified as follows:

$$k_M = \frac{M}{\theta_p} = 2\pi GqH\left[R_O^3\frac{K_2(qR_O)}{K_1(qR_O)} - R_I^3\frac{I_2(qR_I)}{I_1(qR_I)}\right] - \omega^2\pi\rho_p H\frac{R_O^4 - R_I^4}{2}, \tag{60}$$

where k_M is the rotary stiffness of the pile body and a complex function. It can be written as follows:

$$k_M = k_{M1} + ik_{M2}, \tag{61}$$

where k_{M1} is the real part, representing the true stiffness of the pile body rotation, and k_{M2} is the imaginary part, which reflects the dissipation of energy, and it is referred to as the rotational impedance in this article.

Finally, it is pointed out that the proposed method can be extended in principle to the saturated porous medium containing any number of fluid components. Since the analysis process is similar, it will not be repeated here.

4. Degeneration Problems

4.1. Degradation of Saturated Medium. In contrast to unsaturated soil, saturated soil is the special case of unsaturated soils at saturation $S = 1$. In this case, the pores in the soil particles are all filled with incompressible fluid (liquid). The medium degenerates from a three-phase body to a two-phase body. All the effects of the gas phase components on the governing equation can be treated as 0. Equation (29) is changed to the following form (subscript sat indicates saturation):

$$q_{\mathrm{sat}}^2 = \frac{-\omega^2}{G}\left(\rho_{\mathrm{sat}} + \frac{\overline{\rho}_{l\mathrm{sat}}\omega}{ig/(k_l - \omega)}\right). \tag{62}$$

Therefore, when $S = 1$, displacements, shear stresses, and complex stiffness deform as follows:

$$\begin{aligned} u_{I\theta\mathrm{sat}} &= \frac{R_I\theta_p}{I_1(q_{\mathrm{sat}}R_I)}I_1(q_{\mathrm{sat}}r),\\ u_{O\theta\mathrm{sat}} &= \frac{R_O\theta_p}{K_1(q_{\mathrm{sat}}R_O)}K_1(q_{\mathrm{sat}}r),\\ \tau_{I\mathrm{sat}} &= -Gq_{\mathrm{sat}}\frac{R_I\theta_p}{I_1(q_{\mathrm{sat}}R_I)}I_2(q_{\mathrm{sat}}r)e^{i\omega t},\\ \tau_{O\mathrm{sat}} &= -Gq_{\mathrm{sat}}\frac{R_O\theta_p}{K_1(q_{\mathrm{sat}}R_O)}K_2(q_{\mathrm{sat}}r)e^{i\omega t}. \end{aligned} \tag{63}$$

In addition, since liquid phase and gas phase have exactly the same status in equation (27), this degradation mode is also applicable to the situation that the pores in soil particles are completely filled by gas and do not contain water.

4.2. Degradation of Single-Phase Elastic Medium. On the surface, this problem seems very easy. If we start from the equation of the saturated medium in the previous section and just let the porosity $n = 0$, rewrite μ as the shear modulus G, and use the subscript e to be elasticity, equations (29) and (30) change to the following form:

$$q_e^2 = -\frac{\rho_{es}\omega^2}{G}, \tag{64}$$

$$u_{\theta e} = C_{ae} I_1(q_e r) + C_{be} K_1(q_e r). \tag{65}$$

However, the following process of determining the unknown coefficients C_{ea} and C_{eb} encountered difficulties. In the equations (31) and (32) referenced before, z needs to be a complex number whose real part is nonzero. In contrast, the right end of equation (64) is a negative real number without an imaginary part. This means that the independent variable $q_e r$ in equation (65) is a pure imaginary number. The physical explanation is that the absence of fluid results in a pure elastic reaction of solid components in the process of motion without any phase delay or energy dissipation [19]. In this case, equation (32) is valid. For the soil inside the pile, the displacement formula is as follows:

$$u_{I\theta e} = \frac{R_I \theta_p}{I_1(q_e R_I)} I_1(q_e r). \tag{66}$$

But, equation (31) is not valid.

Since the condition that fluid mass is nonzero may lead to the situation that the degradation is unable to be established, we can make the fluid mass tend to a very small positive value. There are two methods: to make the porosity tend to be a very small positive value in the saturated medium; to make the fluid density tend to be a very small positive value. The former is equivalent to assuming that the pores between soil particles in saturated soils are very small, and the latter is equivalent to assuming that the pores of soil particles contain only air and that the density of air is very small. Consider the former approach first, since zero porosity will cause q_e to change from a complex number whose real part is not zero to a pure imaginary number, the porosity of the saturated medium can then be taken to be a very small, but not zero, positive number (for example, 10^{-10000}). So, the function I_1 can be eliminated deterministically. But, at the same time, because the value of porosity is extremely small, it means that the value of the second term in the bracket of the right end of equation (62) will be much smaller than the value of the first term, and its effect on the calculation results of K_1 can be ignored. The displacement of soil outside the pile is calculated in Equation (67).

According to equations (65) and (32) and, at the same time, according to the previous discussion let $z = q_e r$, repeating the previous derivation, we can get

$$u_{O\theta e} = \frac{R_O \theta_p}{K_1(q_e R_O)} K_1(q_e r). \tag{67}$$

The displacement mode of soil inside the pile can be determined by referring to equation (66):

$$u_{I\theta e} = \frac{R_I \theta_p}{I_1(q_e R_I)} I_1(q_e r). \tag{68}$$

The shear stresses of soil inside and outside the pile can be calculated as follows:

$$\begin{aligned} \tau_{Ie} &= -G q_e \frac{R_I \theta_p}{I_1(q_e R_I)} I_2(q_e r) e^{i\omega t}, \\ \tau_{Oe} &= -G q_e \frac{R_O \theta_p}{K_1(q_e R_O)} K_2(q_e r) e^{i\omega t}. \end{aligned} \tag{69}$$

Finally, the formula for calculating the complex stiffness of the rotary vibration of the pipe pile in the single-phase elastic medium under the simultaneous action of pile top force couple and the shear stress of soil outside and inside the pile is as follows:

$$\begin{aligned} k_{Me} = \frac{M}{\theta_p} &= 2\pi G q_e H \left[R_O^3 \frac{K_2(q_e R_O)}{K_1(q_e R_O)} - R_I^3 \frac{I_2(q_e R_I)}{I_1(q_e R_I)} \right] \\ &\quad - \omega^2 \pi \rho_p H \frac{R_O^4 - R_I^4}{2} .a + c. \end{aligned} \tag{70}$$

The results obtained by making the fluid density tend to a very small positive value are identical in form to the above equations.

It must be noted here that there are limits to either reducing porosity or fluid density. The porosity of soils is generally of the order of 0.1, while the density of air at 273.15 K is 1.29 kg/m^3 at one standard atmospheric pressure. But, consider that the density of soil particles is usually 2600~2700 kg/m^3 and three orders of magnitude larger than air. So, maybe the latter approach is more realistic.

4.3. Degradation of Static Problems. The static problem corresponds to that the rotation frequency ω is equal to 0. According to equation (27), the q used in the dimensionless process becomes 0 directly. Thus, in the final analytical expressions of physical quantities of displacements and shear stresses, the limit when $q \longrightarrow 0$ is directly taken to obtain the displacement and shear stresses, respectively, as follows:

$$u_{I\theta \mathrm{st}} = \theta_p r, \tag{71}$$

$$u_{O\theta \mathrm{st}} = \frac{R_O^2 \theta_p}{r}, \tag{72}$$

$$\tau_{I\mathrm{st}} = 0, \tag{73}$$

$$\tau_{O\mathrm{st}} = -\frac{2 G R_O^2 \theta_p}{r^2}. \tag{74}$$

The subscript st stands for static. In equation (55), let $\omega = 0$, we get

$$M_p + M_s = 0. \tag{75}$$

Substitute equations (73) and (74) into equation (57) and substitute $\omega = 0$ into equation (56). By substituting the obtained results into equation (55), the following equation can be obtained:

$$k_{M\text{st}} = \frac{M}{\theta_p} = 4\pi G H R_O^2, \tag{76}$$

where k_M is the rotary stiffness of pile body and is a real number. The fact that there is no imaginary part also indicates that there is no impedance effect in the dynamics of this static problem.

4.4. Degradation of Solid Pile. If the internal radius $R_I = 0$, all equations can be degraded to conclusions for rigid solid friction piles. At this point, the "soil inside the pile" mentioned above does not exist. Since there is no principled difficulty in this degradation, the specific equations are omitted.

5. Calculation Examples and Analysis

In order to analyze the problem in this article specifically, the following example is constructed. The specific parameters are as follows: $\rho_p = 2500\,\text{kg/m}^3$, $\rho_s = 2600\,\text{kg/m}^3$, $\rho_l = 1000\,\text{kg/m}^3$, $p_g = 1.29\,\text{kg/m}^3$, $S = 0.5$, $n = 0.3$, $b_l = 10^7\text{N·s/m}^4$, $b_g = 10^4\text{N·s/m}^4$, $G = 2.5 \times 10^7$ Pa, $R_O = 0.3$ m, $R_I = 0.22$ m, and $H = 20$ m. By adjusting the frequency range, the stiffness and rotational impedance of the pile body obtained by calculation are shown in Figures 2 and 3.

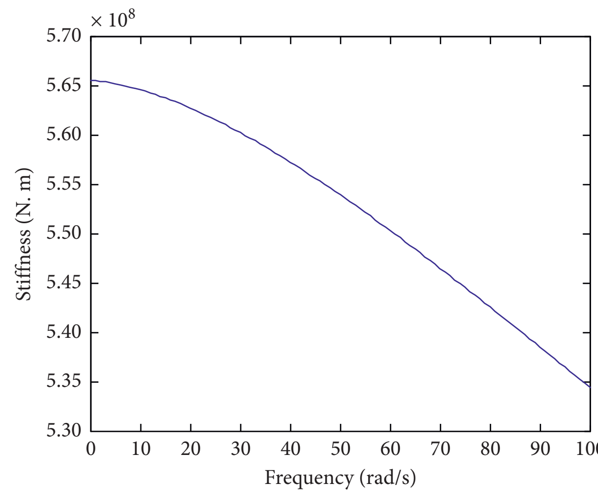

Figure 2: Stiffness varies with frequency.

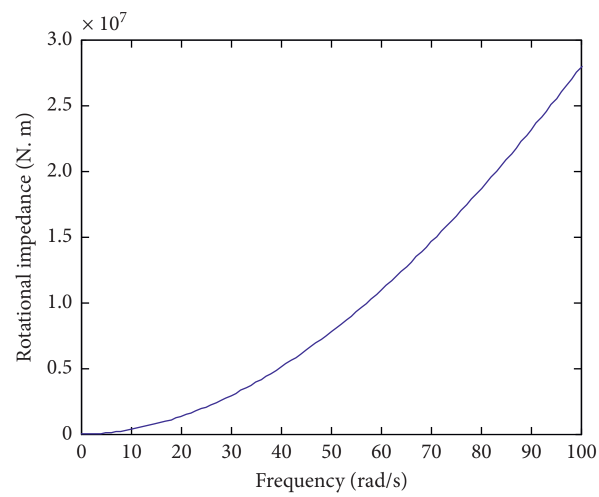

Figure 3: Rotational impedance varies with frequency.

It can be seen from the two figures that, within a given frequency range, the stiffness decreases and the rotational impedance increases while the frequency increases. The decrease of stiffness is not very drastic. When the frequency increases from 0 to 100 rad/s, the stiffness decreases from about 5.65×10^8N·m to about 5.35×10^8N·m, only reducing by about 5%. However, in contrast, the change of rotational impedance is more drastic. When the frequency increases from 0 to 100 rad/s, the rotational impedance increases rapidly from 0 to about 2.7×10^7N·m. Moreover, it can be seen from Figure 3 that the rotational impedance grows faster. However, in this example, even the magnitude of the maximum rotational impedance (about 2.7×10^7N·m) is an order smaller than the stiffness value. In addition, the variation law of the stiffness and rotational impedance in this rotary vibration is basically the same as that of the torsional vibration stiffness and impedance in reference [20].

In addition, the influence of other factors on stiffness and rotational impedance can be further analyzed. Figures 4 and 5 show the calculated results of the stiffness and rotational impedance of porosity $n = 0.26$, 0.28, and 0.30 adjusted, respectively, under the condition of fixed saturation $S = 0.5$ and other parameters unchanged.

It must be pointed out that porosity has a great influence on soil modulus. In this example, the soil modulus is assumed to remain the same during the slight adjustment of porosity in order to avoid the interaction between variables (the analysis object of this example can also be considered as several different types of soil with the same modulus but different porosities). Because the change in porosity in this example is very small, the difference between the three curves is not significant. However, as can be seen from the numerical results, as for the stiffness, its value will increase with the increase of porosity. However, the rotational impedance decreases with the increase of porosity, and the range of rotational impedance changes is larger than that of stiffness. From the qualitative results, it can be seen that the more solid the soil has, the more obvious its elasticity will be. At this time, the external force needed to maintain the periodic elastic vibration of the system is smaller, that is, the stiffness is smaller.

Please notice that the definition of rotary complex stiffness in this article ($k_M = M/\theta_p$) is special. If we apply a similar definition to a periodic natural vibration system without attenuation, the stiffness of the system is zero, and that is because the external force is zero, but the amplitude is nonzero (natural vibration).

Fix porosity $n = 0.3$, other parameters as before, and adjusted saturation of 0.5, 0.7, and 0.9, respectively, to obtain the law of stiffness and rotational impedance changing with saturation, as shown in Figures 6 and 7.

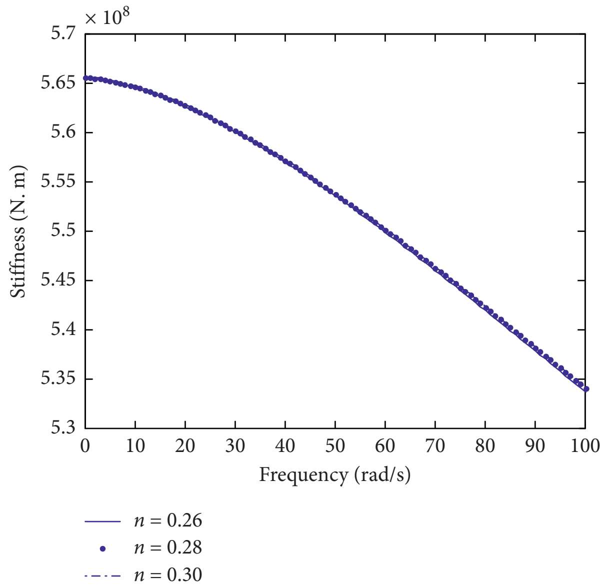

FIGURE 4: Variation curve of stiffness with frequency under different porosities.

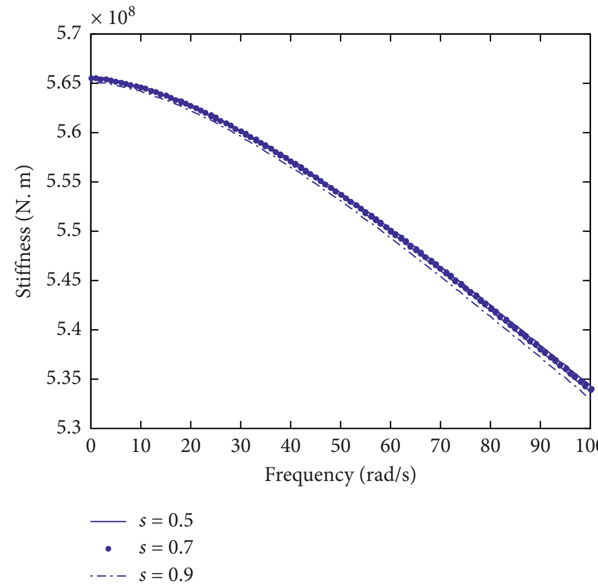

FIGURE 6: Variation curve of stiffness with frequency under different saturations.

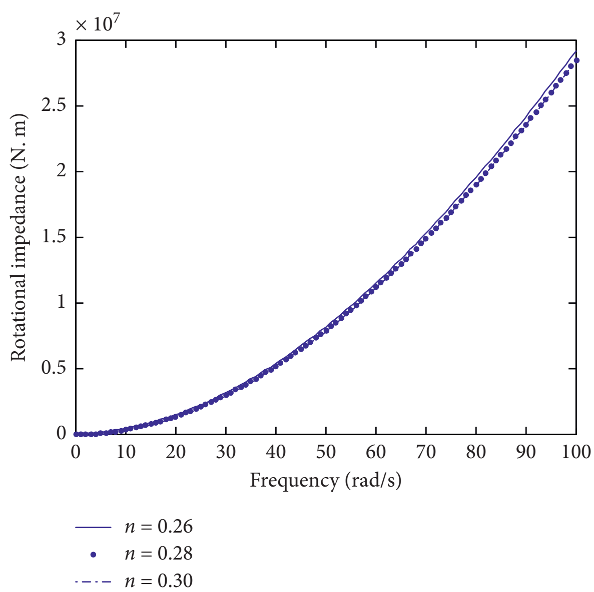

FIGURE 5: Variation curve of rotational impedance with frequency under different porosities.

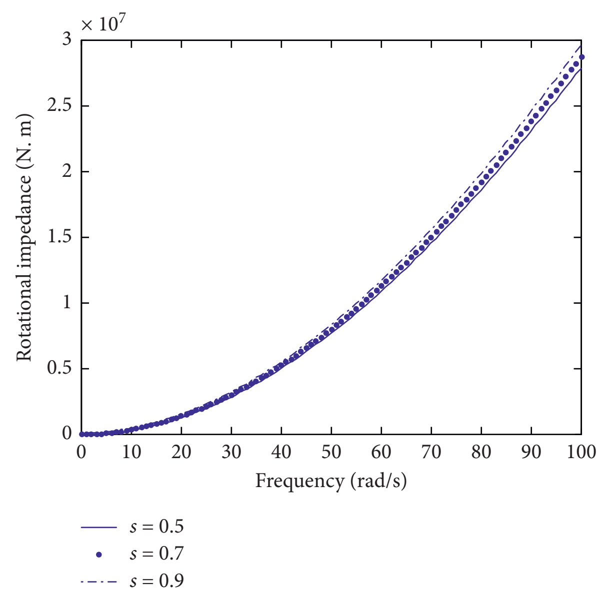

FIGURE 7: Variation curve of rotational impedance with frequency under different saturations.

Figures 6 and 7 have the same problem as of Figures 4 and 5; the variations of data are small, and the reflections in these figures are not obvious. However, it can be seen from the numerical results that the stiffness decreases slightly with the increase of saturation. This is mainly due to the increase of liquid in the pores, which increases the inertia of the rotary vibration and makes the external force slightly reduced. According to Figure 7, the rotational impedance increases with increasing saturation. This shows that liquid dissipates more energy than gas. But, the increase is very small. Based on the above analysis, the contribution of the three components to stiffness is as follows: solid < liquid < gas; contribution to rotational impedance: solid > liquid > gas (again, please note that the more the solid, the greater the elasticity, the smaller the external force required for periodic vibration, and the smaller the stiffness).

By adjusting the value of unit liquid penetration force b_l and other parameters as the first example in this section, the corresponding variation of stiffness and rotational impedance can be obtained, as shown in Figures 8 and 9.

As can be seen from Figures 8 and 9, when b_l increases (i.e., permeability coefficient decreases), the stiffness decreases slightly but has almost no effect. Moreover, when its order of magnitude reaches 10^7, a limit state almost appears, that is, the

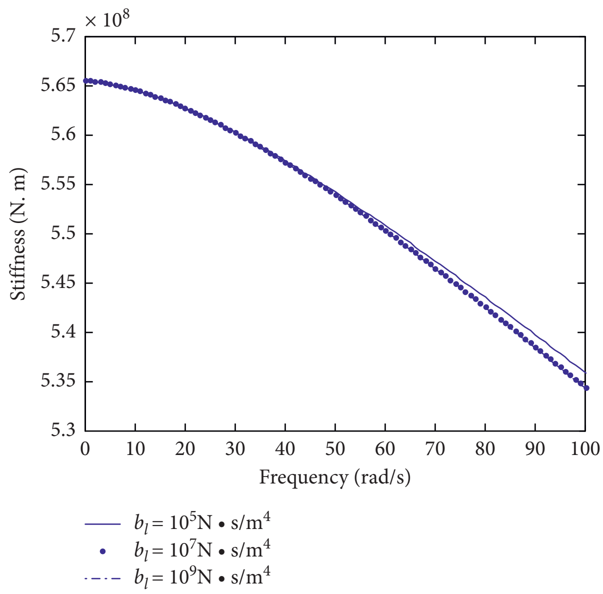

Figure 8: Variation curve of stiffness with frequency under different b_l.

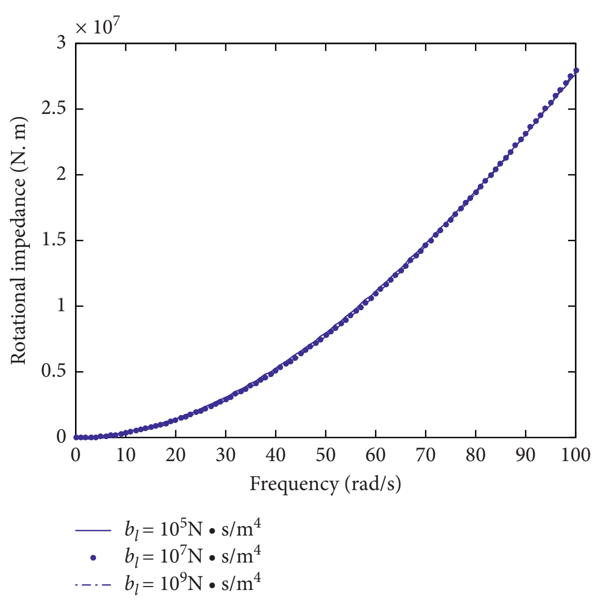

Figure 9: Variation curve of rotational impedance with frequency under different b_l.

further increase of b_l makes stiffness almost unchanged. The rotational impedance is basically the same in the figure and only slightly increases in the result value. Physically, reducing the liquid permeability coefficient is equivalent to making the solid easier to maintain natural vibration, similar to the previous analysis of adjusting porosity. Therefore, it is equivalent to a reduction in stiffness, which corresponds to an increase in the rotational impedance.

By adjusting the value of unit gas penetration force b_g, other parameters as the first example in this section, the corresponding variation of stiffness and rotational impedance can be obtained, as shown in Figures 10 and 11.

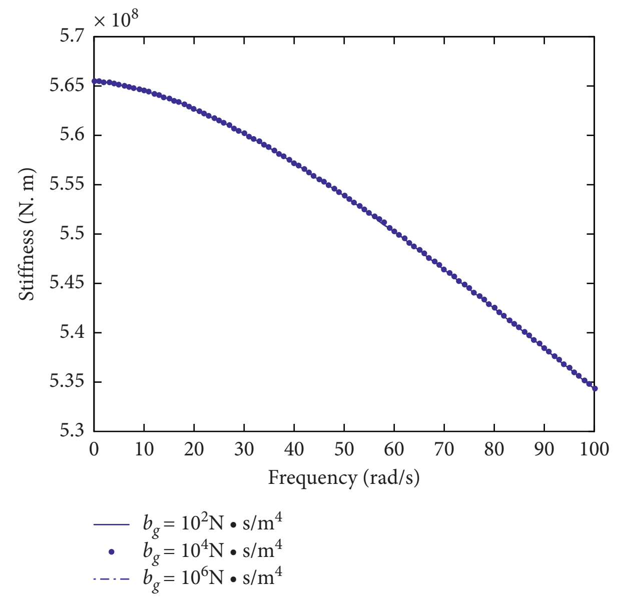

Figure 10: Variation curve of stiffness with frequency under different b_g.

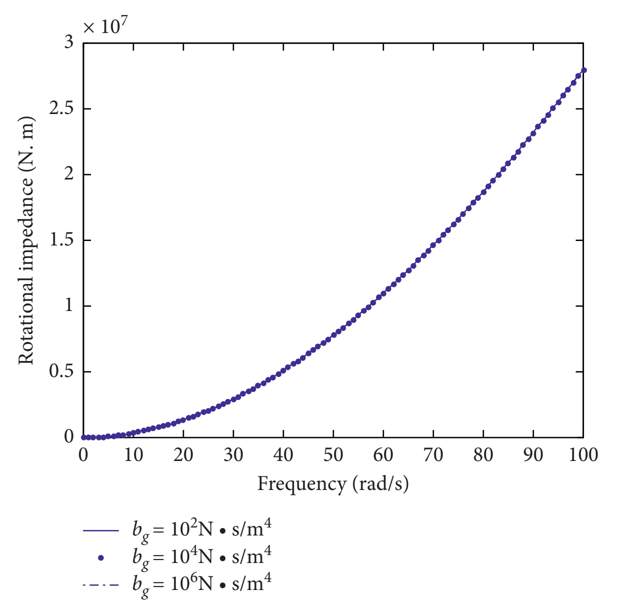

Figure 11: Variation curve of rotational impedance with frequency under different unit gas penetration forces.

As can be seen from Figures 10 and 11, gas permeability has almost no effect on the calculation results. Although it can be seen from the numerical results that the influence trend of gas permeability on stiffness and rotational impedance is similar to the previous results of liquid permeability, from the perspective of engineering, the density of gas itself is too small to affect the dynamic characteristics of the system significantly. Therefore, the effect of gas permeability can be ignored in the analysis of practical engineering.

6. Conclusion

In this article, the mixture theory under the framework of continuum mechanics is used as the basic tool, and the field equations are established by using, Vardoulakis and Beskos' methods. This part of work is based on the previous work of the other scholars. The specific work and conclusions of this article are as follows:

(1) The concept of rotary vibration of rigid friction pile is established. Referring to the analysis method of torsional vibration, the basic governing equations of rotary vibration of rigid friction pile in unsaturated soil are put forward and the model is extended from solid pile to pipe pile.

(2) The solution of the rotary vibration problem of the rigid friction pipe pile in unsaturated soil is obtained. For the soil inside the pile and the soil outside the pile, the analytical expressions of such physical quantities as circumferential displacements, shear stresses, and complex stiffness of rotary vibration are given, respectively.

(3) Based on the extensiveness of the model in this article, four degradations are given: degradation of saturated soil, degradation of single-phase elastic medium, degradation of static problem, and degradation of solid pile. The relevant formulas have been given.

(4) Based on the analysis of the example, the variation laws of stiffness and rotational impedance with frequency, porosity, and saturation are obtained, respectively. The different influences of solid, liquid, and gas components are obtained. Contribution to stiffness is as follows: solid < liquid < gas; contribution to rotational impedance is as follows: solid > liquid > gas. Also, the influence of liquid permeability and gas permeability is analyzed; when the fluid permeability coefficient decreases (fluid penetration force increases), the stiffness decreases and the rotational impedance increases, but the influence is not obvious (especially the gas permeability)

These conclusions have reference significance for the engineering in which pipe piles might be under dynamic loading. In addition, based on the discussion on acceleration in reference [21], the discussion of normal stress of pile and soil can be referred to in reference [22].

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

The authors Fu-yao Zhao and Peng Xiang have contributed equally to this work.

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References

[1] J. K. Wu, M. Z. Wang, and W. Wang, *An Introduction to Mechanics of Elasticity*p. 98, 2001, in Chinese, revised edition.

[2] M. Novak and K. Sachs, "Torsional and coupled vibrations of embedded footings," *Earthquake Engineering & Structural Dynamics*, vol. 2, no. 1, pp. 11–33, 1973.

[3] K. Wang, Z. Zhang, C. J. Leo, and K. Xie, "Dynamic torsional response of an end bearing pile in saturated poroelastic medium," *Computers and Geotechnics*, vol. 35, no. 3, pp. 450–458, 2008.

[4] Z. Q. Zhang, "Study on pile torsional vibration in heterogeneous saturated soils," Ph.D. thesis in Chinese, Zhejiang University, Hangzhou, China, 2008.

[5] M. Y. Fattah, B. S. Zbar, and F. S. Mustafa, "Effect of soil saturation on load transfer in a pile excited by pure vertical vibration," *Proceedings of the Institution of Civil Engineers—Structures and Buildings*, pp. 1–14, 2019.

[6] M. Y. Fattah, R. R. Al-Omari, and S. H. Fadhil, "Load sharing and behavior of single pile embedded in unsaturated swelling soil," *European Journal of Environmental and Civil Engineering*, vol. 24, no. 12, pp. 1967–1992, 2018.

[7] M. Y. Fattah, B. S. Zabar, and F. S. Mustafa, "Effect of saturation on response of a single pile embedded in saturated sandy soil to vertical vibration," *European Journal of Environmental and Civil Engineering*, vol. 24, no. 3, pp. 381–400, 2017.

[8] M. Y. Fattah, B. S. Zbar, and F. S. Mustafa, "Vertical vibration capacity of a single pile in dry sand," *Marine Georesources & Geotechnology*, vol. 35, no. 8, pp. 1111–1120, 2017.

[9] S. C. Chikr, K. Abdelhakim, A. A. Bousahla et al., "A novel four-unknown integral model for buckling response of FG sandwich plates resting on elastic foundations under various boundary conditions using Galerkin's approach," *Geomechanics and Engineering*, vol. 21, no. 5, pp. 471–487, 2020.

[10] R. Salah, A. A. Bousahla, B. Abdelhakim et al., "Effect of hygro-thermo-mechanical conditions on the buckling of FG sandwich plates resting on elastic foundations," *Computers and Concrete*, vol. 25, no. 4, pp. 311–325, 2020.

[11] M. Kaddari, A. Kaci, A. A. Bousahla et al., "A study on the structural behaviour of functionally graded porous plates on elastic foundation using a new quasi-3D model: bending and free vibration analysis," *Computers and Concrete*, vol. 25, no. 1, pp. 37–57, 2020.

[12] M. Rabhi, K. H. Benrahou, A. Kaci, and M. S. A. Houari, "A new innovative 3-unknowns HSDT for buckling and free vibration of exponentially graded sandwich plates resting on elastic foundations under various boundary conditions," *Geomechanics and Engineering*, vol. 22, no. 2, pp. þ119–132, 2020.

[13] A. Tounsi, S. U. Al-Dulaijan, M. A. Al-Osta et al., "A four variable trigonometric integral plate theory for hygro-

thermo-mechanical bending analysis of AFG ceramic-metal plates resting on a two-parameter elastic foundation," *Steel and Composite Structures*, vol. 34, no. 4, pp. 511–524, 2020.
[14] I. Vardoulakis and D. E. Beskos, "Dynamic behavior of nearly saturated porous media," *Mechanics of Materials*, vol. 5, no. 1, pp. 87–108, 1986.
[15] R. M. Bowen, *Theory of Mixtures*, Academic Press, New York, NY, USA, 1976.
[16] P. V. Verruijt and H. J. Luger, "Some remarks on the principle of effective stress," in *Proceedings of the IUTAM Conference on Deformation and Failure of Granular Materials*, p. 167, Delft, Netherlands, August 1982.
[17] Z.-Q. Zhang, K.-H. Wang, and K.-H. Xie, "Torsional vibration of a pile embedded in unsaturated soil," *Chinese Journal of Geotechnical Engineering*, vol. 28, no. 6, pp. 729–734, 2006, in Chinese.
[18] Z. X. Wang and D. R. Guo, *Special Functions*, World Scientific Publishing, Singapore, 1989.
[19] F. Y. Zhao, *Analysis of Rotary Vibration of Rigid Friction Pile and its Extension in Analysis of Torsion Vibration of End-Bearing Pile*, Tsinghua University, Beijing, China, 2015, in Chinese.
[20] Z. Q. Zhang, *Study on Pile Torsional Vibration in Heterogenous Saturated Soils*, Zhejiang University, Hangzhou, China, 2008.
[21] F. Y. Zhao and E.-X. Song, "A note about cylindrical wave equation," *Soil Dynamics and Earthquake Engineering*, vol. 115, no. 4, pp. 933-934, 2018.
[22] F. Y. Zhao, E.-X. Song, and J. Yang, "Stress analysis of rotary vibration of rigid friction pile and stress general solution of central symmetry plane elastic problem," *Mathematical Problems in Engineering*, vol. 2015, Article ID 685941, 12 pages, 2015.