

## Research Article

# Two Types of Synchronization Problems in a New 5D Hyperchaotic System

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This paper investigates the synchronization problem in a new 5D hyperchaotic system. Firstly, the existence of two types of synchronization problems in the new 5D hyperchaotic system is proved. Then, by the dynamic feedback control method, one complete synchronization problem and three coexistence of complete synchronization and antisynchronization problems in such system are realized. Finally, numerical simulations are used to verify the validity and effectiveness of the theoretical results.

## 1. Introduction

Since Lorenz proposed the first chaotic system in 1963, many researchers are stimulated to investigate this chaotic phenomena. Then, lots of chaotic systems and hyperchaotic systems are proposed. Since the OGY method [1] and PC method [2] were first observed for chaos control and chaos synchronization in 1990, respectively, the control problems and synchronization problems of those chaotic systems have become hot topics, see References [13–15] and the references therein. However, there are still some important questions need to be solved completely. For example, the existence of the synchronization problem in a given chaotic or hyperchaotic system, which is a fundamental theoretical base to design a physical controller. Furthermore, how to design a not only simple but also physical controller to realize the synchronization problem is also a question which needs to be solved. For the realization of such synchronization problems, there are many methods to use, linear feedback control method, dynamic feedback control method, sliding mode control method, and so on. Among those methods, the dynamic feedback control method has been applied often in applications, which is also used in this paper.

The new 5D hyperchaotic system was firstly presented in [16], and it has complex dynamics. There are eight

parameters to control with only one equilibrium point. This new dynamics can generate four-wing hyperchaotic and chaotic attractors for some specific parameters and initial conditions. One remarkable feature of the new system is that it can generate double-wing and four-wing smooth chaotic attractors with special appearance. However, the synchronization problem in this system has not been solved. Moreover, for the new 5D hyperchaotic system, the existence of the synchronization problems in such system is a fundamental question, but it still not be solved. Therefore, investigating the synchronization problems in the new 5D hyperchaotic system is very important in both theory and applications, which motivates our work in this paper.

Motivated by the abovementioned conclusions, the synchronization problem in the new 5D chaotic system is studied by the dynamic feedback control method. The main contributions of this paper are given as follows:

- (1) The existence of two types of synchronization problems in the new 5D system is proved
- (2) These two types of synchronization problems in the new 5D system are realized by the dynamic feedback control method

## 2. Problem Formulation

According to [16], the new 5D hyperchaotic system is given as

$$\dot{x} = f(x), \quad (1)$$

where  $x \in R^5$  is the state and

$$f(x) = \begin{pmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \\ f_5(x) \end{pmatrix} = \begin{pmatrix} -10x_1 + x_2x_3 \\ -60x_2 + x_5 \\ -20x_3 + 50x_4 + x_1x_2 \\ 15x_4 - 10x_3 \\ 40x_5 - x_1^2x_2 \end{pmatrix}. \quad (2)$$

Let system (1) be the master system, then the controlled slave system is described as follows:

$$\dot{y} = f(y) + Bu, \quad (3)$$

where  $y \in R^5$  is the state,

$$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (4)$$

and  $u \in R^3$  is the controller to be designed.

This paper investigates the synchronization problem of the master system (1) and the slave system (3) and presents some new results.

For the development of this paper, the dynamic feedback control method is introduced in the next.

## 3. Preliminary

Consider the following controlled chaotic system:

$$\dot{z} = F(z) + Bu, \quad (5)$$

where  $z \in R^n$  is the state,  $F(z) \in R^n$  is continuous vector function with  $F(0) = 0$ ,  $B \in R^{n \times r}$  is a constant matrix, and  $u$  is the designed controller.

**Lemma 1** (see [17]). *Consider system (5), if  $(F(z), B)$  can be stabilized; then, the controller  $u$  is designed as follows:*

$$u = K(t)z, \quad (6)$$

where  $K = k(t)B^T$ , and the feedback gain  $k(t)$  is updated by the following equation:

$$\dot{k}(t) = -z^T z = -\|z\|^2. \quad (7)$$

## 4. Main Results

**4.1. The Existence of Synchronization Problems in the New 5D Hyperchaotic System.** According to the results in [17], the existence of synchronization problems in the 5D hyperchaotic system (1) is proved by the following condition:

$$f(\alpha x) \equiv \alpha f(x), \quad (8)$$

where  $\alpha = \text{Diag}\{\alpha_1, \alpha_2, \dots, \alpha_5\}$ ,  $\alpha_i \neq 0$ ,  $i = 1, 2, \dots, 5$ ,

$$\begin{cases} f_1(\alpha x) - \alpha_1 f_1(x) = (\alpha_2 \alpha_3 - \alpha_1) x_2 x_3 \equiv 0, \\ f_2(\alpha x) - \alpha_2 f_2(x) = (\alpha_2 - \alpha_5) x_5 \equiv 0, \\ f_3(\alpha x) - \alpha_3 f_3(x) = 50(\alpha_3 - \alpha_4) x_4 + (\alpha_1 \alpha_2 - \alpha_3) x_1 x_2 \equiv 0, \\ f_4(\alpha x) - \alpha_4 f_4(x) = -10(\alpha_3 - \alpha_4) x_3 \equiv 0, \\ f_5(\alpha x) - \alpha_4 f_4(x) = -(\alpha_1^2 \alpha_2 - \alpha_5) x_1 \equiv 0. \end{cases} \quad (9)$$

It results in

$$\begin{cases} \alpha_2 \alpha_3 = \alpha_1, \\ \alpha_2 = \alpha_5, \\ \alpha_1 \alpha_2 = \alpha_3, \\ \alpha_3 = \alpha_4, \\ \alpha_1^2 \alpha_2 = \alpha_5. \end{cases} \quad (10)$$

Solving equation (10), we obtain

- (I)  $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 1$ , which implies the complete synchronization problem exists
- (II)  $\alpha_1 = \alpha_2 = \alpha_5 = -1$  and  $\alpha_3 = \alpha_4 = 1$ , which implies the coexistence of complete synchronization and antisynchronization problem exists
- (III)  $\alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = -1$  and  $\alpha_1 = 1$ , which implies the coexistence of complete synchronization and antisynchronization problem exists
- (IV)  $\alpha_1 = \alpha_3 = \alpha_4 = -1$  and  $\alpha_2 = \alpha_5 = 1$ , which implies the coexistence of complete synchronization and antisynchronization problem exists

**4.2. Complete Synchronization in the New 5D Hyperchaotic System.** For the master system (1) and the slave system (3), let  $e = y - x$ , and the error system is given as

$$\dot{e} = f(y) - f(x) + Bu = g(x, e) + Bu, \quad (11)$$

where

$$g(x, e) = \begin{pmatrix} g_1(x, e) \\ g_2(x, e) \\ g_3(x, e) \\ g_4(x, e) \\ g_5(x, e) \end{pmatrix} = \begin{pmatrix} -10e_1 + x_2e_3 + x_3e_2 + e_2e_3 \\ -60e_2 + e_5 \\ -20e_3 + 50e_4 + x_1e_2 + x_2e_1 + e_1e_2 \\ 15e_4 - 10e_3 \\ 40e_5 - 2x_1x_2e_1 - 2x_1e_2e_1 - x_2e_1^2 - e_1^2e_2 \end{pmatrix}, \quad (12)$$

and  $B$  is given in (4).

According to Lemma 1, we propose the following conclusion.

**Theorem 1.** Consider the error system (11). If the controller  $u$  is designed as follows

$$u = Ke, \quad (13)$$

where  $K = k(t)B^T$ ,  $B$  is given in (4), and  $k(t)$  is updated by the following update law:

$$\dot{k} = -\|e\|^2. \quad (14)$$

Then, the master system (1) and the slave system (3) reach complete synchronization.

*Proof.* For the error system (11) with  $u = 0$ , if  $e_3 = e_4 = e_5 = 0$ , then the following subsystem

$$\begin{aligned} \dot{e}_1 &= -10e_1 + x_3e_2, \\ \dot{e}_2 &= -60e_2, \end{aligned} \quad (15)$$

is asymptotically stable.

Thus,  $(g(x, e), B)$  can be stabilized. According to Lemma 1, the error system (11) is stabilized by the abovementioned controller  $u$  given in (13), which completes the proof.  $\square$

In the following, numerical simulation is carried out with the initial conditions:  $x_1(0) = 1, x_2(0) = -2, x_3(0) = 3, x_4(0) = -4, y_1(0) = 5, y_2(0) = -6, y_3(0) = 7, y_4(0) = -8$ , and  $k(0) = -1$ . Figure 1 shows  $e_1, e_2$ , and  $e_3$  are asymptotically stable, Figure 2 shows  $e_4$  and  $e_5$  are asymptotically stable, and Figure 3 shows that the feedback gain  $k(t)$  tends to constant.

#### 4.3. Coexistence of Synchronization and Antisynchronization in the New 5D Hyperchaotic System

*Case 1.*  $\alpha_1 = \alpha_2 = \alpha_5 = -1$  and  $\alpha_3 = \alpha_4 = 1$ .

Let  $E_i = x_i + y_i, i = 1, 2, 5$ , and  $e_i = y_i - x_i, i = 3, 4$ ; then, the sum and error system is described as follows:

$$\begin{pmatrix} \dot{E} \\ \dot{e} \end{pmatrix} = \begin{pmatrix} \dot{E}_1 \\ \dot{E}_2 \\ \dot{E}_5 \\ \dot{e}_3 \\ \dot{e}_4 \end{pmatrix} = \begin{pmatrix} f_1(y) + f_1(x) \\ f_2(y) + f_2(x) \\ f_5(y) + f_5(x) \\ f_3(y) - f_3(x) \\ f_4(y) - f_4(x) \end{pmatrix} + Bu = G(x, E, e) + Bu, \quad (16)$$

where

$$G(x, E, e) = \begin{pmatrix} G_1(x, E, e) \\ G_2(x, E, e) \\ G_5(x, E, e) \\ G_3(x, E, e) \\ G_4(x, E, e) \end{pmatrix} = \begin{pmatrix} -10E_1 - x_2e_3 - x_3E_2 + E_2e_3 \\ -60E_2 + E_5 \\ 40e_5 - 2x_1x_2E_1 + 2x_1E_2E_1 + x_2E_1^2 - E_1^2E_2 \\ -20e_3 + 50e_4 - x_1E_2 - x_2E_1 + E_1E_2 \\ 15e_4 - 10e_3 \end{pmatrix}, \quad (17)$$

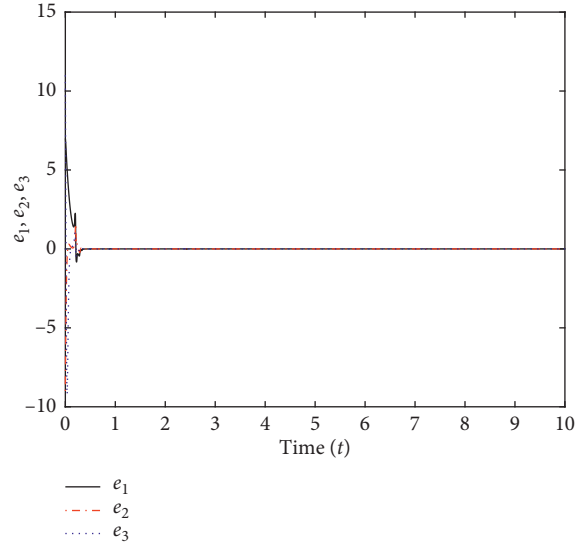


FIGURE 1:  $e_1, e_2$ , and  $e_3$  are asymptotically stable.

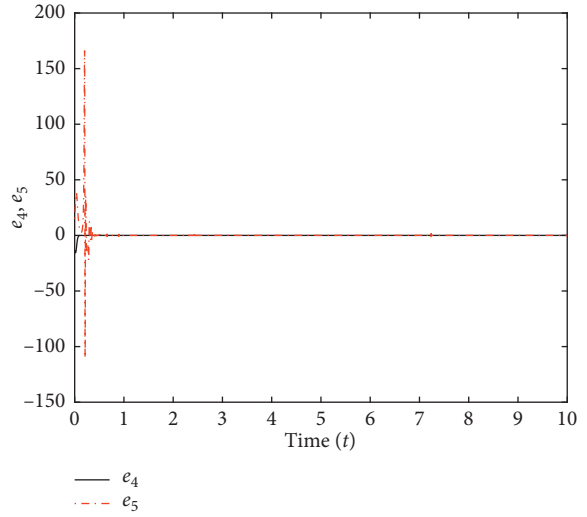


FIGURE 2:  $e_4$  and  $e_5$  are asymptotically stable.

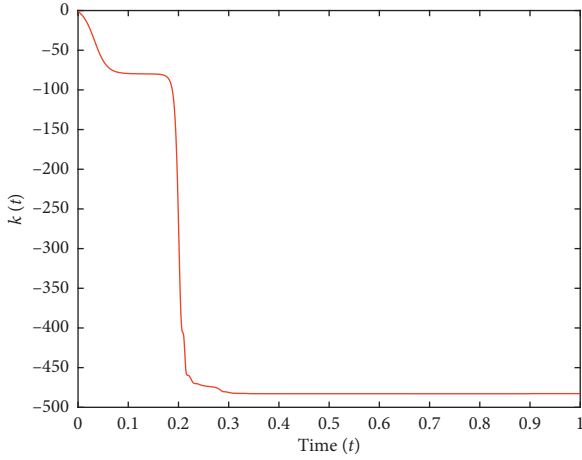


FIGURE 3: The feedback gain  $k(t)$  tends to constant.

and  $B$  is given in (4).

According to Lemma 1, we propose the following conclusion.

**Theorem 2.** Consider the sum and error system (16). If controller  $u$  is designed as follows

$$u = K \begin{pmatrix} E \\ e \end{pmatrix}, \quad (18)$$

where  $K = k(t)B^T$ ,  $B$  is given as (4), and  $k(t)$  is updated by the following update law:

$$\dot{k} = -(\|e\|^2 + \|E\|^2). \quad (19)$$

Then, the master system (1) and the slave system (3) realize the coexistence of complete synchronization and antisynchronization.

$$G^*(x, E, e) = \begin{pmatrix} G_2^*(x, E, e) \\ G_3^*(x, E, e) \\ G_4^*(x, E, e) \\ G_5^*(x, E, e) \\ G_1^*(x, E, e) \end{pmatrix} = \begin{pmatrix} -60E_2 + E_5 \\ -20E_3 + 50E_4 + x_1E_2 - x_2e_1 + e_1E_2 \\ 15E_4 - 10E_3 \\ 40E_5 - 2x_1x_2e_1 - 2x_1E_2e_1 + x_2e_1^2 - e_1^2E_2 \\ -10e_1 + x_2E_3 + x_3E_2 + E_2E_3 \end{pmatrix}, \quad (22)$$

and

$$B^* = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}. \quad (23)$$

According to Lemma 1, we propose the following conclusion.

*Proof.* For the sum and error system (16) with  $u = 0$ , if  $e_3 = e_4 = E_5 = 0$ , then the following subsystem

$$\begin{aligned} \dot{E}_1 &= -10E_1 - x_3E_2, \\ \dot{E}_2 &= -60E_2, \end{aligned} \quad (20)$$

is asymptotically stable.

Thus,  $(G(x, E, e), B)$  can be stabilized. According to Lemma 1, the sum and the error system (16) is stabilized by the abovementioned controller  $u$  given in (18), which completes the proof.  $\square$

In the following, numerical simulation is carried out with the initial conditions:  $x_1(0) = 1$ ,  $x_2(0) = -2$ ,  $x_3(0) = 3$ ,  $x_4(0) = -4$ ,  $y_1(0) = 5$ ,  $y_2(0) = -6$ ,  $y_3(0) = 7$ ,  $y_4(0) = -8$ , and  $k(0) = -1$ . Figure 4 shows  $E_1, E_2$ , and  $e_3$  are asymptotically stable, Figure 5 shows  $e_4$  and  $E_5$  are asymptotically stable, and Figure 6 shows that the feedback gain  $k(t)$  tends to constant.

*Case 2.*  $\alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = -1$  and  $\alpha_1 = 1$ .

Let  $E_i = x_i + y_i, i = 2, 3, 4, 5$ , and  $e_1 = y_1 - x_1$ , and then the sum and error system is described as follows:

$$\begin{pmatrix} \dot{E} \\ \dot{e} \end{pmatrix} = \begin{pmatrix} \dot{E}_2 \\ \dot{E}_3 \\ \dot{E}_4 \\ \dot{E}_5 \\ \dot{e}_1 \end{pmatrix} = \begin{pmatrix} f_2(y) + f_2(x) \\ f_3(y) + f_3(x) \\ f_4(y) + f_4(x) \\ f_5(y) + f_5(x) \\ f_1(y) - f_1(x) \end{pmatrix} + B^*u = G^*(x, E, e) + B^*u, \quad (21)$$

where

**Theorem 3.** Consider the controlled sum and error system (21). If controller  $u$  is designed as follows

$$u = K \begin{pmatrix} E \\ e \end{pmatrix}, \quad (24)$$

where  $K = k(t)B^{*T}$ ,  $B^*$  is given in (23), and  $k(t)$  is updated by the following update law:

$$\dot{k} = -(\|e\|^2 + \|E\|^2). \quad (25)$$

Then, the master system (1) and the slave system (3) realize the coexistence of complete synchronization and antisynchronization.

*Proof.* For the sum and error system (21) with  $u = 0$ , if  $E_3 = E_4 = E_5 = 0$ , then the following subsystem

$$\begin{aligned} \dot{e}_1 &= -10e_1 + x_3E_2, \\ \dot{E}_2 &= -60E_2, \end{aligned} \quad (26)$$

is asymptotically stable.

Thus,  $(G(x, E, e), B)$  can be stabilized. According to Lemma 1, the sum and the error system (21) is stabilized by the abovementioned controller  $u$  given in (24), which completes the proof.  $\square$

In the following, numerical simulation is carried out with the initial conditions:  $x_1(0) = 1, x_2(0) = -2, x_3(0) = 3, x_4(0) = -4, y_1(0) = 5, y_2(0) = -6, y_3(0) = 7, y_4(0) = -8$ , and  $k(0) = -1$ . Figure 7 shows  $e_1, E_2$ , and  $E_3$  are asymptotically stable, Figure 8 shows  $E_4$  and  $E_5$  are asymptotically stable, and Figure 9 shows that the feedback gain  $k(t)$  tends to constant.

*Case 3.*  $\alpha_1 = \alpha_3 = \alpha_4 = -1$  and  $\alpha_2 = \alpha_5 = 1$ .

Let  $E_i = x_i + y_i, i = 1, 3, 4$ , and  $e_i = y_i - x_i, i = 2, 5$ , and then the sum and error system is described as follows:

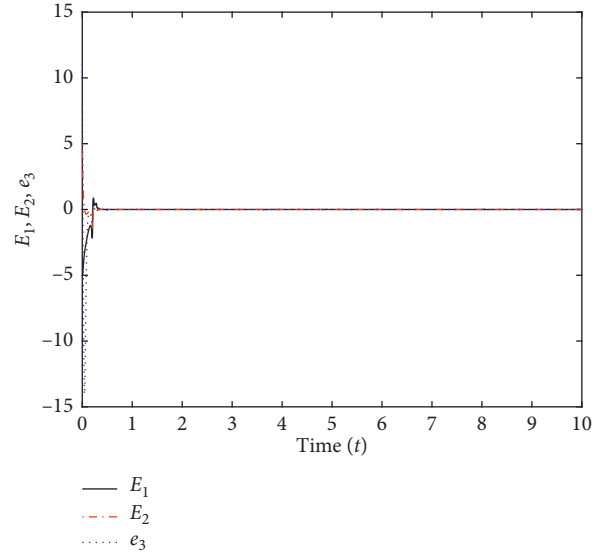


FIGURE 4:  $E_1, E_2$ , and  $e_3$  are asymptotically stable.

$$\begin{pmatrix} \dot{E}_1 \\ \dot{E}_3 \\ \dot{E}_4 \\ \dot{e}_2 \\ \dot{e}_5 \end{pmatrix} = \begin{pmatrix} \dot{E}_1 \\ \dot{E}_3 \\ \dot{E}_4 \\ \dot{e}_2 \\ \dot{e}_5 \end{pmatrix} = \begin{pmatrix} f_1(y) + f_1(x) \\ f_3(y) + f_3(x) \\ f_4(y) + f_4(x) \\ f_2(y) - f_2(x) \\ f_5(y) - f_5(x) \end{pmatrix} + B^{**}u = G^{**}(x, E, e) + B^{**}u, \quad (27)$$

where

$$G^{**}(x, E, e) = \begin{pmatrix} G_1^{**}(x, E, e) \\ G_3^{**}(x, E, e) \\ G_4^{**}(x, E, e) \\ G_2^{**}(x, E, e) \\ G_5^{**}(x, E, e) \end{pmatrix} = \begin{pmatrix} -10E_1 + x_2E_3 - x_3e_2 + e_2E_3 \\ -20E_3 + 50E_4 - x_1e_2 + x_2E_1 + E_1e_2 \\ 15E_4 - 10E_3 \\ -60e_2 + e_5 \\ 40e_5 + 2x_1x_2E_1 + 2x_1e_2E_1 - x_2E_1^2 - E_1^2e_2 \end{pmatrix} \quad (28)$$

and

$$B^{**} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (29)$$

According to Lemma 1, we propose the following conclusion.

**Theorem 4.** Consider the controlled sum and error system (27). If controller  $u$  is designed as follows:

$$u = K \begin{pmatrix} E \\ e \end{pmatrix}, \quad (30)$$

where  $K = k(t)B^{**T}$ ,  $B^{**}$  is given in (29), and  $k(t)$  is updated by the following update law:

$$\dot{k} = -(\|e\|^2 + \|E\|^2). \quad (31)$$

Then, the master system (1) and the slave system (3) realize the coexistence of complete synchronization and antisynchronization.

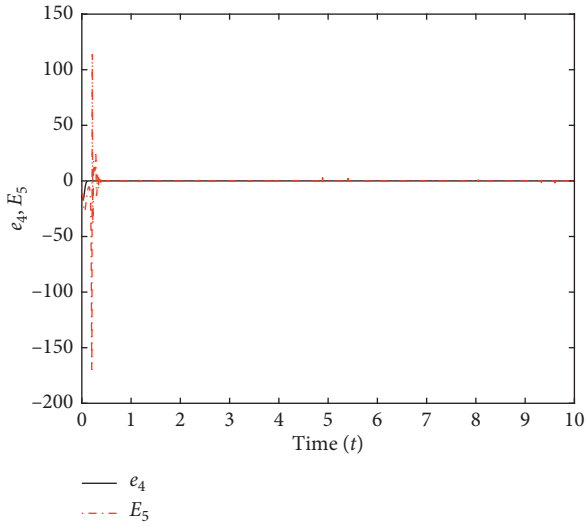


FIGURE 5:  $e_4$ , and  $E_5$  are asymptotically stable.

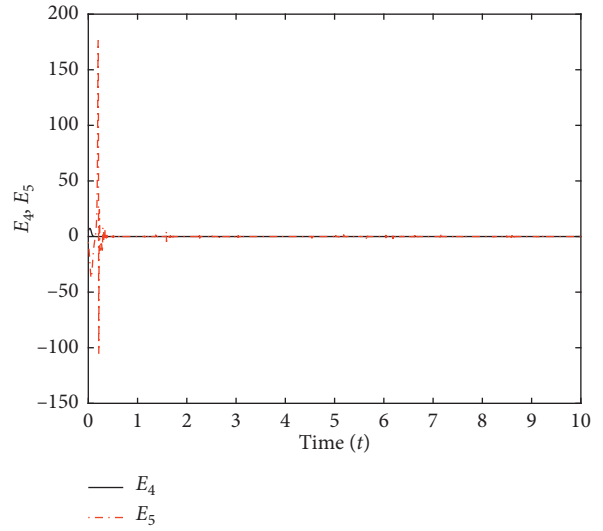


FIGURE 8:  $E_4$  and  $E_5$  are asymptotically stable.

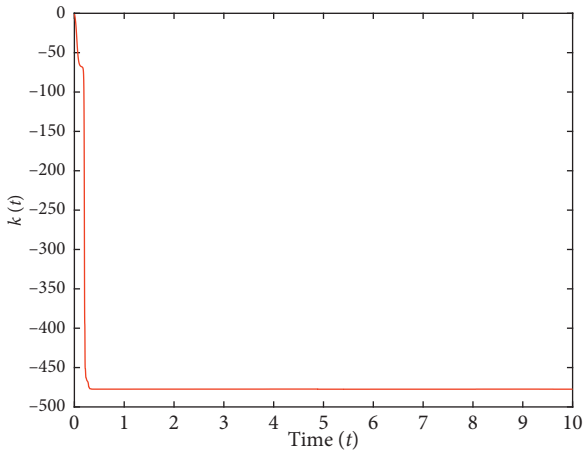


FIGURE 6: The feedback gain  $k(t)$  tends to constant.

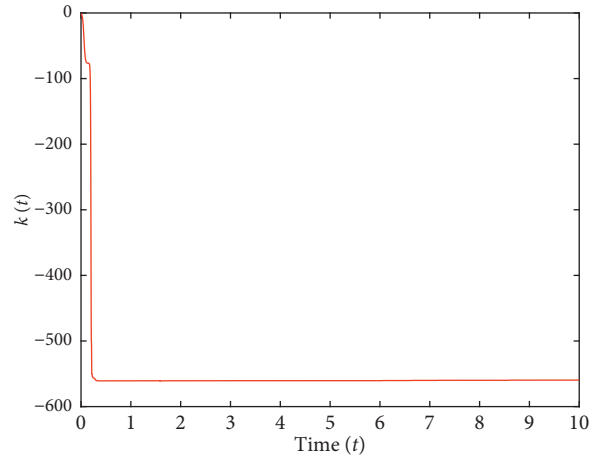


FIGURE 9: The feedback gain  $k(t)$  tends to constant.

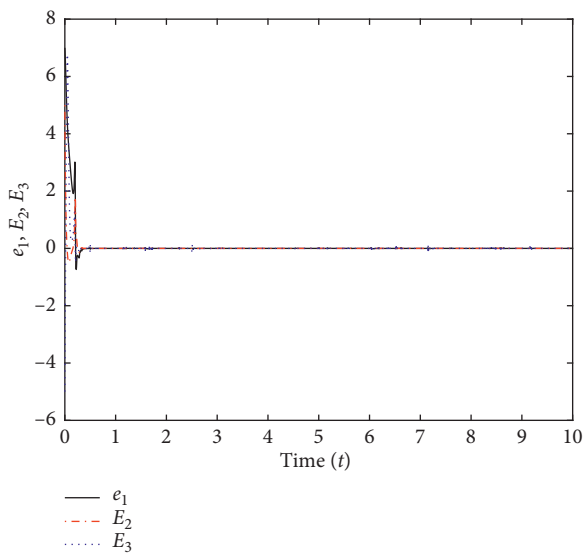


FIGURE 7:  $e_1$ ,  $E_2$ , and  $E_3$  are asymptotically stable.

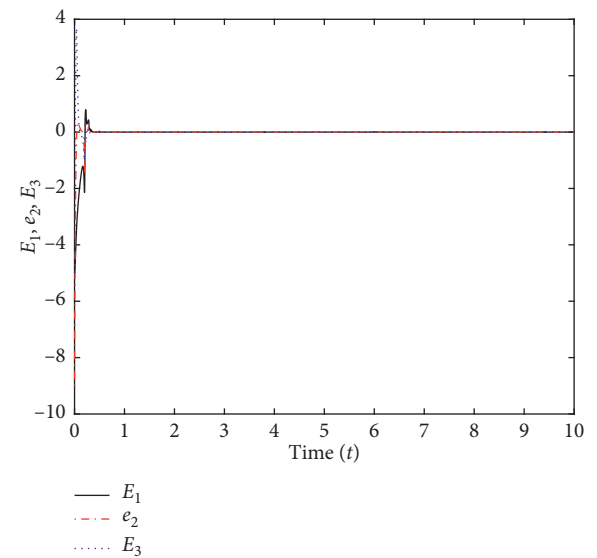


FIGURE 10:  $E_1$ ,  $e_2$ , and  $E_3$  are asymptotically stable.

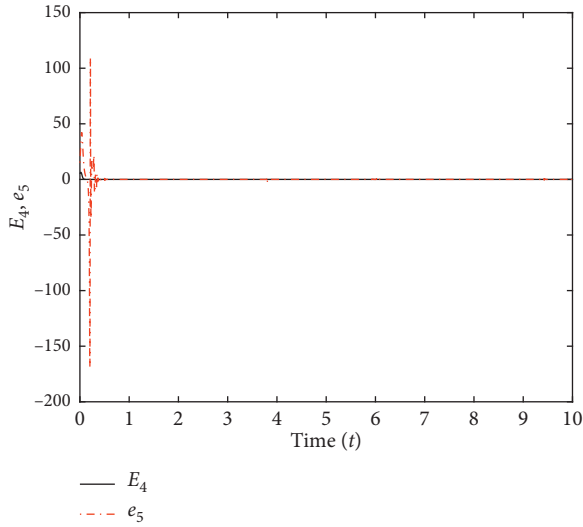


FIGURE 11:  $E_4$  and  $E_5$  are asymptotically stable.

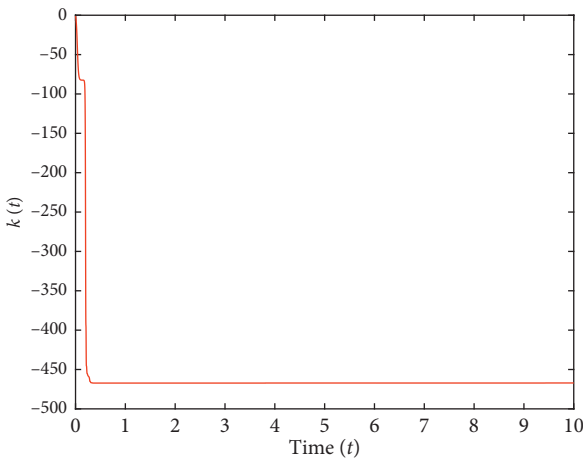


FIGURE 12: The feedback gain  $k(t)$  tends to constant.

*Proof.* For the sum and error system (27) with  $u = 0$ , if  $E_3 = E_4 = E_5 = 0$ , then the following subsystem

$$\begin{aligned} \dot{E}_1 &= -10e_1 - x_3e_2, \\ \dot{e}_2 &= -60e_2, \end{aligned} \quad (32)$$

is asymptotically stable.

Thus,  $(G^{**}(x, E, e), B^{**})$  can be stabilized. According to Lemma 1, the sum and the error system (27) is stabilized by the abovementioned controller  $u$  given in (30), which completes the proof.  $\square$

In the following, numerical simulation is carried out with the initial conditions:  $x_1(0) = 1, x_2(0) = -2, x_3(0) = 3, x_4(0) = -4, y_1(0) = 5, y_2(0) = -6, y_3(0) = 7, y_4(0) = -8$ , and  $k(0) = -1$ . Figure 10 shows  $E_1, e_2$ , and  $E_3$  are asymptotically stable, Figure 11  $E_4$  and  $E_5$  are asymptotically stable, Figure 12 shows that the feedback gain  $k(t)$  tends to constant.

## 5. Conclusions

In conclusion, two types of synchronization problems in the new 5D hyperchaotic system have been investigated. Firstly, the existence of these synchronization problems in such system has been proven. Secondly, those synchronization problems have been realized by the dynamic feedback control method. Finally, numerical simulations have been used to verify the validity and effectiveness of the proposed results.

## Data Availability

In our paper, we only used MATLAB for simulation. Therefore, we could only provide simulation programming which can be obtained from the corresponding author upon request.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

## Acknowledgments

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