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Research Article

Adaptive Dynamic Surface Control for a Class of Nonlinear Pure-Feedback Systems with Parameter Drift

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In order to solve the problem of unknown parameter drift in the nonlinear pure-feedback system, a novel nonlinear pure-feedback system is proposed in which an unconventional coordinate transformation is introduced and a novel unconventional dynamic surface algorithm is designed to eliminate the problem of "calculation expansion" caused by the use of backstepping in the pure-feedback system. Meanwhile, a sufficiently smooth projection algorithm is introduced to suppress the parameter drift in the nonlinear pure-feedback system. Simulation experiments demonstrate that the designed controller ensures the global and ultimate boundedness of all signals in the closed-loop system and the appropriate designed parameters can make the tracking error arbitrarily small.

1. Introduction

In recent years, nonlinear systems have been widely studied by researchers at home and abroad. Isidori [1] proposed a precise linearization technique, Krstic et al. [2] proposed backstepping, and Astolfi et al. [3] put forward adaptive control methods. After that, backstepping adaptive control has been widely developed and applied [4, 5]. In the majority of studies on nonlinear system tracking control problems [6, 7], the global or semiglobal asymptotic stability and the gradual stability of tracking error are considered as the ultimate control target of nonlinear systems. However, at present, there is no uniform method to analyze nonlinear systems in the control theory of nonlinear systems.

Since nonlinear systems are different from linear systems, it is difficult to be analyzed in the way that linear systems are discussed. However, nonlinear pure-feedback systems [8, 9] are more general systems compared with lower triangle nonlinear systems, which can reflect the actual physical situation while the nonaffine pure-feedback system is one of the complicated systems. According to the literature [10, 11], there are two nonaffine functions in the pure-feedback system: system control input and system state variables. Liu [12] proposed a novel coordinate transformation in the research on the tracking problem of pure-feedback systems and achieved global asymptotic stability of tracking error using the backstepping method. Zeng et al. [13] proposed an active sliding mode disturbance rejection control of a single-input single-output nonaffine nonlinear system. However, the research of Zeng et al. is only applicable to a second-order system. For systems larger than second-order, this method cannot work. In this regard, Liu [14] proposed nonconventional coordinate transformation based on [12] and introduced a first-order auxiliary controller to solve the nonaffine control input problem of n-order system. In engineering applications, system parameters change due to various conditions, so system parameters are variables rather than constants. However, in most of the tracking control theory studies of nonlinear systems [15, 16], we often set system parameters as constants, which reduces the practicality of the tracking control theory of nonlinear systems, so it is necessary to study the influence of unknown parameter drift on system stability. For the uncertainty of parameterization, when there is external disturbance or unmodeled dynamics, parameter drift will occur, and the adaptive controller will deteriorate the system and even lead to system instability [17]. In this case, the boundedness of the estimated value of parameters cannot be guaranteed, causing divergence of other signals in the closed-loop system. A common method for dealing with this situation is to add a robust term (leakage term) to the parameter adaptive law or use a projection operator [18].

In this paper, a sufficiently smooth projection algorithm [19] is introduced to deal with the parameter drift in the designed nonlinear pure-feedback system. Since nonaffine functions exist in the system state variables x_i and the system control input u in the nonlinear pure-feedback system, the conventional coordinate transformation [2] will increase the difficulty of designing the controllers, so the nonconventional coordinate transformation proposed by [14] is introduced. Due to the nonaffine structure of the pure-feedback system, it is difficult to design the virtual controllers α_i, and the conventional dynamic surface algorithm [20] is difficult to be directly used in the pure-feedback system. Thus, in this paper, a novel nonconventional dynamic surface algorithm is designed to reduce the complexity of designing the controllers for the nonlinear pure-feedback system. A nonlinear pure-feedback system with unknown parameters θ_i is designed, in which unknown parameters are added compared with [21] in order to solve the control problem of parameter uncertainty in the pure-feedback system, which is a further supplement to nonlinear pure-feedback systems. In [21], the unknown parameters are set to θ^*, which limits the selection of system parameters, thus constraining the whole system. However, the unknown parameters are θ_i in this paper, so the nonlinear pure-feedback system designed is more common. In this paper, backstepping method, nonconventional dynamic surface algorithm, Nussbaum function [22], and sufficiently smooth projection algorithm are combined so that the designed controller can achieve the uniform and ultimate boundedness of all signals in the closed-loop system.

Main contributions of this paper are summarized as follows:

(1) The parametric drift concept is first proposed in pure-feedback systems. In order to solve the parameter drift problem of the nonlinear pure-feedback system, a sufficiently smooth projection algorithm is introduced.

(2) A novel nonconventional dynamic surface algorithm is designed according to the nonconventional coordinate transformation to eliminate the problem of "calculation expansion" in the pure-feedback system.

(3) The virtual controllers α_i, the actual controller w, and the adaptive laws $\widehat{\theta}_i$ are designed so that the tracking error of the nonlinear pure-feedback system with parameter drift converges to a region of the origin, and all signals in the closed-loop system are uniform and ultimately bounded.

Other parts of this paper are arranged as follows: In Section 2, problem statement and preliminaries are presented. In the third section, aiming at the tracking control problem of the nonlinear pure-feedback system with parameter drift, the virtual controllers α_i, the actual controller w, and the parameter adaptive laws $\widehat{\theta}_i$ are designed and semiglobal uniform ultimate boundedness of all signals in the closed-loop system are proved. In Section 4, the correctness of the proposed control scheme is proved by simulation study. The conclusion is presented in Section 5.

2. Problem Statement and Preliminaries

2.1. Systems Description. Consider the following nonlinear pure-feedback system:

$$\begin{cases} \dot{x}_i(t) = f_i\left(x_{i+1}(t), \overline{x}_i(t)\right) + \phi_i^T\left(\overline{x}_i(t), t\right)\theta_i + d_i\left(\overline{x}_i(t), t\right), \\ \dot{x}_n(t) = f_n\left(bu(t), \overline{x}_n(t)\right) + \phi_n^T\left(\overline{x}_n(t), t\right)\theta_n + d_n\left(\overline{x}_n(t), t\right), \\ y(t) = x_1(t), \end{cases} \tag{1}$$

where $1 \le i \le n-1$, $\overline{x}_i = [x_1, x_2, \ldots, x_i]^T \in R^i$ with $i = 1, 2, \ldots, n$, $u \in R$, and $y \in R$ are system state variables, system control input, and system output, respectively. The nonlinear functions $f_i(x_{i+1}, \overline{x}_i)$ are smooth nonaffine known functions; the nonlinear function $f_n(bu, \overline{x}_n)$ is the smooth nonaffine known function with control input coefficient b, among which b is a unknown nonzero constant; $\phi_i(\overline{x}_i, t)$ are known smooth functions, where $\phi_i(0, \ldots, 0) = 0$; $\theta_i = [\theta_1, \ldots, \theta_h]^T \in R^h$ are unknown parameters; and $d_i(x_1, \ldots, x_i)$ are unmodeled dynamics and external disturbances related to state variables.

In system (1), the control coefficient b is added compared with [11, 12]. The unknown parameters $\theta_i \in R^h$ are added in system (1) compared with [21], where i represents the ith order subsystem and h represents the number of unknown parameters in the ith subsystem. In [21], the number of parameters in each subsystem is equal, while in actual engineering systems, the number of parameters in each subsystem cannot be equal. The unknown parameters $\theta_i \in R^h$ designed in this paper just solve this defect, that is, different h is set according to different i.

For easy analysis and calculation, $f_i(x_{i+1}, \overline{x}_i)$ are simplified as f_i and $f_n(bu, \overline{x}_n)$ is simplified as f_n.

In order to avoid the control problem of system (1) in the design of the controller, the following assumptions are made.

Remark 1. Since the control coefficient b is an unknown nonzero constant, the positive and negative signs are unknown, i.e., the control direction is unknown.

Assumption 1. The following equation is made workable at any time:

$$\begin{cases} \dfrac{\partial f_i}{\partial x_{i+1}} \neq 0, \quad i = 1, \ldots, n, \\ \\ \dfrac{\partial f_n}{\partial u} \neq 0. \end{cases} \tag{2}$$

Remark 2. Assumption 1 is weaker than the assumed condition $\exists\kappa \in R^+$, $|\partial f_i(x_{i+1}, \overline{x}_i)/\partial x_{i+1}| > \kappa$ in [11].

Because the input u is characterized by being nonaffine. Therefore, the first-order auxiliary system of the controller is designed as follows:

$$\frac{\partial f_n}{\partial x_{n+1}}\dot{u} = w, \tag{3}$$

where w is the undetermined function and $x_{n+1} = u$.

Assumption 2. Unknown parameters are bounded and there is a bounded set Ω

$$\Omega = \{\theta\colon \|\theta\| \le \theta_0\}, \tag{4}$$

where θ_0 is a known positive constant.

Assumption 3. $d_i(\overline{x}_i, t)$ are bounded and their known upper bounds satisfy

$$\left|d_i(\overline{x}_i, t)\right| \le \overline{d}_i(\overline{x}_i), \quad i = 1, 2, \ldots, n, \tag{5}$$

where $\overline{d}_i(\overline{x}_i)$ are positive constants correlated with $\overline{x}_i$.

Assumption 4. Smooth nonaffine functions $f_i(x_{i+1}, \overline{x}_i)$ meet

$$0 < \left|\frac{\partial f_i(x_{i+1}, \overline{x}_i)}{\partial x_{i+1}}\right| \le A_i, \quad i = 1, 2, \ldots, n-1, \tag{6}$$

where A_i are positive constants.

Lemma 1. *If smooth bounded functions $f_i(x_{i+1}, \overline{x}_i)$ meet assumption 4, x_{i+1} are bounded.*

Proof. According to mean value theorem, the following equation is obtained:

$$f_i(x_{i+1}, \overline{x}_i) - f_i(0, \overline{x}_i) = \frac{\partial f_i(\varsigma, \overline{x}_i)}{\partial \varsigma}(x_{i+1} - 0), \tag{7}$$

where ς is the arbitrary point of $(0, x_{i+1})$ interval. It can be known from assumption 4 that $\partial f_i(x_{i+1}, \overline{x}_i)/\partial x_{i+1}$ are bounded and $f_i(x_{i+1}, \overline{x}_i)$ are also bounded; thus, x_{i+1} are bounded with $i = 1, 2, \ldots, n$. □

Assumption 5. The smooth reference trajectory r is bounded and the $n+1$th order is derivable.

2.2. Preliminaries. In this paper, a sufficiently smooth projection algorithm [19] is introduced to the nonlinear pure-feedback system, as shown in (8)–(11).

$$\begin{aligned}
\mathrm{Proj}_d(\mu, \widehat{\theta}) &= \mu - \frac{\eta_1\eta_2}{4(\sigma^2 + 2\sigma\theta_0)^{m+1}\theta_0^2}\nabla p_d(\widehat{\theta}),\\
p_d(\widehat{\theta}) &= \widehat{\theta}^T\widehat{\theta} - \theta_0^2,\\
\eta_1 &= \begin{cases} p_d^{m+1}(\widehat{\theta}), & p_d(\widehat{\theta}) > 0,\\ 0, & \text{others},\end{cases}\\
\eta_2 &= \frac{1}{2}\nabla p_d(\widehat{\theta})^T\mu + \sqrt{\left(\frac{1}{2}\nabla p_d(\widehat{\theta})^T\mu\right)^2 + \delta^2},
\end{aligned} \tag{8}$$

where $\sigma > 0$ and $\delta > 0$ are arbitrary positive constants.

If $\widehat{\theta}(0) \in \Omega$, the algorithm has the following properties:

(1) $\|\widehat{\theta}(t)\| \le \theta_0 + \sigma$, $\forall t \ge 0$

(2) $\widetilde{\theta}^T\mathrm{Proj}_d(\mu, \widehat{\theta}) \ge \widetilde{\theta}^T\mu$

(3) $\mathrm{Proj}_d(\mu, \widehat{\theta})$ is m-order differentiable

Definition 1. If the continuous function $N(\cdot)$ satisfies (9), it is called the Nussbaum function [22].

$$\begin{cases} \lim\limits_{s\longrightarrow+\infty}\sup\dfrac{1}{s}\displaystyle\int_0^s N(\xi)d\xi = +\infty,\\ \lim\limits_{s\longrightarrow+\infty}\inf\dfrac{1}{s}\displaystyle\int_0^s N(\xi)d\xi = -\infty,\end{cases} \tag{9}$$

where $\xi^2\cos(\xi)$ is adopted as the Nussbaum function in this paper.

Lemma 2 (see [23]). *If $V(\cdot)$ and $\xi(\cdot)$ are smooth functions defined in $[0, t_f)$ and $V(t) \ge 0$ with $\forall t \in [0, t_f)$, $N(\cdot)$ is even smooth Nussbaum function, if the following inequality holds:*

$$V(t) \le C_0 + e^{-C_1 t}\int_0^t (N(\xi(\tau)) + 1)\dot{\xi}(\tau)e^{C_1\tau}\mathrm{d}\tau, \quad \forall t \in \left[0, t_f\right), \tag{10}$$

where $C_1 > 0$ and C_0 are certain appropriate constants, then $V(t)$, $\xi(t)$ and $\int_0^t (N(\xi(\tau)) + 1)\dot{\xi}(\tau)d\tau$ are bounded in $[0, t_f)$.

3. Main Results

The backstepping design is based on the following nonconventional coordinate transformation [14]:

$$\begin{cases} z_1 = x_1 - r,\\ z_i = \dot{x}_{i-1} - q_{i-1},\end{cases} \tag{11}$$

where $i = 2, 3, \ldots, n+1$, $z_1 \in R$, r, and q_{i-1} are the tracking error, smooth reference trajectory signal, and error state variables, respectively.

Remark 3. In the conventional coordinate transformation [2], x_{i+1} are made as the virtual controller of the ith order subsystem, as shown in (15).

$$\begin{cases} z_1 = x_1 - r, \\ z_i = x_i - a_{i-1}. \end{cases} \tag{12}$$

Unlike the standard dynamic surface algorithm [20], the nonconventional dynamic surface algorithm designed in this paper is as follows.

Define the first-order low-pass filter formula.

$$\begin{cases} \tau_1 \dot{q}_1 + q_1 = \alpha_1, \\ \dfrac{\partial f_{i-1}}{\partial x_i}\left(\tau_i \dot{q}_i + \dot{q}\right) = \alpha_i, \\ q_1(0) = \alpha_1(0), \\ q_i(0) = \alpha_i(0), \end{cases} \tag{13}$$

where $i = 2, \ldots, n$, τ_i are the filter time constants.

The filter error equation is defined as follows:

$$\begin{cases} y_1 = q_1 - \alpha_1, \\ \dfrac{\partial f_{i-1}}{\partial x_i} y_i = \dfrac{\partial f_{i-1}}{\partial x_i} q_i - \alpha_i, \end{cases} \tag{14}$$

where $i = 2, \ldots, n$, α_i are the ith virtual controllers. The nonconventional coordinate transformation cooperates with the nonconventional dynamic surface algorithm to solve the problem of "calculation expansion" caused by the application of the nonconventional coordinate transformation in the nonlinear pure-feedback systems.

Remark 4. The first-order low-pass filter of standard dynamic surface algorithm [20]

$$\tau_{i+1} \dot{q}_{i+1} + q_{i+1} = \alpha_i. \tag{15}$$

The filter error of standard dynamic surface algorithm

$$y_i = q_i - a_{i-1}. \tag{16}$$

Step 1. According to (1) and (11), we obtain

$$\dot{z}_1 = \dot{x}_1 - \dot{r} = z_2 + y_1 + \alpha_1 - \dot{r}. \tag{17}$$

Design the virtual controller α_1 as

$$\alpha_1 = -c_1 z_1 + \dot{r}, \tag{18}$$

where c_1 is a positive design constant.

According to (14), the following equation is obtained:

$$y_1 = q_1 - \alpha_1. \tag{19}$$

By deriving y_1 with respect to time, we can obtain

$$\begin{aligned} \dot{y}_1 &= \dot{q}_1 - \dot{\alpha}_1 \\ &= -\frac{y_1}{\tau_1} + c_1\left(z_2 + y_1 + \alpha_1 - \dot{r}\right) - \ddot{r} \\ &= -\frac{y_1}{\tau_1} + B_1\left(z_1, z_2, y_1, r, \dot{r}, \ddot{r}\right), \end{aligned} \tag{20}$$

where $B_1(z_1, z_2, y_1, r, \dot{r}, \ddot{r})$ is a continuous function, the maximum of $|B_1(\cdot)|$ is set to be D_1.

The Lyapunov function is defined as

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} y_1^2. \tag{21}$$

According to (17), (20) and Young's inequality, we have

$$\dot{V}_1 \leq -\left(c_1 - \frac{1}{2}\right) z_1^2 - \left(\frac{1}{\tau_1} - \frac{1+\delta}{2}\right) y_1^2 + z_1 z_2 + \frac{D_1^2}{2\delta}, \tag{22}$$

where $|y_1| \cdot |B_1| \leq (\delta y_1^2/2) + (D_1^2/2\delta)$, δ is a positive constant.

Step 2. According to (1) and (11), we have

$$\begin{aligned} \dot{z}_2 &= \frac{\partial f_1}{\partial x_2}\dot{x}_2 + \frac{\partial f_1}{\partial x_1}\dot{x}_1 + \frac{\partial \phi_1}{\partial x_1}\dot{x}_1 \theta_1 + \frac{\partial d_1}{\partial x_1}\dot{x}_1 - \dot{q}_1 \\ &= \frac{\partial f_1}{\partial x_2}\left(\frac{z_3 + y_2 + \alpha_2}{(\partial f_1/\partial x_2)}\right) + \frac{\partial f_1}{\partial x_1}\dot{x}_1 + \frac{\partial \phi_1}{\partial x_1}\dot{x}_1 \theta_1 + \frac{\partial d_1}{\partial x_1}\dot{x}_1 - \dot{q}_1. \end{aligned} \tag{23}$$

Design the virtual controller α_2 as

$$\alpha_2 = -c_2 z_2 - z_1 - \frac{\partial f_1}{\partial x_1}\dot{x}_1 - \frac{\partial \phi_1}{\partial x_1}\dot{x}_1 \hat{\theta}_1 - \frac{\overline{d}(x_1)_1^2}{2\varepsilon} z_2 + \dot{q}_1, \tag{24}$$

where c_2 is a positive design constant, $\hat{\theta}_1$ is the estimation of θ_1, and $\overline{d}(x_1)_1^2 z_2/2\varepsilon$ is a nonlinear damping term which is used to process the disturbance term in (23).

Design the adaptive law as

$$\dot{\hat{\theta}}_1 = \gamma \mathrm{Proj}_d\left(\mu_1, \hat{\theta}_1\right), \tag{25}$$

$$\mu_1 = \frac{\partial \phi_1}{\partial x_1}\dot{x}_1 z_2. \tag{26}$$

Substituting (24) into (23) yields

$$\begin{aligned} \dot{z}_2 = {} & -c_2 z_2 - z_1 + \frac{\partial f_1}{\partial x_2} z_3 + \frac{\partial f_1}{\partial x_2} y_2 + \frac{\partial \phi_1}{\partial x_1}\dot{x}_1 \tilde{\theta}_1 \\ & + \frac{\partial d_1}{\partial x_1}\dot{x}_1 - \frac{\overline{d}(x_1)_1^2}{2\varepsilon} z_2, \end{aligned} \tag{27}$$

where $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$.

According to (14), we can obtain

$$\frac{\partial f_1}{\partial x_2} y_2 = \frac{\partial f_1}{\partial x_2} q_2 - \alpha_2. \tag{28}$$

By deriving y_2 with respect to time, we can obtain

$$\dot{y}_2 = -\frac{y_2}{\tau_2} + B_2, \quad (29)$$

where $B_2(z_1, z_2, z_3, y_1, y_2, r, \dot{r}, \ddot{r})$ is a continuous function and the maximum of $|B_2(\cdot)|$ is set to be D_2.

The Lyapunov function is defined as

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2\gamma}\widetilde{\theta}_1^T\widetilde{\theta}_1 + \frac{1}{2}y_2^2. \quad (30)$$

According to (22), (25), (27), (28), and Young's inequality, we have

$$\dot{V}_2 \le \dot{V}_1 - c_2 z_2^2 - z_1 z_2 + \frac{\partial f_1}{\partial x_2} z_2 z_3 + \frac{\partial f_1}{\partial x_2} y_2 z_2 + \frac{\varepsilon}{2} + \widetilde{\theta}_1\left(\frac{\partial \phi_1}{\partial x_1}\dot{x}_1 z_2 - \mathrm{Proj}_d\left(\mu_1, \widehat{\theta}_1\right)\right) - \frac{y_2^2}{\tau_2} + \frac{\delta y_2^2}{2} + \frac{D_2^2}{2\delta}. \quad (31)$$

According to the properties (2) of the sufficiently smooth projection algorithm [20], the following equation is obtained:

$$\dot{V}_2 \le -\left(c_1 - \frac{1}{2}\right)z_1^2 - \left(c_2 - \frac{A_1^2}{2}\right)z_2^2 - \sum_{i=1}^{2}\left(\frac{1}{\tau_i} - \frac{1+\delta}{2}\right)y_i^2 + \sum_{i=1}^{2}\frac{D_i^2}{2\delta} + \frac{\partial f_1}{\partial x_2} z_2 z_3 + \frac{\varepsilon}{2}, \quad (32)$$

where $|y_2| \cdot |B_2| \le (\delta y_2^2/2) + (D_2^2/2\delta)$ and ε is a positive constant.

Step i $(3 \le i \le n-1)$. According to (1) and (11), we can obtain

$$\begin{aligned}\dot{z}_i &= \frac{\partial f_{i-1}}{\partial x_i}\dot{x}_i + \sum_{j=1}^{i-1}\frac{\partial f_{i-1}}{\partial x_j}\dot{x}_j + \sum_{j=1}^{i-1}\frac{\partial \phi_{i-1}}{\partial x_j}\dot{x}_j\theta_{i-1} + \sum_{j=1}^{i-1}\frac{\partial d_{i-1}}{\partial x_j}\dot{x}_j - \dot{q}_{i-1} \\ &= \frac{\partial f_{i-1}}{\partial x_i}\left(\frac{z_{i+1} + y_i + \alpha_i}{\partial f_{i-1}/\partial x_i}\right) + \sum_{j=1}^{i-1}\frac{\partial f_{i-1}}{\partial x_j}\dot{x}_j + \sum_{j=1}^{i-1}\frac{\partial \phi_{i-1}}{\partial x_j}\dot{x}_j\theta_{i-1} \\ &\quad + \sum_{j=1}^{i-1}\frac{\partial d_{i-1}}{\partial x_j}\dot{x}_j - \dot{q}_{i-1}.\end{aligned} \quad (33)$$

Design the virtual controller α_i as

$$\alpha_i = -c_i z_i - \frac{\partial f_{i-2}}{\partial x_{i-1}} z_{i-1} - \sum_{j=1}^{i-1}\frac{\partial f_{i-1}}{\partial x_j}\dot{x}_j - \sum_{j=1}^{i-1}\frac{\partial \phi_{i-1}}{\partial x_j}\dot{x}_j\widehat{\theta}_{i-1} - (i-1)\frac{\overline{d}(x_{i-1})_{i-1}^2}{2\varepsilon}z_i + \dot{q}_{i-1}, \quad (34)$$

where c_i is a positive design constant, $\widehat{\theta}_{i-1}$ is the estimation of θ_{i-1}, and $(i-1)z_i\overline{d}(x_{i-1})_{i-1}^2/2\varepsilon$ is the nonlinear damping term which is used to process the disturbance term in (33).

Design the adaptive law as

$$\dot{\widehat{\theta}}_{i-1} = \gamma\mathrm{Proj}_d\left(\mu_{i-1}, \widehat{\theta}_{i-1}\right), \quad (35)$$

$$\mu_{i-1} = \sum_{j=1}^{i-1}\frac{\partial \phi_{i-1}}{\partial x_j}\dot{x}_j z_i. \quad (36)$$

Substituting (34) into (33) yields

$$\dot{z}_i = -c_i z_i - \frac{\partial f_{i-2}}{\partial x_{i-1}} z_{i-1} + \frac{\partial f_{i-1}}{\partial x_i} z_{i+1} + \frac{\partial f_{i-1}}{\partial x_i} y_i + \sum_{j=1}^{i-1}\frac{\partial \phi_{i-1}}{\partial x_j}\dot{x}_j\widetilde{\theta}_{i-1} + \sum_{j=1}^{i-1}\frac{\partial d_{i-1}}{\partial x_j}\dot{x}_j - (i-1)\frac{\overline{d}(x_{i-1})_{i-1}^2}{2\varepsilon}z_i, \quad (37)$$

where $\widetilde{\theta}_{i-1} = \theta_{i-1} - \widehat{\theta}_{i-1}$.

Based on the filter error equation (14), we can obtain

$$\frac{\partial f_{i-1}}{\partial x_i} y_i = \frac{\partial f_{i-1}}{\partial x_i} q_i - \alpha_i. \quad (38)$$

By deriving y_i with respect to time, the following equation is obtained:

$$\dot{y}_i = -\frac{y_i}{\tau_i} + B_i, \quad (39)$$

where $B_i(z_1, \ldots, z_{i+1}, y_1, \ldots, y_i, r, \dot{r}, \ddot{r})$ is a continuous function and the maximum of $|B_i(\cdot)|$ is set to be D_i.

The Lyapunov function is defined as $V_i = V_{i-1} + (z_i^2/2) + (\widetilde{\theta}_{i-1}^T\widetilde{\theta}_{i-1}/2\gamma) + (y_i^2/2)$.

According to (35), (37), (39) and Young's inequality, we have

$$\begin{aligned}\dot{V}_i &\le \dot{V}_{i-1} - c_i z_i^2 - \frac{\partial f_{i-2}}{\partial x_{i-1}} z_{i-1} z_i + \frac{\partial f_{i-1}}{\partial x_i} z_i z_{i+1} + \frac{\partial f_{i-1}}{\partial x_i} y_i z_i \\ &\quad + \widetilde{\theta}_{i-1}\left(\sum_{j=1}^{i-1}\frac{\partial \phi_{i-1}}{\partial x_j}\dot{x}_j z_i - \mathrm{Proj}_d\left(\mu_{i-1}, \widehat{\theta}_{i-1}\right)\right) \\ &\quad + \sum_{j=1}^{i-1}\frac{\partial d_{i-1}}{\partial x_j}\dot{x}_j z_i - (i-1)\frac{\overline{d}(\overline{x}_{i-1})_{i-1}^2}{2\varepsilon}z_i^2 - \frac{y_i^2}{\tau_i} + \frac{\delta y_i^2}{2} + \frac{D_i^2}{2\delta}.\end{aligned} \quad (40)$$

By the properties (2) of the sufficiently smooth projection algorithm [20], the following equation is obtained:

$$\dot{V}_i \le -\left(c_1 - \frac{1}{2}\right)z_1^2 - \sum_{j=2}^{i}\left(c_j - \frac{A_j^2}{2}\right)z_j^2 - \sum_{j=1}^{i}\left(\frac{1}{\tau_j} - \frac{1+\delta}{2}\right)y_i^2 + \sum_{j=1}^{i}\frac{D_j^2}{2\delta} + \frac{\partial f_{i-1}}{\partial x_i} z_i z_{i+1} + \frac{i(i-1)}{4}\varepsilon, \quad (41)$$

where $|y_i| \cdot |B_i| \le (\delta y_i^2/2) + (D_i^2/2\delta)$.

Step n. According to (1) and (11), we can obtain

$$\dot{z}_n = \frac{\partial f_{n-1}}{\partial x_n}\dot{x}_n + \sum_{j=1}^{n-1}\frac{\partial f_{n-1}}{\partial x_j}\dot{x}_j + \sum_{j=1}^{n-1}\frac{\partial \phi_{n-1}}{\partial x_j}\dot{x}_j\theta_{n-1} + \sum_{j=1}^{n-1}\frac{\partial d_{n-1}}{\partial x_j}\dot{x}_j - \dot{q}_{n-1}$$
$$= \frac{\partial f_{n-1}}{\partial x_n}\left(\frac{z_{n+1} + y_n + \alpha_n}{\partial f_{n-1}/\partial x_n}\right) + \sum_{j=1}^{n-1}\frac{\partial f_{n-1}}{\partial x_j}\dot{x}_j + \sum_{j=1}^{n-1}\frac{\partial \phi_{n-1}}{\partial x_j}\dot{x}_j\theta_{n-1}$$
$$+ \sum_{j=1}^{n-1}\frac{\partial d_{n-1}}{\partial x_j}\dot{x}_j - \dot{q}_{n-1}. \tag{42}$$

Design the virtual controller α_n as

$$\alpha_n = -c_n z_n - \frac{\partial f_{n-2}}{\partial x_{n-1}}z_{n-1} - \sum_{j=1}^{n-1}\frac{\partial f_{n-1}}{\partial x_j}\dot{x}_j - \sum_{j=1}^{n-1}\frac{\partial \phi_{n-1}}{\partial x_j}\dot{x}_j\widehat{\theta}_{n-1}$$
$$- (n-1)\frac{\overline{d}(x_{n-1})_{n-1}^2}{2\varepsilon}z_n + \dot{q}_{n-1}, \tag{43}$$

where c_n is a positive design constant, $\widehat{\theta}_{n-1}$ is the estimation of θ_{n-1}, $(n-1)z_n\overline{d}(x_{n-1})_{n-1}^2/2\varepsilon$ is the nonlinear damping term which is used to process the disturbance term in (42).

Design the adaptive law as

$$\dot{\widehat{\theta}}_{n-1} = \gamma \mathrm{Proj}_d\left(\mu_{n-1}, \widehat{\theta}_{n-1}\right), \tag{44}$$

$$\mu_{n-1} = \sum_{j=1}^{n-1}\frac{\partial \phi_{n-1}}{\partial x_j}\dot{x}_j z_n. \tag{45}$$

Substituting (43) into (42) yields

$$\dot{z}_n = -c_n z_n - \frac{\partial f_{n-2}}{\partial x_{n-1}}z_{n-1} + \frac{\partial f_{n-1}}{\partial x_n}z_{n+1} + \frac{\partial f_{n-1}}{\partial x_n}y_n$$
$$+ \sum_{j=1}^{n-1}\frac{\partial \phi_{n-1}}{\partial x_j}\dot{x}_j\widetilde{\theta}_{n-1} + \sum_{j=1}^{n-1}\frac{\partial d_{n-1}}{\partial x_j}\dot{x}_j - (n-1)\frac{\overline{d}(x_{n-1})_{n-1}^2}{2\varepsilon}z_n, \tag{46}$$

where $\widetilde{\theta}_{n-1} = \theta_{n-1} - \widehat{\theta}_{n-1}$.

According to (14), we can obtain

$$\frac{\partial f_{n-1}}{\partial x_n}y_n = \frac{\partial f_{n-1}}{\partial x_n}q_n - \alpha_n, \tag{47}$$

By deriving y_n with respect to time, the following equation is obtained:

$$\dot{y}_n = -\frac{y_n}{\tau_n} + B_n, \tag{48}$$

where $B_n(z_1, \ldots, z_{n+1}, y_1, \ldots, y_n, r, \dot{r}, \ddot{r})$ is a continuous function and the maximum of $|B_n(\cdot)|$ is set to be D_n.

The Lyapunov function is defined as $V_n = V_{n-1} + (z_n^2/2) + (\widetilde{\theta}_{n-1}^T\widetilde{\theta}_{n-1}/2\gamma) + (y_n^2/2)$.

According to (44), (46), (48), and Young's inequality, we have

$$\dot{V}_n \leq \dot{V}_{n-1} - c_n z_n^2 - \frac{\partial f_{n-2}}{\partial x_{n-1}}z_{n-1}z_n + \frac{\partial f_{n-1}}{\partial x_n}z_n z_{n+1}$$
$$+ \frac{\partial f_{n-1}}{\partial x_n}y_n z_n + \frac{\delta y_n^2}{2} + \frac{D_n^2}{2\delta}$$
$$+ \widetilde{\theta}_{n-1}\left(\sum_{j=1}^{n-1}\frac{\partial \phi_{n-1}}{\partial x_j}\dot{x}_j z_n - \mathrm{Proj}_d\left(\mu_{n-1}, \widehat{\theta}_{n-1}\right)\right) \tag{49}$$
$$+ \sum_{j=1}^{n-1}\frac{\partial d_{n-1}}{\partial x_j}\dot{x}_j z_n - (n-1)\frac{\overline{d}(\overline{x}_{n-1})_{n-1}^2}{2\varepsilon}z_n^2 - \frac{y_n^2}{\tau_n}.$$

By the properties (2) of the sufficiently smooth projection algorithm [20], the following equation is obtained:

$$\dot{V}_n \leq -\left(c_1 - \frac{1}{2}\right)z_1^2 - \sum_{j=2}^{n}\left(c_j - \frac{A_j^2}{2}\right)z_j^2 - \sum_{j=1}^{n}\left(\frac{1}{\tau_j} - \frac{1+\delta}{2}\right)y_j^2$$
$$+ \sum_{j=1}^{n}\frac{D_j^2}{2\delta} + \frac{\partial f_{n-1}}{\partial x_n}z_n z_{n+1} + \frac{n(n-1)}{4}\varepsilon, \tag{50}$$

where $|y_n| \cdot |B_n| \leq (\delta y_n^2/2) + (D_n^2/2\delta)$.

Step $n+1$. According to (1) and (11), we can obtain

$$\dot{z}_{n+1} = \frac{\partial f_n}{\partial x_{n+1}}\dot{x}_{n+1} + \sum_{j=1}^{n}\frac{\partial f_n}{\partial x_j}\dot{x}_j + \sum_{j=1}^{n}\frac{\partial \phi_n}{\partial x_j}\dot{x}_j\theta_n + \sum_{j=1}^{n}\frac{\partial d_n}{\partial x_j}\dot{x}_j - \dot{q}_n$$
$$= \frac{\partial f_n}{\partial x_{n+1}}\dot{u} + \sum_{j=1}^{n}\frac{\partial f_n}{\partial x_j}\dot{x}_j + \sum_{j=1}^{n}\frac{\partial \phi_n}{\partial x_j}\dot{x}_j\theta_n + \sum_{j=1}^{n}\frac{\partial d_n}{\partial x_j}\dot{x}_j - \dot{q}_n. \tag{51}$$

Remark 5. The control coefficient b is included in $\partial f_n/\partial x_{n+1}$, so it is unnecessary to analyze the unknown parameter b independently, which is also the advantage of introducing nonconventional coordinate transformation in this paper.

Design the actual controller as

$$w = N(\xi)\alpha_{n+1}, \tag{52}$$

$$\alpha_{n+1} = c_{n+1}z_{n+1} + \frac{\partial f_{n-1}}{\partial x_n}z_n + \sum_{j=1}^{n}\frac{\partial f_n}{\partial x_j}\dot{x}_j + \sum_{j=1}^{n}\frac{\partial \phi_n}{\partial x_j}\dot{x}_j\widehat{\theta}_n$$
$$+ n\frac{\overline{d}(x_n)_n^2}{2\varepsilon}z_{n+1} - \dot{q}_n, \tag{53}$$

where c_{n+1} is a positive design constant, $\widehat{\theta}_n$ is the estimation of θ_n, and $nz_{n+1}\overline{d}(x_n)_n^2/2\varepsilon$ is the nonlinear damping term which is used to process the disturbance term in (51).

The adaptive law of the parameter ξ is designed as

$$\dot{\xi} = z_{n+1}\alpha_{n+1}. \tag{54}$$

The adaptive law of parameter $\widehat{\theta}_n$ is designed to be

$$\dot{\widehat{\theta}}_n = \gamma \mathrm{Proj}_d\left(\mu_n, \widehat{\theta}_n\right), \tag{55}$$

$$\mu_n = \sum_{j=1}^{n} \frac{\partial \phi_n}{\partial x_j} \dot{x}_j z_{n+1}. \tag{56}$$

Substituting (52) into (51) yields

$$\dot{z}_{n+1} = N(\xi)\alpha_{n+1} + \sum_{j=1}^{n} \frac{\partial f_n}{\partial x_j}\dot{x}_j + \sum_{j=1}^{n} \frac{\partial \phi_n}{\partial x_j}\dot{x}_j \theta_n + \sum_{j=1}^{n} \frac{\partial d_n}{\partial x_j}\dot{x}_j - \dot{q}_n. \tag{57}$$

where $\widetilde{\theta}_n = \theta_n - \widehat{\theta}_n$.

Theorem 1. *For system (1), under the condition that Assumptions 1–5 are satisfied, the first-order low-pass filter (13), the control law (52), and the parameter adaptive laws (26), (35), (44), (54), and (55) are adopted to make the tracking error of the system converge to a region of the origin, and all signals in the closed-loop system are uniformly and ultimately bounded.*

Proof. The Lyapunov function is defined as $V = V_n + (z_{n+1}^2/2) + (\widetilde{\theta}_n^T \widetilde{\theta}_n / 2\gamma)$.

According to (50), (55), (57) and Young's inequality, we can get

$$\begin{aligned}
\dot{V} &= \dot{V}_n + \dot{\xi} - \dot{\xi} + z_{n+1} N(\xi)\alpha_{n+1} + \sum_{j=1}^{n} \frac{\partial f_n}{\partial x_j}\dot{x}_j z_{n+1} \\
&\quad + \sum_{j=1}^{n} \frac{\partial \phi_n}{\partial x_j}\dot{x}_j \theta_n z_{n+1} + \sum_{j=1}^{n} \frac{\partial d_n}{\partial x_j}\dot{x}_j z_{n+1} - \dot{q}_n z_{n+1} - \frac{1}{\gamma}\widetilde{\theta}_n^T \dot{\widehat{\theta}}_n \\
&\le -\left(c_1 - \frac{1}{2}\right) z_1^2 - \sum_{j=2}^{n} \left(c_j - \frac{A_j^2}{2}\right) z_j^2 - \sum_{j=1}^{n} \left(\frac{1}{\tau_j} - \frac{1+\delta}{2}\right) y_j^2 \\
&\quad - c_{n+1} z_{n+1}^2 + \sum_{j=1}^{n} \frac{D_j^2}{2\delta} + \frac{n(n-1)}{4}\varepsilon + N(\xi)\dot{\xi} + \dot{\xi} + \widetilde{\theta}_n \\
&\quad \cdot \left(\sum_{j=1}^{n} \frac{\partial \phi_n}{\partial x_j}\dot{x}_j z_{n+1} - \mathrm{Proj}_d\left(\mu_n, \widehat{\theta}_n\right)\right) + \frac{n\varepsilon}{2}.
\end{aligned} \tag{58}$$

Based on the property (2) of the sufficiently smooth projection algorithm, we can obtain

$$\begin{aligned}
\dot{V} &\le -\left(c_1 - \frac{1}{2}\right) z_1^2 - \sum_{j=2}^{n} \left(c_j - \frac{A_j^2}{2}\right) z_j^2 - \sum_{j=1}^{n} \left(\frac{1}{\tau_j} - \frac{1+\delta}{2}\right) y_j^2 \\
&\quad + \sum_{j=1}^{n} \frac{D_j^2}{2\delta} + \frac{n(n+1)}{4}\varepsilon + N(\xi)\dot{\xi} + \dot{\xi} - c_{n+1} z_{n+1}^2,
\end{aligned} \tag{59}$$

where the parameter is designed to satisfy

$$\begin{cases}
\delta > 0, & \varepsilon > 0, \\
c_1 > \dfrac{1}{2}, & \\
c_i > \dfrac{A_i^2}{2}, & i = 2, \ldots, n, \\
\dfrac{1}{\tau_i} > \dfrac{1+\delta}{2}, & \tau_i > 0,\ i = 1, \ldots, n.
\end{cases} \tag{60}$$

Make

$$a_0 = \min \left\{ \begin{array}{c} c_1 - \dfrac{1}{2}, c_j - \dfrac{A_j^2}{2}, c_{n+1} \\ \dfrac{1}{\tau_1} - \dfrac{1+\delta}{2}, \dfrac{1}{\tau_j} - \dfrac{1+\delta}{2} \end{array} \right\}, \quad j = 2, \ldots, n,$$

$$b_0 = \sum_{j=1}^{n} \frac{D_j^2}{2\delta}, \tag{61}$$

then (59) can be rewritten to

$$\begin{aligned}
\dot{V} &\le -a_0 \left(\sum_{i=1}^{n+1} z_i^2 + \sum_{i=1}^{n} y_i^2\right) + b_0 + \frac{n(n+1)}{4}\varepsilon + N(\xi)\dot{\xi} + \dot{\xi} \\
&\le -k_0 V + \sum_{i=1}^{n} \frac{\theta_i^T \theta_i}{2\gamma} + \frac{n(n+1)}{4}\varepsilon + N(\xi)\dot{\xi} + \dot{\xi} + b_0,
\end{aligned} \tag{62}$$

where $k_0 = \min\{2a_0, 1\}$.

When both ends of (62) are multiplied by the exponent $e^{k_0 t}$, we can get

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(e^{k_0 t} V(t)\right) \le \left(\sum_{i=1}^{n} \frac{\theta_i^T \theta_i}{2\gamma} + \frac{n(n+1)}{4}\varepsilon + N(\xi)\dot{\xi} + \dot{\xi} + b_0\right) e^{k_0 t}. \tag{63}$$

After the integration of both ends of (63) in $[0, t]$, we can get

$$\begin{aligned}
V(t) &\le e^{-k_0 t} V(0) + e^{-k_0 t} \int_0^t \\
&\quad \cdot \left(\sum_{i=1}^{n} \frac{\theta_i^T \theta_i}{2\gamma} + \frac{n(n+1)}{4}\varepsilon + (N(\xi) + 1)\dot{\xi} + b_0\right) e^{k_0 \tau} \mathrm{d}\tau \\
&= e^{-k_0 t} V(0) + e^{-k_0 t} \int_0^t (N(\xi) + 1)\dot{\xi} e^{k_0 \tau} \mathrm{d}\tau \\
&\quad + \int_0^t \left(\sum_{i=1}^{n} \frac{\theta_i^T \theta_i}{2\gamma} + \frac{n(n+1)}{4}\varepsilon + b_0\right) e^{-k_0 (t-\tau)} \mathrm{d}\tau.
\end{aligned} \tag{64}$$

Since the adaptive law $\dot{\hat{\theta}}_i$ utilizes a sufficiently smooth projection algorithm, it can be known from the properties of a sufficiently smooth projection algorithm (1) that $\hat{\theta}_i$ is bounded and thus $\tilde{\theta}_i$ is also bounded, which means the $\int_0^t (\sum_{i=1}^n \theta_i^T \theta_i / 2\gamma + ((n(n+1)))/4\varepsilon + b_0) e^{-k_0(t-\tau)} d\tau$ is bounded.

It is noted that (64) can be written as

$$V(t) \leq V(0) + e^{-k_0 t} \int_0^t (N(\xi) + 1)\dot{\xi} e^{k_0 \tau} d\tau + C_0. \tag{65}$$

According to Lemma 2, it can be known that in the interval $[0, t_f)$, $V(t)$, $\xi(t)$, and $e^{-k_0 t} \int_0^t (N(\xi) + 1)\dot{\xi} e^{k_0 \tau} d\tau$ are bounded, and it can be known from Lemma 1 that $f_1, \ldots, f_n$ are bounded and satisfies assumption 4; thus, $x_1, \ldots, x_{n+1}$ are bounded. When t_f tends to ∞, all signals in the closed-loop system are uniformly and ultimately bounded. The upper bound of $e^{-k_0 t} \int_0^t (N(\xi) + 1)\dot{\xi} e^{k_0 \tau} d\tau$ is set to be C_N, then (64) can be written as

$$V(t) \leq e^{-k_0 t} V(0) + C_N + C_0. \tag{66}$$

For arbitrarily given $v \geq \sqrt{2(V(0)e^{-k_0 t} + C_N + C_0)}$, there is T; make it when $t \geq T$, $\|z\| \leq v$, $z = [z_1, \ldots, z_{n+1}]^T$. Appropriate parameters are selected so that the tracking error z_1 can be adjusted to small region of the origin:

$$|z_1| \leq \sqrt{2(V(0)e^{-k_0 t} + C_N + C_0)}. \tag{67}$$

□

4. Illustrative Example

Consider the following second-order nonlinear pure-feedback system:

$$\begin{cases} \dot{x}_1 = x_1 + x_2 + \dfrac{x_2^3}{5} + \theta_{11} x_1^2 + \theta_{12} x_1^3 + d_1(\overline{x}_1, t), \\ \dot{x}_2 = x_1 x_2 + b\left(u + \dfrac{u^3}{7}\right) + \theta_{21} x_1 x_2 + d(\overline{x}_2, t), \\ z_1 = x_1 - r. \end{cases} \tag{68}$$

Corresponding to system (1), we have

$$\begin{aligned} \phi_1^T(\overline{x}_1, t) &= [x_1^2, x_1^3], \\ \theta_1 &= [\theta_{11}; \theta_{12}], \\ \phi_2^T(\overline{x}_2, t) &= [x_1 x_2^2, 0], \\ \theta_2 &= [\theta_{21}; \theta_{22}], \\ d_1(\overline{x}_1, t) &= 0.5 x_1^2 \cos(1.5t), \\ d_2(\overline{x}_2, t) &= 0.7(x_1^2 + x_2^2)\sin^3(t), \\ r &= 0.5\sin(\pi t) + 0.5\sin(t). \end{aligned} \tag{69}$$

Set the design parameters in the above control scheme as

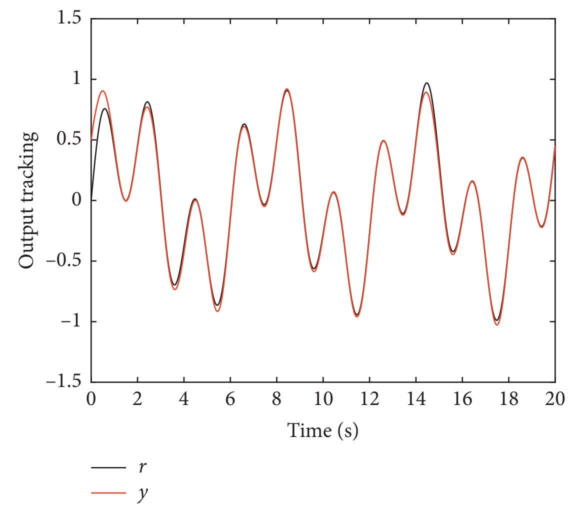

Figure 1: The curve of tracking performance.

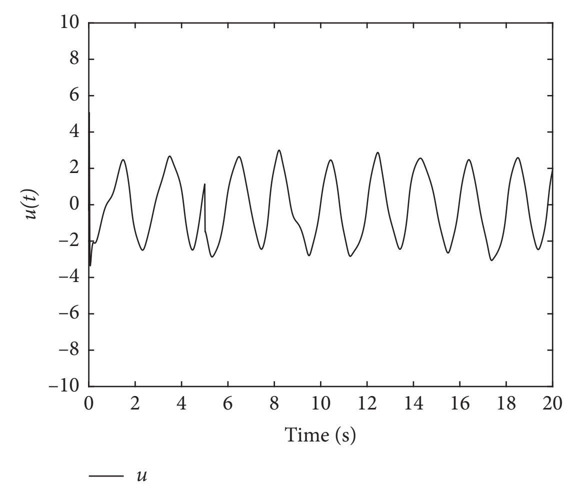

Figure 2: Actual control law $u(t)$.

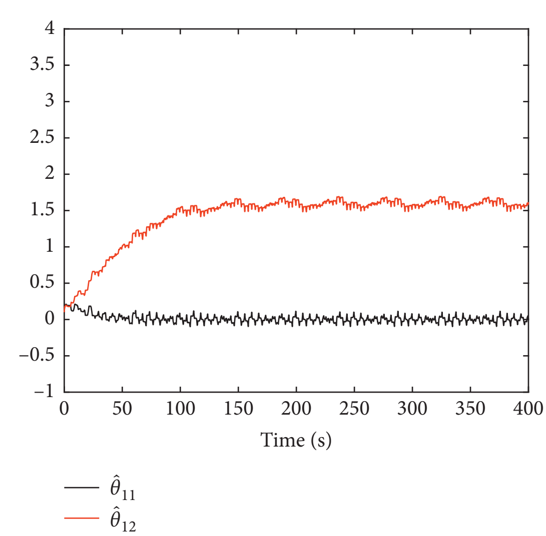

Figure 3: The estimated curve of parameter θ_1.

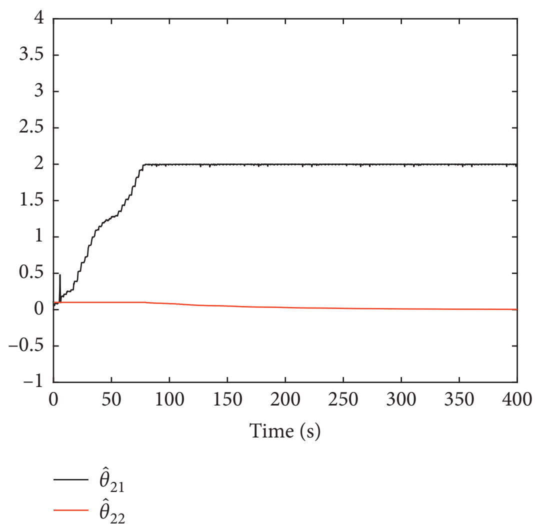

Figure 4: The estimated curve of parameter θ_2.

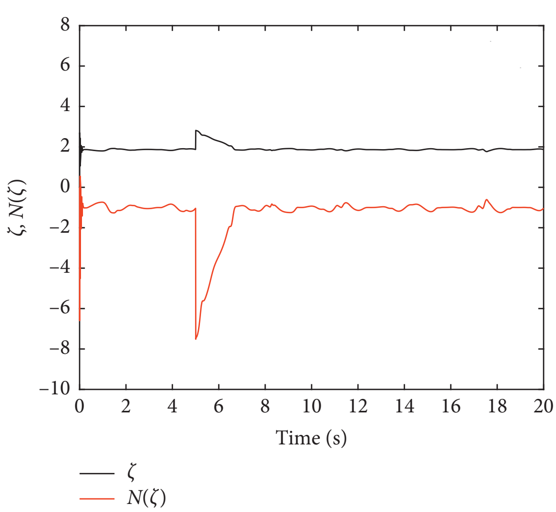

Figure 5: The curve of ξ and $N(\xi)$.

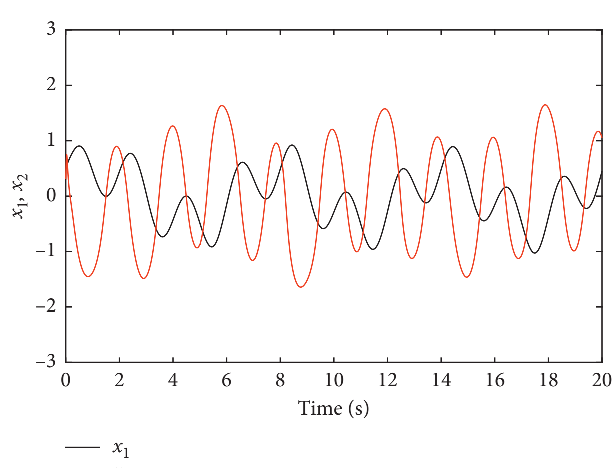

Figure 6: The curve of system state variables x_1 and x_2.

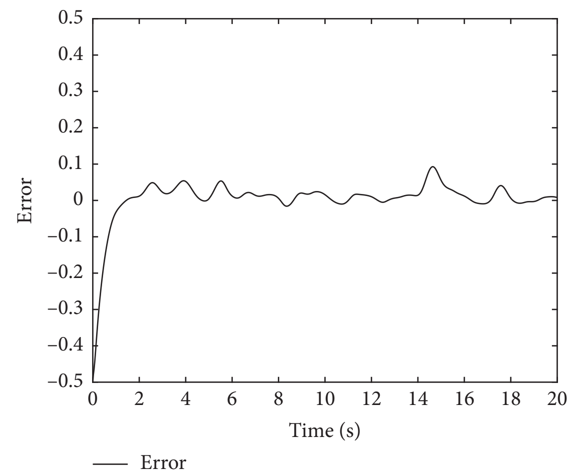

Figure 7: The curve of system error.

$$
\begin{aligned}
c_1 &= 2.2,\\
c_2 &= 5,\\
c_3 &= 5,\\
\tau_1 &= 0.01,\\
\tau_2 &= 0.01,\\
\delta &= 0.1,\\
\varepsilon &= 0.1,\\
A_1 &= 2,\\
D_1 &= D_2 = 2,\\
x_1(0) &= 0.5,\\
x_2(0) &= 0.3,\\
\theta_0 &= 2,\\
\theta_1 &= [0.2\sin(t); 0.2\cos(1.5t)],\\
\theta_2 &= [0.1\sin(t); 0.1],\\
\hat{\theta}_1(0) &= [0.1; 0.1],\\
\hat{\theta}_2(0) &= [0.1; 0.1],\\
\zeta(0) &= 0,\\
u(0) &= 0,\\
q_1(0) &= 0,\\
q_2(0) &= 0.
\end{aligned}
\tag{70}
$$

The control coefficient is set to be $b = \begin{cases} 1, & t \le 5 \\ -1, & \text{others} \end{cases}$.

The simulation results are shown in the following figures. Figure 1 shows the system output y and the reference signal r. It can be seen that the system output y can track the reference signal r well even when the parameters θ_1, θ_2, and control coefficient b change. Figure 2 shows the control input u, indicating that the system input curve is asymptotically bounded. Figure 3 shows the estimated curve of the parameter θ_1. Figure 4 shows the estimated curve of the parameter θ_2. Figure 5 shows that functions ξ and $N(\xi)$ are bounded. Figure 6 shows the system state variables x_1 and x_2. Figure 7 shows the error between the system output y and the reference signal r.

It can be seen from Figures 1–7 that system state variables x_1 and x_2 are bounded, actual controller u is bounded, $\widehat{\theta}_{11}$ and $\widehat{\theta}_{12}$ are bounded, $\widehat{\theta}_{21}$ and $\widehat{\theta}_{22}$ are bounded, ξ and $N(\xi)$ are bounded, and error is bounded, so all signals in the closed-loop system are bounded.

5. Conclusion

In this paper, the adaptive tracking control for a class of nonlinear pure-feedback systems is studied to solve the parameter drift problem. Different from the conventional backstepping method, the nonconventional coordinate transformation is introduced and the nonconventional dynamic surface algorithm is designed to solve the problem "calculation expansion" in the pure-feedback system, and the sufficiently smooth projection algorithm is introduced to solve the unknown parameter drift problem in the pure-feedback system so that the designed controller can make all signals of the closed-loop system globally bounded. Finally, the correctness of the algorithm in this paper is verified by simulation. The nonlinear pure-feedback system considered in this paper has the defect, that is, $f_i(x_{i+1}, \overline{x}_i)$ are known functions, but $f_i(x_{i+1}, \overline{x}_i)$ are often not accurately modeled in engineering applications, so $f_i(x_{i+1}, \overline{x}_i)$ are smooth unknown functions. In future, the radial basis function (RBF) will be used to approximate the unknown functions $f_i(x_{i+1}, \overline{x}_i)$, so that the adaptive control for a class of nonlinear pure-feedback systems with parameter drift is more extensive and practical.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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