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Research Article

Alternative Model to Determine the Optimal Government Subsidies in Construction Stage of PPP Rail Transit Projects under Dynamic Uncertainties

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Urban rail transit is a quasioperational project and its net cash inflow can hardly cover the investment expenditure. It is essential to determine an acceptable amount of government subsidy to ensure the financial viability of the PPP projects, so as to encourage the entry of the private partner. The partners involved in PPPs have common interests but conflict regarding the value of government subsidy. Considering the uncertainty characteristic by PPPs and information incompleteness in the decision-making process, this study presents a methodology to calculate the equitable subsidy ratio favored by both participants. This study divides the decision process into two steps. First, this study constructs a financial model and introduces an acceptable range of subsidy ratio by using the Monte Carlo simulation method. Second, this study uses the bargaining game theory to determine a particular subsidy ratio under incomplete information. To verify the applicability of the presented model, the researchers invoke an illustrative example for model validation. This research provides a referential and operational method for the government and private sectors to make government subsidy decisions for quasioperational projects.

1. Introduction

With the accelerating urbanization, residents are increasingly inclined to choose urban rail transit for travelling [1]. A great demand exists for constructing the urban rail transit due to its advantages in solving the surging passenger volume and reducing urban pollution [2, 3]. Given its high cost and the significant burden on the government's finance, the traditional financing mode, which mainly relies on the government's fund, fails to meets the target demand for urban construction and development [4, 5]. Public–private partnership (PPP) is a significant financing means for the government to encourage the private sectors to involve in providing public infrastructures [6, 7], which can reduce the heavy burdens on the government and promotes the transformation of its functions [8, 9]. It is gradually becoming a powerful means for the fast-growing of rail traffic [10, 11]. However, the urban rail transit PPP project requires substantial investment and long payback periods; hence, the situation of payments disequilibrium is prone to occur during the project operation [5, 11–13]. Hosting governments typically offer subsidies to maintain the investment interest for the private sector when the project is economically viable but financially nonviable [3, 14]. Certain famous rail transit cases in Beijing and Hangzhou in China began to adopt a different derivative mode of subsidizing in building-operate-transfer (SBOT).

Subsidies generally include financial and asset subsidies [15]. The financial subsidy is divided into direct financial and indirect supports, such as a guarantee in passenger flow volume [16, 17], demand [18], and revenue [19–24]. Asset subsidy is divided into tangible asset compensation and the provision of management rights applied to the subway projects in Hong Kong and Shenzhen. The amount of government subsidies related to the investors' participation is one of the critical clauses in the negotiations between the private investor and government. Expectations for cash flow are the basis for calculating the amount of government subsidy, significantly affecting the participants' will. Attitudes toward subsidy ratio differ among parties involved in PPP projects [17]. The objective of the private sector is to obtain the maximum value of the investment, so they expect a higher level of subsidies. The government is inclined to reduce the subsidy ratio to relieve financial burdens to ensure social benefit simultaneously. However, projects with low subsidy could impact the private sector's enthusiasm to invest and the quality of service that the company provides for the consumers. Also, an excessive subsidy may impede the efficiency of private entities in management, which may harm government departments [25–27]. Only when both participants agree on the proportion of government subsidies, the project can proceed successfully, which means that the determination of government subsidy proportion needs to meet the win-win principle. Therefore, to determine the appropriate subsidy ratio to ensure a win-win situation is crucial to the success of the PPP rail transit project.

This research provides an effective method to determine the optimal government subsidy ratio, which satisfies the private investor and government considering the impact of dynamic uncertainty in the financial evaluation procedure and information asymmetry on the negotiation process. The proposed methodology not only can generate a feasible interval of subsidy ratio in compliance with the public–private win-win principle but also can determine the optimal value of a subsidy ratio to maximize the interests of both PPP players. The remainder of the paper is sectioned as follows. Section 2 presents the literature study on government subsidies. Section 3 provides the methodology. Section 4 demonstrates and discusses. Section 5 concludes the paper.

2. Literature Review

In the investment decision-making stage in engineering construction of urban rail transit PPP projects, three major concerns must be considered when the government grants these subsidies.

First, the fact that PPP rail transit projects remain various sources of uncertainty leads to many risks, usually in the form of demand, cost, revenue, or other parameters [24]. Without a rational analysis of such risk, the decision-making of government subsidy is likely to be defective. The demand risk mainly depends on the uncertainty of the passenger flow volume. Many studies have investigated the demand risks, and a corresponding guarantee mechanism has been set up to share the risk [16, 19]. The revenue risk depends critically on the uncertainty of the passenger flow volume [17, 28–31], toll price [32, 33], or combinations of the two [24]. On the other hand, cost risk has been the popular choice for the researchers to study, which originates from the uncertainty of the construction costs [34, 35] and maintenance and rehabilitation costs [36]. Furthermore, some scholars began to focus on the risk of other parameters, such as the interest rate risks [37] and the inflation rate [26]. It is clear that toll revenue, cost, and interest rate are the main drivers of uncertainty for the project, and the Monte Carlo simulation method remains the favored object in the uncertainty analysis. However, when reviewing rail transit PPP projects' uncertainty, substantial attention is often given to passenger flow volume, interest rate, and the capital investment uncertainty was rarely mentioned.

Second, information is one of the decisive aspects of negotiating a subsidy agreement for a rail transit PPP project. Rail transit PPP projects involve two key stakeholders. The government sector is much more concerned about social benefits, whereas the private investor focuses more on its profitability in the negotiation phase. Participants have distinct understandings of the game information; hence, the negotiation process is typically incomplete information [38]. Neglect of information asymmetry can reduce the likelihood of the successful fulfillment of agreements [39]. Consequently, a need persists in formulating the subsidy agreement under incomplete information so that the government can determine an appropriate subsidy plan. However, as Feng et al. [40] mentioned, a critical assumption exists that the negotiators are well informed of one another to design a guarantee agreement accordingly in the previous studies. Few studies have handled the problem of incomplete information while determining the amount of subsidy.

Third, define a reasonable level of subsidy to be sufficient to make the project financially viable but low enough to guarantee that government benefits are of utmost importance. A win-win solution must consider the interest of both of the partners involved in PPPs [21, 37]. In other words, the government subsidy can be enough to meet the condition of maximizing the benefits of both parties. Most of the literature determines the government subsidy ratio considering only the private sector perspective. For example, Chen and Subprasom [41] quantified the minimum level of subsidies sufficient to guarantee the lowest interest of private investors. Liou et al. [26] provided a methodology for subsidy level decision making from the perspective of private sectors and obtained the minimum subsidy ratio when the PPP project is not financially viable. However, the condition accepted by the private entities in this study is net present value (NPV) is not less than 0, which collided with the willingness of the private investor. The private investor usually takes the NPV of the concession period not less than the product of the total amount of investment and the expected return on investment as the benchmark condition for its investment in PPP projects [9, 14]. Recently, some researchers proposed the win-win principle, which considers the maximization of both partners' benefits. Wu et al. [42] increased the consideration of government perspective when calculating the investment boundary conditions and established a Monte Carlo simulation model of the government subsidies and obtained the feasible region of the amount of government subsidies. Sun et al. [43] introduced the real option method when calculating the subsidy level of rail transit PPP projects. These studies started to implement the win-win principle in recent years. However, these studies only found a feasible region rather than a specific value.

Drawing from the previous literature, a growing literature on subsidy contracts in PPP rail transit projects prevails. Although scholars have conducted numerous studies to research the benefits of government subsidies and further determine the subsidy ratio, three major deficiencies exist in determining the subsidy amount for rail transit PPP projects:

(1) The neglect of capital investment uncertainty for rail transit PPP project: Shen and Wu [44] proposed the uncertainty of capital investment and estimated that the total capital investment was subject to a triangular probability distribution. However, present studies related to government subsidy concentrate on the uncertainty of demand and revenue, while the uncertain capital investment is rarely considered. As an urban infrastructure construction project, for the rail transit PPP project to achieve the revenue growth efficiency as expected by investors at an early stage is difficult and is greatly affected by the external environment, which will significantly affect investors' investment decisions and further increase the uncertainty of the project capital investment.

(2) Improper treatment of the information asymmetry in the process of negotiation: previous studies related to government subsidies mostly assumed that the negotiating parties are susceptible to complete information. However, participants generally have a poor understanding of other participants' characteristics, strategy space, and benefit function information in the actual negotiation process [38]. Information asymmetry leads to speculation and makes it difficult for the success of the contract [22, 39].

(3) No method is available on how to select the specific subsidy ratio within the closed interval. Scholars' current research are only the minimum subsidy ratio or the feasible region of subsidy ratio. Time and cost losses may occur for the participants of both sides during the long-term negotiation progress over subsidy [11]. Therefore, studies on the approaches of how to determine optimal subsidy ratio are significant and necessary.

3. Methodology

Reasonable compensation in the construction stage is the key to ensure the successful operation of rail transit PPP projects. Following the win-win principle, the optimal government subsidy should not only meet the investment threshold of the private sector to ensure the minimum return rate of investment but also meet the investment threshold of the public sector to maximize social welfare. To study how to determine the optimal government subsidy under dynamic uncertainty, this paper proposed an alternative method based on Monte Carlo simulation and bargaining game theory. The specific research process is as follows.

First, this study performed an uncertainty analysis to identify the certain and uncertain parameters invoked in the financial model. Then, the researchers established the financial model and employed a Monte Carlo simulation to generate an effective subsidy interval for the PPP players to negotiate in the first stage. Finally, the researchers used a game model to define the optimal government subsidy ratio, which satisfied both sides of the negotiation. Figure 1 demonstrates the framework of methodology.

3.1. Uncertainty Analysis. In line with the operational characteristics of rail transit PPP project, parameters invoked in the model can be subdivided into certain and uncertain parameters.

3.1.1. Certain Parameters. This study lists certain parameters of which value tends to be deterministic in the financial model as follows:

(1) Ticket rates: the ticket rates policy of rail transit in China mainly incorporates the initial price set under macroeconomic conditions and the ticket price adjustment coefficient in subsequent years. The starting price should consider operating cost, residents' consumption level, non–rail transit project fee standard, and other factors. The authority of government departments and the need to consume certain administrative costs in the pricing process can be adjusted every three years.

(2) Return on investment: the participation of private investors is typical economic behavior for rail transit PPP projects. However, attracting investors with a low return on investment is difficult. The prevailing practice in China is to improve the benchmark interest rate of bank loans by 2–4 percentage points during the same period [45].

(3) Discount rate: the discount rate reflects the minimum profit level of the project investment and is the basis for calculating the project's NPV and the investment payback period. Normally, it can be adjusted in accordance with the capital asset pricing model [19].

(4) Concession period: the concession period is an agreement reached by the private and government sectors. The contract shall be completed when the concession period expires. The private sector must unconditionally transfer operating assets to the host government [46].

3.1.2. Uncertain Parameters. Considering all kinds of uncertain risks in implementing such projects, the model gives different probability distribution for these uncertain factors instead of deterministic values.

(1) Annual passenger flow volume: as a basis for calculating cash inflow and the cash outflow, the annual passenger flow volume fluctuates significantly in PPP rail transit projects. Previous literature [42, 44, 47] mentioned that the passenger flow volume generally follows a normal distribution.

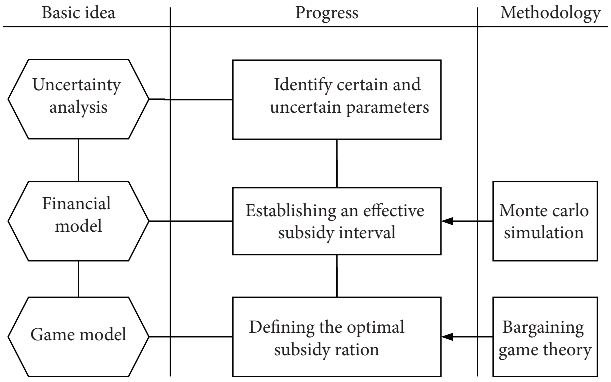

Figure 1: Framework of methodology.

(2) Capital investment: as an urban infrastructure construction project, PPP rail transit projects find it difficult to achieve the revenue growth efficiency expected by investors in the short term, and it has numerous stakeholders and is greatly affected by the external environment. Such investment can considerably influence investment planning and investors' final decision and further increase the uncertainty of the project investment amount. The total investment in the construction period conforms to the triangular probability distribution [44].

3.2. Financial Model. Suppose a PPP rail transit project exists whose gross investment is I, which will be input in lump-sum at the beginning of the construction period. The ratio of government subsidy is λ; the initial capital input of the private sector is $I_1 = (1-\lambda)I$. Negotiators may be incompatible in the process of negotiating the appropriate subsidy ratio. The private sectors prefer earning economic profits, whereas the public sectors focus more on social benefits [48].

The following conditions are considered during the model development and construction process:

The net present value (NPV) of private investor is determined by

$$\mathrm{NPV}^{(1)} = \sum_{t=1}^{T} \mathrm{NPV}_t - I_1 = \sum_{t=1}^{T} \frac{Y_t - C_t - Z}{(1+r_1)^t} - I_1, \tag{1}$$

where $\mathrm{NPV}^{(1)}$is the NPV of the private investor from the first year to the last year, NPV_t is the NPV in year t, Y_t is the operating incomes in year t, C_t is the operating costs in year t, I_1 is the initial input of the private sector, T is the concession period, Z is the rent paid by the private sector to the government during this period, and r_1 is the discount rate of the private investor.

How to determine the discount rate is a key step in calculating NPV. Generally, the capital sources of private sector investment in PPP rail transit projects include equity and debt capitals. Thus, weighted average cost of capital (WACC) is regarded as the new discount rate in evaluating the value of the project. WACC can be expressed as

$$\mathrm{WACC} = \frac{I_e R_f + I_l R_l (1-f)}{I_e + I_l}, \tag{2}$$

where I_e and I_l is the equity investment and debt financing of the private sector, respectively. R_f and R_l are the risk-free interest and loan interest rates, respectively. The corporate income tax rate is represented by f.

In practice, the public sector takes into account the positive externality of PPP rail transit projects. Therefore, economic returns and social welfare are incorporated into consideration for the government. Social benefits here mainly refer to the social costs reduced to environmental and traffic accident losses of urban rail transit projects compared with non–rail transit, denoted as $V(t)$, which can be expressed as

$$V(t) = \sum C_i \times L \times Q_t, \tag{3}$$

where C_i is the social cost of the project compared with the nonrail transit Q_t is the passenger flow volume transferred from nonrail transit to rail transit project in year t, and L represents the average passenger distance.

Therefore, the NPV of government could be expressed as

$$\begin{aligned}\mathrm{NPV}^{(2)} &= \sum_{t=1}^{n} \mathrm{NPV} - I_2 = \sum_{t=1}^{T} \frac{Z}{(1+r_2)^t} \\ &\quad + \sum_{t=T+1}^{n} \frac{Y_t - C_t}{(1+r_2)^t} + \sum_{t=1}^{n} \frac{V(t)}{(1+r_2)^t} - I_2.\end{aligned} \tag{4}$$

In expression (4), n represents the service life of the project by year, I_2 represents the amount of government capital subsidy, $I_2 = \lambda I$. r_2 represents the public sector discount rate, which should take into account the bank's loan interest and inflation rates. Therefore, its calculation formula can be expressed as

$$r_2 = \frac{1+i}{1+I_{nf}} - 1, \tag{5}$$

where I_{nf} and i are the rates of inflation and the interest rate on bank loans, respectively. The definition of other parameters is similar to equation (1).

Proposition 1. *From the private sector perspective, the top priority is financial efficiency. If the NPV of the project fails to meet the expectation of the private sector, it will not be favored by investors [14]. Therefore, when* $\mathrm{NPV}^{(1)} \le I_c R$, *where Ic stands for the total expenditure of the private sector during the construction and operation period and R represents the return on investment required of the private sector, the host government should compensate the PPP rail transit project. If the government adopts the capital compensation model, its compensation ratio must be satisfied with the following equation:*

$$\lambda \geq 1 - \frac{1}{(1+R)I}\left[\sum_{t=1}^{T}\frac{Y_t - (1+R)(C_t + Z)}{(1+\mathrm{WACC})^t}\right]. \quad (6)$$

Proposition 2. *From the public sector perspective, striving for social benefits, such as traffic accidents reduction and environmental protection is the primary target [3, 14]. Therefore, when $NPV^{(2)} \geq 0$, the willingness of the public sector to invest is satisfied.*

Moreover, for the sake of engaging the private sector to participate in the construction of the project, the ratio of government subsidy must meet the following requirement:

$$\lambda \leq \frac{1}{I}\left[\sum_{t=1}^{T}\frac{Z}{(1+r_2)^t} + \sum_{t=T+1}^{n}\frac{Y_t - C_t}{(1+r_2)^t} + \sum_{t=1}^{n}\frac{V(t)}{(1+r_2)^t}\right]. \quad (7)$$

The interested parties involved in the project are the private investor and the government. Project cash flow must meet the benchmark conditions for both sides to spend on the rail transit PPP project. Thus, Proposition 3 can be achieved.

Proposition 3. *Only when conditions $NPV^{(1)} \leq I_c R$ and $NPV^{(2)} \geq 0$ are satisfied, both participants would be pleased to cooperate on investing in the project. Therefore, for project success, the feasible government subsidy ration interval is shown as follows:*

$$1 - \frac{1}{(1+R)I}\left[\sum_{t=1}^{T}\frac{Y_t - (1+R)(C_t + Z)}{(1+\mathrm{WACC})^t}\right] \leq \lambda \leq \frac{1}{I}\left[\sum_{t=1}^{T}\frac{Z}{(1+r_2)^t} + \sum_{t=T+1}^{n}\frac{Y_t - C_t}{(1+r_2)^t} + \sum_{t=1}^{n}\frac{V(t)}{(1+r_2)^t}\right]. \quad (8)$$

With a probability distribution of uncertain variables resulting from the simulation and applying the principles of equation (8), the subsidy boundaries $[\lambda_{\min}, \lambda_{\max}]$ can be obtained. The feasible range of subsidy is able to ensure a win-win condition for both parties involved.

3.3. Game Model. The amount of government capital compensation is one of the core clauses in the PPP project franchise agreement, which is related to the expected income of public and private sectors investing in PPP projects. If the two sides cannot agree on the amount of government subsidies, the PPP project will be difficult to carry out successfully. Under the condition that the feasible region $[\lambda_{\max}, \lambda_{\min}]$ of subsidy ratio is known, both sides of the negotiation will bargain over the government subsidy ratio. From the private sector's perspective, they will ask the government to increase the subsidy proportion as much as possible to protect their operating costs and profits. In contrast, from the government's perspective, they will not provide more subsidies to meet the feasibility of social evaluation of rail transit PPP projects. That is, government compensation should not exceed the external effect of the rail transit PPP project.

3.3.1. Problem Description. As shown in Figure 2, the maximum proportion of government subsidy ratio ($\lambda_{\max}$) and the minimum proportion of subsidy ratio ($\lambda_{\min}$) are known. The value of i is directly proportional to the government subsidy ratio (λ). In this study, it is defined as the subsidy decision coefficient, and the explicit subsidy decision coefficient is determined by the game between the negotiating parties.

Thus, the following formula can be obtained:

$$\lambda = \lambda_{\min} + i(\lambda_{\max} - \lambda_{\min}), \quad (0 \leq i \leq 1). \quad (9)$$

The participants will then bargain around the value of i. Normally the offer is first put forward by the government; the private sector can choose to accept or reject. In the first round, the government introduces its own decision coefficient as i_1; the negotiation will be reached and ended if the private sector accepts the proposal. On the other hand, if the private sector rejects the offer, the decision coefficient of the government will be valued as i_2 by the private sector in the second round. The agreement will be reached if the government accepts the proposal whereas the government will introduce its own decision coefficient as i_3 in the third round. By analogy, only when the two sides of the investment do reach a consensus on the scheme proposed in a certain round, can the negotiation be completed. The bargaining process of the value of i is shown in Figure 3.

3.3.2. Model Assumption. For the subsidy decision coefficient, both sides of the negotiation obtain the final equilibrium solution of the subsidy decision coefficient through the bargaining model considering their respective interests and influencing factors. Four hypotheses are proposed before the bargaining game model is constructed:

Assumption 1. The private sector (P) and government (G) are rational; neither side wants the talks to break down while seeking to maximize their own interests.

Assumption 2. The game is in a state of incomplete information, which indicates that the information, such as the strategic and cost-benefit function of the negotiations is not fully disclosed. The probability that the government is in a strong position is q_1 while the probability that the government department is in the opposite state is q_2. Obviously, $q_1 + q_2 = 1$.

Assumption 3. The subsidy decision coefficient given by both sides of the game is independent of each other. For a specific subsidy decision coefficient, the decision coefficient occupied by the government sector is i_g while the private sector is $1 - i_g$.

Assumption 4. In the game process, the negotiation needs to cost. The negotiation loss coefficient of the government sector is δ_g while the private sector is δ_p. In addition, $1 < \delta_g < \delta_p$.

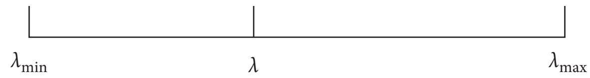

Figure 2: The scope of government contribution.

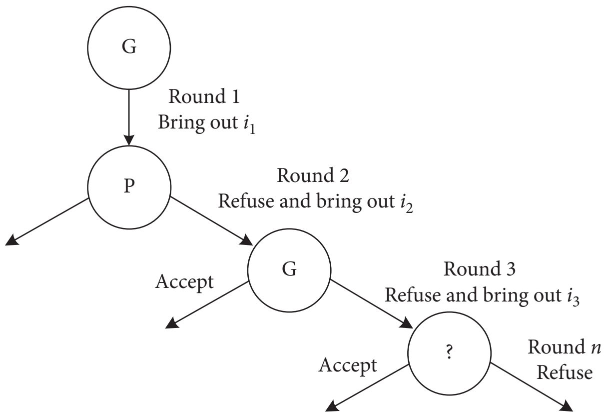

Figure 3: An extended representation of the government and private sector bargaining model.

3.3.3. Model Established. According to the Harsanyi transformation, a virtual player-nature (N) is introduced to convert "incomplete information" into complete but imperfect information and the static game of incomplete information into a two-stage dynamic game. Therefore, this study divided the bargaining game of each round of subsidy decision coefficient into two procedures. In the first stage, the government is in a strong position with q_1 probability. In the second stage, the government remains in a weak position with q_2 probability. These two stages are transpiring in different rounds.

During the first phase of the first round: with probability q_1 in a strong position, the government sector introduces its own decision coefficient as i_1. The share transferred to the private investor is η_1; then the final compensation decision coefficient of both sides in the negotiation is

$$\begin{aligned} G_{11} &= q_1 (i_1 - \eta_1), \\ p_{11} &= q_1 (1 - i_1 + \eta_1). \end{aligned} \tag{10}$$

During the second phase of the first round: the government remains in a weak position and the probability is q_2. The compensation decision-making coefficient of both sides in the negotiation is

$$\begin{aligned} G_{12} &= q_2 i_1, \\ p_{12} &= q_2 (1 + i_1). \end{aligned} \tag{11}$$

Therefore, in the first round of negotiations, the expectation of the compensation decision coefficient after bargaining is as follows:

$$\begin{aligned} G_1 &= G_{11} + G_{12} = q_1 (i_1 - \eta_1) + q_2 i_1, \\ P_1 &= P_{11} + P_{12} = q_1 (1 - i_1 + \eta_1) + q_2 (1 - i_1). \end{aligned} \tag{12}$$

If the government's proposal is not accepted by the private sector, the negotiations proceed.

During the first phase of the second round: the private sector puts forward i_2 is the decision coefficient of government sector while its own decision coefficient is $1 - i_2$. Meanwhile, the public sector remains in a strong position with probability q_1 and transfers its share η_2 of decision-making coefficient to the private sector. In addition, from the second round onwards, negotiations must consume costs, so the final decision coefficient of compensation of both sides in the negotiation is

$$\begin{aligned} G_{21} &= q_1 \delta_g (i_2 - \eta_2), \\ P_{21} &= q_1 \delta_p (1 - i_2 + \eta_2). \end{aligned} \tag{13}$$

During the second phase of the second round: the government remains in a weak position and the probability is q_2. The compensation decision-making coefficient of both sides in the negotiation is

$$\begin{aligned} G_{22} &= q_2 \delta_g i_2, \\ P_{22} &= q_2 \delta_p (1 + i_2). \end{aligned} \tag{14}$$

If the game ends in the second round, the expected compensation decision coefficient of both sides is

$$\begin{aligned} G_2 &= G_{21} + G_{22} = q_1 \delta_g (i_2 - \eta_2) + q_2 \delta_g i_2, \\ P_2 &= P_{21} + P_{22} = q_1 \delta_p (1 - i_2 + \eta_2) + q_2 \delta_p (1 - i_2). \end{aligned} \tag{15}$$

If the subsidy decision factor proposed by the private sector does not fulfill the minimum expectations of the government, the negotiations move to the next round.

During the first phase of the third round: with probability q_1 in a strong position, the government sector introduces its own decision coefficient as i_3. The share transferred to the private investor is η_3; then the final compensation decision coefficient of both sides in the negotiation is

$$\begin{aligned} G_{31} &= q_1 \delta_g^2 (i_3 - \eta_3), \\ P_{31} &= q_1 \delta_p^2 (1 - i_3 + \eta_3). \end{aligned} \tag{16}$$

During the second phase of the first round: the government remains in a weak position and the probability is q_2. The compensation decision-making coefficient of both sides in the negotiation is

$$\begin{aligned} G_{32} &= q_2 \delta_g^2 i_2, \\ P_{32} &= q_2 \delta_p^2 (1 - i_3). \end{aligned} \tag{17}$$

If the game ends in the third round, the expected compensation decision coefficient of both sides is

$$\begin{aligned} G_3 &= G_{31} + G_{32} = q_1 \delta_g^2 (i_3 - \eta_3) + q_2 \delta_g^2 i_3, \\ P_3 &= P_{31} + P_{32} = q_1 \delta_p^2 (1 - i_3 + \eta_3) + q_2 \delta_p^2 (1 - i_3). \end{aligned} \tag{18}$$

Likewise, in the n round, the expected compensation decision coefficient of both sides is

$$G_n = G_{n1} + G_{n2} = q_1\delta_g^{n-1}(i_n - \eta_n) + q_2\delta_g^{n-1}i_n,$$
$$P_n = P_{n1} + P_{n2} = q_1\delta_p^{n-1}(1 - i_n + \eta_n) + q_2\delta_p^{n-1}(1 - i_n). \tag{19}$$

3.3.4. Model Solution. For an endless round of bargaining games, conventional thinking is that you cannot find the equilibrium solution. According to the theory introduced by Shaked and Sutton, the end result of each round is common to an infinite round of bargaining game. Therefore, this model can take the third round as being the basis of the inverse and use the inverse induction method to solve the problem.

Analysis of Round 3: for the government sector, the expected decision coefficient of possession compensation is G_3 while the private sector is P_3.

Analysis of Round 2: from the perspective of the government sector to avoid unnecessary losses, the government sector will not drag the negotiation to the next round as long as the expectation of the decision-making coefficient of the second round obtained by the government is not less than the value of the next round. From the private sector perspective, the optimal strategy of the bidder in their own best interests should meet the minimum expectation of the government on the compensation decision coefficient ($G_2 = G_3$).

Thus, it can be obtained that

$$q_1\delta_g(i_2 - \eta_2) + q_2\delta_g i_2 = q_1\delta_g^2(i_3 - \eta_3) + q_2\delta_g^2 i_3,$$
$$P_2 = \delta_p\left(1 - \delta_g i_3 - q_1\delta_g i_3\right). \tag{20}$$

A comparison is made between the second and third rounds of the private sector expectations. Ultimately, to maximize their own interests, neither party will choose to drag the negotiation into the third round.

Analysis of Round 3: similarly, the negotiators are reluctant to push the negotiations into a second round. Therefore, when $P_2 \leq P_1$, the private sector will not reject the compensation decision factor proposed by the government in this round. Consequently, to meet the expectations of the private investor and make their own expectations higher, the optimal decision of the government is as follows: $P_1 = P_2$.

It can be obtained that

$$i_1 = 1 + q_1\eta_1 - \delta_p\left(1 - \delta_g i_3 + q_1\delta_g i_3\right). \tag{21}$$

For an infinite round bargaining game, according to Shaked and Sutton, the first and third round results should be identical; hence, the following results can be obtained:

$$i_3 = i_1 = 1 + q_1\eta_1 - \delta_p\left(1 - \delta_g i_3 + q_1\delta_g i_3\right),$$
$$1 - i_3 = \frac{\left[\delta_p\left(\delta_g - 1\right) - q_1\left(\delta_g\delta_p\eta_3 - \eta_1\right)\right]}{\left(\delta_g\delta_p - 1\right)}. \tag{22}$$

Table 1: Operating cost measurement results.

Year	Operation cost (ten thousand yuan)
2010	53658
2011	54971
2012	56362
2013	57778
2014	59277
2015	60942
2016	60024
2017	59251
2018	58473
2019	57696
2025	53708
2026	53161
2027	52671
2028	52187
2029	51853
2020	57421
2021	56418
2022	55496
2023	54594
2024	53673
2030	51735
2031	54167
2032	56572
2033	59018
2034	60795
2035	61264
2036	62583
2037	64386
2038	66394
2039	67978

Table 2: The value of the game parameters.

Game parameter	Symbol	Value
The probability that the public sector is in a strong position	q_1	0.642
The probability that the public sector is in a weak position	q_2	0.358
Loss factor for the government sector	δ_g	1.101
Loss factor for the private sector	δ_p	1.165
Transfer of capital compensation decision coefficient share	η	0.086

If the fraction of the compensation decision coefficient of the transfer is set to a constant,

$$\eta_i = \eta. \tag{23}$$

Thus, the Nash equilibrium solution of this model can be obtained:

$$i^* = \frac{\left(\delta_p - 1\right)}{\left(\delta_g\delta_p - 1\right)} + q_1\eta, \tag{24}$$

$$1 - i^* = \frac{\left(\delta_g\delta_p - \delta_p\right)}{\left(\delta_g\delta_p - 1\right)} - q_1\eta. \tag{25}$$

Table 3: Unit social cost of various modes of transportation.

Transport (yuan/person • km)	Bus	Bicycle	Taxi	Subway	Light rail
Own costs	0.144	0.0105	0.393	0.375	0.125
Road occupancy costs	0.01	0.061	0.11	0	0
Parking costs	0.00736	0.0526	0.0092	0	0
Traffic accident damage costs	0.0012	0.006	0.007	≈0	≈0
Environmental costs	0.121	0	0.175	0.00966	0.0126
Time value	0.867	1.079	0.325	0.325	0.325
Summation	1.151	1.209	1.019	0.71	0.463

Source: [49, 50].

Table 4: The probability distribution of uncertain parameters.

Variables	Probability distribution
Annual traffic volume	Normal distribution ($\mu = 75.41$, $\sigma = 76.77$)
Capital investment	Triangular distribution (minimum—15.2 billion,most likely—15.3 billion, and maximum—15.4 billion)

Combining with the obtained feasible region interval, equation (24) is replaced by the text into equation (9) to obtain the actual capital compensation ratio of the government.

4. Illustrative Example

4.1. Profile Project M. This study introduced an illustrative example of line M rail transit in China to illustrate how the proposed model is employed to determine the optimal government subsidy ratio. This was a PPP rail transit project with an estimated total investment of RMB15.3 billion and was used in September 2009. The length of the concession is 30 years while the service life is 50 years. The rent during the concession period is RMB42.5 million and the private sector expects a 15% return on investment. Table 1 presents the operational costs and Table 2 shows the relevant game parameters in the project capital compensation decision. The traffic volume is usually stable when the concession period expires and the operating cost increases at an annual rate of 5%. Table 3 exhibits that the social benefits created by the project are calculated with the data in the literature [49, 50]. This study is calculated built on the bus. Table 4 illustrates the probability distribution of uncertain parameters.

4.2. Model Validation

4.2.1. Government Subsidy Range. This study used the Crystal Ball 7.3.1 software package to simulate each factor and set the simulation times to 10,000. Figure 4 and Table 5 present the probability distribution and simulated value of the government's maximum subsidy ratio ($\lambda_{\max}$), and Figure 5 and Table 6 show the probability distribution and the simulated value of the government's minimum subsidy ratio ($\lambda_{\min}$).

In accordance with the results of Monte Carlo simulation, the mean value of $\lambda_{\max}$ in this rail transit PPP project is 0.79, and the mean value of $\lambda_{\min}$ is 0.54; that is, the government subsidy ratio in the closed interval [0.54, 0.79] cannot merely meet expected return of the private investor but also achieve the expected benefits of the public sector.

4.2.2. Government's Optimal Subsidy Ratio. After an effective subsidy interval for the PPP players is simulated in the financial model, the researchers substituted the simulation results into the bargaining game model so that the final equilibrium solution of the subsidy ratio can be obtained: $\lambda = 0.565$. Thus, the government's contribution ratio is 0.565, whereas the private sector's contribution ratio is 0.435.

4.3. Discussion. To further discuss how parameters in the game influence the equilibrium solution of the optimal government subsidy ratio for PPP projects under the condition that other parameters remain unchanged, this study analyses the impact of the parameters, δ_g, δ_p, and q_1 on the optimal government subsidy ratio, as shown in Figure 6. The simulation proceeds by changing the sensitivity factor from −40 to +40% at 10% intervals.

The equilibrium solution of the optimal government subsidy ratio (λ) of PPP projects is escalating with the increase of the loss factors for the private investor (δ_p) under the condition that other variables are consistent. This suggests that high negotiation costs will make it convenient for the private sector to compromise on the proportion of subsidies offered by the government rather than drag the negotiations to the next round.

Similarly, the equilibrium solution of the optimal government subsidy ratio (λ) of PPP projects is increasing with the probability that the public sector is in a strong position (q_1) under the condition that other variables are constant. However, compared with the loss factors for the private investor (δ_p), the effect of the probability that the public sector is in a strong position (q_1) on the optimal government subsidy ratio (λ) is comparatively low.

Disparately, a turning point exists for the effect of loss factor for the government sector (δ_g) on the government optimal subsidy ratio (λ). Initially, with the loss factor for the government sector (δ_g) increased, the government optimal subsidy ratio (λ) decreased. If the loss factor for the government sector (δ_g) continues to escalate, the government optimal subsidy ratio (λ) begins to increase instead. This suggests that the government is willing to

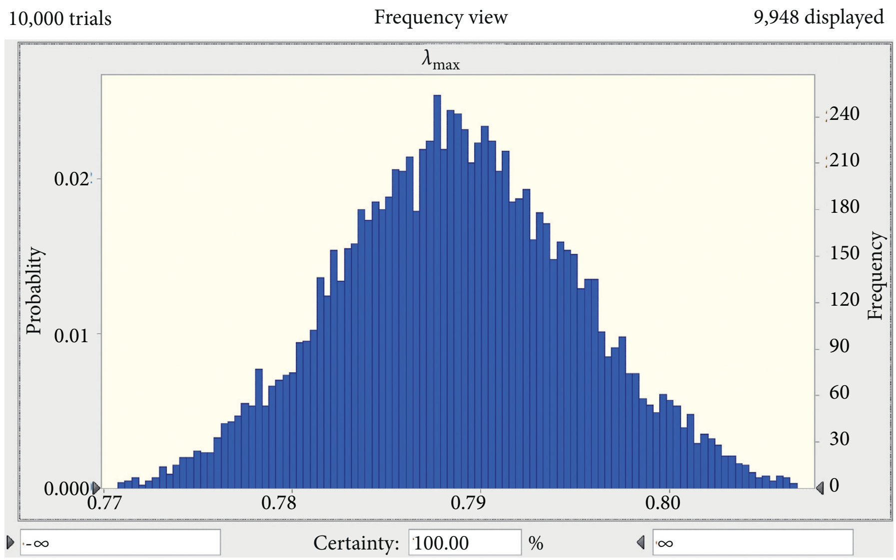

Figure 4: Probability distribution of the government's maximum subsidy ratio.

Table 5: The simulated value of the government's maximum subsidy ratio.

Statistics	Forecast values
Trials	10,000
Mean	0.79
Median	0.79
Standard deviation	0.01
Skewness	0.00
Kurtosis	2.99
Coeff. of variability	0.0082
Minimum	0.76
Maximum	0.81
Range width	0.05
Mean std. error	0.00

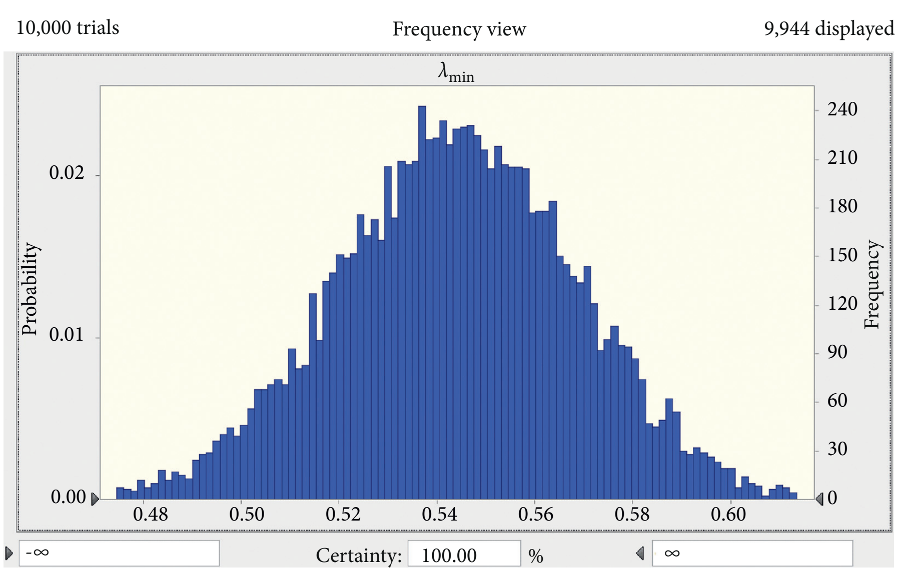

Figure 5: Probability distribution of the government's minimum subsidy ratio.

Table 6: The simulated value of the government's minimum subsidy ratio.

Statistics	Forecast values
Trials	10,000
Mean	0.54
Median	0.54
Standard deviation	0.02
Skewness	0.0436
Kurtosis	3.00
Coeff. of variability	0.0453
Minimum	0.46
Maximum	0.64
Range width	0.18
Mean std. error	0.00

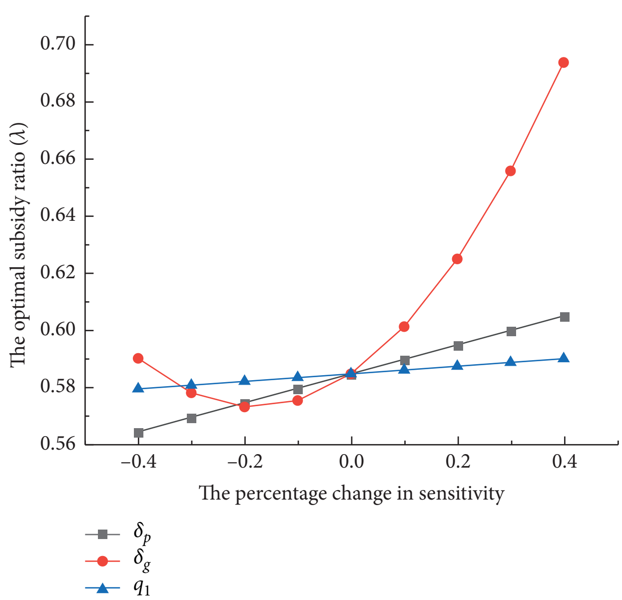

Figure 6: Sensitivity analysis of uncertain parameters.

continue negotiating when the initial costs are low but prefers to end the negotiations early when the negotiation costs are high. Thus, this study can conclude from the sensitivity analysis of the game parameters that the loss factor of both participants during the negotiation significantly affects the equilibrium of the optimal government subsidy ratio (λ).

5. Conclusions

The massive investment scale, long payback period, and nonprofit characteristics of urban rail transit projects make them unqualified for market operation. To encourage the private sector involved in constructing the PPP projects, the government usually provides subsidies to gratify the profitability expectations of the investors. The amount of government subsidies plays an important part in the PPP project of rail transit. It cannot satisfy the rational constraint of investors when the value is low. Conversely, it cannot achieve the expected social benefits of the government. How to determine an equitable subsidy ratio is critical to the implementation of the project when the project is economically viable but financially nonviable. Considering the impact of various uncertainties in the financial evaluation stage and information asymmetry on the negotiation process, this study takes rail transit PPP project as the research object and solves the optimal government subsidy ratio in two steps. First, following the win-win principle and taking Monte Carlo simulation as the main research method, this paper establishes the feasible range of government subsidy ratio in the financial model. Second, the paper can obtain the equilibrium solution of government subsidy rate based on the bargaining model of government subsidies during the construction period.

Our contribution is mainly in three aspects as follows: First, a systematic approach is introduced to calculate the feasible range of government subsidy ratio in the construction stage of PPP rail transit projects considering the dynamic uncertainties. Second, a bargaining game model is proposed to determine a particular government subsidy ratio in the construction stage of PPP rail transit projects under incomplete information. Finally, this paper focuses on the issue of government subsidies in the construction stage of PPP rail transit project, and the method proposed can provide a useful reference for the government subsidies in the construction stage of other quasioperational PPP projects, which enriches the relevant theories and methods of government subsidy research.

Notably, this study has management implications. First, when conducting subsidies in the construction stage of quasioperational PPP projects, the government needs to consider not only its investment boundary but also the investment requirements of the private sector. Second, there are upper and lower limits of government subsidies for quasioperational PPP projects, and the uncertainty of PPP projects should also be considered when making decisions on their feasible ranges. Finally, information uncertainty should be taken into account in the decision-making process. Our findings can better provide a basis for the government and the private sector to compensate in the construction period.

In practice, the government should not only consider the balance of the interests of the partners involved in PPPs but also juggle construction period subsidy and operating period subsidy. Future research shall be strengthened on the combined compensation and further balance the government's current and future finances in diffident PPP projects.

Data Availability

Some or all data, models, or codes that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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