Research Article

Faster Calculation of the Low-Frequency Radiated Sound Power of Underwater Slender Cylindrical Shells

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Based on the fact that beam-type modes play the main role in determining the sound radiation from an underwater thin slender (length-to-radius ratio $L/a > 20$) elastic cylindrical shell, an equivalent-beam method is proposed for calculating the low-frequency radiated sound power of underwater thin slender unstiffened and stiffened cylindrical shells. The natural bending frequencies of the cylindrical shell are calculated by analytical and numerical methods and used to solve equivalent Young's modulus of the equivalent beam. This approach simplifies the vibration problem of the three-dimensional cylindrical shell into that of a two-dimensional beam, which can be used to simplify the calculation process of radiated sound power. Added mass is used to approximate the fluid-structure coupling, further simplifying the calculation process. Calculation examples of underwater simply supported unstiffened and stiffened cylindrical shells verify the proposed method by comparison with analytical and numerical results. Finally, the effects of the size and spacing of the stiffeners on the sound radiation characteristics of underwater free-free stiffened cylindrical shells are discussed. The proposed method can be extended to the rapid calculation of the sound radiation characteristics of underwater slender complex cylindrical shells in the low-frequency range.

1. Introduction

Cylindrical shells are regarded as typical models of submarine structures whose radiated noise in the low-frequency range plays an important role in the overall mechanical noise. Therefore, studying the low-frequency vibroacoustic responses of cylindrical shells is important for reducing the sound radiation from submarine structures, and a suitable metric for quantifying the sound field is the radiated sound power.

For the sound radiation characteristics of unstiffened cylindrical shells, Junger [1, 2] developed the velocity distribution and pressure field of an infinite fluid-loaded shell excited by a line force. It was found that interaction between the cylindrical shell and the surrounding water is equivalent to an added mass and a damping force for the cylindrical shell vibrating in air. Thus, the natural frequency of the

cylindrical shell in water is lower than that in air. Forsberg [3] investigated the accuracy and limitations of beam and bar approximations for predicting shell behavior in air. By comparing the frequencies computed from both the shell and the beam equations, it was found that the simple beam approximation gives good results only for thin cylindrical shells with a large length-to-radius ratio ($L/a > 20$). It is worth noting that if transverse displacement is considered and the coupling between displacements in different directions is neglected in the theoretical beam model, then only the first-order beam-type frequency of the cylindrical shell is predicted precisely using the theoretical beam model. Using Green's functions and Fourier integrals, Stepanishen [4, 5] evaluated the radiation impedance of, and radiated power from, the nonuniform harmonically vibrating surface of an infinite cylindrical shell. He then evaluated the pressure field and vibratory response of a finite fluid-loaded

cylindrical shell with infinite rigid extensions and presented low-frequency and asymptotic expressions for both the self-impedance and interaction impedance. Zhang et al. [6] studied the sound and vibration of a finite cylindrical shell with rigid baffled ends. They used general elasticity theory along with the Helmholtz equation to simulate the interaction between the cylinder and the surrounding fluid, and the method is verified by comparing with Flügge shell theory. Zhang et al. [7] used analytical, hybrid analytical/numerical, and fully coupled numerical approaches to predict the radiated sound power of a fluid-loaded finite cylindrical shell with simply supported boundary conditions. The results calculated using the analytical and hybrid analytical/numerical approaches agree perfectly, but those for the cylinder structural responses obtained analytically and numerically differ slightly. They attributed those errors to the truncation of the order of the circumferential modes used in the analytical model. Liu et al. [8] used the surface contribution method to predict the radiated sound power of a fluid-loaded finite cylindrical shell with hemispherical end closures. They compared the radiated sound power obtained from both the surface contribution and active intensity methods, and the numerical results showed that the surface contribution method can be applied at low frequencies.

In engineering practice, stiffened cylindrical shells are used widely as structural components because of their excellent structural strength and stiffness. The sound radiation characteristics of underwater stiffened cylindrical shells have received extensive attention. Using the Sanders–Koiter shell equations, Harari et al. [9] predicted the sound radiation from a finite ring-stiffened cylindrical shell with plates at both ends and submerged in fluid, and the results were confirmed experimentally. Based on Flügge theory, Laulagnet and Guyader [10] used modal analysis to study the sound radiation of a finite ring-stiffened cylindrical shell. They gave a general expression for the radiated sound power and investigated how the stiffener parameters affect the sound radiation characteristics. Yan et al. [11] used space-harmonic analysis to study the radiated sound power of an infinite submerged periodically stiffened cylindrical shell excited by a radial cosine harmonic line force. It was found that the radiated sound power of the stiffened shell is weaker than that of the unstiffened shell at all frequencies because of the presence of the stiffeners. Meixia et al. [12] studies the vibrational behavior and far-field sound radiation of a submerged stiffened conical shell at low frequencies. It is shown that the added stiffness has little effect on the radiated pressure and the amplitudes of the pressure from conical shells with and without ring stiffeners are similar.

Many researchers have investigated approximate methods to solve the acoustic radiation from elastic structures. Finite element methods were used in some early papers [13, 14], but the boundary integral equation formulation has become more popular. A good review on the evolution of boundary element techniques is given by Chien et al. [15]. Coupled finite element/boundary element methods (FEM/BEM) have been presented by Jeans and Mathews [16–18] to study the vibrations of structures in an infinite acoustic medium. Caresta and Kessissoglou [19]

studied the vibroacoustic behaviors of stiffened conical-cylindrical-conical shells. It was shown that the importance of the $n=1$ bending modes when evaluating the sound pressure radiated by a submarine under harmonic excitation from the propulsion system. Li et al. [20] proposed a passive noise control method to suppress sound radiation from a submarine hull structure. It is found that in the case of vertical excitation, the sound power in lower frequency range is mainly contributed by the $n=1$ modes.

For calculating the radiated sound field of underwater cylindrical shells, the analytical approach is limited by the simply supported boundary conditions, and the accuracy and efficiency of the numerical approach are limited by the fluid-structure coupling and the number of fluid elements. Many authors have calculated the vibroacoustic responses of unstiffened and stiffened cylindrical shells, but the literature contains very few cases of calculating rapidly either analytically or numerically the radiated sound power of underwater stiffened cylindrical shells. In the present paper, a numerical study is presented showing that beam-type modes play the main role in determining the sound radiation of underwater thin slender (length-to-radius ratio $L/a > 20$) elastic cylindrical shells. The paper proposed an equivalent-beam method using low-frequency vibroacoustic responses of the equivalent beam to approximate that of an underwater thin slender cylindrical shell. Note that the low-frequency range in this paper means the first five natural bending frequencies of the underwater unstiffened or stiffened cylindrical shell. The effectiveness of the equivalent-beam method is verified by comparison with the results of analytical and numerical methods. The advantages of the proposed method is that acoustic radiation problems of three-dimensional (3D) cylindrical shells is transformed into that of two-dimensional (2D) beams. Thereby, the equivalent-beam method simplifies the low-frequency acoustic radiation problems of slender cylindrical shells and greatly improves the calculation efficiency.

2. Theory and Method

2.1. Sound Radiation Characteristics of Slender Cylindrical Shells. As shown in Figure 1, we consider a simply supported finite thin cylindrical shell with radius a , thickness h , and length L . The ends of the cylindrical shell are connected to rigid circular extensions. For the equations of motion of the underwater cylindrical shell, the Donnell–Mushtari thin-shell theory is used. Applying simply supported boundary conditions, the sound pressure and sound radiation impedance are obtained [21]. The radiated sound power W_{crad} is obtained by integrating the surface sound intensity around the shell:

$$W_{\text{crad}} = \frac{S}{4} \operatorname{Re} \left\{ -i\omega \sum_n \sum_m \frac{1}{\varepsilon_n} W_{nm} Z_{nmm} W_{nm}^* \right\}, \quad (1)$$

where W_{nm} is the amplitude of the surface displacement of the shell, Z_{nmm} is the sound radiation impedance, $*$ indicates the complex conjugate, ω is the angular frequency of harmonic vibration, and S is the superficial area of the shell and $\varepsilon_n = 1$ when $n = 0$ or $\varepsilon_n = 2$ when $n \geq 1$.

FIGURE 1: Model of the cylindrical shell. Schematic diagram of rectangular coordinate to cylindrical coordinate system is given.

The effects of beam-type modes ($n = 1$) on the low-frequency radiated sound power of a thin slender cylindrical shell are illustrated by the following example. The dimensions of the cylindrical shell are $L = 40$ m, $a = 1.75$ m, and $h = 0.015$ m. The material is assumed to be steel with density $\rho = 7,800$ kg/m³, Young's modulus $E = 2.06 \times 10^{11}$ Pa, and Poisson's ratio $\gamma = 0.3$. The cylindrical shell is submerged in water of density $\rho_w = 1,000$ kg/m³ and sound speed $c_w = 1,500$ m/s. A unit radial harmonic point force is located at $L/2$ along the length of the cylindrical shell. With the numerical calculation having converged, the radiated sound power calculated using beam-type modes ($n = 1$) and that calculated using all circumferential modes ($n = 0, 1, 2, \dots$) are shown in Figure 2.

Figure 2 shows that the results calculated using beam-type modes and those calculated using all the circumferential modes agree well in the low-frequency range. From left to right, the frequencies of the three peaks in the curve correspond to the first-, third-, and fifth-order natural bending frequencies, respectively, of the beam-type modes of the cylindrical shell. Below the fifth-order natural bending frequency, the contribution of beam-type modes to the radiated sound power is clearly much greater than that of the other circumferential modes. Thus, the beam-type modes play the main role in determining the low-frequency sound radiation of an underwater thin slender cylindrical shell, and it is feasible to use the beam-type modes alone to calculate the radiated sound power.

For a thin slender cylindrical shell, the beam-type modes correspond essentially to beam-type motion of the cylindrical shell. Such beam-type motion occurs when the circumferential displacement and the radial displacement are essentially equal but of opposite sign, thereby representing a rigid-body translation of the shell cross section. Simple beam theory assumes that the circumferential displacement and the radial displacement are equal. Thus, we use beam theory to calculate approximately the low-frequency radiated sound power of an underwater thin slender cylindrical shell, and we propose an equivalent-beam method that transforms 3D acoustic radiation problems of cylindrical shells into 2D acoustic radiation problems of beams, thereby improving the computational efficiency.

2.2. Vibroacoustic Responses of Underwater Beam. In this subsection, the vibroacoustic responses of an underwater beam are reviewed. Using the rectangular coordinate system

FIGURE 2: Radiated sound power calculated using beam-type modes (blue) and all the circumferential modes (red).

shown in Figure 1, the equation of motion for an underwater Euler-Bernoulli beam can be expressed as

$$D \frac{d^4 y}{dx^4} - \gamma \frac{\partial^2 y}{\partial t^2} = f(x, t), \quad (2)$$

where $y(x, t)$ is the displacement, $D = EI$ is the flexural rigidity, $\gamma = \rho_b S + M$ is the mass per unit area, E is Young's modulus, I is the second moment of area, ρ_b is the density of the beam, S is its cross-sectional area, M is the added mass per unit area, and $f(x, t)$ is the harmonic force acting on the beam. Letting $f(x, t) = 0$ and $y(x, t) = Y(x)e^{-i\omega t}$, the governing differential equation for the free vibrations of the underwater beam is

$$D \frac{d^4 Y}{dx^4} - \gamma \omega^2 Y = 0. \quad (3)$$

With simply supported boundary conditions, the natural frequencies of the underwater beam are

$$\omega_m = \left(\frac{EI}{\rho_b S + M} \right)^{1/2} \left(\frac{m\pi}{L} \right)^2, \quad m = 1, 2, 3, \dots, \quad (4)$$

where L is the length of the beam. Applying simply supported boundary conditions and the orthogonality of the modes, the displacement is obtained as

$$y(x) = \sum_{m=1}^{\infty} \frac{-2f_m}{(\rho_b S + M)L} \frac{\sin(m\pi x/L)}{\omega^2 - \omega_m^2}, \quad (5)$$

$$f_m = \int_0^L f(x) \sin \frac{m\pi x}{L} dx. \quad (6)$$

For a beam of circular cross section, the far-field sound pressure contributed by a length element dx can be approximated in the low-frequency range by means of the familiar expression for the pressure radiated by active elements on a cylindrical baffle. Changing the previous rectangular coordinates into spherical coordinates, the sound pressure can be written as [22].

$$p(r, \theta, \phi) = -\frac{ie^{ikr} S \sin \theta \cos \phi}{r\lambda} \cdot \int_0^L \ddot{y}(x) e^{-ikx \cos \theta} dx, \quad \lambda \gg a, \quad (7)$$

where $\ddot{y}(x)$ is the transverse acceleration of the beam cross section, r is measured from the element dx , and λ and k are the wavelength and wave number, respectively, of the sound. The radiated sound power W_{brad} can be calculated by integrating the average sound intensity [23]:

$$W_{\text{brad}} = \frac{r^2}{2\rho_w c_w} \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} |p(r, \theta, \phi)|^2 \sin \theta d\theta d\phi, \quad (8)$$

where ρ_w and c_w are the density and sound speed, respectively, of water.

2.3. Equivalent-Beam Method. The underwater dynamic motion of the cylindrical shell and that of the beam will be similar if their structural shapes and modes are similar in air. To establish a relationship between the cylindrical shell and the beam, equivalent coefficients of Young's modulus are proposed to ensure that the natural bending frequencies of the cylindrical shell and the equivalent beam in air are the same. The added mass of the cylindrical shell is then applied to the equivalent beam to approximate the fluid-structure coupling. Then, according to the natural bending frequencies of the cylindrical shell in air as calculated either analytically or numerically, the equivalent natural bending frequencies of the underwater cylindrical shell are obtained and used to calculate the vibroacoustic responses of the underwater equivalent beam based on simple beam theory. The details are as follows.

The subscript m and the superscripts b and c denote parameters of the m th mode, the beam, and the cylindrical shell, respectively. From equation (4), the natural bending frequencies of the beam with simply supported boundary conditions in air are obtained as

$$\omega_m^b = \left(\frac{E^b I^b}{\rho^b S^b} \right)^{1/2} \left(\frac{m\pi}{L} \right)^2, \quad m = 1, 2, 3, \dots \quad (9)$$

For the equivalent beam, the structural parameters except for Young's modulus remain the same as those for the cylindrical shell, namely,

$$\begin{aligned} \rho^b &= \rho^c, \\ S^b &= S^c, \\ I^b &= I^c. \end{aligned} \quad (10)$$

The natural bending frequencies of the cylindrical shell in air can be obtained either analytically or numerically. To ensure that each natural bending frequency of the equivalent beam is the same as the corresponding natural bending frequency of the cylindrical shell, appropriate Young's modulus is needed in equation (9), and the equivalent coefficients of Young's modulus are calculated. Thus, the natural bending frequencies of the cylindrical shell with simply supported boundary conditions in air can be expressed in the form of the equivalent beam as

$$\omega_m^c = \left(\frac{\bar{E}_m I^c}{\rho^c S^c} \right)^{1/2} \left(\frac{m\pi}{L} \right)^2, \quad m = 1, 2, 3, \dots, \quad (11)$$

where $\bar{E}_m$ is the equivalent Young's modulus given by

$$\bar{E}_m = C_m E, \quad (12)$$

$$C_m = \frac{\rho^c S^c (\omega_m^c)^2}{I^c (m\pi/L)^4} \cdot \frac{1}{E}, \quad (13)$$

where C_m is the equivalent coefficient of Young's modulus. As the flow around the cylindrical shell is 3D flow, the added mass per unit area of the cylindrical shell is approximately the mass of water pushed by unit area and modified by a 3D-flow correction factor k_m [24]. Thus, the modal added mass of the equivalent beam should be expressed as

$$M_m = k_m \rho_w \pi a^2, \quad m = 1, 2, 3, \dots \quad (14)$$

We assume that the added mass affects only the mass per unit area of the equivalent beam. Substituting equations (12)–(14) into equation (4), the equivalent natural bending frequencies of the underwater cylindrical shell are obtained as

$$\bar{\omega}_m = \left(\frac{\bar{E}_m I^c}{\rho^c S^c + M_m} \right)^{1/2} \left(\frac{m\pi}{L} \right)^2, \quad m = 1, 2, 3, \dots \quad (15)$$

According to equations (5) and (6), if the radial harmonic point force is located at $x = x_0$ and its amplitude is F , then the displacement is obtained as

$$\bar{y}(x) = \sum_{m=1}^{\infty} \frac{-2F \sin(m\pi x_0/L) \sin(m\pi x/L)}{(\rho^c S^c + M_m)L} \cdot \frac{1}{\omega^2 - \bar{\omega}_m^2}. \quad (16)$$

Substituting equation (16) into equations (7) and (8), the sound pressure and the radiated sound power can be obtained.

2.4. General Method for Unstiffened Cylindrical Shell. From the analyses in Sections 2.2 and 2.3, the general method for an underwater unstiffened thin slender cylindrical shell in the low-frequency range is as follows. (1) The natural bending

frequencies of the cylindrical shell in air are obtained either analytically or numerically. With simply supported boundary conditions, the analytical method is used, but with other boundary conditions, numerical methods such as the finite-element method (FEM) or the boundary-element method (BEM) are used. (2) The equivalent beam is established with structural parameters (except for Young's modulus) that are the same as those of the cylindrical shell. (3) The natural bending frequencies of the cylindrical shell in air obtained in step 1 are substituted into equation (13) to calculate the equivalent coefficients of Young's modulus. (4) The equivalent coefficients of Young's modulus and the added mass are used to calculate the equivalent natural bending frequencies of the underwater cylindrical shell by equations (12)–(15). (5) The equivalent natural bending frequencies of the underwater cylindrical shell are substituted into equation (16) to calculate the displacement. (6) Finally, the displacement in step 5 is used to calculate the radiated sound power by equations (7) and (8).

2.5. General Method for Stiffened Cylindrical Shell. The general method for an underwater thin slender stiffened cylindrical shell in the low-frequency range is discussed in this subsection. A ring-stiffened cylindrical shell is shown in Figure 3. The flexural rigidity D and the mass per unit area γ of the stiffened parts are larger than those of the unstiffened parts. Due to the presence of stiffeners, when a stiffened cylindrical shell is equivalent to a beam directly, the stiffened cylindrical shell will be equivalent to a nonuniform equivalent beam, but the latter cannot be calculated analytically as in Section 2.2. Thus, an appropriate equivalent method must be proposed.

The integral bending vibration contributes most to the low-frequency sound radiation characteristics of a stiffened cylindrical shell. Therefore, to simplify the calculation, the effects of the stiffeners on the cylindrical shell are incorporated in the equivalent beam in the form of the average added flexural rigidity and the average added mass per unit area. In other words, the flexural rigidity D in equation (2) is changed to

$$D = E \cdot \frac{1}{L} (I_{\text{unstiffened}} \cdot L_{\text{unstiffened}} + I_{\text{stiffened}} \cdot L_{\text{stiffened}}). \quad (17)$$

The mass per unit area γ in equation (2) is changed to

$$\gamma = \rho_b \cdot \frac{1}{L} (S_{\text{unstiffened}} \cdot L_{\text{unstiffened}} + S_{\text{stiffened}} \cdot L_{\text{stiffened}}) + M, \quad (18)$$

where the subscripts unstiffened and stiffened denote parameters of the unstiffened parts and stiffened parts, respectively.

The problem of solving for a nonuniform equivalent beam is thereby transformed into that of solving for a uniform equivalent beam. Then, using the general method in Section 2.4, the low-frequency radiated sound power of an underwater slender stiffened cylindrical shell is calculated.

FIGURE 3: Model of the ring-stiffened cylindrical shell.

3. Results and Discussion

Herein, the results of using the proposed method are compared with analytical and numerical results, the latter obtained using the ANSYS software. The parameters of the cylindrical shell that is considered are listed in Table 1. The material parameters of the stiffeners are the same as those of the cylindrical shell, and the density and sound speed of water are $1,000 \text{ kg/m}^3$ and $1,500 \text{ m/s}$, respectively. The reference sound power level is $0.67 \times 10^{-18} \text{ W}$.

3.1. Validity of Equivalent-Beam Method. We begin by assessing the validity of using the equivalent-beam method to calculate the low-frequency radiated sound power of underwater unstiffened and stiffened thin slender cylindrical shells. For comparison with the analytical results, the boundary conditions for the unstiffened and stiffened cylindrical shells are simply supported ones, and the ends of the cylindrical shell are connected to rigid circular extensions.

First, we use the general method in Section 2.4 to investigate the low-frequency radiated sound power of an underwater thin slender unstiffened cylindrical shell. A unit radial harmonic point force is located at $L/2$ along the length of the cylindrical shell. In Table 2, we list (i) the first five natural bending frequencies f_m^c of the cylindrical shell in air as calculated analytically, (ii) the added mass M_m , (iii) the equivalent coefficients of Young's modulus C_m , (iv) the first five natural bending frequencies of the underwater cylindrical shell as calculated analytically f_{mW}^c , and (v) the first five natural bending frequencies of the underwater cylindrical shell as calculated the proposed method $\bar{f}_m$. The results in Table 2 show that in the low-frequency range, the proposed method calculates accurately the natural bending frequencies of the underwater thin slender unstiffened cylindrical shell.

Using the natural bending frequencies of the underwater cylindrical shell in Table 2, we calculate the vibroacoustic response of underwater cylindrical shell both analytically and using the equivalent-beam method, as shown in Figure 4. The displacement responses at the excitation point are shown in Figure 4(a). As the excitation position is a node of the vibration motion, resonance occurs when the excitation frequency approaches the first-, third-, and fifth-order natural bending frequencies. As shown, the displacement responses calculated analytically agree well with those calculated using the proposed method.

In Figure 4(b), we compare the curves of radiated sound power level calculated analytically with the equivalent-beam method. The two curves clearly have similar patterns and

TABLE 1: Parameters of the cylindrical shell.

Parameter	L (m)	a (m)	h (m)	E (Pa)	γ	ρ (kg/m ³)
Value	40	1.75	0.015	2.06×10^{11}	0.3	7,800

TABLE 2: Calculated parameters of equivalent-beam method and the first five natural bending frequencies of the underwater unstiffened cylindrical shell.

Order m	f_m^c (Hz)	M_m (kg/m)	C_m	f_{mW}^c (Hz)	$\bar{f}_m$ (Hz)
1	6.09	9.39×10^3	0.9501	2.10	2.09
2	22.73	8.88×10^3	0.8286	8.04	8.03
3	46.54	8.23×10^3	0.6862	17.02	17.02
4	74.42	7.76×10^3	0.5550	27.90	27.89
5	104.2	7.51×10^3	0.4457	39.63	39.58

their peaks are almost the same. However, the discrepancies between the two curves are because the proposed method considers only transverse displacement and neglects any effect of the vibration responses in the other directions on the radiated sound power. Nevertheless, we consider that the results shown in Figure 4 validate the proposed method for calculating the vibration response and radiated sound power of an underwater unstiffened cylindrical shell with $L/a > 20$ in the low-frequency range.

Next, using the general method in Section 2.5, we investigate the low-frequency radiated sound power of an underwater thin slender stiffened cylindrical shell. The cylindrical shell stiffened with periodically arranged rings is shown in Figure 3. Each ring stiffener has a uniform rectangular cross section with height $w = 0.125$ m and width $v = 0.007$ m, and the stiffener spacing is $u = 0.4$ m. A unit radial harmonic point force is located at $L/6$ along the length of the cylindrical shell.

The first five natural bending frequencies f_m^c of the stiffened cylindrical shell in air as calculated numerically are listed in Table 3. The added mass is the same as that in Table 2 because it relates only to the surface structure and not the internal structure. The added mass and the equivalent coefficients of Young's modulus are listed in Table 3. The first five natural bending frequencies of the underwater stiffened cylindrical shell as calculated numerically f_{mB}^c and with the proposed method $\bar{f}_m$ are listed in Table 3, wherein the two sets of results agree well.

The curves of radiated sound power level calculated analytically, numerically, and with the equivalent-beam method are shown in Figure 5. The overall trends of the three curves are clearly the same, in which (i) the radiated sound power level calculated analytically and with the proposed method agree well, but (ii) there are some discrepancies with the results obtained numerically using a fully coupled FEM/BEM method. These discrepancies would decrease if we increased the radius of the fluid domain and the number of fluid elements, but doing so would decrease the computational efficiency of the numerical method. Nevertheless, we consider that the results as presented validate the proposed method for calculating the low-frequency radiated sound power of underwater thin slender stiffened cylindrical shell.

3.2. Effects of Stiffener Parameters on Sound Radiation Characteristics. Next, we use the equivalent-beam method to investigate how the size and spacing of the stiffeners affect the sound radiation characteristics in the low-frequency range. To simplify the calculations, the boundary conditions for the stiffened cylindrical shell are free-free ones and the ends of the cylindrical shell are connected to rigid circular extensions. A unit radial harmonic point force is located at $L/6$ along the length of the cylindrical shell.

First, we compare the low-frequency radiated sound power of an underwater unstiffened cylindrical shell with that of an underwater stiffened cylindrical shell. For the latter, the height, width, and spacing of the stiffeners are 0.125 m, 0.007 m, and 0.4 m, respectively. The first five natural bending frequencies of the underwater unstiffened and stiffened cylindrical shells are listed in Table 4, wherein the two sets of results are close. This can be attributed to the presence of the stiffeners, which increase not only the flexural rigidity but also the mass per unit area of the equivalent beam. Furthermore, the bending frequencies of the underwater unstiffened and stiffened cylindrical shells calculated by the proposed method are proportional to the square root of the ratio of the flexural rigidity to the mass per unit area. The curves of radiated sound power level of the underwater unstiffened and stiffened cylindrical shells are shown in Figure 6. The two curves agree well below the first natural bending frequency, but above it the radiated sound power of the underwater stiffened cylindrical shell is weaker than that of the underwater unstiffened cylindrical shell because of the stiffeners. Nevertheless, the two curves have similar patterns in general, and the differences between them are less obvious in the low-frequency range. However, the stiffeners influence the natural bending frequencies of the cylindrical shell and affect the contribution of the natural bending frequencies to the radiated sound power.

Next, to investigate how the stiffener size affects the radiated sound power, we select widths of 0.007 m and 0.01 m while fixing the height as 0.125 m and the spacing as 0.4 m. Furthermore, we select heights of 0.125 m and 0.08 m while fixing the width as 0.007 m and the spacing as 0.4 m. The first five natural bending frequencies of the underwater stiffened cylindrical shell with different stiffener sizes are listed in Table 4. The natural bending frequencies clearly decrease with both stiffener width and height. The curves of radiated sound power level of the underwater stiffened cylindrical shell with the aforementioned different widths and heights are shown in Figure 7, where it can be seen that the respective curves have similar patterns. In general, changing the stiffener size has a slight effect on the low-frequency radiated sound power of the underwater free-free stiffened cylindrical shell. In this paper, structural damping is ignored, and this would lead to resonance and infinite vibroacoustic response if the frequency of excitation was exactly equal to one of the natural bending frequencies of the underwater cylindrical shell. The obvious maximum value in Figure 7(b) is because the corresponding frequency of excitation is extremely close to one of the natural bending frequencies of the underwater stiffened cylindrical shell.

FIGURE 4: The vibroacoustic response of the underwater cylindrical shell. (a) Displacement responses of excitation point. (b) Radiated sound power level.

TABLE 3: Calculated parameters of the equivalent-beam method and the first five natural bending frequencies of the underwater stiffened cylindrical shell.

Order m	f_m^c (Hz)	M_m (kg/m)	C_m	f_{mB}^c (Hz)	$\bar{f}_m$ (Hz)
1	5.715	9.39×10^3	0.8459	2.09	2.08
2	21.329	8.88×10^3	0.7364	8.16	8.00
3	43.626	8.23×10^3	0.6086	17.55	16.89
4	69.692	7.76×10^3	0.4914	28.86	27.64
5	97.535	7.51×10^3	0.3942	41.32	39.24

TABLE 4: The first five natural bending frequencies of underwater cylindrical shell with different stiffener parameters.

Stiffener parameters		Order m				
Height (m)	Width (m)	1 (Hz)	2 (Hz)	3 (Hz)	4 (Hz)	5 (Hz)
0.125	0.007	5.17	13.94	25.34	38.35	52.09
	0.01	5.16	13.89	25.27	38.21	51.86
0.08	0.007	5.18	13.99	25.43	38.50	52.29
Without stiffeners		5.20	13.44	25.18	37.80	51.01

FIGURE 5: Radiated sound power level of the underwater stiffened cylindrical shell.

FIGURE 6: Comparison of radiated sound power between underwater unstiffened (red) and stiffened (blue) cylindrical shells.

FIGURE 7: Effect of stiffener width/height on radiated sound power of the underwater stiffened cylindrical shell. (a) Different stiffener width. (b) Different stiffener height.

TABLE 5: The first five natural bending frequencies of the underwater stiffened cylindrical shell with different stiffener spacings.

Stiffener parameters (m)	Spacing (m)	Order m				
		1 (Hz)	2 (Hz)	3 (Hz)	4 (Hz)	5 (Hz)
Height 0.125	0.2	5.16	13.83	25.16	38.02	51.48
Width 0.007	0.4	5.17	13.94	25.34	38.35	52.09
	0.8	5.17	14.00	25.45	38.56	52.39

Similarly, to investigate how the stiffener spacing affects the radiated sound power, we select spacings of 0.2 m, 0.4 m, and 0.8 m while fixing the width as 0.007 m and the height as 0.125 m. The first five natural bending frequencies of the underwater stiffened cylindrical shell with different stiffener spacings are listed in Table 5. The natural bending frequencies clearly increase with stiffener spacing. The curves of radiated sound power level of the underwater stiffened cylindrical shell with different stiffener spacings are shown in Figure 8, where it can be seen that the curves have similar patterns. In general, changing the stiffener spacing has a slight effect on the low-frequency radiated sound power of the underwater free-free stiffened cylindrical shell. As mentioned above, the stiffener spacing influences the natural bending frequencies and affects their contribution to the radiated sound power.

4. Conclusion

Beam-type modes play the main role in determining the sound radiation of an underwater thin slender cylindrical shell. The paper proposed an equivalent-beam method to calculate the low-frequency radiated sound power of underwater thin slender unstiffened and stiffened cylindrical shells. The equivalent-beam method uses low-frequency vibroacoustic responses of the equivalent beam to approximate that of an underwater thin slender cylindrical shell. In this paper, calculation examples of underwater simply supported slender unstiffened and stiffened cylindrical shells

FIGURE 8: Effect of stiffener spacing on radiated sound power of the underwater stiffened cylindrical shell.

are given, and equivalent-beam method is validated by comparing its results with analytical and numerical results.

The equivalent-beam method is used to analyze how the stiffener size and spacing affect the low-frequency radiated sound power of an underwater free-free slender stiffened cylindrical shell. The curves of radiated sound power level of

the underwater free-free unstiffened and stiffened cylindrical shells are found to have similar patterns, the differences being less obvious in the low-frequency range. However, the stiffeners influence the natural bending frequencies of the cylindrical shell and affect how those frequencies contribute to the radiated sound power. The natural bending frequencies decrease with both stiffener width and height but increase with stiffener spacing. However, although changing the stiffener size and spacing changes the natural bending frequencies of an underwater free-free stiffened cylindrical shell, the curves of radiated sound power level exhibit similar trends. Overall, the effects of changing the stiffener size and spacing on the amplitude of the radiated sound power in the low-frequency range are slight.

The equivalent-beam method can transform 3D acoustic radiation problems of cylindrical shells into 2D acoustic radiation problems of beams. Besides, the proposed method uses added mass to approximate the fluid-structure coupling, which greatly reduces the amount of computation required for such coupling. Thereby, the equivalent-beam method simplifies the low-frequency acoustic radiation problem of slender cylindrical shells and greatly improves the calculation efficiency. Moreover, the analytical method for acoustic radiation problems of underwater thin slender cylindrical shells is only applicable to simple-supported boundary condition, while the equivalent-beam method is applicable to arbitrary boundary conditions. The proposed method has reference significance for the calculation of low-frequency vibroacoustic responses of complex slender shell structures.

Data Availability

The data used support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

- [1] M. C. Junger, "Vibrations of elastic shell in a fluid medium and the associated radiation of sound," *Journal of Applied Mechanics*, vol. 74, pp. 439–445, 1952.
- [2] M. C. Junger, "The physical interpretation of the expression for an outgoing wave in cylindrical coordinates," *The Journal of the Acoustical Society of America*, vol. 25, no. 1, pp. 40–47, 1953.
- [3] K. Forsberg, "Axisymmetric and beam-type vibrations of thin cylindrical shells," *AIAA Journal*, vol. 7, no. 2, pp. 221–227, 1969.
- [4] P. R. Stepanishen, "Radiated power and radiation loading of cylindrical surfaces with nonuniform velocity distributions," *The Journal of the Acoustical Society of America*, vol. 63, no. 2, pp. 328–338, 1978.
- [5] P. R. Stepanishen, "Modal coupling in the vibration of fluid-loaded cylindrical shells," *The Journal of the Acoustical Society of America*, vol. 71, no. 4, pp. 813–823, 1982.
- [6] J. Zhang, C. Yuan, X. Zhu, and T. Li, "A general method to study the sound radiation of a finite cylindrical shell based on elastic theory," *Journal of Marine Science and Application*, vol. 11, no. 2, pp. 258–264, 2012.
- [7] Y. Zhang, W. Jiang, H. Peters, and N. Kessissoglou, "Analytical and numerical approaches to predict radiated sound power of fluid-loaded cylindrical shell," in *Proceedings of INTER-NOISE and NOISE-CON Congress and Conference*, vol. 249, pp. 2728–2734, Melbourne, Australia, 2014.
- [8] D. Liu, H. Peters, N. Kessissoglou, and S. Marburg, "Prediction of the radiated sound power from a fluid-loaded finite cylinder using the surface contribution method," in *Proceedings of INTER-NOISE and NOISE-CON Congress and Conference*, vol. 249, pp. 760–767, Melbourne, Australia, 2014.
- [9] A. Harari, B. E. Sandman, and J. A. Zaldonis, "Analytical and experimental determination of the vibration and pressure radiation from a submerged, stiffened cylindrical shell with two end plates," *The Journal of the Acoustical Society of America*, vol. 95, no. 6, pp. 3360–3368, 1994.
- [10] B. Laulagnet and J. L. Guyader, "Sound radiation by finite cylindrical ring stiffened shells," *Journal of Sound and Vibration*, vol. 138, no. 2, pp. 173–191, 1990.
- [11] J. Yan, T. Y. Li, J. X. Liu, and X. Zhu, "Space harmonic analysis of sound radiation from a submerged periodic ring-stiffened cylindrical shell," *Applied Acoustics*, vol. 67, no. 8, pp. 743–755, 2006.
- [12] C. Meixia, Z. Cong, T. Xiangfan et al., "Structural and acoustic responses of a submerged stiffened conical shell," *Shock and Vibration*, vol. 2014, Article ID 954253, 14 pages, 2014.
- [13] R. J. Alfredson, "A note on the use of the finite difference method for predicting steady state sound fields," *Acta Acustica United with Acustica*, vol. 28, pp. 296–301, 1973.
- [14] P. A. Laura, P. L. Arlett, A. K. Bahrani, and O. C. Zienkiewicz, "Application of finite elements to the solution of Helmholtz's equation," *Proceedings of the Institution of Electrical Engineers*, vol. 116, no. 7, p. 1168, 1969.
- [15] C. C. Chien, H. Rajiyah, and S. N. Atluri, "An effective method for solving the hyper-singular integral equations in 3-D acoustics," *The Journal of the Acoustical Society of America*, vol. 88, no. 2, pp. 918–937, 1990.
- [16] I. C. Mathews, "Numerical techniques for three-dimensional steady-state fluid-structure interaction," *The Journal of the Acoustical Society of America*, vol. 79, no. 5, pp. 1317–1325, 1986.
- [17] R. Jeans and I. C. Mathews, "A unique coupled boundary element/finite element method for the elastoacoustic analysis of fluid-filled thin shells," *The Journal of the Acoustical Society of America*, vol. 94, no. 6, pp. 3473–3479, 1993.
- [18] R. A. Jeans and I. C. Mathews, "Solution of fluid-structure interaction problems using a coupled finite element and variational boundary element technique," *The Journal of the Acoustical Society of America*, vol. 88, no. 5, pp. 2459–2466, 1990.
- [19] M. Caresta and N. J. Kessissoglou, "Acoustic signature of a submarine hull under harmonic excitation," *Applied Acoustics*, vol. 71, no. 1, pp. 17–31, 2010.
- [20] C. Li, J. Su, J. Wang, and H. Hua, "Effects of symmetrical foundation on sound radiation from a submarine hull structure," *The Journal of the Acoustical Society of America*, vol. 138, no. 5, pp. 3195–3201, 2015.

- [21] M. C. Junger and D. Feit, *Sound, Structure, and Their Interaction*, MIT Press, Cambridge, UK, 1986.
- [22] M. C. Junger, "Sound radiation by resonances of free-free beams," *The Journal of the Acoustical Society of America*, vol. 52, no. 1B, pp. 332–334, 1972.
- [23] H. Zheng and C. Cai, "Minimization of sound radiation from baffled beams through optimization of partial constrained layer damping treatment," *Applied Acoustics*, vol. 65, no. 5, pp. 501–520, 2004.
- [24] X. Lu, X. Jin, and Y. Liu, *Ship Vibration Theory*, National Defense Industry Press, Beijing, China, 1980.