Research Article

Information Investment and Sharing in a Two-Echelon Supply Chain under Government Subsidy and Consumer Preference for Energy-Saving Products

Jinxi Li, Yuyin Yi, and Haishen Yang

1 Dongwu Business School, Soochow University, Suzhou 215006, China
2 Management School, Jinan University, Guangzhou 510632, China
3 Institute of International Studies, Guangdong Academy of Social Sciences, Guangzhou 510635, China

Correspondence should be addressed to Yuyin Yi; yiyuyin2001@sina.com

Received 17 April 2020; Revised 4 June 2020; Accepted 10 June 2020; Published 12 July 2020

1.Introduction

With the development of the global economy, energy shortage, climate warming, and species extinction become more and more serious [1–5]. Among these, energy shortage is the most pressing issues. To address it, apart from seeking new energy sources, developing energy-saving products (ESPs) is an essential and effective method at present [6, 7]. Therefore, governments in various countries have taken a series of measures to encourage manufacturers to research and develop ESPs [8–11]. For example, America has offered a federal subsidy programme for electric vehicles. Consumers can receive a one-time bonus of a maximum of $3400 (USD) upon purchasing a hybrid electric vehicle, and the incentive for plug-in electric vehicles and battery electric vehicles is between $2500 and $7500 [12]. Similarly, China has implemented public-benefit projects for consumers who purchase ESPs including inverter air conditioners and liquid crystal televisions (LCTs) [13, 14]. Apart from powerful support from government, consumers’ environmental awareness also increases [15, 16]. For example, in 2005, the proportion of the European population that was willing to purchase green products was 31%, and this rose to 75% by 2008 [17]. When purchasing products, people prefer ESPs, and therefore energy-saving air conditioners and refrigerators have been considered as the standard configuration of ordinary families [18, 19]. So, under the dual influence of government subsidy and consumer preference for ESPs, the manufacturers have to make energy-saving R&D to improve products efficiency [20–22].
Besides that, information technology has developed rapidly [23]. With the application of big data, the downstream retailers close to consumers can obtain a large amount of market data [24, 25]. Through the processing and analysis of these data, the retailer has better forecast than the upstream manufacturer [26]. For example, the retail magnate Walmart needs to deal with several terabytes of new data and petabytes of historical data. These data cover millions of products and hundreds of millions of customers. By analysing the data, Walmart optimises the sales decisions on ESPs [27]. The automobile sales service shop 4S, such as Tesla and BYD, registers potential electric vehicle buyers to allow buyer-tracking and subsequently record transaction information once they become customers. Afterwards, the related data are transmitted to the manufacturer so that the upstream manufacturer can make better production decisions [28]. However, in an era of information explosion, sometimes data play an insignificant role and may even be useless. For instance, the e-retailer RadioShack and the toys retailer RU stated that they possess a long-term customer database albeit that is unhelpful [29].

Motivated by the reasons in the introduction above, we want to research these following questions:

(1) If there is investment risk (collection data were invalid), whether retailer will invest in information acquisition techniques or not? How do government subsidy and consumer preference influence the investment decision?

(2) After collecting the data, will the retailer share the market information with the manufacturer? What is the impact of government subsidy and consumer preference on information sharing?

(3) How does information sharing influence the energy-saving R&D and the profits of supply chain members?

To answer these questions, we establish a two-echelon supply chain with one manufacturer and one retailer. The manufacturer produces a kind of ESPs, such as energy-saving air conditioners or refrigerators, and the government provides subsidies and consumers prefer ESPs. The retailer sells the ESPs to consumers, and it is superior to grasp market data when she invests in information acquisition techniques. However, the data may be unhelpful. So the retailer firstly determines whether, or not, to invest in information acquisition techniques and then decides whether, or not, to share demand information with the manufacturer. Afterwards, the manufacturer determines the wholesale price and energy-saving level of ESPs. Finally, the retailer determines the retail price.

The research mainly contributes to the state-of-the-art in the following: Firstly, the retailer’s income from investment (IFI) is unrelated to the government subsidy and consumer preference when the retailer conducts information investment while not sharing demand information. However, the IFI is influenced by government subsidy and consumer preference as well as correlated with the probability of a high demand when the retailer shares information. Secondly, whether, or not, the retailer agreeing to share information is closely related to government subsidy and the probability of a high demand. Thirdly, whether, or not, the retailer’s information sharing improves the manufacturer’s profit depends on the probability of a high demand and manufacturing cost. Fourthly, the manufacturer can motivate the retailer to share information through sharing the information investment cost.

The rest of this paper is organised as follows: Section 2 contains a literature review; Section 3 covers model description and basic hypotheses; Section 4 studies the retailer’s information investment and sharing decision and the influence on the manufacturer’s energy-saving R&D and profit; Section 5 covers the numerical analysis and validates the main conclusions; Finally, conclusions and future research directions are given in Section 6. All proofs of the main results are put in the Appendix for clarity.

2. Literature Review

This study is related to these areas of the literature: demand information asymmetry and government subsidies for ESPs.

Information asymmetry is the primary topic related to this study: a majority of the research on this topic has been conducted on the basis of cost [30–32] or demand information asymmetry. The researches about demand information asymmetry are directly related to our study. Lee et al. [33] established a two-echelon supply chain composed of a retailer and a manufacturer, in which the retailer has market information advantage. They found that the sharing of demand information can reduce the manufacturer’s inventory cost. Guo [34] found that invisibility of market information acquisition will decrease the retailer’s loss and information sharing is conducive to manufacturer profits. Taylor and Xiao [35] explored the influence of the retailer’s predictive ability on manufacturer’s profit. They suggested that, if and only if the retailer’s predictive ability is strong, the manufacturer can benefit; otherwise, the manufacturer will suffer economic loss. Moreover, if the marginal profit of products is high, it is unfavourable for the manufacturer if the retailer improves its predictive ability. Chu et al. [36] showed that the retailer does not provide true market information unless sharing information returns greater profit. Zhang et al. [37] investigates manufacturer encroachment with both endogenous quality decision and asymmetric demand information to examine the effects of encroachment and information structure on product quality and profits of chain members. Li and Zhang [38] investigated a two-echelon supply chain composed of a manufacturer and multiple retailers, in which each retailer attains private information about market demand. They found that whether, or not, retailers agree to share information depends on whether the manufacturer causes information leakage. Shin and Tunca [39] investigated the influence of incentive contracts (wholesale-price contract and two-part tariff contract) on retailers’ investment in demand forecasting. Yu and Li [40] investigated information sharing issues in different supply chain structures under the cap-and-trade policy.
Above literatures focus on whether the retailer shares market information or not and the influence of information sharing on the supply chain. They consider market demand as a linear function of price while failing to consider the impact of energy-saving level of products. But in reality, people prefer ESPs when purchasing as consumer’s environmental awareness is growing constantly [41]. Therefore, the energy-saving characteristics of products can also influence market demand [42]. Since considering the energy-saving factor, some new questions need to be solved: after acquiring information about market demand, should retailers share it with manufacturers or not? How does information sharing (or not) influence the manufacturers’ energy-saving R&D and profit?

Another topic directly related to our study is government subsidies for ESPs. To encourage enterprises developing ESPs, the government has provided energy-saving subsidy. Therefore, many studies investigate energy-saving R&D from the perspective of subsidy policy. Huang et al. [43] consider government subsidy for electric vehicles and showed that the subsidy policy can promote the development of electric vehicles and improve the environment, but the role is limited. Luo et al. [44] supposed that government provides a price discount subsidies for consumers while there is a ceiling. The result reveals that when manufacturing cost is high, the influence of the subsidy ceiling on wholesale price is more significant than that of the discount rate. Zhou and Huang [45] compared two subsidy policies: price discount subsidy and fixed amount subsidy. They found that, when government budgets are low, the fixed amount subsidy is more preferable than the price discount subsidy, and vice versa. Shao et al. [46] further presented a case of duopoly competition: they suggested that fixed amount subsidy and price discount subsidy show the same influence on consumer surplus, market demand, and environmental and social welfare. However, fiscal expenditure is reduced if the government adopts fixed amount subsidy policy. Li et al. [47] studied the impacts of government consumption subsidy and replacement subsidy towards environmentally-friendly products in a dual-channel supply chain. These literatures only considered price influences demand while the influence of energy-saving level of products on demand is ignored. In recent years, scholars have gradually begun to consider market demand is related to the price and energy-saving level of products. For example, Xie [48] investigated the impact of policy makers when they set a threshold value of energy-saving level and examined the impact on energy-saving level and price decided by green supply chains within two different structures, i.e., vertical integration and decentralized setting. Afterwards, Xie [49] explored the energy-saving level of the suppliers and enterprises’ profits under different modes of cooperation. Hafezalkotob [50] developed four competition and cooperation models for the supply chains and four government targets (energy-saving efforts, net revenue, social welfare, and sustainable development). After that, Hafezalkotob [51] considered six regulatory policies furthermore: deregulation, direct tariff, direct limitation, government certification, government permit, and cooperative energy saving. He found that all intervention policies were more advantageous than a deregulation policy, and cooperative energy-saving policy yielded the highest social utility and energy savings, while also entailing the highest government investment.

The aforementioned literatures suppose that government subsidy is independent of energy-saving level. However, in practice, the amounts of government subsidy are determined by it. For example, China subsidises 240–400 CNY for an inverter air conditioner and 100–400 CNY for a liquid crystal display (LCD) television by the energy efficiency index [45, 46]. Additionally, the aforementioned researches were conducted under perfect information conditions: in reality, enterprises tend to face problems with incomplete information. For instance, retailers often possess better demand-forecast compared with manufacturers as they are close to consumers. However, the information retailers possessed may be inaccurate. So should the less-informed manufacturers follow the retailers’ forecast to make decisions? How government subsidy and consumer preference for ESPs affect retailers’ decisions on information investment and sharing? These questions are all considered in this paper.

3. Model Description and Hypotheses

A two-echelon supply chain composed of a manufacturer and a retailer, in which the manufacturer produces a kind of ESPs (such as energy-saving air-conditioner or refrigerator), with a marginal production cost $c$. The manufacturer sells the ESPs to the retailer at a wholesale price $w$, and the retailer resells it to consumers at a retail price $p$.

3.1. Energy Saving. To encourage enterprises to produce ESPs, government delegates a third party to conduct certification (free of charge). According to the certified energy-saving level, the degree to which the products can save, government provides subsidy to the manufacturer. The amounts of subsidy for unit product is $s_g$, that is, the higher the energy-saving level, the greater amounts of the subsidy.

$$s = s_g \text{ for } (s > 0)$$

$s$ refers to the subsidy coefficient and $g$ denotes the energy-saving level of products. The manufacturer’s energy-saving R&D cost is $(1/2)bg^2$, where $b$ represents the coefficient of energy-saving R&D cost. To ensure the existence of the equilibrium solutions, we need $b > (k + s)^{3/2}$ (for proof thereof, see Lemma 1).

3.2. Market Demand. Many countries have implemented a labelling system showing the energy-saving level of ESPs to inform consumers. Therefore, market demand is influenced by not only the price but also the energy-saving level of products. According to [48–50], the demand function is supposed as follows:

$$q = A - p + kg,$$  \hfill (1)

where $A$ is the market potential, $p$ is the retail price of the products, and $k$ is the sensitivity of consumers to energy-saving level. To capture the uncertainty about market demand, we assume that the market potential is a constant plus
a random factor and can take two possible values, \( A_H = a (1 + d) \) or \( A_L = a (1 - d) \), which means the high or low market potentials, respectively. The parameter \( a (a > 0) \) is the average market size, and \( d (0 < d < 1) \) is the uncertainty level of market potential. Similar setups have been used in [27, 34].

3.3. Demand Forecasting. It is necessary for the retailer to estimate whether, or not, to invest in information acquisition techniques (such as UPC scanning system). If the retailer refuses to invest in the technique, the prior probabilities of the retailer and the manufacturer reacting to potential market demand are \( \Pr (A = A_H) = \rho \) or \( \Pr (A = A_L) = 1 - \rho (0 < \rho < 1) \). If the retailer chooses to invest in information acquisition techniques, the investment cost is \( F \) and the retailer can acquire large amounts of market data from which to forecast potential market demand. However, due to influences of some factors including errors in data processing and limited ability of analysts, the retailer always incurs some forecast error. It is assumed that the forecast accuracy of the retailer is \( \sigma (0 < \sigma < 1) \). After assessing potential market demand, the retailer decides whether, or not, to share her forecast information. If the retailer refuses to share information, the manufacturer’s judgment of the potential market demand remain \( \Pr (A = A_H) = \rho \) or \( \Pr (A = A_L) = 1 - \rho \). Reference to [34, 35], we suppose the retailer delivers true demand information owing to the retailer and the manufacturer being long-term co-operative partners. The retailer delivers signals \( S_H \) and \( S_L \), corresponding to high and low market demand to the manufacturer. Afterwards, the manufacturer rebuilds the belief on market demand after receiving the signals.

3.4. Game Sequence. We model the interaction between the two firms as a four-stage Stackelberg game (illustrated in Figure 1). In the first stage, the retailer determines whether, or not, to invest in the information acquisition techniques; in the second stage, the retailer decides whether, or not, to share market information; in the third stage, the manufacturer determines the energy-saving level and wholesale price according to received signals; in the fourth stage, the retailer determines the retail price.

We use the superscript \( N \) to denote the scenario that the retailer does not invest in information acquisition techniques. The superscript \( IN \) represents the scenario that the retailer invests in information acquisition techniques while refusing to share the market information with the manufacturer. The superscript \( IS \) means that the retailer invests in information acquisition techniques and shares the market information with the manufacturer. The notations are presented in Table 1.

4. Model Analysis

4.1. No Information Investment (N). First, we analyse the scenario that the retailer does not invest in information acquisition techniques. So the retailer’s expected profit is expressed as follows:

\[
\pi_r^N = (p - w)[\rho q_H + (1 - \rho)q_L].
\]  

(2)

The expected profit of the manufacturer is calculated as follows:

\[
\pi_m^N = \rho [(w - c) + s]q_H + (1 - \rho) [(w - c) + s]q_L - \frac{1}{2} b g^2.
\]  

(3)

According to the principle of backward induction, Lemma 1 follows.

Lemma 1. In the model of no information investment, the expected wholesale price, retail price, energy-saving level, and profits of the manufacturer and the retailer are, respectively, displayed as follows:

\[
w_{N*}^* = c + \frac{[2b - (k + s)][1 - d + 2 dp]a - c]}{4b - (k + s)^2},
\]

\[
\rho_{N*}^* = w_{N*}^* + \frac{b[(1 - d + 2 dp)a - c]}{4b - (k + s)^2},
\]

\[
g_{N*}^* = \frac{(k + s)[1 - d + 2 dp]a - c]}{4b - (k + s)^2},
\]

\[
\pi_m^{N*} = \frac{b[(1 - d + 2 dp)a - c]^2}{2[4b - (k + s)^2]},
\]

\[
\pi_r^{N*} = \frac{b^2[(1 - d + 2 dp)a - c]^2}{[4b - (k + s)^2]^2}.
\]  

(4)
4.2. Information Investment but No Sharing (IN). In this subsection, we assume that the retailer invests in information acquisition techniques while refusing to share market information with the manufacturer. Similar to the analysis of Lemma 1, Lemma 2 is shown as follows:

Lemma 2. In the model of information investment but no sharing, the expected wholesale price, retail price, energy-saving level, and the expected profits of the manufacturer and the retailer are, respectively, displayed as follows:

\[ w^{\text{IN}*} = c + \frac{[2b - s(k + s)][(1 - d + 2dp)a - c]}{4b - (k + s)^2}, \]

\[ p^{\text{IN}*} = w^{\text{IN}*} + \frac{b[(1 - d + 2dp)a - c]}{4b - (k + s)^2}, \]

\[ g^{\text{IN}*} = \frac{(k + s)[(1 - d + 2dp)a - c]}{4b - (k + s)^2}, \] (5)

\[ \pi^{\text{IN}*}_m = \frac{b[(1 - d + 2dp)a - c]}{2[4b - (k + s)^2]}, \]

\[ \pi^{\text{IN}*}_r = \frac{b^2[(1 - d + 2dp)a - c]^2}{4b - (k + s)^2} + F_1 - F, \]

where \( F_1 = \alpha^2d^2(1 - \rho)\sigma \) represents the income from investment (IFI) in the case of information investment but no sharing.

By comparing the results of Lemmas 1 and 2, Proposition 1 can be obtained:

Proposition 1

(i) \( w^{\text{IN}*} = w^{\text{N}*}, \quad p^{\text{IN}*} = p^{\text{N}*}, \quad g^{\text{IN}*} = g^{\text{N}*}, \quad \pi^{\text{IN}*}_m = \pi^{\text{N}*}_m \)

(ii) \( \pi^{\text{IN}*}_r > \pi^{\text{N}*}_r \) if \( F < F_1 \), \( \pi^{\text{IN}*}_r < \pi^{\text{N}*}_r \) otherwise

(iii) \( \frac{\partial F_1}{\partial \sigma} = \frac{\partial F_1}{\partial \beta} = 0 \)

Proposition 1 implies the following:

(i) The expected wholesale price, retail price, energy-saving level, and manufacturer’s profit obtained when the retailer invests in information acquisition techniques while refusing to share information are the same as those when the retailer does not invest in information acquisition techniques.

(ii) The retailer will choose to invest in information acquisition techniques while refusing to share information if the IFI (\( F_d \)) exceeds the investment cost. Otherwise, the retailer does not make any information investment. The reason is that investing in information acquisition techniques can bring market information to the retailer, thus reducing the demand uncertainty. When the benefit from information investment can compensate for the cost, the retailer will invest in information acquisition techniques, and vice versa.

(iii) The IFI (\( F_d \)) does not change with government subsidy and consumer preference. This reveals that the government subsidy and consumer preference for ESPs do not influence the retailer’s decision-making on information investment if the retailer refuses to share information.

4.3. Information Investment and Sharing (IS). In this subsection, we assume that the retailer invests in information acquisition techniques and shares the market information with the manufacturer.

1. If the retailer fails to acquire valid data, the analysis is same as the case of no information investment.
2. If the data are valid, the retailer can forecast the market demand \( A_i \) (\( i = H, L \)) and truly deliver the signal \( S_i \) to the manufacturer. Afterwards, the manufacturer rebuilds the belief of potential market demand after receiving the signal.

We obtain the following Lemma 3.

Lemma 3. \( \Pr(A = A_H \mid S = S_H) = \frac{(1 + \sigma)p/(2\sigma - \sigma + 1)}{(1 + \sigma + 2\sigma p)/(1 + \sigma)}, \quad \Pr(A = A_L \mid S = S_H) = \frac{(1 - \sigma)p/(\sigma + 1 - 2\sigma p)}{(1 - \sigma)/(\sigma + 1 - 2\sigma p)}, \quad \text{and} \quad \Pr(A = A_L \mid S = S_L) = 1 - \frac{(1 - \sigma)p/(\sigma + 1 - 2\sigma p)}{(1 - \sigma)/(\sigma + 1 - 2\sigma p)} \).

In Lemma 3, if \( \sigma = 0 \), it indicates that the forecast information is completely inaccurate, and the retailer and the manufacturer still show the prior probabilities \( \Pr(A = A_H) = \rho \) and \( \Pr(A = A_L) = 1 - \rho \). If \( \sigma = 1 \), it indicates that the forecast information is completely accurate. In this case, \( \Pr(A = A_H \mid S = S_H) = 1, \quad \Pr(A = A_L \mid S = S_H) = 0, \quad \text{and} \quad \Pr(A = A_H \mid S = S_L) = 0, \quad \text{and} \quad \Pr(A = A_L \mid S = S_L) = 0. \)

Similar to the analysis of Lemma 2, Lemma 4 is obtained:

Lemma 4. In the model of information investment and sharing, the expected wholesale price, retail price, energy-
savings level, and profits of the manufacturer and the retailer are, respectively, calculated as follows:

\[
\begin{align*}
\omega_{IS}^* &= c + \frac{[2b - s(k + s)][(1 - d + 2 dp)a - c]}{4b - (k + s)^2} + \frac{4a d}[2b - s(k + s)](1 - \rho)(2\rho - 1)(1 - \sigma)\sigma^2 \\
\rho_{IS}^* &= c + \frac{[3b - s(k + s)][(1 - d + 2 dp)a - c]}{4b - (k + s)^2} + \frac{2ad}[2b + k^2 - \sigma^2](1 - \rho)(2\rho - 1)(1 - \sigma)\sigma^2 \\
g_{IS}^* &= \frac{(k + s)[(1 - d + 2 dp)a - c]}{4b - (k + s)^2} + \frac{4ad(k + s)(1 - \rho)(2\rho - 1)(1 - \sigma)\sigma^2}{[4b - (k + s)^2][1 - (1 - 2\rho)^2\sigma^2]}, \\
\pi_{Ie}^{15*} &= \frac{b[(1 - d + 2 dp)a - c]}{4b - (k + s)^2} + \frac{4ba d(1 - \rho)\sigma^2}{2[4b - (k + s)^2]}B, \\
\pi_{Ie}^{15*} &= \frac{b^2[(1 - d + 2 dp)a - c]}{4b - (k + s)^2} + F_2 - F, \\
\end{align*}
\]

where

\[
B = 2[(1 - d + 2 dp)a - c]\left(\frac{\rho}{1 - \sigma + 2\rho\sigma} + \frac{1 - \rho}{-1 + \sigma + 2\rho\sigma}\right) + 4ad(1 - \rho)\rho\sigma\left[\frac{\rho}{(1 - \sigma + 2\rho\sigma)^2} + \frac{1 - \rho}{(-1 - \sigma + 2\rho\sigma)^2}\right] \\
F_2 = \frac{4ad^2\rho(1 - \rho)\sigma^2}{4b - (k + s)^2}B + \rho(1 - \rho)\sigma(1 - \sigma)ad\left(\frac{2b}{4b - (k + s)^2}\right) \\
\cdot \left\{[(1 - d + 2 dp)a - c]\left(\frac{1}{1 - \sigma + 2\rho\sigma} + \frac{1}{-1 + \sigma + 2\rho\sigma}\right) + 4ad(1 - \rho)\rho\left[\frac{1}{(1 - \sigma + 2\rho\sigma)^2} + \frac{1}{(-1 - \sigma + 2\rho\sigma)^2}\right]\right\} \\
+ ad(1 - \sigma)\left[\frac{1 - \rho}{(1 - \sigma + 2\rho\sigma)^2} + \frac{\rho}{(-1 - \sigma + 2\rho\sigma)^2}\right], \\
\]

which represents the IFI in the case of information investment and sharing.

By comparing the retailer’s profit with information investment and sharing with that without information investment and analysing influences of government subsidy and consumer preference on the \(F_2\), Proposition 2 is obtained.

**Proposition 2**

(i) \(\pi_{Ie}^{15*} > \pi_{Ie}^{15*}\) if \(F < F_2\), \(\pi_{Ie}^{15*} < \pi_{Ie}^{15*}\) otherwise

(ii) When \(\rho \geq (1/2)\), \(\frac{\partial F_2}{\partial \sigma} = (\frac{\partial F_2}{\partial k}) > 0\); when \(\rho < (1/2)\), \(\frac{\partial F_2}{\partial \sigma} = (\frac{\partial F_2}{\partial k}) < 0\), \(s < s_1\) ...

where

\[
s_1 = \sqrt{\frac{16abd(1 - \rho)\rho[2 - \sigma - (1 - 2\rho)^2\sigma^2]}{[1 - (1 - 2\rho)^2\sigma^2][1 - (1 - 2\rho)^2\sigma^2]} - k, \\
M = (a - ad + 2ad\rho - c)\frac{2(1 - 2\rho)\sigma}{1 - (1 - 2\rho)^2\sigma^2} + 4ad(1 - \rho)\rho\left[\frac{2[1 + (1 - 2\rho)^2\sigma^2]}{[1 - (1 - 2\rho)^2\sigma^2]^2}\right]. \\
\]

Proposition 2 shows the following:

(i) Similar to Proposition 1, the profit when the retailer conducts information investment and sharing is
Proposition 3. When \( \rho \geq 1/2 \), \( w^{IS} > w^{IN} \), \( p^{IS} > p^{IN} \), \( g^{IS} \geq g^{IN} \); when \( \rho < 1/2 \), \( w^{IS} < w^{IN} \), \( p^{IS} < p^{IN} \), \( g^{IS} < g^{IN} \).

Proposition 3 indicates that the wholesale price, retail price, and energy-saving level when the retailer shares information are no lower than those without information sharing when the probability of a high demand is not less than 50%, and vice versa. The reason is that the retailer’s information sharing enables the manufacturer to make a decision adaptive to the high market demand when the prior probability of a market boom is high, i.e., setting a higher wholesale price and energy-saving level which in turn affect the retailer’s benefit. This implies that, when the probability of a market boom is high, information sharing can promote energy-saving level and prices.

Proposition 4

(i) When \( \rho \geq 1/2 \), \( \pi^{IS} < \pi^{IN} \).

(ii) When \( \rho < 1/2 \), \( \pi^{IS} < \pi^{IN} \), \( s < s_2 \).

where \( s_2 = \sqrt{(H/F)} - k \), \( J = a d (1 - \rho) \rho [2 - \sigma - (1 - 2\rho)^2 \sigma^3] H = a c - a d + 2a d p\beta (2\rho - 1)(1 - \sigma) \sin^{-1}\left[1 - (1 - 2\rho)^2 \sigma^2\right] + a d (1 - \rho) \rho 2b [4 - \sigma + (1 - 2\rho)^2 \sigma^2]

Proposition 4 shows the following:

(i) The retailer’s information sharing will reduce her own profits when the probability of a high demand is no less than 50%. This is because the manufacturer and the retailer, as individuals to make decisions, both pursue the maximisation of their own profits. Therefore, when the retailer shares information, the manufacturer makes full use of his first-mover advantage to squeeze the retailer’s profit margin by increasing the wholesale price, thus leading to a profit loss of retailer. We emphasize that government subsidy do not influence the retailer’s information sharing decision in this case.

(ii) The retailer’s information sharing is influenced by the government subsidy when the probability of a high demand is less than 50%. The retailer refuses to share information with the manufacturer if the government subsidy is low. The retailer will share information if the government subsidy is high. The reason is that, with a high probability of a depressed market, on the one hand, information sharing enables the manufacturer to formulate a low wholesale price; on the other hand, a high government subsidy facilitates the manufacturer to improve the energy-saving level of their products. Therefore, a high energy-saving level and a low retail price will promote the demand and promote retailer’s profits, and the retailers enjoy sharing information.

Proposition 4 implies that the retailer’s information sharing will damage her own profits if the probability of a market boom is high. In contrast, the retailer is willing to share market demand information unless the government subsidy is high enough if the probability of a depressed market is high.

By analysing Propositions 1, 2, and 4, ded 1 can be obtained.

Deduction 1

(i) When \( \rho \geq 1/2 \) or \( \rho < 1/2 \) and \( s < s_2 \), we have

\[
\begin{align*}
\pi^{IN} > \pi^{IS} > \pi^{rN}, & \quad F < F_2, \\
\pi^{IN} > \pi^{IS} > \pi^{rN}, & \quad F_2 < F < F_1, \\
\pi^{IN} > \pi^{IS} > \pi^{rN}, & \quad F > F_1,
\end{align*}
\]

(ii) When \( \rho < 1/2 \) and \( s > s_2 \), we have

\[
\begin{align*}
\pi^{IS} > \pi^{IN} > \pi^{rN}, & \quad F_1 < F < F_2, \\
\pi^{IS} > \pi^{IN} > \pi^{rN}, & \quad F_2 < F < F_1, \\
\pi^{IS} > \pi^{IN} > \pi^{rN}, & \quad F > F_2.
\end{align*}
\]

ded 1 shows that the optimal strategy for the retailer is to invest in information acquisition techniques but not to share information with the manufacturer when the market booms and the investment cost is low, or the market is depressed and government subsidy and investment cost are both low. The optimal strategy for the retailer is to invest and share information when the market is depressed and the investment cost is low while there is a high government subsidy.
optimal strategy for the retailer is making no information investment regardless of government subsidy and market state on condition that the investment cost is very high.

**Proposition 5**

(i) When \( \rho \geq (1/2) \), \( \pi_m^{IS} > \pi_m^{IN} \)

(ii) When \( \rho < (1/2) \), \( \pi_m^{IS} < \pi_m^{IN} \), \( c < c_1 \)

\[ \pi_m^{IS} = \pi_m^{IN} + (1 - 2\rho)^3 \sigma (2 - \rho)(1 - \rho)|1 - (2\rho)^3 \sigma|^2 \]

where \( c_1 = (1 - d + 2d\rho)a + (2d \cdot (1 - \rho)\rho \sigma [1 - 2(1 - 2\rho)^3 \sigma]}/(2p - 1)(1 - (1 - 2(1 - 2\rho)^3 \sigma)] \).

**Proposition 5** shows the following:

(i) The manufacturer can obtain a higher profit when the retailer shares information if the probability of a high demand is no less than 50%. Based on information shared by the retailer, the manufacturer can clearly judge the market state and utilise first-mover advantage to acquire more profit by setting a high wholesale price.

(ii) The retailer’s information sharing may decrease the manufacturer's profit if the probability of a high demand is less than 50% and the production cost is low, and vice versa. It is because, in the case that the probability of a depressed market is high, on the one hand, the manufacturer with a low production cost shows a strong ability to deal with the risk of market fluctuations; on the other hand, the manufacturer has to reduce the wholesale price and decrease its profits if the retailer shares information. So the low-production-cost manufacturer is unfavourable for information sharing when the probability of a depressed market is high. On the contrary, if the production cost is high, the manufacturer’s ability to deal with the risk of market fluctuations is weak. Even slight market fluctuations may lead to a huge loss. So, the manufacturer expects the retailer to share information to help him forecast the market better.

Proposition 5 implies that the retailer’s information sharing is favourable to the manufacturer if the probability of a market boom is high. The retailer’s information sharing is not beneficial to the manufacturer unless a high production cost is incurred if the market is more likely to be depressed.

4.4. Sharing Information Investment Costs. From Propositions 4 and 5, it can be seen that the retailer sharing information will harm her own profits, but the manufacturer can benefit when the probability of market boom is high. Besides that, the retailer will refuse to share information when the market is very likely to be depressed and the level of government subsidy is low. However, the manufacturer with a high production cost can benefit from the information sharing. Therefore, under the two aforementioned conditions, the manufacturer has an incentive to motivate the retailer to share information.

We assume that the manufacturer shares the retailer’s investment cost for information sharing and suppose the sharing ratio is \( \gamma (0 < \gamma < 1) \). The conditions of incentive compatibility are as follows:

(i) The retailer is willing to invest in information acquisition techniques, i.e., \( \pi_r^{IS} > \pi_r^{IN} \)

(ii) The retailer’s profit when it shares information is higher than that when it does not, i.e., \( \pi_r^{IS} > \pi_r^{IN} \)

(iii) The manufacturer gains more profit when the retailer shares than that if the retailer shares no information, i.e., \( \pi_m^{IS} > \pi_m^{IN} \)

By conducting analysis and simplifying the constraint conditions, the following results can be attained:

\[ F_2 > F, \]

\[ F_2 - (1 - \gamma)F > a^2 \sigma^2 (1 - \rho)(1 - \rho)F, \]

\[ 4ba\ dp(1 - \rho)\sigma^2 \]

\[ \frac{2}{\left(4b - (k + s)^2\right)}B - \gamma F > 0. \]

According to the above conditions (11)–(13), Proposition 6 obtained.

**Proposition 6**

(i) When \( \rho \geq (1/2) \), \( (a^2 \cdot \sigma)^2 (1 - \rho)\sigma - F_2 < F < F_2 \), \( ((a^2 \cdot \sigma^2 (1 - \rho)\sigma - F_2)/F) < \gamma < (4ba\ dp(1 - \rho)\sigma^2 B/2 \cdot \left[4b - (k + s)^2\right])F \), such that the retailer invests in information acquisition techniques and shares market information with the manufacturer.

(ii) When \( a^2 \cdot \sigma^2 (1 - \rho)\sigma - F_2 < F < \min \left\{(4ba\ dp(1 - \rho)\sigma^2 B/2 \cdot \left[4b - (k + s)^2\right])B, F_2\right\} \), \( (a^2 \cdot \sigma^2 (1 - \rho)\sigma - F_2)/F \) \( \gamma < 1 \), the retailer will invest in and share information.

According to Proposition 6, whether the market booms or not, there is a cost sharing ratio that can lead the retailer to invest in and share market information. The range of sharing ratio is wider if the information investment cost is lower and therefore the probability that the manufacturer cooperates with the retailer is higher.

5. Numerical Analysis

To validate the aforementioned conclusions, no loss of generality, we suppose \( a = 13, c = 3, k = 2, b = 5, \sigma = 0.8, \) and \( d = 0.2 \). Owing to the effect of government subsidy being consistent with that of consumer preference, only government subsidy is subjected to analysis.

As shown in Figure 2, when \( \rho = 0.5 \), the retailer’s IFI \( F_r \) is always larger than \( F_2 \). Moreover, the IFI \( F_1 \) without information sharing does not vary with the change in government subsidy, while the IFI \( F_2 \) with information sharing grows with increasing government subsidy.

In Figure 2, the graph is partitioned into three zones by the IFIs \( F_1 \) and \( F_2 \). In zone I, the investment cost is low
h; the retailer’s profit when she does not share information is higher than that when information is being sharing, and she acquires the lowest profit when she refuses to invest in the information acquisition techniques. In zone II, the investment cost is moderate \(F_2 < F < F_1\); the retailer’s profit from information investment while no information is shared is higher than that without information investment, and she obtains the lowest profit if she shares information. In zone III, the investment cost is high \(F > F_1\); the retailer’s profit is the highest without information investment while the lowest profit accrues when the retailer shares market information.

When \(\rho > 0.5\), the conclusions conform to those in Figure 3, and associated discussion is not repeated.

Figure 3 shows that, when \(\rho = 0.1\) \((< 0.5)\), the IFI \((F_1)\) is larger than \(F_2\) if the government subsidy is low \((s < s_2)\). In contrast, if the government subsidy is high \((s > s_2)\), the IFI \((F_1)\) is higher. Similar to the case of \(\rho = 0.5\), the retailer’s IFI \((F_1)\) does not vary with changing government subsidy; however, the IFI \((F_1)\) rises at first and then declines with the increasing government subsidy.

In Figure 3, the graph is divided into six zones by the IFIs \((F_1\) and \(F_2\)) and the critical point \((s_2)\) of government subsidy. (i) When government subsidy is low \((s < s_2)\), zones I, II, and III have the same meaning with those in Figure 2. (ii) When government subsidy is high \((s > s_2)\), in zone IV, the retailer’s profit from information sharing is higher than that without information sharing if the investment cost is low \((F < F_1)\). Moreover, the lowest profit accrues when the retailer does not invest in information acquisition techniques. In zone V, if the investment cost is moderate \((F_1 < F < F_2)\), the profit obtained when the retailer conducts information investment and shares market information is higher than that without information investment; moreover, the profit when not sharing information is the lowest. In zone VI, if the investment cost is high \((F > F_2)\), the retailer can obtain the highest profit when it does not take information investment while the lowest profit accrues without information sharing.

Figure 4 shows that if \(\rho \geq 0.5\), the wholesale price when the retailer invests in and shares information is not less than that when making information investment but not sharing information. If \(\rho < 0.5\), the wholesale price in the case of a retailer sharing information is lower. The conclusions in Figures 5 and 6 are consistent with those in Figure 4, which, respectively, represent the retail price and energy-saving level.

Figure 7 shows that when \(\rho = 0.5\), the manufacturer’s profit when the retailer shares information is always higher than that without information sharing. When \(\rho > 0.5\), the conclusion conforms to that at \(\rho = 0.5\).

Figure 8 shows that when \(\rho = 0.1\) \((< 0.5)\), the manufacturer’s profit when the retailer refuses to share information is higher than that when sharing information if the production cost is low; however, if the production cost is high, the manufacturer’s profit when the retailer shares information is higher than that when the retailer does not share information.
6. Conclusions

This paper establishes a two-echelon supply chain with one manufacturer who invests in energy-saving products (ESPs) and one retailer who sells the products and possesses better demand-forecast information. Considering government subsidy and consumer preference for ESPs, we study the information investment and sharing strategy of the retailer and investigate how the retailer’s sharing strategy affect the manufacturer’s decision-making and profit. Compared with the previous research, the conclusions we get are richer and contribute significantly to the state of the art.

Firstly, Xie [48] investigated the pricing and energy-saving level decisions of a supply chain under complete information while ignoring the fact that the retailer tends to have superiority in acquiring market information. When considering the retailer’s market information advantage and sharing strategy, we find that the wholesale price, retail price, and energy-saving level when the retailer shares information are lower than those when the retailer does not share information if the probability of a high demand is less than 50%, and vice versa.

Secondly, Taylor and Xiao [35] explored the impact of the retailer’s ability to predict market information on the manufacturer’s profit; however, they did not consider the influences of government subsidy and consumer preference on the retailer’s income from investment (IFI). This paper shows that the IFI ($F_1$) is not influenced by government subsidy and consumer preference if the retailer refuses to share market information. On the contrary, the IFI ($F_2$) will increase with the government subsidy and consumer preference on condition that the probability of a high demand is no less than 50% if the retailer shares information. In the event that the probability of a high demand is less than 50%, the IFI ($F_2$) grows with the improvement of government subsidy and consumer preference if the government subsidy is low; oppositely, the IFI ($F_2$) declines with the increasing government subsidy.
subsidy and consumer preference if the government subsidy is high; this means excessive government subsidy will weaken the retailer’s IFI (F_r).

Thirdly, Guo [34] investigated whether, or not, the retailer with demand advantage will share information with the manufacturer. Differing from his hypothesis, we suppose that the demand function is not only influenced by price but also by energy-saving level and consumer preference. Additionally, we also consider that government provides subsidy for ESPs and the prior probability of potential market demand is not Pr(A_H) = (1/2) assumed by [31] but generalized Pr(A_H) = ρ. It can be found that whether the retailer sharing information or not is influenced by various factors: the retailer sharing information will impair its own profit while benefiting the manufacturer when the probability of a high demand is no less than 50%. The optimal strategy for the retailer is sharing information if the probability of a high demand is less than 50%, and the government subsidy is high, while the manufacturer with low production cost does not expect the retailer to share. On the contrary, the optimal strategy for the retailer is avoiding sharing if the probability of a high demand is less than 50% and the government subsidy is low, while the manufacturer with high production cost expects sharing. Moreover, the manufacturer can motivate the retailer to change his/her decision by sharing the information investment cost.

This paper presents significant managerial insights: according to the aforementioned conclusions, it can be seen that the probability of a high demand and government subsidy can influence the retailer’s decisions on information investment and sharing. Therefore, apart from investigating market quotation, the retailer also needs pay attention to the government’s policy for ESPs and timely adjust her information sharing strategy. Additionally, whether the retailer shares information or not directly affects the manufacturer’s profits. Therefore, to acquire more profits, the manufacturer should actively cooperate with the retailer and share the information investment cost. However, sharing market demand information is not beneficial to the manufacturer when the probability of a market depression is high and the production cost is low. In this case, the manufacturer should refuse to cooperate with the retailer in information sharing.

There are still many issues need to be resolved in the future: (1) if there is a potential manufacturer in the market, whether the in situ retailer only shares market information with the in situ manufacturer or publicize information? And, how does the sharing strategy influence the in situ and potential manufacturer’s decisions on energy-saving level and profits? It is a very interesting study. (2) In reality, the retailer may fail to estimate the consumer preference for ESPs which affect the market demand significantly. Therefore, studying how the incomplete information of consumer preference for ESPs influences the supply chain is another interesting study. (3) As a producer, the manufacturer generally has a better advantage of its own production cost and energy-saving R&D cost, so exploring asymmetry of cost information may prove fruitful.

### Appendix

#### A. Proof of Lemma 1

Using backward induction, we first solve the retail price:

\[
\frac{\partial \pi_r^N}{\partial p} = a - ad + gk - 2p + w + 2adp. \quad (A.1)
\]

Equating \((\partial \pi_r^N/\partial p)\) to 0, we get

\[
p^* = (1/2)(a - a + gk + w + 2a dp). \quad \text{And, the profit } \pi_r^N \text{ is a concave function of } p \text{ since } (\partial^2 \pi_r^N/\partial p^2) = -2 < 0.
\]

Next, we solve the wholesale price and energy-saving level:

\[
\frac{\partial \pi_m^N}{\partial w} = \frac{1}{2} (a + c - a d + gk - gs - 2w + 2a dp),
\]

\[
\frac{\partial \pi_m^N}{\partial g} = \frac{1}{2} (-2bg - c k + a d s + 2gks + kw - sw + 2a ds \rho).
\]

(A.2)

His corresponding Hessian matrix is

\[
F(w, g) = \begin{bmatrix}
-1 & \frac{((k - s)/2)}{(k - s)/2} + ks - b \\
\frac{(k - s)/2}{ks - b} & \frac{2}{2k - b}
\end{bmatrix}. \quad \text{By the condition of } 2b > (k + s)^2, \text{ we have } ks - b < 0. \text{ Since the first-order principal minor } -1 < 0, \text{ ks } - b < 0, \text{ and second-order principal minor } b - \frac{((k + s)/2)^2}{2} > 0, \text{ the profit } \pi_m^N \text{ is concave in } w \text{ and } g.
\]

Equating \((\partial \pi_m^N/\partial w)\) and \((\partial \pi_m^N/\partial g)\) to 0, the simultaneous equations can be solved and the equilibrium solutions are

\[
w^N = c + \frac{[2b - s(k + s)][(1 - d + 2dp)a - c]}{4b - (k + s)^2},
\]

\[
g^N = \frac{(k + s)[(1 - d + 2dp)a - c]}{4b - (k + s)^2}. \quad (A.3)
\]

Finally, substituting the equilibrium solutions to the function of \(\pi_r^N\) and \(\pi_m^N\), we get Lemma 1.

#### B. Proof of Lemma 2

① If the data are invalid, both the retailer and the manufacturer judge the potential market demand Pr(A = A_H) = ρ and Pr(A = A_r) = 1 − ρ. Similar to the case of no information investment, we have

\[
w^{INI}_r = w^{N*} + \frac{[2b - s(k + s)][(1 - d + 2dp)a - c]}{4b - (k + s)^2},
\]

\[
p^{INI}_r = p^{N*} + \frac{b[(1 - d + 2dp)a - c]}{4b - (k + s)^2},
\]

\[
g^{INI}_r = g^{N*} + \frac{[(k + s)[(1 - d + 2dp)a - c]]}{4b - (k + s)^2},
\]

\[
\pi_r^{INI} = \frac{b[(1 - d + 2dp)a - c]}{2[4b - (k + s)^2]} - F. \quad \text{(B.1)}
\]
If the data are valid, the retailer knows the potential market demand $A_i (i = H, L)$. But the retailer does not share market information, so the manufacturer’s belief of the potential market demand remains $\Pr(A = A_H) = \rho$ and $\Pr(A = A_L) = 1 - \rho$. Then, the profit functions of the retailer and manufacturer are

$$\max_{p_i} \pi_{i}^{NA} = (p - w)q_H - F,$$

$$\max_{(w,g)} \pi_{i}^{NA} = \rho[w - c + sg]q_H + (1 - \rho)[w - c + sg]q_L - \frac{1}{2}bg^2.$$

(B.2)

Based on backward induction, we get $w^{INAs} = c + \frac{[2b - s(k + s)][(1 - d + 2dp)a - c]}{4b - (k + s)^2}$, $p^{INAs} = \frac{(k + s)}{(4b - (k + s)^2)}p^{INAs} + \frac{b(1 - d + 2dp)a - c]}{4b - (k + s)^2}$, $\pi_{i}^{INAs} = \frac{b[(1 - d + 2dp)a - c]}{4b - (k + s)^2}$, $\pi_{i}^{INAS} = \frac{b^2[(1 - d + 2dp)a - c]}{4b - (k + s)^2} + a^2d^2(1 - \rho)\rho - F$.

Synthesizing (i) and (ii), Lemma 2 can be obtained.

C. Proof of Proposition 1

(i) By Lemmas 1 and 2, we have

$$w^{INAs} = w^{N*} + \frac{[2b - s(k + s)][(1 - d + 2dp)a - c]}{4b - (k + s)^2},$$

$$p^{INAs} = p^{N*} + \frac{b[(1 - d + 2dp)a - c]}{4b - (k + s)^2}.$$

(D.1)

where $S = N$ means that the retailer does not collect valid data, $S = Y$ means the retailer collecting valid data.

According to the Bayesian formula, we have

$$\Pr(\{A = A_H\} | S = S_H) = \frac{\Pr(\{A = A_H\} | S = S_H) \cdot \Pr(S = S_H)}{\Pr(\{A = A_H\} | S = S_H) \cdot \Pr(S = S_H)}$$

$$= \frac{\Pr(\{S = S_H | A = A_H\} * \Pr(\{A = A_H\})}{\Pr(\{S = S_H | A = A_H\} * \Pr(\{A = A_H\}) + \Pr(\{S = S_H | A = A_L\} * \Pr(\{A = A_L\})$$

$$= \frac{\Pr(\{S = S_H | S = Y\} * \Pr(S = Y) + \Pr(\{S = S_H | S = S_H\} * \Pr(S = N) + \Pr(\{S = S_H | S = N\} * \Pr(S = N) + \Pr(\{S = S_H | S = N\} * \Pr(S = N))}{\Pr(\{S = S_H | A = A_H\} * \Pr(\{A = A_H\}) + \Pr(\{S = S_H | A = A_H\} * \Pr(\{A = A_H\}) + \Pr(\{S = S_H | S = S_H\} * \Pr(S = Y)) + \Pr(\{S = S_H | S = S_H\} * \Pr(S = N) + \Pr(\{S = S_H | S = N\} * \Pr(S = N)$$

$$= \frac{1 + \sigma + (1/2)(1 - \sigma)\rho}{1 + \sigma + (1/2)(1 - \sigma)\rho} = \frac{2\rho}{2\rho - \sigma + 1}.$$
Pr(\(A = A_H|S = S_L\)) = \frac{Pr(A = A_H, S = S_L)}{Pr(S = S_L)}

= \frac{Pr(\{S = S_L|A = A_H\}) * Pr(A = A_H)}{Pr(\{S = S_L|A = A_H\}) * Pr(A = A_H) + Pr(\{S = S_L|A = A_L\}) * Pr(A = A_L)}

= \frac{\left[ Pr(\{S = S_L|S = Y\} \right) * Pr(S = Y) + Pr(\{S = S_L|S = N\} \right) * Pr(S = N) ] * Pr(A = A_H)}{\left[ Pr(\{S = S_L|A = A_H\}) * Pr(A = A_H) + \left[ Pr(\{S = S_L|S = Y\} \right) * Pr(S = Y) + Pr(\{S = S_L|S = N\} * Pr(S = N) \right] * Pr(A = A_L)}

= \frac{[0*\sigma + (1/2) (1 - \sigma)]}{[0*\sigma + (1/2) (1 - \sigma)] \rho + [1*\sigma + (1/2) (1 - \sigma)] (1 - \rho)} = \frac{(1 - \sigma) \rho}{\sigma + 1 - 2\rho \sigma}

Pr(\{A = A_L|S = S_L\}) = 1 - \frac{(1 - \sigma) \rho}{\sigma + 1 - 2\rho \sigma}

\(\text{(D.3)}\)

**E. Proof of Lemma 4**

\(\text{① The data are valid}\)

\(\text{(i) If the retailer observes that the market demand is high and transmits signal } S_H, \text{ then}\)

\[
\max_p \pi^\text{ISA*}_{A_H} = (p - w)q_H - F. \tag{E.1}
\]

Solving it, we get \(\pi^\text{ISA*}_{H} = (1/2)(gk + w + a + a d)\)

After receiving the signal \(S_H\), the manufacturer updating the beliefs of the potential market demand are

\[
Pr(\{A = A_H|S = S_H\}) = \frac{1 + \sigma \rho}{2\rho \sigma - \sigma + 1}. \tag{E.2}
\]

Thus, the manufacturer’s profit is

\[
\pi^\text{ISA*}_{H}(\{w, g|S_H\}) = \frac{1 + \sigma \rho}{2\rho \sigma - \sigma + 1} (w - c + sg)q_H + \left(1 - \frac{(1 + \sigma \rho)}{2\rho \sigma - \sigma + 1}\right)(w - c + sg)q_L - \frac{1}{2}bg^2. \tag{E.3}
\]

Solving it, we get

\[
\omega^\text{ISA*}_{H} = \frac{c (2b - k (k + s)) (1 - 1 + 2\rho \sigma) + a (2b - s (k + s)) (1 + 1 - 2\rho \sigma + d (-1 + 2\rho + s))}{4b - (k + s)^2} \cdot (1 + 1 - 2\rho \sigma),
\]

\[
\theta^\text{ISA*}_{H} = \frac{(k + s) [(1 - d + 2d \rho)a - c]}{4b - (k + s)^2} + \frac{4a d (k + s) (1 - \rho) \rho \sigma}{4b - (k + s)^2} \cdot \frac{1 + (2\rho - 1) \sigma}{[1 + (2\rho - 1) \sigma]^2} \tag{E.4}
\]

\[
\phi^\text{ISA*}_{H} = \frac{c (b - k (k + s)) + a (b (3 + d + 2d \rho) + (k + s) (-s + d (k (-1 + \rho) - sp)))}{4b - (k + s)^2} + \frac{2a d (2b + k^2 - s^2) (1 - \rho) \rho \sigma}{4b - (k + s)^2} \cdot (1 + 1 - 2\rho \sigma).
\]

Substituting the above value into the profit functions of the retailer and manufacturer, we get
\[ n_{mH}^{ISA} = \frac{b[(1 - d + 2d\rho)a - c][1 + (2\rho - 1)\sigma + 4a d (1 - \rho)\rho\sigma]^2}{2[4b - (k + s)^2] [1 + (2\rho - 1)\sigma]^2} , \]

\[ n_{rH}^{ISA} = \frac{b[(1 - d + 2d\rho)a - c][1 + (2\rho - 1)\sigma + a d (1 - \rho) (1 - \sigma)[4b - (k + s)^2] + 4a db (1 - \rho)\rho\sigma]^2}{[4b - (k + s)^2]^2 [1 + (2\rho - 1)\sigma]^2} , \]  
\[(E.5)\]

(ii) Similarly, if the retailer observes that the market demand is low and transmits signal \( S_L \), we can get:

\[ w_{ISA}^{ISI} = \frac{-\alpha(2b - s(k + s))(1 + d(-1 + 2\rho - \sigma) + \sigma - 2\rho\sigma) + c(2b - k(k + s))(-1 + (-1 + 2\rho)\sigma)}{(4b - (k + s)^2)(-1 + (-1 + 2\rho)\sigma)} \]

\[ g_{ISA}^{ISI} = \frac{(k + s)[(1 - d + 2d\rho)a - c]}{4b - (k + s)^2} + \frac{4a d (k + s)(1 - \rho)\rho\sigma}{[4b - (k + s)^2] [-1 + (2\rho - 1)\sigma]^2} \]

\[ p_{ISA}^{ISI} = \frac{c(b - k(k + s) + a(b + d(-3 + 2\rho) + (k + s)(dk \rho + s(-1 + d - dp)))}{4b - (k + s)^2} + \frac{2a d(2b + k^2 - s^2)(1 - \rho)\rho\sigma}{(4b - (k + s)^2)(-1 + (-1 + 2\rho)\sigma)} \]

\[ n_{mISA}^{ISI} = \frac{b[(1 - d + 2d\rho)a - c][-1 + (2\rho - 1)\sigma + 4a d (1 - \rho)\rho\sigma]^2}{2[4b - (k + s)^2][-1 + (2\rho - 1)\sigma]^2} \]

\[ n_{rISA}^{ISI} = \frac{b[(1 - d + 2d\rho)a - c][-1 + (2\rho - 1)\sigma + a d (1 - \sigma)[4b - (k + s)^2] + 4a db (1 - \rho)\rho\sigma]^2}{[4b - (k + s)^2]^2 [-1 + (2\rho - 1)\sigma]^2} - F. \]  
\[(E.6)\]

Therefore, combining (i) and (ii) and when the data are valid, the expected wholesale price, retail price, energy-saving level, and profit of the retailer and the manufacturer are as follows:

\[ w^{ISA} = \rho w_H^{ISA} + (1 - \rho)w_L^{ISA} , \]

\[ p^{ISA} = \rho p_H^{ISA} + (1 - \rho)p_L^{ISA} , \]

\[ g^{ISA} = \rho g_H^{ISA} + (1 - \rho)g_L^{ISA} , \]

\[ n_{mISA}^{ISA} = \rho n_{mH}^{ISA} + (1 - \rho)n_{ml}^{ISA} , \]

\[ n_{rISA}^{ISA} = \rho n_{rH}^{ISA} + (1 - \rho)n_{rL}^{ISA} . \]  
\[(E.7)\]

When the data are invalid, it is similar to the case of no information investment:

\[ w^{ISI} = c + \frac{[2b - s(k + s)][1 - d + 2d\rho)a - c]}{4b - (k + s)^2} , \]

\[ p^{ISI} = \frac{b[(1 - d + 2d\rho)a - c]}{4b - (k + s)^2} , \]

\[ g^{ISI} = \frac{(k + s)[(1 - d + 2d\rho)a - c]}{4b - (k + s)^2} , \]

\[ n_{mISI}^{ISI} = \frac{b[(1 - d + 2d\rho)a - c]^2}{2[4b - (k + s)^2]} , \]

\[ n_{rISI}^{ISI} = \frac{b^2[(1 - d + 2d\rho)a - c]^2}{[4b - (k + s)^2]^2} - F. \]  
\[(E.8)\]
Finally, substituting the equilibrium solutions to the functions

\[ w^{IS}_{IS} = \omega w^{JSS}_{JSS} + (1 - \sigma) w^{JIS}_{JIS}, \]

\[ p_{r}^{IS} = \sigma p_{r}^{JSS} + (1 - \sigma) p_{r}^{JIS}, \]

\[ g_{r}^{IS} = \sigma g_{r}^{JSS} + (1 - \sigma) g_{r}^{JIS}, \]

and

\[ \pi_{r}^{IS} = \sigma \pi_{r}^{JSS} + (1 - \sigma) \pi_{r}^{JIS}, \]

we obtain Lemma 4.

### F. Proof of Proposition 2

(i) \( \pi_{r}^{IS} - \pi_{r}^{N^*} = F_{z} - F_{x} \), where

\( F_{z} = \frac{4a \rho}{4b \sigma} \left[ \frac{1}{2} k^2 (k + s)^2 \right] \),

\[ F_{x} = \left\{ \begin{array}{l}
2 [(1 - d + 2 dp)a - c] \left[ \frac{\rho}{1 - \sigma + 2 \rho \sigma} + \frac{1 - \rho}{-1 - \sigma + 2 \rho \sigma} \right] \\
+ 4a \left( 1 - \rho \right) \rho \sigma \left[ \frac{\rho}{1 - \sigma + 2 \rho \sigma} + \frac{1 - \rho}{-1 - \sigma + 2 \rho \sigma} \right]
\end{array} \right. \]

\[ + \rho \left( 1 - \rho \right) \sigma (1 - \sigma) a d \left( \frac{2b}{4b \sigma} \right) \\
+ \rho \left( 1 - \rho \right) \sigma (1 - \sigma) a d \left( \frac{2b}{4b \sigma} \right) \\
+ \frac{1}{1 - \sigma + 2 \rho \sigma} + \frac{1 - \rho}{-1 - \sigma + 2 \rho \sigma} \right] \}

\[ + 4a \left( 1 - \rho \right) \sigma \left[ \frac{1}{1 - \sigma + 2 \rho \sigma} + \frac{1 - \rho}{-1 - \sigma + 2 \rho \sigma} \right] \}

\[ + a \left( 1 - \sigma \right) \left[ \frac{1}{1 - \sigma + 2 \rho \sigma} + \frac{1 - \rho}{-1 - \sigma + 2 \rho \sigma} \right]. \]  

(F.1)

Therefore, \( \pi_{r}^{IS} > \pi_{r}^{N^*} \) when \( F < F_{z} \), and \( \pi_{r}^{IS} < \pi_{r}^{N^*} \) when \( F > F_{z} \).

(ii) \( \partial F/\partial s = \partial F/\partial k = (4a \rho b \sigma(W) \left[ (1 - (1 - 2 \rho)^2 \sigma^2) - (2(1 - 2 \rho)(a - c - a d + 2a d \rho)) \right] \\
/ (1 - (1 - 2 \rho)^2 \sigma^2) \sigma (1 - \sigma) (k + s)^2 W = 2b(1 - \rho) \left( 2 - \sigma \right) - (k + s)^2 (1 - \rho) (1 - \sigma) [1 + (1 - 2 \rho)^2 \sigma^2]. \)

By

\[ (1 - \rho) \left[ 2 - \sigma - (1 - 2 \rho)^2 \sigma^2 \right] - (1 - \rho) \sigma \left[ 1 + (1 - 2 \rho)^2 \sigma^2 \right] = (1 - \rho) \left[ 1 - (1 - 2 \rho)^2 \sigma^2 \right] > 0 \]

and \( 2b > (k + s)^2 \), we have \( W > 0 \). To make the equilibrium solutions meaningful, it is necessary to make \( (1 - d + 2 dp)a - c > 0 \).

Then, we have

\[ \frac{2 (1 - 2 \rho)(a - c - a d + 2a d \rho)}{1 - (1 - 2 \rho)^2 \sigma^2} \sigma (1 - \sigma) (k + s)^2 \leq 0, \]  

(F.2)

when \( \rho \geq (1/2) \). Sequentially, we get \( D > 0 \), i.e., \( \partial F_{z}/\partial s = \partial F_{x}/\partial k > 0 \).

To facilitate the analysis, we simplify \( D \) to a quadratic function of \( s \). Then we have

\[ D = \frac{16ab \left( 1 - \rho \right) \rho \sigma \left[ 2 - \sigma - (1 - 2 \rho)^2 \sigma^2 \right]}{1 - (1 - 2 \rho)^2 \sigma^2}, \]

(F.3)

where

\[ M = (a - a d + 2a d \rho - c) \frac{2 (1 - 2 \rho)^2 \sigma^2}{1 - (1 - 2 \rho)^2 \sigma^2} + 4a \left( 1 - \rho \right) \rho \sigma \left[ (1 + (1 - 2 \rho)^2 \sigma^2) \right] \]

(F.4)

where

\[ M = (a - a d + 2a d \rho - c) \]

Obviously, \( M > 0 \) when \( \rho < (1/2) \), i.e., \( D \) is opening downward of \( s \). Equating \( D = 0 \), we have

\[ s_{1} = \frac{16ab \left( 1 - \rho \right) \rho \sigma \left[ 2 - \sigma - (1 - 2 \rho)^2 \sigma^2 \right]}{1 - (1 - 2 \rho)^2 \sigma^2} - k, \]

(F.5)

\[ s_{2} = \frac{16ab \left( 1 - \rho \right) \rho \sigma \left[ 2 - \sigma - (1 - 2 \rho)^2 \sigma^2 \right]}{1 - (1 - 2 \rho)^2 \sigma^2} - k < 0 \] (remove).

According to the properties of quadratic function, we have \( D > 0 \), i.e., \( \partial F_{z}/\partial s = \partial F_{x}/\partial k > 0 \) when \( s < s_{1} \), and \( \partial F_{z}/\partial s = \partial F_{x}/\partial k < 0 \) when \( s > s_{1} \).

Based on the above proof, it can be concluded that \( \partial F_{z}/\partial s = \partial F_{x}/\partial k > 0 \) when \( \rho \geq (1/2) \), \( \partial F_{z}/\partial s = \partial F_{x}/\partial k > 0 \) when \( \rho < (1/2) \) and \( s < s_{1} \), \( \partial F_{z}/\partial s = \partial F_{x}/\partial k < 0 \) when \( \rho < (1/2) \) and \( s > s_{1} \).

### G. Proof of Proposition 3

\[ w^{IS}_{IS} - w^{JN^*}_{JN^*} = 4a \left( 2b - s \right) \left( k + s \right) \left( 1 - \rho \right) \rho \left( 2 - 1 \right) \left( 1 - \sigma \right) \sigma^2 \\
/ \left[ 4b \left( k + s \right)^2 \right] \left( 1 - (1 - 2 \rho)^2 \sigma^2 \right] \]

(G.1)

By the condition of \( 2b > (k + s)^2 \), \( 0 < \sigma < 1 \), and \( 0 < \rho < 1 \), we have \( 2b - s \left( k + s \right) > 0 \) and \( 1 - (1 - 2 \rho)^2 \sigma^2 > 0 \). Therefore, \( w^{IS}_{IS} \geq w^{JN^*}_{JN^*} \) when \( \rho \geq (1/2) \), and \( w^{IS}_{IS} < w^{JN^*}_{JN^*} \) when \( \rho < (1/2) \):

\[ p^{IS}_{IS} - p^{JN^*}_{JN^*} = 2a \left( 2b + k^2 - s^2 \right) \left( 1 - \rho \right) \rho \left( 2 - 1 \right) \left( 1 - \sigma \right) \sigma^2 \\
/ \left[ 4b \left( k + s \right)^2 \right] \left( 1 - (1 - 2 \rho)^2 \sigma^2 \right] \]

(G.2)

Similarly, we get \( p^{IS}_{IS} \geq p^{JN^*}_{JN^*} \) when \( \rho \geq (1/2) \), and \( p^{IS}_{IS} < p^{JN^*}_{JN^*} \) when \( \rho < (1/2) \):

\[ g^{IS}_{IS} - g^{JN^*}_{JN^*} = 4a \left( 1 - \rho \right) \rho \left( 2 - 1 \right) \left( 1 - \sigma \right) \sigma^2 \\
/ \left[ 4b \left( k + s \right)^2 \right] \left( 1 - (1 - 2 \rho)^2 \sigma^2 \right] \]

(G.3)

We also get \( g^{IS}_{IS} \geq g^{JN^*}_{JN^*} \) when \( \rho \geq (1/2) \), and \( g^{IS}_{IS} < g^{JN^*}_{JN^*} \) when \( \rho < (1/2) \).
H. Proof of Proposition 4

\[ n_{iS}^* - n_{IN}^* = F_2 - F_1 = \]
\[ \frac{2b - (k + s)^2}{4b - (k + s)^2} \cdot \frac{16 \rho \sigma^2}{(1 - 2\rho)^2 \sigma^2} X, \]
where

\[ X = \left( \frac{a - c - a d + 2a \rho d b (2b - 1) (1 - \sigma) [1 - (1 - 2\rho)^2 \sigma^2] + a d (1 - \rho) [2 - \sigma - (1 - 2\rho)^2 \sigma^2] + a d (1 - \rho) [2 - \sigma - (1 - 2\rho)^2 \sigma^2]}{[1 - (1 - 2\rho)^2 \sigma^2]} \right) Y, \]
and

\[ Y = 2b \left[ 4 - 2(1 - 2\rho)^2 \sigma^2 - (1 - 2\rho)^2 \sigma^2 \right] - \left[ 2 - \sigma - (1 - 2\rho)^2 \sigma^2 \right] > 0 \text{ and } 2b > (k + s)^2, \]
we have \( Y > 0 \). Since \( (1 - d + 2d \rho a - c > 0, 0 < \sigma < 1, \)
we have \( (a - c - a d + 2a \rho d) b (2b - 1) (1 - \sigma) [1 - (1 - 2\rho)^2 \sigma^2] \geq 0 \) when \( \rho \geq (1/2) \).
This, we get \( X > 0 \), i.e., \( n_{iS}^* < n_{IN}^* \) when \( \rho \geq (1/2) \).
\( (a - c - a d + 2a \rho d) b (2b - 1) (1 - \sigma) [1 - (1 - 2\rho)^2 \sigma^2] < 0 \) when \( \rho < (1/2) \). Therefore, whether \( X \) is greater than \( 0 \) or not needs to be discussed. To facilitate the analysis, we simplify \( -X \) to a quadratic function of \( s \), i.e., \( -X = ad (1 - \rho) [2 - \sigma - (1 - 2\rho)^2 \sigma^2] + a d (1 - \rho) [2 - \sigma - (1 - 2\rho)^2 \sigma^2] + a \]
\( b (2b - 1) (1 - \sigma) \left[ 1 - (1 - 2\rho)^2 \sigma^2 \right] + a d (1 - \rho) [2 - \sigma - (1 - 2\rho)^2 \sigma^2] + a d (1 - \rho) [2 - \sigma - (1 - 2\rho)^2 \sigma^2] \). Equating \( X = 0 \), we have \( s_2 = \sqrt{(H/F)} - k \) and \( s_1 = -\sqrt{(H/F)} - k < 0 \) (removable), where \( J = ad (1 - \rho) [2 - \sigma - (1 - 2\rho)^2 \sigma^2] \) and \( H = (a - c - a d + 2a \rho d) b (2b - 1) (1 - \sigma) \left[ 1 - (1 - 2\rho)^2 \sigma^2 \right] + a d (1 - \rho) [2 - \sigma - (1 - 2\rho)^2 \sigma^2] + a d (1 - \rho) [2 - \sigma - (1 - 2\rho)^2 \sigma^2] \). As \( X \) is opening upward, we can get \( -X < 0 \), i.e., \( n_{iS}^* < n_{IN}^* \) when \( s < s_2 \), and \( n_{iS}^* > n_{IN}^* \) when \( s > s_2 \).

I. Proof of Proposition 5

\[ n_{IS}^* - n_{IN}^* = \frac{4ba dp (1 - \rho) \sigma^2}{2[4b - (k + s)^2]} B, \]
where

\[ B = -2 \left( \frac{\rho}{1 - \sigma + 2\rho} + \frac{1 - \rho}{-1 - \sigma + 2\rho} \right) \]
\[ + a \left[ \frac{4d (1 - \rho) \sigma}{(1 - \sigma + 2\rho)^2} + \frac{1 - \rho}{(-1 - \sigma + 2\rho)^2} \right] \]
\[ + 2 \left( 1 - d + 2d \rho \right) \left( \frac{\rho}{1 - \sigma + 2\rho} + \frac{1 - \rho}{-1 - \sigma + 2\rho} \right), \]
and regarding \( B \) as a linear function of \( c \). Obviously, \( n_{IS}^* - n_{IN}^* \) is positively correlated with \( B \):

\[ n_{IS}^* - n_{IN}^* = \frac{a^2bd \sigma^3}{8b - 2(k + s)^2} > 0, \]
i.e., \( n_{IS}^* > n_{IN}^* \) when \( \rho = (1/2) \):

\[ \frac{\rho}{1 - \sigma + 2\rho} + \frac{1 - \rho}{-1 - \sigma + 2\rho} = \frac{2(2\rho - 1)(1 - \sigma)}{(1 - 2\rho)^2 \sigma^2} < 0, \]

(1.4)
when \( \rho > (1/2) \). So the coefficient of \( c \) is less than \( 0 \), i.e., \( B \) is a monotone decreasing function of \( c \). Equating \( B = 0 \), we have
\[ c = c_1 = \left( \frac{a(2 - d(-1 + \rho) \sigma^2 (1 - 1 + 2\rho)(1 - 1 + \rho)(1 + (1 - 2\rho)^2(2 + \sigma) \sigma^2) + (-1 + 2\rho)(1 + (1 - 2\rho)^2(2 + \sigma) \sigma^2)}{(1 - 1 + \rho)(1 - 1 + \rho)(1 + (1 - 2\rho)^2(2 + \sigma) \sigma^2)} \right) / [(1 - 2\rho) (1 - 1 + \rho)(1 - 1 + \rho)(1 + (1 - 2\rho)^2 \sigma^2)]. \]
By the condition of
\[ (1 - d + 2d \rho)a - c > 0, \]
\[ c_1 - (1 - d + 2d \rho)a + 2a d (1 - \rho) - \frac{2(2\rho - 1)(1 - \sigma)}{(1 - 2\rho)^2 \sigma^2} > 0, \]
(1.5)
when \( \rho > (1/2) \). Therefore, we have \( c_1 > (1 - d + 2d \rho)a > c_1 \).
According to the properties of linear function, we have \( B > 0 \), i.e., \( n_{IS}^* > n_{IN}^* \) when \( \rho > (1/2) \):

\[ -2 \left( \frac{\rho}{1 - \sigma + 2\rho} + \frac{1 - \rho}{-1 - \sigma + 2\rho} \right) = -2 \left( \frac{2(2\rho - 1)(1 - \sigma)}{(1 - 2\rho)^2 \sigma^2} \right) > 0, \]
(1.6)
when \( \rho < (1/2) \), i.e., \( B \) is a monotone increasing function of \( c \). So by the properties of linear function, we have \( B < 0 \), i.e., \( n_{IS}^* < n_{IN}^* \) when \( c < c_1 \), and \( n_{IS}^* > n_{IN}^* \) when \( c > c_1 \).

J. Proof of Proposition 6

By the conditions of (11) – (13), we have

\[ \frac{a^2d^2 (1 - \rho) \sigma^2 - F_2}{2[4b - (k + s)^2]} \]
(1.1)

Consequently, there is a sharing ratio ((\( a^2d^2 (1 - \rho) \sigma^2 - C)/F) < \( y < (4ba dp (1 - \rho) \sigma^2 B/[2(4b - (k + s)^2)] ) \)) to make the retailer to invest and share information when \( (4ba dp (1 - \rho) \sigma^2 B/[2(4b - (k + s)^2)] ) B < F < F_2 \). And, the sharing ratio is \( ((a^2d^2 (1 - \rho) \sigma^2 - C)/F) < \( y < 1 \) when \( a^2d^2 (1 - \rho) \sigma^2 - F_2 < F < \min [(4ba dp (1 - \rho) \sigma^2 B/[2(4b - (k + s)^2)] ), F_2] \).

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.
Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant 71571086).

References


