

Research Article

Inversion of the Equivalent Electric Dipole Moment of Ship's Corrosion-Related Static Electric Field in Frequency Domain

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When evaluating the ship's underwater electric field stealth, the underwater electric potential or the underwater electric field is often used, but it is easily affected by the environment conditions. As a result, the evaluating accuracy is not high. To solve this problem, the equivalent electric dipole moment is used as the evaluating factor in this paper. Firstly, the method of inverting the equivalent electric dipole moment in the frequency domain is proposed. However, the limited measuring range will also lead to some errors based on the proposed method. As a result, we improve the proposed method by applying an integral correction which uses a standard dipole source. To test the effectiveness of this method, a simulation experiment is carried out, and the results show that the method has high inversion accuracy even in a low signal-to-noise ratio (SNR) environment. This method has provided a new technological approach for evaluating the ship's corrosion-related static electric field.

1. Introduction

The corrosion-related static electric field is an important exposure for warships and attacking source for underwater weapons [1]. To avoid being attacked by mines equipped with electric field fuse, the electric field decreasing technology has been generally used in the ship design and building in various countries [2–4]. When evaluating the performance of decreasing static electric field and determining the optimal tactics in stealth control, the electric potential difference U or the peak-peak value of electric field E below the ship has usually been the evaluating factor [5]. However, the peak-peak value of electric field is easily affected by the background noise. As a result, it will cause some error in evaluating the ship's electric field stealth performance.

To solve the problem mentioned above, the equivalent electric dipole moment can be used as the evaluating factor because the ship's static electric field can be approximately viewed as a horizontal electric dipole field [6]. Therefore, the key of solution is to calculate the equivalent electric dipole moment accurately. It can be calculated by the forward

method (in which the ship surface electric potential, shaft current, and etc., should be measured) or the inversion method (in which only the underwater electric field should be measured) [7–9].

The ship's underwater electric field changes with temperature, salinity, and oxygen level of seawater, as well as the ship coating status [10]. As a result, the forward method is mostly used in software with the finite element method (FEM), boundary element method (BEM), or generalized finite difference method (GFDM) [11, 12]. To get the ship surface electric potential, a large number of electric field sensors should be installed on the ship surface. It is obvious that this method is difficult to implement in practice. However, the inversion method only needs to measure the underwater electric field. Then, the ship's electric model is established based on the relationship between the equivalent sources (e.g., point charges, dipoles, and current lines) and the underwater electric field. When the mean square error between the measured value and the calculated value reaches its minimal value, the equivalent electric dipole moment can be obtained [6, 7, 13]. The inversion method can be employed in real-time computation in various sea areas and

states, but it needs many underwater measurement nodes, which make it difficult to use the electric field measurement system to obtain a fast evaluation of the ship's electric field stealth performance. As a result, it is necessary to explore a new method of inverting the ship's equivalent electric dipole moment.

2. The Inversion Method of the Equivalent Electric Dipole Moment in the Frequency Domain

Under the three-layered medium model of "air-seawater-seabed," the electric potential at any field point $P(x, y, z)$ produced by the current line (whose coordinate is $(-l, l, y_0, z_0)$ as shown in Figure 1) is as follows:

$$U = \frac{1}{4\pi\sigma} \int_{-l}^l K(\xi, P) \rho(\xi) d\xi, \quad (1)$$

where $\rho(\xi)$ represents the current line density, σ represents the electrical conductivity, ξ represents the location of the source point, $R^2 = (\xi - x)^2 + (y_0 - y)^2$, and $K(\xi, P)$ represents the distance function between the source point and the field point.

$K(\xi, P)$ can be expressed as

$$K(\xi, P) = \frac{1}{\sqrt{R^2 + (z_0 - z)^2}} + \frac{1}{\sqrt{R^2 + (z_0 + z)^2}} + \sum_{m=1}^{\infty} k^m \sum_{p=1, -1} \sum_{q=1, -1} \frac{1}{\sqrt{R^2 + (2mH + pz_0 + qz)^2}}, \quad (2)$$

where k represents the seabed reflection coefficient [14, 15].

Through the Fourier transform of the electric potential in the direction of x -axis, there is the following equation:

$$\varphi(j\omega) = \int_{-\infty}^{+\infty} U \cdot e^{-j\omega x} dx. \quad (3)$$

Substituting equation (1) into equation (3) and changing the order of integral calculus make equation (3) become the following equation:

$$\varphi(j\omega) = \frac{1}{4\pi\sigma} \int_{-l}^{+l} \int_{-\infty}^{+\infty} K(\xi, P) \cdot e^{-j\omega x} dx \rho(\xi) d\xi. \quad (4)$$

For $K_0(\cdot)$ is the zero-order modified second-kind Bessel function and $\tau = \sqrt{(y - y_0)^2 + (z - z_0)^2}$ in the following equation:

$$\int_{-\infty}^{+\infty} \frac{e^{-j\omega x}}{\sqrt{R^2 + (z - z_0)^2}} dx = 2e^{-j\xi\omega} K_0(\omega\tau). \quad (5)$$

Substituting the mathematical relationship of equation (5) into equation (4),

$$\varphi(j\omega) = \frac{E(\omega)}{2\pi\sigma} \int_{-l}^{+l} e^{-j\xi\omega} \rho(\xi) d\xi, \quad (6)$$

where

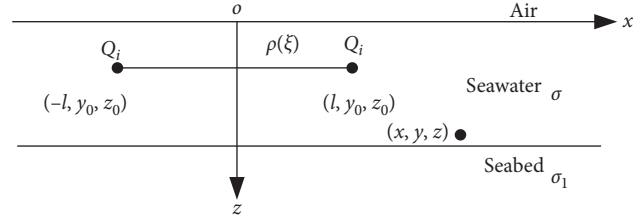


FIGURE 1: The current line model under the three-layered medium.

$$E(\omega) = K_0(\omega R_1) + K_0(\omega R_2) + \sum_{m=1}^{\infty} k^m [K_0(\omega R_3) + K_0(\omega R_4) + K_0(\omega R_5) + K_0(\omega R_6)],$$

$$\left\{ \begin{array}{l} R_1 = \sqrt{(y - y_0)^2 + (z - z_0)^2}, \\ R_2 = \sqrt{(y - y_0)^2 + (z + z_0)^2}, \\ R_3 = \sqrt{(y - y_0)^2 + (2mH + z - z_0)^2}, \\ R_4 = \sqrt{(y - y_0)^2 + (2mH + z + z_0)^2}, \\ R_5 = \sqrt{(y - y_0)^2 + (2mH - z - z_0)^2}, \\ R_6 = \sqrt{(y - y_0)^2 + (2mH - z + z_0)^2}, \end{array} \right. \quad (7)$$

Transforming $\varphi(j\omega)$ into the sum of real part and imaginary part, it can be expressed as

$$\varphi(j\omega) = A(\omega) + B(\omega)j, \quad (8)$$

where $A(\omega)$ is the result of Fourier cosine transform and $B(\omega)$ is the result of Fourier sine transform.

Transforming $\sin(\omega\xi)$ into the sum of power series, $B(\omega)$ can be expressed as

$$B(\omega) = \frac{\omega E(\omega)}{2\pi\sigma} \sum_{n=0}^{\infty} \frac{(-1)^n (\omega)^{2n}}{(2n+1)!} \int_{-l}^{+l} (\xi)^{2n+1} \rho(\xi) d\xi. \quad (9)$$

When $\omega \rightarrow 0$, equation (10) can be obtained as follows:

$$2\pi\sigma \lim_{\omega \rightarrow 0} \frac{B(\omega)}{\omega E(\omega)} = \int_{-l}^{+l} \xi \rho(\xi) d\xi. \quad (10)$$

As the integral term in equation (10) exactly corresponds to the equivalent electric dipole moment P_x of the current line, so we can get

$$P_x = 2\pi\sigma \lim_{\omega \rightarrow 0} \frac{B(\omega)}{\omega E(\omega)}. \quad (11)$$

It can be found in equation (11) that the equivalent electric dipole moment P_x can be estimated if we calculate $B(\omega)/\omega E(\omega)$ with the measured data under the condition of $\omega \rightarrow 0$. In fact, P_x can also be calculated in the way as follows.

Define

$$C(\omega) = 2\pi\sigma \frac{B(\omega)}{\omega E(\omega)}. \quad (12)$$

Equation (13) can be obtained from equation (9):

$$C(\omega) = P_x + \omega^2 D + O(\omega^4). \quad (13)$$

In equation (13), $O(\omega^4)$ is the minimum in higher order of ω^4 , and D is the coefficient of ω^2 .

Defining $\omega_2 = 2\omega_1$ and substituting ω_2 and ω_1 into equation (13), there is the following equation:

$$P_x = \frac{4C(\omega_1) - C(\omega_2)}{3}. \quad (14)$$

It proves in equation (14) that the equivalent electric dipole moment of a current line can be calculated by Fourier transform of the electric potential of a measured line in different frequencies of ω_2 and ω_1 .

3. Integral Correction Method

Section 2 indicates that the equivalent electric dipole moment of the current line can be calculated with the sine transform $B(\omega)$ and $\omega E(\omega)$ in different frequencies of ω_2 and ω_1 . However, it can be known from equation (3) that the integral range is $[-\infty, +\infty]$, but the measured range is limited within $[-L, +L]$ in the actual measurement. In other words, the calculation of $B(\omega)$ based on the measured data should be as follows:

$$B_1(\omega) = \int_{-L}^{+L} U \cdot \sin(\omega x) dx. \quad (15)$$

In fact, $C(\omega)$ in equation (13) can be expressed as

$$C_1(\omega) = 2\pi\sigma \frac{B_1(\omega)}{\omega E(\omega)}, \quad (16)$$

which leads to some errors in the inversion process.

Defining $\Delta(\omega)$ as the error between $C(\omega)$ and $C_1(\omega)$, $\Delta(\omega)$ can be expressed as

$$\begin{aligned} \Delta(\omega) &= \frac{2\pi\sigma}{\omega K_0(\omega\tau)} \int_{-\infty}^{-L} U_g \cdot \sin(\omega x) dx \\ &+ \frac{2\pi\sigma}{\omega K_0(\omega\tau)} \int_{+L}^{+\infty} U_g \cdot \sin(\omega x) dx, \end{aligned} \quad (17)$$

where U_g is the electric potential beyond the measuring range.

In the distant field, the ship's static electric field can be viewed as an electric dipole field. The difference $f(\omega)$ between $C_1(\omega)$ and $C(\omega)$ caused by a unit dipole moment can be obtained by using a unit standard dipole source moment by building the calculation model beyond the measuring range and implementing sine transform. According to the linearity principle, $\Delta(\omega)$ caused by the ship whose equivalent electric dipole moment is P_x can be calculated with

$$\Delta(\omega) = P_x f(\omega). \quad (18)$$

Therefore, equation (13) can be transformed as follows when the measuring range is limited:

$$C_1(\omega) + P_x f(\omega) = P_x + \omega^2 D + O(\omega^4). \quad (19)$$

Defining $\omega_2 = 2\omega_1$ and substituting ω_2 and ω_1 into equation (19), we can get

$$P_x = \frac{4C(\omega_1) - C(\omega_2)}{4[1 - f(\omega_1)] - [1 - f(\omega_2)]}. \quad (20)$$

In the infinite space medium, the electric potential at the field point (x, y, z) produced by the unit electric dipole moment whose coordinate is $(0, y_0, z_0)$ should be calculated with

$$U_g = \frac{1}{4\pi\sigma} \frac{x}{(x^2 + \tau^2)^{3/2}}. \quad (21)$$

Substituting equation (21) into equation (17), equation (22) can be obtained:

$$f(\omega) = \frac{1}{\omega E(\omega)} \int_{+L}^{+\infty} \frac{x}{(x^2 + \tau^2)^{3/2}} \cdot \sin(\omega x) dx. \quad (22)$$

Equation (22) indicates that, as the measuring range is limited, the standard electric dipole moment can be used to build the model. Then, the errors caused by the limited measuring range can be compensated by using the potential continuation.

4. The Specific Inversion Procedure

From the analysis of Section 3, the equivalent electric dipole moment inversion procedure is concluded as follows:

- (1) Calculate the sine transform of the measured electric potential in the range of $[-L, +L]$. And the sine transform frequencies are ω_1 and $\omega_2 = 2\omega_1$; then, $B_1(\omega_1)$ and $B_1(\omega_2)$ can be obtained.
- (2) Calculate $E(\omega_1)$ and $E(\omega_2)$ in the three-layered model medium with the data of the measuring points. Then, $C_1(\omega_1)$ and $C_1(\omega_2)$ can be obtained with equation (16).
- (3) Calculate $f(\omega_1)$ and $f(\omega_2)$ with equation (22), and calculate the equivalent electric dipole moment P_x with equation (20).

It is noteworthy that the frequency of ω_1 should be relatively low according to equation (11). However, when the frequency of ω_1 is too low, its sine transform will be interfered easily in the actual measurement. As a result, the appropriate setting is as equation (23) shows:

$$\begin{aligned} \omega_1 &= \frac{\pi}{2L}, \\ \omega_2 &= \frac{\pi}{L}. \end{aligned} \quad (23)$$

5. Simulation Example Verification

It is difficult to obtain the accurate equivalent electric dipole moment of a real warship. Therefore, the calculation results from BEASY software (which is widely used in the electromagnetic field analysis domain [16]) are used to verify the effectiveness of the proposed method.



FIGURE 2: Structural sketch of a surface warship.

5.1. The Warship Model. The length L of the surface warship is 136m, and the width B is 16 m. The warship draught T is 4.5 m. The materials of the hull, shaft bracket, and rudder plate are steel with a coating damage rate of 2%, and the screw propeller is nickel-aluminum bronze (as shown in Figure 2). The seawater conductivity $\sigma = 4 \text{ S/m}$, the seabed conductivity $\sigma_1 = 0.1 \text{ S/m}$, and the seawater depth is 200 m.

The hull is divided into 11 parts from the bow to the stern (each surface part is equal to an equivalent point charge), and all the sections are defined as hull 1, hull 2, . . . , hull 11. The coordinates and current of these equivalent point charges are given as Table 1. In Table 1, the positive and negative current corresponds to the cathode and anode condition, respectively.

The underwater electric potential at the depth of $1.0 B$ is shown in Figure 3, where the stern is at the depth of -68 m , the range of longitudinal measuring points is $(-160 \text{ m} \sim 160 \text{ m})$, the range of lateral measuring points is $(-80 \text{ m} \sim 80 \text{ m})$, and there are 321 calculation points in each measuring line.

5.2. The Inversion Results. According to the definition of the equivalent electric dipole, the longitudinal equivalent electric dipole moment of the warship can be calculated approximately with the following equation:

$$P_x = \sum_{i=1}^l I_i \cdot x_i, \quad (24)$$

where x_i and I_i are the coordinate and current of the equivalent point charge of number i .

According to the calculation results in Table 1, the equivalent electric dipole moment of the warship can be approximately expressed as $P_x = -297.18 \text{ A} \cdot \text{m}$. It is worth noting that there are some errors between the equivalent electric dipole moment calculated by equation (24) and the actual value of the warship, which are mainly caused by a relatively small number of divided sections. Based on the inversion steps in Section 4, we get the relationship between the inversed electric dipole moment and the transverse distance of measuring line, see in Figure 4.

Defining $P_x = -297.18 \text{ A} \cdot \text{m}$ as the truth value, some conclusions can be drawn from Figure 4: (1) there is little difference between the equivalent electric dipole moment inversed by the proposed method and the equivalent electric dipole moment inversed by the equivalent point charge method, which proves the validity of the proposed method; (2) the inversed equivalent electric dipole moment changes with different transverse distances. When the transverse distance is less than 10 m, the inverted equivalent electric dipole moment under different transverse distances is close to the value estimated by point charges (with the difference less than 2%). When the transverse distance is more than 10 m, the inverted equivalent electric dipole moment tends

TABLE 1: Coordinates and current of the equivalent point charges of a surface warship.

No.	Section	x (m)	y (m)	z (m)	Current (mA)
1	Hull 1	64.6	0	3.54	$-7.49E+01$
2	Hull 2	108.8	0	2.29	$-3.59E+02$
3	Hull 3	40.8	0	2.29	$-3.91E+02$
4	Hull 4	27.2	0	2.29	$-4.49E+02$
5	Hull 5	13.6	0	2.29	$-5.00E+02$
6	Hull 6	0	0	2.29	$-5.75E+02$
7	Hull 7	-13.6	0	2.29	$-6.40E+02$
8	Hull 8	-27.2	0	2.29	$-7.67E+02$
9	Hull 9	-40.8	0	2.29	$-9.82E+02$
10	Hull 10	-54.4	0	2.29	$-1.92E+03$
11	Hull 11	-64.6	0	3.91	$-7.89E+02$
12	Rudder plate	-63.7	0	1.52	$-3.33E+02$
13	Shaft bracket	-57.6	0	1.83	$-4.45E+02$
14	Screw propeller	-59.09	0	0.15	$8.35E+03$
15	Fin stabilizer I	16.8	0	1.75	$-2.62E+01$
16	Fin stabilizer II	-11.63	0	0.85	$-4.67E+01$

to increase gradually. The reason for this phenomenon is that the electric field strength in the limited measuring range gradually decreases as the transverse distance increases. In other words, the error caused by the limited measuring range becomes bigger, thus leading to a bigger error in the equivalent electric dipole moment calculation. In conclusion, when calculating the equivalent electric dipole moment, the distance between the measuring line and the warship should be small so as to avoid the inversion error caused by the limited measuring range.

5.3. The Influence of Measurement Noise. It should be taken into consideration that there exists some measurement noise in the actual experiment. In order to study the effect of measurement noise on the proposed method, we add white Gaussian noise signals on the basis of Section 5.1. The inversed equivalent electric dipole moment under different SNR conditions is shown in Table 2. And the SNR of the added noise is calculated with the electric potential signal exactly below the keel, and the result is the average value of 20 simulated calculations. It can be known from Table 2 that (1) defining $P_x = -297.18 \text{ A} \cdot \text{m}$ as the truth value, the relative error is 8.75% when the SNR = -10.14 dB , and the relative error is no more than 5% when the SNR is more than -4 dB ; (2) the inversed equivalent electric dipole moment differs slightly when the SNR range is within $-10.14 \text{ dB} \sim 20.13 \text{ dB}$; this is because the FFT transform is used in the inversion process as FFT transform is good at suppressing the noise.

5.4. Evaluation of Ship's Electric Field Stealth. To further examine whether the inversed equivalent electric dipole moment can evaluate the ship's electric field stealth

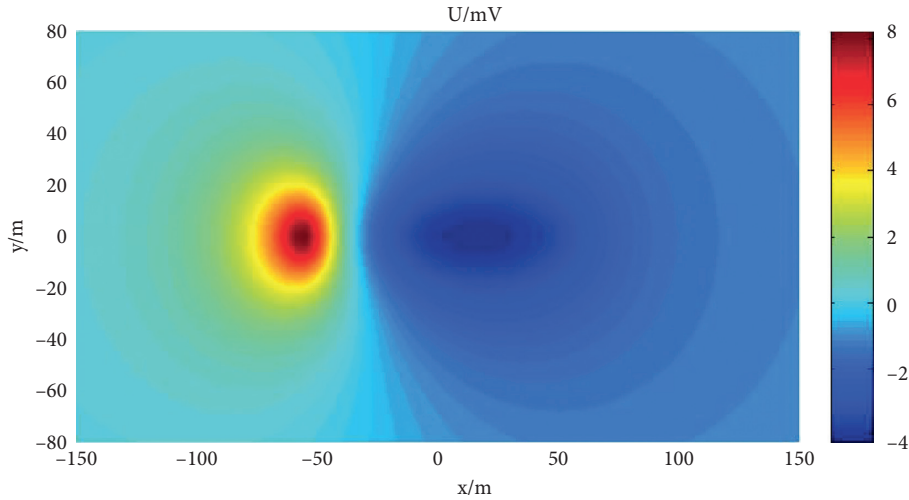


FIGURE 3: Underwater electric potential at the depth of 1.0 B.

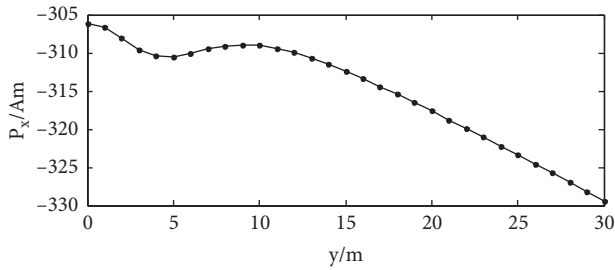


FIGURE 4: Equivalent electric dipole moment inversed under different transverse distances.

TABLE 2: The equivalent electric dipole moment under different SNR conditions.

No.	SNR (dB)	P_x (A · m)	No.	SNR (dB)	P_x (A · m)
1	20.13	-307.64	5	-2.32	-309.23
2	10.59	-309.08	6	-4.93	-303.94
3	1.73	-301.48	7	-8.34	-310.33
4	-0.43	-305.21	8	-10.14	-323.98

performance, two compensatory anodes have been installed right above the propellers based on the impressed current compensation method [17]. When anodes emit different output currents, the underwater electric field peak-peak values at the depth of 1.0 B and the inversed equivalent electric dipole moment are shown in Figure 5. It can be seen in Figure 5 that the inversed equivalent electric dipole moment can reflect the electric field stealth; in other words, a smaller inverted equivalent electric dipole moment refers to a weaker underwater electric potential. Figure 6 shows the equivalent electric dipole moment inversed by the proposed method and point charge method (proposed by [7] under different output currents). It can be found from Figure 6 that the difference between these two methods is relatively small, that is, these two methods show a good uniformity.

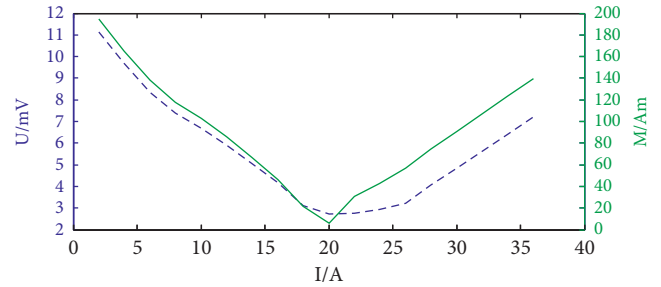


FIGURE 5: Relationship between the peak-peak value of underwater electric field and the inversed equivalent electric dipole moment.

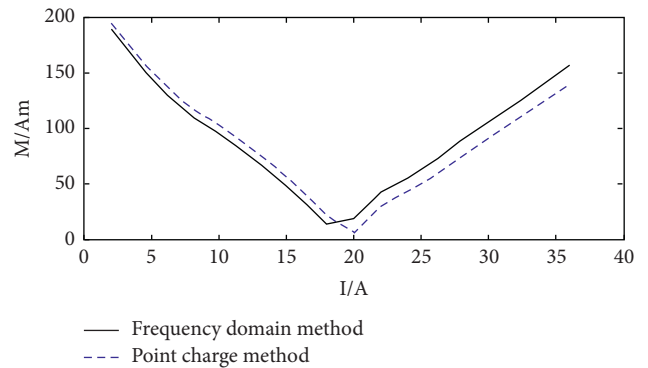


FIGURE 6: Equivalent electric dipole moment inversed by the proposed method and point charge method under different output currents.

6. Conclusions

A new inversion method in the frequency domain is proposed to evaluate the equivalent electric dipole moment of a ship. Its basic principle is to decrease the error caused by the limited measuring range with an integral correction approach based on Fourier transform. The proposed method has advantages of less measuring points, simple calculation,

and high calculation accuracy, which provide a new technological approach for the evaluation of ship's electric field stealth performance. The next step of this research is to verify the effectiveness of the proposed method with real ship experiment data.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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References

- [1] C. Lin and S. Gong, *Physical Fields of Ships*, Publishing House of Ordnance Industry, Beijing, China, 2007, in Chinese.
- [2] E. S. Diaz and R. Adey, "Optimising the location of anodes in cathodic protection systems to smooth potential distribution," *Advances in Engineering Software*, vol. 36, no. 9, pp. 591–598, 2005.
- [3] J. Parlongue and L. Bortels, *A Numerical Modeling Technique to Evaluate and Control the Electromagnetic Signature*, UDT 2010, Hamburg, Germany, 2010.
- [4] S. Xing, J. Wu, and Y. Yan, "Optimization of a ship's ICCP system to minimize electrical and magnetic signature by mathematical simulation," *Electrochemical Process Simulation III*, vol. 65, p. 11169, 2009.
- [5] P. Yu, J. Cheng, and R. Jiang, "Research of ship's stealth methods in static electric field," in *Proceedings of the Advances in Materials, Machinery, Electrical Engineering (AMMEE 2017)*, Atlantis Press, Tianjin, China, June 2017.
- [6] A. Poikonen, I. Suppala, and K. Sulkanen, "Studies on underwater electric potential (UEP)," in *Proceedings. Marine Electromagnetics (Marelec97)*, pp. 23–27, London, UK, June 1997.
- [7] R. X. Jiang, L. Chun-sheng, and S. G. Gong, "Electrostatic electric field inversion method for ship based on point charge source model," *Acta Electronica Sinica*, vol. 36, no. 3, pp. 545–551, 2015, in Chinese.
- [8] Y. Peng, J. F. Cheng, and R. X. Jiang, *Inversion of UEP Signatures Induced by Ships Based on PSO Method*, Defence Technology, DRDO Bhavan, New Delhi, 2019.
- [9] D. Schäfer, J. Doose, M. Pichlmaier, A. Rennings, and D. Erni, "Comparability of UEP signatures measured under varying environmental conditions," in *Proceedings of the 8th International Marine Electromagnetics Conference, MARELEC*, PA, USA, July 2013, pp. 16–19.
- [10] H. Claesson and P. Krylstedt, "On the environmental impact on the UEP signature from submarines," in *Proceedings of the International Conference on Marine Electromagnetics MAR-ELEC 2011*, pp. 20–23, PA, USA, June 2011.
- [11] Y. Wang, Y. Gu, and J. Liu, "A domain-decomposition generalized finite difference method for stress analysis in three-dimensional composite materials," *Applied Mathematics Letters*, vol. 104, 2020.
- [12] Y. Gu and H.G. Sun, "A meshless method for solving three dimensional time fractional diffusion equation with variable-order derivatives," *Applied Mathematical Modelling*, vol. 78, 2019.
- [13] C. Cong, Y. Wei, Y. Lu-feng et al., "Estimation and simulation analysis of the submarine magnetic field based on current-line mode," *Journal of Electronics & Information Technology*, vol. 37, no. 2, pp. 461–467, 2015, in Chinese.
- [14] D. L. Jones and C. P. Burke, "The DC field components of horizontal and vertical electric dipole sources immersed in three-layered stratified media," *Annales Geophysicae*, vol. 154, pp. 503–510, April 1997.
- [15] R. W. P. King, "The electromagnetic field of a horizontal electric dipole in the presence of a three-layered region: Supplement," *Journal of Applied Physics*, vol. 74, no. 8, pp. 4845–4848, 1993.
- [16] Z. A. Daya, D. L. Hutt, and T. C. Richards, *Maritime Electromagnetism and DRDC Management Research*, Defence R&D Canada-Atlantic, NS Canada, 2015.
- [17] Y. Qin, R. Jiang, J. Cheng et al., "Ship's static electric field reduction method based on impressed current compensation," *Mine Warfare & Ship Self-Defence*, vol. 24, no. 1, pp. 14–18, 2016, in Chinese.