Research Article

Adaptive Dynamic Surface Decentralized Output Feedback Control for Switched Nonstrict Feedback Large-Scale Systems with Unknown Dead Zones

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This paper investigates a novel adaptive output feedback decentralized control scheme for switched nonstrict feedback large-scale systems with unknown dead zones. A decentralized linear state observer is designed to estimate the unmeasurable states of subsystems. The dead zone inverse technique is used to compensate the effect of the unknown dead zone. A variable separation approach is applied to deal with the nonstrict feedback problem. Moreover, dynamic surface control and minimal parameter learning technology are adopted to reduce the computation burden. The proof of stability and the arbitrary switching are obtained by the common Lyapunov method. Finally, simulation results are given to show the effectiveness of the proposed control scheme.

1. Introduction

With the development of science and technology, the complexity of control plant is increasing, and more and more large-scale interconnected systems appear. It is well known that the decentralized control can reduce the computational burden compared with the centralized control strategy. Therefore, as an effective control scheme, the decentralized control has been widely used for large-scale nonlinear system control. Jain and Khorrami first discussed the decentralized adaptive control of a class of large-scale interconnected nonlinear systems in [1], which did not require the unknown interconnected terms of the system to meet the so-called matching conditions. Sun et al. [2] studied the fuzzy adaptive decentralized optimal control problem of the strict feedback large-scale system. Yoo and Park [3] proposed an adaptive decentralized output feedback control method for a large-scale system with unknown time-varying delayed interactions. In [4], a decentralized adaptive output feedback controller was designed for a stochastic nonlinear interconnected system. In [5], based on backstepping technology, an adaptive decentralized neural H_∞ tracking

control scheme was proposed for a class of large-scale interconnected systems. In [6], to deal with the output feedback control problem of the stochastic large-scale system with unknown dead zone, an adaptive fuzzy decentralized control method based on state observer was proposed. In [7], a decentralized control scheme was discussed for a SISO large-scale system with strong interconnection. Huang and Wang [8] extended the results in [7] to control a class of large-scale MIMO systems with strong interconnection and successfully applied the control method to the automated highway systems.

The aforementioned results all focus on large-scale uncertain nonlinear systems. In the actual engineering, a lot of systems not only contain many low-dimensional subsystems but also have switched properties. That is to say, these systems are not only large-scale interconnected systems but also switched systems. For example, the robot cooperative control system, spacecraft control system, power system, large-scale traffic control system, communication system, and so on are all switched large-scale systems. Recently, the control problem of large-scale interconnected switched nonlinear systems has aroused the researchers'

strong interest, and a small number of relevant results have appeared. Wang and Tong [9] studied the adaptive output feedback control problem for a class of switched Takagi–Sugeno fuzzy large-scale systems with unmeasured states. Li and Tong [10] investigated the adaptive fuzzy output feedback control problem of a switched large-scale system with nonstrict feedback structure. In [11], by adopting switched Lyapunov function with H_∞ performance, the decentralized control was designed for switched fuzzy large-scale Takagi–Sugeno systems. However, there are no results on the control problems of large-scale switched nonstrict feedback systems with input constraints.

Many actuators used for industrial control, such as the servo motor, reducer, gear, hydraulic proportional servo system, cylinder, and so on, have input constraints due to mechanical structure design, manufacturing, and friction damping [12, 13]. Unknown dead zone will seriously affect the control performance of the control system, such as reduce the control accuracy, or even completely fail or produce safety accident caused by self-excited oscillation. In recent years, in order to solve the control problem caused by the dead zone of actuators, some researchers in the field of control have carried out more in-depth research and achieved some research results. In [14], an adaptive fuzzy tracking control was designed for a class of pure feedback nonlinear systems with unknown dead zone. In [15], based on backstepping control, a fuzzy output feedback tracking control method was proposed for the strict feedback system with unknown dead zone. In [16], by using dynamic surface control, an adaptive control problem was studied for a pure feedback system with unknown dead zone. Wang and Chen [17] studied the problem of adaptive fuzzy dynamic surface control of nonstrict feedback nonlinear systems with unknown virtual control coefficients, dead zone, and full state constraints. However, none of the above results have considered the control problem of switched systems and switched large-scale systems. In [18], Tong et al. proposed an adaptive fuzzy decentralized tracking control scheme based on state observer for switched large-scale systems with unknown dead zone. But the system in [18] was strict feedback form, and there existed the problem of “explosion of complexity.”

This paper will study the control problem of large-scale switched nonlinear systems with nonstrict feedback structure and use the common Lyapunov function method to overcome the arbitrary switching problem. Moreover, adaptive parameters will be designed to identify the unknown dead zone parameters through dead zone inverse technology. Compared with the previous literature, the main advantages of the proposed control scheme are listed as follows:

- (1) For the first time, the control problem of large-scale switched systems with nonstrict feedback structure and unknown dead zones is studied. The proposed control scheme is quite different from the existing results. The control method in [19–24] cannot be used to control switched systems. In [25–28], the control problem of switched systems was studied,

whereas these methods cannot control switched large-scale systems. Moreover, unknown dead zone was not considered. Though switched large-scale systems with dead zones were considered in [18, 29], they were strict feedback form, rather than nonstrict feedback system in this paper. The system in [10] was a switched large-scale nonstrict feedback system, but it was without a dead zone, and the control method had the drawback of “explosion of complexity.”

- (2) The control method proposed in this paper does not need n order differentiable and bounded conditions of input signals and a monotonically increasing condition of unknown functions. However, these strict assumptions are common in the existing literature [30, 31]. Because of using dynamic surface control, “explosion of complexity” problem is avoided in this paper. Moreover, by applying linear state observer and minimal parameter learning technology, the proposed control strategy only contains $6M$ online learning parameters for the switched large-scale system, where M is the number of interconnected subsystems. Therefore, the computation burden is reduced significantly.
- (3) The results in this paper are developed with full consideration of the dead zone problem. By constructing a dead zone compensator, the proposed adaptive fuzzy decentralized output feedback control approach can overcome the unknown dead zone problem of uncertain switched nonlinear large-scale nonstrict feedback systems.

2. Problem Description and Preliminaries

The switched nonstrict feedback large-scale system considered in this paper has M interconnected subsystems. k th subsystem $\sum_k$ ($k = 1, 2, \dots, M$) is shown as follows [10]:

$$\begin{cases} \dot{x}_{k,1} = x_{k,2} + f_{k,1}^{\sigma(t)}(\underline{x}_k) + h_{k,1}^{\sigma(t)}(\underline{y}), \\ \dot{x}_{k,2} = x_{k,3} + f_{k,2}^{\sigma(t)}(\underline{x}_k) + h_{k,2}^{\sigma(t)}(\underline{y}), \\ \vdots \\ \dot{x}_{k,m_k-1} = x_{k,m_k} + f_{k,m_k-1}^{\sigma(t)}(\underline{x}_k) + h_{k,m_k-1}^{\sigma(t)}(\underline{y}), \\ \dot{x}_{k,m_k} = D_k u_k + f_{k,m_k}^{\sigma(t)}(\underline{x}_k) + h_{k,m_k}^{\sigma(t)}(\underline{y}), \\ y_k = x_{k,1}, \end{cases} \quad (1)$$

where $\underline{x}_k = [x_{k,1}, x_{k,2}, \dots, x_{k,m_k}]^T \in R^{m_k}$ is the state vector of $\sum_k$, and only $x_{k,1}$ can be measured. $\underline{y} = [y_1, \dots, y_M] \in R^M$ is the output vector of the large-scale system. $f_{k,l}^{\sigma(t)}(\underline{x}_k)$ and $h_{k,l}^{\sigma(t)}(\underline{y})$ ($1 \leq k \leq M$, $1 \leq l \leq m_k$) are unknown smooth functions, and $h_{k,l}^{\sigma(t)}(\underline{y})$ represents the coupling effect between M subsystems of the large-scale system. $\sigma(t): [0, \infty) \rightarrow N = \{1, 2, \dots, n\}$ is the switching signal, which is a piecewise continuous function of time from the right. The sth subsystem is active when $\sigma(t) = s$. $u_k \in R$ is the actual control input of $\sum_k$. $D_k u_k \in R$ is the output of the unknown dead zone with input u_k , which can be represented as [32]

$$D_k u_k = \begin{cases} m_{k,r}(u_k - d_{k,r}), & u_k \geq d_{k,r}, \\ 0, & -d_{k,l} < u_k < d_{k,r}, \\ m_{k,l}(u_k + d_{k,l}), & u_k \leq -d_{k,l}, \end{cases} \quad (2)$$

where $m_{k,r}$ and $m_{k,l}$ denote the unknown slope of the dead zone and $d_{k,l}$ and $d_{k,r}$ denote the unknown width parameters of the dead zone.

Assumption 1 (see [9, 10]). For $1 \leq k \leq M$ and $1 \leq l \leq m_k$, there exists unknown smooth function $\tilde{h}_{k,l,i}^s(y_i)$, such that

$$|h_{k,l}^s(\underline{y})|^2 \leq \sum_{i=1}^M (y_i \tilde{h}_{k,l,i}^s(y_i))^2, \quad (3)$$

where $s \in N$.

Assumption 2 (see [9, 10]). There exists unknown smooth function $\tilde{f}_{k,l}^s(\underline{x}_k)$, such that $f_{k,l}^s(\underline{x}_k) = x_{k,1} \tilde{f}_{k,l}^s(\underline{x}_k)$, where $l = 1, 2, \dots, m_k$.

Assumption 3. The reference signal $y_{k,d}(t)$ and its first derivative exist and are bounded.

Assumption 4 (see [32]). The output of dead zone $D_k u_k \in R$ is unmeasurable, and the parameters $m_{k,r}$, $m_{k,l}$, $d_{k,l}$, and $d_{k,r}$ are unknown, whereas the signs are known such that $m_{k,r} > 0$, $m_{k,l} > 0$, $d_{k,l} \geq 0$, and $d_{k,r} \geq 0$, respectively.

Assumption 5 (see [32]). The slopes of the dead zone are bounded by constants $m_{k,r \min}$, $m_{k,r \max}$, $m_{k,l \min}$, and $m_{k,l \max}$ and satisfy $0 < m_{k,r \min} \leq m_{k,r} \leq m_{k,r \max}$ and $0 < m_{k,l \min} \leq m_{k,l} \leq m_{k,l \max}$.

Remark 1. It should be mentioned that if for $t \in [0, \infty)$, the switching signal $\sigma(t) = s$, then system (1) represents a class of nonswitched nonlinear large-scale systems in nonstrict feedback form, which is investigated commonly in the existing literature.

Remark 2. It is worth pointing out that the switched systems are different from those nonswitched systems, which brings more difficulties in the control design of switched systems than those of nonswitched systems.

In this paper, the dead zone inverse technique will be used to compensate the unknown dead zone effect. Defining $u_{k,d} \in R$ as the control input from the controller to achieve the control objective for the plant without a dead zone, $u_k \in R$ can be generated according to the certainty equivalence dead zone inverse [29, 32]:

$$u_k = D_k^{-1}(u_{k,d}) = \frac{u_{k,d} + \hat{d}_{k,mr}}{\hat{m}_{k,r}} \delta_k + \frac{u_{k,d} - \hat{d}_{k,ml}}{\hat{m}_{k,l}} (1 - \delta_k), \quad (4)$$

where $\hat{m}_{k,r}$, $\hat{m}_{k,l}$, $\hat{d}_{k,mr}$, and $\hat{d}_{k,ml}$ are the estimations of $m_{k,r}$, $m_{k,l}$, $d_{k,r}$, and $d_{k,l}$, respectively. Moreover, δ_k satisfies

$$\delta_k = \begin{cases} 1, & u_{k,d} \geq 0, \\ 0, & u_{k,d} < 0. \end{cases} \quad (5)$$

Then, we can have the error between $D_k(u_k)$ and $u_{k,d}$:

$$\begin{aligned} D_k(u_k) - u_{k,d} &= \left(\tilde{d}_{k,mr} - \frac{u_{k,d} + \hat{d}_{k,mr}}{\hat{m}_{k,r}} \tilde{m}_{k,r} \right) \delta_k \\ &+ \left(-\tilde{d}_{k,ml} - \frac{u_{k,d} - \hat{d}_{k,ml}}{\hat{m}_{k,l}} \tilde{m}_{k,l} \right) (1 - \delta_k) + \varepsilon_{k,d}, \end{aligned} \quad (6)$$

where $\tilde{m}_{k,r} = \hat{m}_{k,r} - m_{k,r}$, $\tilde{m}_{k,l} = \hat{m}_{k,l} - m_{k,l}$, $\tilde{d}_{k,mr} = \hat{d}_{k,mr} - d_{k,mr}$, and $\tilde{d}_{k,ml} = \hat{d}_{k,ml} - d_{k,ml}$ are the parameter estimate errors; $\varepsilon_{k,d} = -m_{k,r} \chi_{k,r}(u_k - d_{k,r}) - m_{k,l} \chi_{k,l}(u_k + d_{k,l})$ is bounded, where $\chi_{k,r}$ and $\chi_{k,l}$ are as follows:

$$\begin{aligned} \chi_{k,r} &= \begin{cases} 1, & 0 \leq u_k < d_{k,r}, \\ 0, & u_k < 0, u_k \geq d_{k,r}, \end{cases} \\ \chi_{k,l} &= \begin{cases} 1, & -d_{k,l} \leq u_k < 0, \\ 0, & u_k < -d_{k,l}, u_k \geq 0. \end{cases} \end{aligned} \quad (7)$$

Control Objective. The control objective is to design an adaptive output feedback decentralized control scheme to keep all the outputs $y_k(t)$ of the subsystem tracking the desired trajectories $y_{k,d}(t)$ under unknown dead zones, respectively. Moreover, the tracking errors can be kept as small as possible and all the signals of the closed system are bounded.

3. The Approximation of Fuzzy Logic Systems

A fuzzy logic system can be written as $\hat{f}(x|\theta) = \theta^T \xi(x)$, where $\xi(x)$ is the fuzzy basis function vector and $\theta \in R^N$ is the adjustable weight parameter vector.

Lemma 1 (see [33–35]). If $f(x)$ is a continuous function defined on the compact set Ω , then for any given small constant $\varepsilon > 0$, there exists a fuzzy logic system such that $\sup_{x \in \Omega} |f(x) - \theta^T \xi(x)| \leq \varepsilon$.

4. Observer-Based Adaptive Control Scheme Design

In order to estimate the unmeasured states, we design a linear state observer for k th as follows [10]:

$$\begin{cases} \dot{\hat{x}}_{k,1} = \hat{x}_{k,2} - \ell_{k,1}(\hat{x}_{k,1} - y_k), \\ \dot{\hat{x}}_{k,2} = \hat{x}_{k,3} - \ell_{k,2}(\hat{x}_{k,1} - y_k), \\ \vdots \\ \dot{\hat{x}}_{k,m_k-1} = \hat{x}_{k,m_k} - \ell_{k,m_k-1}(\hat{x}_{k,1} - y_k), \\ \dot{\hat{x}}_{k,m_k} = D_k u_k - \ell_{k,m_k}(\hat{x}_{k,1} - y_k), \end{cases} \quad (8)$$

where $\hat{x}_{k,l}$ ($l = 1, 2, \dots, m_k$) is the estimation of $x_{k,l}$. $\ell_{k,1}, \dots, \ell_{k,m_k}$ are the observer design parameters. Suppose that sth switched subsystem is active. Define observer error vector as $\tilde{\underline{x}}_k = \underline{x}_k - \hat{\underline{x}}_k$, where $\underline{x}_k = [x_{k,1}, x_{k,2}, \dots, x_{k,m_k}]^T$, $\hat{\underline{x}}_k = [\hat{x}_{k,1}, \hat{x}_{k,2}, \dots, \hat{x}_{k,m_k}]^T$.

Then, from (1) and (8), we can have

$$\dot{\tilde{\mathbf{x}}}_k = A_k \tilde{\mathbf{x}}_k + F_k^s + H_k^s, \quad (9)$$

where $A_k = \begin{bmatrix} -\ell_{k,1} & & \\ \vdots & I_{(m_k-1) \times (m_k-1)} & \\ -\ell_{k,m_k} & 0 & 0 \end{bmatrix}$, $F_k^s = [f_{k,1}^s(\mathbf{x}_k), \dots, f_{k,m_k}^s(\mathbf{x}_k)]^T$, $H_k^s = [h_{k,1}^s(y), \dots, h_{k,m_k}^s(y)]^T$. We can choose appropriate parameters $\ell_{k,1}, \dots, \ell_{k,m_k}$ to ensure A_k is a Hurwitz matrix, that is, for any given positive definite matrix $Q_k = Q_k^T > 0$, there exists a positive definite matrix $P_k = P_k^T > 0$, such that

$$A_k^T P_k + P_k A_k = -Q_k. \quad (10)$$

Choose Lyapunov function candidate as [10]

$$V_0 = \sum_{k=1}^M \tilde{\mathbf{x}}_k^T P_k \tilde{\mathbf{x}}_k. \quad (11)$$

Then, we can obtain the time derivative of V_0 :

$$\dot{V}_0 = - \sum_{k=1}^M \tilde{\mathbf{x}}_k^T Q_k \tilde{\mathbf{x}}_k + \sum_{k=1}^M 2 \tilde{\mathbf{x}}_k^T P_k (F_k^s + H_k^s). \quad (12)$$

According to Young's inequality, we have

$$2 \tilde{\mathbf{x}}_k^T P_k F_k^s \leq \|\tilde{\mathbf{x}}_k\|^2 + \|P_k\|^2 \sum_{j=1}^{m_k} (f_{k,j}^s(\mathbf{x}_k))^2. \quad (13)$$

Based on Assumption 2, $f_{k,j}^s(\mathbf{x}_k)$ satisfies

$$f_{k,j}^s(\mathbf{x}_k) = y_k \bar{f}_{k,j}^s(\mathbf{x}_k), \quad (14)$$

where $\bar{f}_{k,j}^s(\mathbf{x}_k)$ is the unknown nonlinear function which can be approximated by a fuzzy logic system.

From (13) and (14), we have

$$\begin{aligned} \|P_k\|^2 \sum_{j=1}^{m_k} (f_{k,j}^s(\mathbf{x}_k))^2 &= \|P_k\|^2 \sum_{j=1}^{m_k} y_k^2 (\bar{f}_{k,j}^s(\mathbf{x}_k))^2 \\ &= y_k \|P_k\|^2 \sum_{j=1}^{m_k} y_k (\bar{f}_{k,j}^s(\mathbf{x}_k))^2. \end{aligned} \quad (15)$$

Based on Assumption 1, we obtain

$$\begin{aligned} \sum_{k=1}^M 2 \tilde{\mathbf{x}}_k^T P_k H_k^s &\leq \sum_{k=1}^M \frac{1}{\lambda} \|\tilde{\mathbf{x}}_k\|^2 + \sum_{k=1}^M \lambda \|P_k\|^2 \|H_k^s\|^2 \\ &\leq \frac{1}{\lambda} \sum_{k=1}^M \|\tilde{\mathbf{x}}_k\|^2 + \bar{\lambda} \sum_{k=1}^M \sum_{l=1}^{m_k} \sum_{i=1}^M (y_i h_{i,l,k}^s(y_k))^2 \\ &\leq \frac{1}{\lambda} \sum_{k=1}^M \|\tilde{\mathbf{x}}_k\|^2 + \bar{\lambda} \sum_{i=1}^M \sum_{l=1}^{m_k} \sum_{k=1}^M (y_i h_{i,l,k}^s(y_k))^2 \\ &\leq \frac{1}{\lambda} \sum_{k=1}^M \|\tilde{\mathbf{x}}_k\|^2 + \bar{\lambda} \sum_{k=1}^M \sum_{i=1}^M \sum_{l=1}^{m_k} y_k^2 (h_{i,l,k}^s(y_k))^2 \\ &\leq \frac{1}{\lambda} \sum_{k=1}^M \|\tilde{\mathbf{x}}_k\|^2 + \sum_{k=1}^M y_k^2 \sum_{i=1}^M \sum_{l=1}^{m_k} \bar{\lambda} (h_{i,l,k}^s(y_k))^2, \end{aligned} \quad (16)$$

where $\lambda > 0$ is the design parameter and $\bar{\lambda} = \lambda \max_{1 \leq k \leq M} (\|P_k\|)^2$.

Substituting equations (13)–(16) into (12) results in

$$\begin{aligned} \dot{V}_0 &\leq \sum_{k=1}^M \left\{ -\tilde{\mathbf{x}}_k^T Q_k \tilde{\mathbf{x}}_k + \left(1 + \frac{1}{\lambda}\right) \|\tilde{\mathbf{x}}_k\|^2 + y_k^2 \sum_{i=1}^M \sum_{l=1}^{m_k} \bar{\lambda} (h_{i,l,k}^s(y_k))^2 + y_k \|P_k\|^2 \sum_{j=1}^{m_k} y_k (\bar{f}_{k,j}^s(\mathbf{x}_k))^2 \right\} \\ &\leq \sum_{k=1}^M \left\{ -\tilde{\mathbf{x}}_k^T Q_k \tilde{\mathbf{x}}_k + \left(1 + \frac{1}{\lambda}\right) \|\tilde{\mathbf{x}}_k\|^2 + y_k \left[\sum_{i=1}^M \sum_{l=1}^{m_k} \bar{\lambda} (h_{i,l,k}^s(y_k))^2 + \|P_k\|^2 \sum_{j=1}^{m_k} y_k (\bar{f}_{k,j}^s(\mathbf{x}_k))^2 + \sum_{i=1}^M y_k (h_{i,1,k}^s(y_k))^2 \right] - \sum_{i=1}^M y_k^2 (h_{i,1,k}^s(y_k))^2 \right\}. \end{aligned} \quad (17)$$

Let $\Phi_{k,s}(\mathbf{x}_k) = y_k \sum_{i=1}^M \sum_{l=1}^{m_k} \bar{\lambda} (h_{i,l,k}^s(y_k))^2 + \|P_k\|^2 \sum_{j=1}^{m_k} y_k (\bar{f}_{k,j}^s(\mathbf{x}_k))^2 + \sum_{i=1}^M y_k (h_{i,1,k}^s(y_k))^2$. By using fuzzy system $\hat{f}_{k,s}(\mathbf{x}_k | \theta_{k,s}) = \theta_{k,s}^T \xi_{k,s}(\mathbf{x}_k)$ to approximate $\Phi_{k,s}(\mathbf{x}_k)$, there exists optimal parameter vector

$$\theta_{k,s}^* = \arg \min_{\theta_{k,s} \in \Omega_{k,s}} \left[\sup_{\mathbf{x}_k \in U_k} |\hat{f}_{k,s}(\mathbf{x}_k | \theta_{k,s}) - \Phi_{k,s}(\mathbf{x}_k)| \right], \quad (18)$$

where $\Omega_{k,s}$ and U_k are the compact sets of $\theta_{k,s}$ and $\mathbf{x}_k$, respectively. Then, the minimal error can be written as

$$\varepsilon_{k,s}(\mathbf{x}_k) = \Phi_{k,s}(\mathbf{x}_k) - \hat{f}_{k,s}(\mathbf{x}_k | \theta_{k,s}^*), \quad (19)$$

where $|\varepsilon_{k,s}(\mathbf{x}_k)| \leq \varepsilon_{k,s}^*$ and $\varepsilon_{k,s}^*$ is the unknown positive constant.

From (18) and (19), we can obtain

$$\dot{V}_0 \leq \sum_{k=1}^M \left\{ -\left(\lambda_{\min}(Q_k) - 1 - \frac{1}{\lambda} \right) \|\tilde{\mathbf{x}}_k\|^2 + y_k [\theta_{k,s}^{*T} \xi_{k,s}(\mathbf{x}_k) + \varepsilon_{k,s}(\mathbf{x}_k)] - \sum_{i=1}^M y_k^2 (h_{i,1,k}^s(y_k))^2 \right\}, \quad (20)$$

where $\lambda_{\min}(Q_k)$ is the minimal eigenvalue of matrix Q_k .

5. Adaptive Control Law Design

Define the tracking error $z_{k,1}$, virtual error $z_{k,l}$, virtual control law $\alpha_{k,l-1}$, and first-order filters as

$$\begin{cases} z_{k,1} = y_k - y_{k,d}, \\ z_{k,l} = \hat{x}_{k,l} - \omega_{k,l-1}, \\ v_{k,l-1} \dot{\omega}_{k,l-1} + \omega_{k,l-1} = \alpha_{k,l-1}, \\ \omega_{k,l-1}(0) = \alpha_{k,l-1}(0), \end{cases} \quad (21)$$

where $l = 2, \dots, m_k$. $v_{k,l-1}$ is the time constant of the filter, that is, by letting $\alpha_{k,l-1}$ pass through a filter that has the time constant $v_{k,l-1}$, we can obtain $\omega_{k,l-1}$.

Step 1. Define $\omega_{k,0} = y_{k,d}$. From (21), we have

$$\begin{aligned} v_{k,1} \dot{\omega}_{k,1} + \omega_{k,1} &= \alpha_{k,1}, \\ \omega_{k,1}(0) &= \alpha_{k,1}(0). \end{aligned} \quad (22)$$

Define $e_{k,1}$ as the first filter output error; then, we can obtain $e_{k,1} = \omega_{k,1} - \alpha_{k,1}$ and $\dot{\omega}_{k,1} = -(e_{k,1}/v_{k,1})$.

The time derivative of $z_{k,1}$ is as follows:

$$\begin{aligned} \dot{z}_{k,1} &= x_{k,2} + f_{k,1}^s + h_{k,1}^s - \dot{\omega}_{k,1} \\ &= z_{k,2} + e_{k,1} + \alpha_{k,1} + \tilde{x}_{k,2} + f_{k,1}^s(\underline{x}_k) + h_{k,1}^s(\underline{y}) - \dot{y}_{k,d}. \end{aligned} \quad (23)$$

By using fuzzy system $\hat{F}_{k,s}(\underline{x}_k | \varphi_{k,s}) = \varphi_{k,s}^T \zeta_{k,s}(\underline{x}_k)$ to approximate $f_{k,1}^s(\underline{x}_k)$ and defining $\varphi_{k,s}^*$ as the optimal parameter vector, we have

$$\varphi_{k,s}^* = \arg \min_{\varphi_{k,s} \in \bar{\Omega}_{k,s}} \left[\sup_{\underline{x}_k \in U_k} |\hat{F}_{k,s}(\underline{x}_k | \varphi_{k,s}) - f_{k,1}^s(\underline{x}_k)| \right], \quad (24)$$

where $\bar{\Omega}_{k,s}$ is the compact set of $\varphi_{k,s}$. Then, the minimal approximation error is as follows:

$$\delta_{k,s}(\underline{x}_k) = f_{k,1}^s(\underline{x}_k) - \hat{F}_{k,s}(\underline{x}_k | \varphi_{k,s}^*), \quad (25)$$

where $|\delta_{k,s}(\underline{x}_k)| \leq \delta_{k,s}^*$ and $\delta_{k,s}^*$ is the unknown positive constant.

Choose Lyapunov function candidate as

$$V_1 = V_0 + \sum_{k=1}^M \left[\frac{1}{2} z_{k,1}^2 + \frac{1}{2} e_{k,1}^2 + \frac{1}{2\gamma_{k,1}} \tilde{\omega}_{k,1}^2 + \frac{1}{2\gamma_{k,2}} \tilde{\omega}_{k,2}^2 \right], \quad (26)$$

where $\gamma_{k,1} > 0$ and $\gamma_{k,2} > 0$ are design parameters, $\bar{\omega}_{k,1}^* = \max_{s \in N} \{\theta_{k,s}^{*T} \theta_{k,s}^*\}$, $\bar{\omega}_{k,2}^* = \max_{s \in N} \{\varphi_{k,s}^{*T} \varphi_{k,s}^*\}$, and $\bar{\omega}_{k,1}$ and $\bar{\omega}_{k,2}$ are estimations of $\bar{\omega}_{k,1}^*$ and $\bar{\omega}_{k,2}^*$, respectively. Define $\tilde{\omega}_{k,1} = \bar{\omega}_{k,1}^* - \bar{\omega}_{k,1}$, $\tilde{\omega}_{k,2} = \bar{\omega}_{k,2}^* - \bar{\omega}_{k,2}$.

Now, we can infer that $\dot{V}_1$ satisfies

$$\begin{aligned} \dot{V}_1 &= \dot{V}_0 + \sum_{k=1}^M \left[z_{k,1} \dot{z}_{k,1} + e_{k,1} \dot{e}_{k,1} - \frac{1}{\gamma_{k,1}} \tilde{\omega}_{k,1} \dot{\omega}_{k,1} - \frac{1}{\gamma_{k,2}} \tilde{\omega}_{k,2} \dot{\omega}_{k,2} \right] \\ &= \dot{V}_0 + \sum_{k=1}^M \left[z_{k,1} (z_{k,2} + e_{k,1} + \alpha_{k,1} + \tilde{x}_{k,2} + f_{k,1}^s(\underline{x}_k) + h_{k,1}^s(\underline{y}) - \dot{y}_{k,d}) + e_{k,1} \dot{e}_{k,1} - \frac{1}{\gamma_{k,1}} \tilde{\omega}_{k,1} \dot{\omega}_{k,1} - \frac{1}{\gamma_{k,2}} \tilde{\omega}_{k,2} \dot{\omega}_{k,2} \right]. \end{aligned} \quad (27)$$

According to Young's inequality and Assumption 1, we have

$$\sum_{k=1}^M z_{k,1} h_{k,1}^s(\underline{y}) \leq \sum_{k=1}^M \frac{z_{k,1}^2}{4} + \sum_{k=1}^M (h_{k,1}^s(\underline{y}))^2 \leq \sum_{k=1}^M \frac{z_{k,1}^2}{4} + \sum_{k=1}^M \sum_{i=1}^M \gamma_k^2 (h_{k,1,i}^s(y_k))^2. \quad (28)$$

Substituting (28) into (27), we obtain

$$\begin{aligned} \dot{V}_1 &= \dot{V}_0 + \sum_{k=1}^M \left[z_{k,1} \dot{z}_{k,1} + e_{k,1} \dot{e}_{k,1} - \frac{1}{\gamma_{k,1}} \tilde{\omega}_{k,1} \dot{\omega}_{k,1} - \frac{1}{\gamma_{k,2}} \tilde{\omega}_{k,2} \dot{\omega}_{k,2} \right] \leq \sum_{k=1}^M \left\{ -\left(\lambda_{\min}(Q_k) - 1 - \frac{1}{\lambda} \right) \|\underline{x}_k\|^2 + y_k [\theta_{k,s}^{*T} \xi_{k,s}(\underline{x}_k) + \varepsilon_{k,s}(\underline{x}_k)] + \frac{z_{k,1}^2}{4} \right\} \\ &\quad + \sum_{k=1}^M \left[z_{k,1} (z_{k,2} + e_{k,1} + \alpha_{k,1} + \tilde{x}_{k,2} + \varphi_{k,s}^{*T} \xi_{k,s}(\underline{x}_k) + \delta_{k,s}(\underline{x}_k) - \dot{y}_{k,d}) + e_{k,1} \dot{e}_{k,1} - \frac{1}{\gamma_{k,1}} \tilde{\omega}_{k,1} \dot{\omega}_{k,1} - \frac{1}{\gamma_{k,2}} \tilde{\omega}_{k,2} \dot{\omega}_{k,2} \right]. \end{aligned} \quad (29)$$

Based on $\xi_{k,s}^T(\cdot)\xi_{k,s}(\cdot) \leq 1$ and Young's inequality, we can infer the following inequalities:

$$\begin{aligned} y_k \left[\theta_{k,s}^{*T} \xi_{k,s}(\underline{x}_k) + \varepsilon_{k,s}(\underline{x}_k) \right] &\leq z_{k,1} \left[\theta_{k,s}^{*T} \xi_{k,s}(\underline{x}_k) + \varepsilon_{k,s}(\underline{x}_k) \right] + y_{k,d} \left[\theta_{k,s}^{*T} \xi_{k,s}(\underline{x}_k) + \varepsilon_{k,s}(\underline{x}_k) \right] \\ &\leq \frac{z_{k,1}^2 \bar{\omega}_{k,1}^*}{4\tau_k} + \frac{z_{k,1}^2}{4} + \frac{y_{k,d}^2}{2} + \bar{\omega}_{k,1}^* + 2\varepsilon_k^{*2} + \tau_k, \end{aligned} \quad (30)$$

$$z_{k,1} \left[\varphi_{k,s}^{*T} \xi_{k,s}(\underline{x}_k) + \delta_{k,s}(\underline{x}_k) \right] \leq \frac{z_{k,1}^2 \bar{\omega}_{k,2}^*}{4\tau_k} + \frac{z_{k,1}^2}{4} + \delta_k^{*2} + \tau_k, \quad (31)$$

$$z_{k,1} \bar{x}_{k,2} \leq \frac{1}{\lambda_k} \|\bar{\underline{x}}_k\|^2 + \lambda_k \frac{z_{k,1}^2}{4}, \quad (32)$$

where $\varepsilon_k^* = \max_{s \in N} \{\varepsilon_{k,s}^{*2}\}$, $\delta_k^* = \max_{s \in N} \{\delta_{k,s}^{*2}\}$, and $\tau_k > 0$ and $\lambda_k > 0$ are design parameters.

Substituting (30)–(32) into (29) results in

$$\begin{aligned} \dot{V}_1 &\leq \sum_{k=1}^M \left\{ - \left(\lambda_{\min}(Q_k) - 1 - \frac{1}{\lambda} - \frac{1}{\lambda_k} \right) \|\bar{\underline{x}}_k\|^2 \right\} \\ &\quad + \sum_{k=1}^M \left[z_{k,1} \left(z_{k,2} + e_{k,1} + \alpha_{k,1} + \frac{z_{k,1} \bar{\omega}_{k,1}^*}{4\tau_k} + \frac{z_{k,1} \bar{\omega}_{k,2}^*}{4\tau_k} + \frac{(3 + \lambda_k) z_{k,1}}{4} - \dot{y}_{k,d} \right) + e_{k,1} \dot{e}_{k,1} \right. \\ &\quad \left. - \frac{1}{\gamma_{k,1}} \bar{\omega}_{k,1} \dot{\omega}_{k,1} - \frac{1}{\gamma_{k,2}} \bar{\omega}_{k,2} \dot{\omega}_{k,2} - \frac{\bar{y}_{k,d}^2}{2} + \bar{\omega}_{k,1}^* + 2\varepsilon_k^{*2} + \delta_k^{*2} + 2\tau_k \right], \end{aligned} \quad (33)$$

where $\bar{y}_{k,d} = \max_{t \in [0, +\infty)} \{y_{k,d}(t)\}$ is a constant. Then, (33) can be rearranged as

$$\begin{aligned} \dot{V}_1 &\leq \sum_{k=1}^M \left\{ -q_{k,1} \|\bar{\underline{x}}_k\|^2 \right\} + \sum_{k=1}^M \left[z_{k,1} \left(z_{k,2} + e_{k,1} + \alpha_{k,1} + \frac{z_{k,1} \bar{\omega}_{k,1}^*}{4\tau_k} + \frac{z_{k,1} \bar{\omega}_{k,2}^*}{4\tau_k} + \frac{(3 + \lambda_k) z_{k,1}}{4} - \dot{y}_{k,d} \right) \right. \\ &\quad \left. + \frac{1}{\gamma_{k,1}} \bar{\omega}_{k,1} \left(\frac{\gamma_{k,1} z_{k,1}^2}{4\tau_k} - \dot{\omega}_{k,1} \right) + \frac{1}{\gamma_{k,2}} \bar{\omega}_{k,2} \left(\frac{\gamma_{k,2} z_{k,1}^2}{4\tau_k} - \dot{\omega}_{k,2} \right) + e_{k,1} \dot{e}_{k,1} + d_{k,1} \right], \end{aligned} \quad (34)$$

where $q_{k,1} = \lambda_{\min}(Q_k) - 1 - (1/\lambda) - (1/\lambda_k)$ and $d_{k,1} = (\bar{y}_{k,d}^2/2) + \bar{\omega}_{k,1}^* + 2\varepsilon_k^{*2} + \delta_k^{*2} + 2\tau_k$.

Design $\alpha_{k,1}$, $\bar{\omega}_{k,1}$, and $\bar{\omega}_{k,2}$ as follows:

$$\begin{aligned} \alpha_{k,1} &= -c_{k,1} z_{k,1} - \frac{z_{k,1} \bar{\omega}_{k,1}}{4\tau_k} - \frac{z_{k,1} \bar{\omega}_{k,2}}{4\tau_k} \\ &\quad - \frac{(3 + \lambda_k) z_{k,1}}{4} + \dot{y}_{k,d}, \end{aligned} \quad (35)$$

$$\dot{\omega}_{k,1} = \frac{\gamma_{k,1} z_{k,1}^2}{4\tau_k} - 2\sigma_{k,1} \bar{\omega}_{k,1}, \quad (36)$$

$$\dot{\omega}_{k,2} = \frac{\gamma_{k,2} z_{k,1}^2}{4\tau_k} - 2\sigma_{k,2} \bar{\omega}_{k,2}, \quad (37)$$

where $c_{k,1} > 0$, $\sigma_{k,1} > 0$, and $\sigma_{k,2} > 0$ are design parameters.

Substituting (35)–(37) into (34) results in

$$\begin{aligned} \dot{V}_1 &\leq \sum_{k=1}^M \left\{ -q_{k,1} \|\tilde{\mathbf{x}}_k\|^2 \right\} + \sum_{k=1}^M \left[-c_{k,1} z_{k,1}^2 + z_{k,1} z_{k,2} + z_{k,1} e_{k,1} + \frac{2\sigma_{k,1}}{\gamma_{k,1}} \tilde{\omega}_{k,1} \omega_{k,1} + \frac{2\sigma_{k,2}}{\gamma_{k,2}} \tilde{\omega}_{k,2} \omega_{k,2} + e_{k,1} \left(-\frac{e_{k,1}}{v_{k,1}} - \dot{\alpha}_{k,1} \right) + d_{k,1} \right] \\ &\leq \sum_{k=1}^M \left\{ -q_{k,1} \|\tilde{\mathbf{x}}_k\|^2 \right\} + \sum_{k=1}^M \left[-c_{k,1} z_{k,1}^2 + z_{k,1} z_{k,2} + z_{k,1} e_{k,1} + \frac{2\sigma_{k,1}}{\gamma_{k,1}} \tilde{\omega}_{k,1} \omega_{k,1} + \frac{2\sigma_{k,2}}{\gamma_{k,2}} \tilde{\omega}_{k,2} \omega_{k,2} - \frac{e_{k,1}^2}{v_{k,1}} + e_{k,1}^2 + \frac{1}{4} \psi_{k,1}^2 + d_{k,1} \right], \end{aligned} \quad (38)$$

where $\psi_{k,1}$ is the maximum absolute value of $\dot{\alpha}_{k,1}$.

According to Young's inequality,

$$\frac{2\sigma_{k,1}}{\gamma_{k,1}} \tilde{\omega}_{k,1} \omega_{k,1} \leq -\frac{\sigma_{k,1}}{\gamma_{k,1}} \omega_{k,1}^2 + \frac{\sigma_{k,1}}{\gamma_{k,1}} \omega_{k,1}^{*2}, \quad (39)$$

$$\frac{2\sigma_{k,2}}{\gamma_{k,2}} \tilde{\omega}_{k,2} \omega_{k,2} \leq -\frac{\sigma_{k,2}}{\gamma_{k,2}} \omega_{k,2}^2 + \frac{\sigma_{k,2}}{\gamma_{k,2}} \omega_{k,2}^{*2}, \quad (40)$$

$$z_{k,1} e_{k,1} \leq z_{k,1}^2 + \frac{1}{4} e_{k,1}^2. \quad (41)$$

Substituting (39)–(41) into (38) results in

$$\begin{aligned} \dot{V}_1 &\leq \sum_{k=1}^M \left\{ -q_{k,1} \|\tilde{\mathbf{x}}_k\|^2 \right\} \\ &\quad + \sum_{k=1}^M \left[-\left(c_{k,1} - 1 \right) z_{k,1}^2 + z_{k,1} z_{k,2} + \left(\frac{5}{4} - \frac{1}{v_{k,1}} \right) e_{k,1}^2 \right. \\ &\quad \left. - \sum_{i=1}^2 \frac{\sigma_{k,i}}{\gamma_{k,i}} \omega_{k,i}^2 + \sum_{i=1}^2 \frac{\sigma_{k,i}}{\gamma_{k,i}} \omega_{k,i}^{*2} + \frac{1}{4} \psi_{k,1}^2 + d_{k,1} \right]. \end{aligned} \quad (42)$$

Substituting the inequality

$$-\frac{\sigma_{k,i}}{2\gamma_{k,i}} \tilde{\omega}_{k,i}^2 \geq -\frac{\sigma_{k,i}}{\gamma_{k,i}} \omega_{k,i}^2 - \frac{\sigma_{k,i}}{\gamma_{k,i}} \omega_{k,i}^{*2}, \quad (43)$$

into equation (42) results in

$$\begin{aligned} \dot{V}_1 &\leq \sum_{k=1}^M \left\{ -q_{k,1} \|\tilde{\mathbf{x}}_k\|^2 \right\} \\ &\quad + \sum_{k=1}^M \left[-\left(c_{k,1} - 1 \right) z_{k,1}^2 + z_{k,1} z_{k,2} + \left(\frac{5}{4} - \frac{1}{v_{k,1}} \right) e_{k,1}^2 \right. \\ &\quad \left. - \sum_{i=1}^2 \frac{\sigma_{k,i}}{2\gamma_{k,i}} \tilde{\omega}_{k,i}^2 + \sum_{i=1}^2 \frac{2\sigma_{k,i}}{\gamma_{k,i}} \omega_{k,i}^{*2} + \frac{1}{4} \psi_{k,1}^2 + d_{k,1} \right]. \end{aligned} \quad (44)$$

Step 2. Define $z_{k,2} = \hat{x}_{k,2} - \omega_{k,1}$ and $z_{k,3} = \hat{x}_{k,3} - \omega_{k,2}$. From (21), we have

$$\begin{aligned} v_{k,2} \dot{\omega}_{k,2} + \omega_{k,2} &= \alpha_{k,2}, \\ \omega_{k,2}(0) &= \alpha_{k,2}(0). \end{aligned} \quad (45)$$

Define $e_{k,2}$ as the filter output error; then, we have $e_{k,2} = \omega_{k,2} - \alpha_{k,2}$ and $\dot{\omega}_{k,2} = -(e_{k,2}/v_{k,2})$.

The time derivative of $z_{k,2}$ is as follows:

$$\begin{aligned} \dot{z}_{k,2} &= \hat{x}_{k,3} - \ell_{k,2}(\hat{x}_{k,1} - y_k) - \dot{\omega}_{k,1} \\ &= z_{k,3} + \alpha_{k,2} + e_{k,2} - \ell_{k,2}(\hat{x}_{k,1} - y_k) - \dot{\omega}_{k,1}. \end{aligned} \quad (46)$$

Choose Lyapunov function candidate as

$$V_2 = V_1 + \sum_{k=1}^M \left[\frac{1}{2} z_{k,2}^2 + \frac{1}{2} e_{k,2}^2 \right]. \quad (47)$$

The time derivative of V_2 is as follows:

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + \sum_{k=1}^M (z_{k,2} \dot{z}_{k,2} + e_{k,2} \dot{e}_{k,2}) \\ &= \dot{V}_1 + \sum_{k=1}^M \left[z_{k,2} (z_{k,3} + \alpha_{k,2} + e_{k,2} - \ell_{k,2}(\hat{x}_{k,1} - y_k) - \dot{\omega}_{k,1}) + e_{k,2} \dot{e}_{k,2} \right] \leq \dot{V}_1 \\ &\quad + \sum_{k=1}^M \left[z_{k,2} (z_{k,3} + \alpha_{k,2} + e_{k,2} - \ell_{k,2}(\hat{x}_{k,1} - y_k) - \dot{\omega}_{k,1}) - \frac{e_{k,2}^2}{v_{k,2}} + e_{k,2}^2 + \frac{1}{4} \psi_{k,2}^2 \right], \end{aligned} \quad (48)$$

where $\psi_{k,2}$ is the maximum absolute value of $\dot{\alpha}_{k,2}$.

According to Young's inequality, we have

$$\begin{aligned}
\dot{V}_2 \leq & \dot{V}_1 + \sum_{k=1}^M \left[z_{k,2} (z_{k,3} + \alpha_{k,2} - \ell_{k,2} (\hat{x}_{k,1} - y_k) - \dot{w}_{k,1}) + z_{k,2}^2 + \frac{1}{4} e_{k,2}^2 - \frac{e_{k,2}^2}{v_{k,2}} + e_{k,2}^2 + \frac{1}{4} \psi_{k,2}^2 \right] \leq \sum_{k=1}^M \left\{ -q_{k,1} \|\tilde{x}_k\|^2 \right\} \\
& + \sum_{k=1}^M \left[- (c_{k,1} - 1) z_{k,1}^2 + z_{k,1} z_{k,2} + \left(\frac{5}{4} - \frac{1}{v_{k,1}} \right) e_{k,1}^2 - \sum_{i=1}^2 \frac{\sigma_{k,i}}{2\gamma_{k,i}} \tilde{w}_{k,i}^2 + \sum_{i=1}^2 \frac{2\sigma_{k,i}}{\gamma_{k,i}} \tilde{w}_{k,i}^{*2} + \frac{1}{4} \psi_{k,2}^2 + d_{k,1} \right] \\
& + \sum_{k=1}^M \left[z_{k,2} (z_{k,3} + \alpha_{k,2} - \ell_{k,2} (\hat{x}_{k,1} - y_k) - \dot{w}_{k,1}) + z_{k,2}^2 + \left(\frac{5}{4} - \frac{1}{v_{k,2}} \right) e_{k,2}^2 + \frac{1}{4} \psi_{k,2}^2 \right].
\end{aligned} \tag{49}$$

Design $\alpha_{k,2}$ as follows:

$$\alpha_{k,2} = -c_{k,2} z_{k,2} - z_{k,1} + \ell_{k,2} (\hat{x}_{k,1} - y_k) - \frac{\omega_{k,1} - \alpha_{k,1}}{v_{k,1}}, \tag{50}$$

where $c_{k,2} > 0$ is the design parameter.

Substituting (50) into (49) leads to

$$\begin{aligned}
\dot{V}_2 \leq & \sum_{k=1}^M \left\{ -q_{k,1} \|\tilde{x}_k\|^2 \right\} + \sum_{k=1}^M \left[- (c_{k,1} - 1) z_{k,1}^2 + z_{k,1} z_{k,2} + \left(\frac{5}{4} - \frac{1}{v_{k,1}} \right) e_{k,1}^2 - \sum_{i=1}^2 \frac{\sigma_{k,i}}{2\gamma_{k,i}} \tilde{w}_{k,i}^2 + \sum_{i=1}^2 \frac{2\sigma_{k,i}}{\gamma_{k,i}} \tilde{w}_{k,i}^{*2} + \frac{1}{4} \psi_{k,1}^2 + d_{k,1} \right] \\
& + \sum_{k=1}^M \left[z_{k,2} z_{k,3} - z_{k,2} z_{k,1} - (c_{k,2} - 1) z_{k,2}^2 + \left(\frac{5}{4} - \frac{1}{v_{k,2}} \right) e_{k,2}^2 + \frac{1}{4} \psi_{k,2}^2 \right] \leq \sum_{k=1}^M \left\{ -q_{k,1} \|\tilde{x}_k\|^2 \right\} \\
& + \sum_{k=1}^M \left[- \sum_{i=1}^2 (c_{k,i} - 1) z_{k,i}^2 + z_{k,2} z_{k,3} + \sum_{i=1}^2 \left(\frac{5}{4} - \frac{1}{v_{k,i}} \right) e_{k,i}^2 - \sum_{i=1}^2 \frac{\sigma_{k,i}}{2\gamma_{k,i}} \tilde{w}_{k,i}^2 + \sum_{i=1}^2 \frac{2\sigma_{k,i}}{\gamma_{k,i}} \tilde{w}_{k,i}^{*2} + \sum_{i=1}^2 \frac{1}{4} \psi_{k,i}^2 + d_{k,1} \right].
\end{aligned} \tag{51}$$

Step l th ($l = 3, \dots, m_k - 1$): define $z_{k,l} = \hat{x}_{k,l} - \omega_{k,l-1}$ and $z_{k,l+1} = \hat{x}_{k,l+1} - \omega_{k,l}$. From (21), we have

$$\begin{aligned}
v_{k,l} \dot{\omega}_{k,l} + \omega_{k,l} &= \alpha_{k,l}, \\
\omega_{k,l}(0) &= \alpha_{k,l}(0).
\end{aligned} \tag{52}$$

Define the filter output error as $e_{k,l}$; then, we obtain $e_{k,l} = \omega_{k,l} - \alpha_{k,l}$ and $\dot{\omega}_{k,l} = -(e_{k,l}/v_{k,l})$.

The time derivative of $z_{k,l}$ is as follows:

$$\begin{aligned}
\dot{z}_{k,l} &= \hat{x}_{k,l+1} - \ell_{k,l} (\hat{x}_{k,1} - y_k) - \dot{\omega}_{k,l-1} \\
&= z_{k,l+1} + \alpha_{k,l} + e_{k,l} - \ell_{k,l} (\hat{x}_{k,1} - y_k) - \dot{\omega}_{k,l-1}.
\end{aligned} \tag{53}$$

Choose Lyapunov function candidate as

$$V_l = V_{l-1} + \sum_{k=1}^M \left[\frac{1}{2} z_{k,l}^2 + \frac{1}{2} e_{k,l}^2 \right]. \tag{54}$$

The time derivative of V_l is as follows:

$$\begin{aligned}
\dot{V}_l &= \dot{V}_{l-1} + \sum_{k=1}^M (z_{k,l} \dot{z}_{k,l} + e_{k,l} \dot{e}_{k,l}) \\
&= \dot{V}_{l-1} + \sum_{k=1}^M \left[z_{k,l} (z_{k,l+1} + \alpha_{k,l} + e_{k,l} - \ell_{k,l} (\hat{x}_{k,1} - y_k) - \dot{\omega}_{k,l-1}) + e_{k,l} \dot{e}_{k,l} \right].
\end{aligned} \tag{55}$$

Design $\alpha_{k,l}$ as follows:

$$\alpha_{k,l} = -c_{k,l} z_{k,l} - z_{k,l-1} + \ell_{k,l} (\hat{x}_{k,1} - y_k) - \frac{\omega_{k,l-1} - \alpha_{k,l-1}}{v_{k,l-1}}, \tag{56}$$

where $c_{k,l} > 0$ is the design parameter.

Using the same recursive method in Step 2, we can get

$$\dot{V}_l \leq \sum_{k=1}^M \left\{ -q_{k,1} \|\tilde{x}_k\|^2 \right\} + \sum_{k=1}^M \left[- \sum_{i=1}^l (c_{k,i} - 1) z_{k,i}^2 + z_{k,l} z_{k,l+1} + \sum_{i=1}^l \left(\frac{5}{4} - \frac{1}{v_{k,i}} \right) e_{k,i}^2 - \sum_{i=1}^2 \frac{\sigma_{k,i}}{2\gamma_{k,i}} \tilde{w}_{k,i}^2 + \sum_{i=1}^2 \frac{2\sigma_{k,i}}{\gamma_{k,i}} \tilde{w}_{k,i}^{*2} + \sum_{i=1}^l \frac{1}{4} \psi_{k,i}^2 + d_{k,1} \right]. \tag{57}$$

Step m_k th: define $z_{k,m_k} = \hat{x}_{k,m_k} - \omega_{k,m_k-1}$, and we can get the time derivative of z_{k,m_k} as follows:

$$\begin{aligned}\dot{z}_{k,m_k} &= D_k u_k - \ell_{k,m_k} (\hat{x}_{k,1} - y_k) - \dot{\omega}_{k,m_k-1} \\ &= u_{k,d} + \left(\tilde{d}_{k,mr} - \frac{u_{k,d} + \hat{d}_{k,mr}}{\hat{m}_{k,r}} \tilde{m}_{k,r} \right) \delta_k - \left(\tilde{d}_{k,ml} + \frac{u_{k,d} - \hat{d}_{k,ml}}{\hat{m}_{k,l}} \tilde{m}_{k,l} \right) (1 - \delta_k) + \varepsilon_{k,d} - \ell_{k,m_k} (\hat{x}_{k,1} - y_k) - \dot{\omega}_{k,m_k-1}.\end{aligned}\quad (58)$$

Choose Lyapunov function candidate as

$$\begin{aligned}V_{m_k} &= V_{m_k-1} + \sum_{k=1}^M \left[\frac{1}{2} z_{k,m_k}^2 + \frac{1}{2\eta_{k,1}} \tilde{m}_{k,r}^2 \right. \\ &\quad \left. + \frac{1}{2\eta_{k,2}} \tilde{m}_{k,l}^2 + \frac{1}{2\eta_{k,3}} \tilde{d}_{k,mr}^2 + \frac{1}{2\eta_{k,4}} \tilde{d}_{k,ml}^2 \right],\end{aligned}\quad (59)$$

where $\eta_{k,1} > 0$, $\eta_{k,2} > 0$, $\eta_{k,3} > 0$, and $\eta_{k,4} > 0$ are design parameters.

Now, we can have the time derivative of V_{m_k} as follows:

$$\begin{aligned}\dot{V}_{m_k} &= \dot{V}_{m_k-1} + \sum_{k=1}^M \left(z_{k,m_k} \dot{z}_{k,m_k} + \frac{1}{\eta_{k,1}} \tilde{m}_{k,r} \dot{\tilde{m}}_{k,r} + \frac{1}{\eta_{k,2}} \tilde{m}_{k,l} \dot{\tilde{m}}_{k,l} + \frac{1}{\eta_{k,3}} \tilde{d}_{k,mr} \dot{\tilde{d}}_{k,mr} + \frac{1}{\eta_{k,4}} \tilde{d}_{k,ml} \dot{\tilde{d}}_{k,ml} \right) \\ &= \dot{V}_{m_k-1} + \sum_{k=1}^M \left[z_{k,m_k} \left(u_{k,d} + \left(\tilde{d}_{k,mr} - \frac{u_{k,d} + \hat{d}_{k,mr}}{\hat{m}_{k,r}} \tilde{m}_{k,r} \right) \delta_k - \left(\tilde{d}_{k,ml} + \frac{u_{k,d} - \hat{d}_{k,ml}}{\hat{m}_{k,l}} \tilde{m}_{k,l} \right) (1 - \delta_k) + \varepsilon_{k,d} - \ell_{k,m_k} (\hat{x}_{k,1} - y_k) - \dot{\omega}_{k,m_k-1} \right) \right. \\ &\quad \left. + \frac{1}{\eta_{k,1}} \tilde{m}_{k,r} \dot{\tilde{m}}_{k,r} + \frac{1}{\eta_{k,2}} \tilde{m}_{k,l} \dot{\tilde{m}}_{k,l} + \frac{1}{\eta_{k,3}} \tilde{d}_{k,mr} \dot{\tilde{d}}_{k,mr} + \frac{1}{\eta_{k,4}} \tilde{d}_{k,ml} \dot{\tilde{d}}_{k,ml} \right] \\ &= \dot{V}_{m_k-1} + \sum_{k=1}^M \left[z_{k,m_k} (u_{k,d} + \varepsilon_{k,d} - \ell_{k,m_k} (\hat{x}_{k,1} - y_k) - \dot{\omega}_{k,m_k-1}) + \frac{1}{\eta_{k,1}} \tilde{m}_{k,r} \left(\dot{\tilde{m}}_{k,r} - \frac{u_{k,d} + \hat{d}_{k,mr}}{\hat{m}_{k,r}} z_{k,m_k} \eta_{k,1} \delta_k \right) + \frac{1}{\eta_{k,2}} \tilde{m}_{k,l} \left(\dot{\tilde{m}}_{k,l} - \frac{u_{k,d} - \hat{d}_{k,ml}}{\hat{m}_{k,l}} z_{k,m_k} \eta_{k,2} (1 - \delta_k) \right) \right. \\ &\quad \left. + \frac{1}{\eta_{k,3}} \tilde{d}_{k,mr} (\dot{\tilde{d}}_{k,mr} + z_{k,m_k} \eta_{k,3} \delta_k) + \frac{1}{\eta_{k,4}} \tilde{d}_{k,ml} (\dot{\tilde{d}}_{k,ml} - z_{k,m_k} \eta_{k,4} (1 - \delta_k)) \right].\end{aligned}\quad (60)$$

Because

$$z_{k,m_k} \varepsilon_{k,d} \leq z_{k,m_k}^2 + \frac{1}{4} \varepsilon_{k,d}^2, \quad (61)$$

we can obtain

$$\begin{aligned}\dot{V}_{m_k} &\leq \dot{V}_{m_k-1} + \sum_{k=1}^M \left[z_{k,m_k} (u_{k,d} + z_{k,m_k} - \ell_{k,m_k} (\hat{x}_{k,1} - y_k) - \dot{\omega}_{k,m_k-1}) + \frac{1}{\eta_{k,1}} \tilde{m}_{k,r} \left(\dot{\tilde{m}}_{k,r} - \frac{u_{k,d} + \hat{d}_{k,mr}}{\hat{m}_{k,r}} z_{k,m_k} \eta_{k,1} \delta_k \right) + \frac{1}{\eta_{k,2}} \tilde{m}_{k,l} \left(\dot{\tilde{m}}_{k,l} - \frac{u_{k,d} - \hat{d}_{k,ml}}{\hat{m}_{k,l}} z_{k,m_k} \eta_{k,2} (1 - \delta_k) \right) + \frac{1}{\eta_{k,3}} \tilde{d}_{k,mr} (\dot{\tilde{d}}_{k,mr} + z_{k,m_k} \eta_{k,3} \delta_k) \right. \\ &\quad \left. + \frac{1}{\eta_{k,4}} \tilde{d}_{k,ml} (\dot{\tilde{d}}_{k,ml} - z_{k,m_k} \eta_{k,4} (1 - \delta_k)) + \frac{1}{4} \varepsilon_{k,d}^2 \right].\end{aligned}\quad (62)$$

Design $u_{k,d}$, $\hat{m}_{k,r}$, $\hat{m}_{k,l}$, $\hat{d}_{k,mr}$, and $\hat{d}_{k,ml}$ as follows:

$$\begin{aligned}u_{k,d} &= -c_{k,m_k} z_{k,m_k} - z_{k,m_k-1} + \ell_{k,m_k} (\hat{x}_{k,1} - y_k) \\ &\quad - \frac{\omega_{k,m_k-1} - \alpha_{k,m_k-1}}{v_{k,m_k-1}},\end{aligned}\quad (63)$$

$$\dot{\tilde{m}}_{k,r} = \frac{u_{k,d} + \hat{d}_{k,mr}}{\hat{m}_{k,r}} z_{k,m_k} \eta_{k,1} \delta_k + 2\mu_{k,1} \tilde{m}_{k,r}, \quad (64)$$

$$\dot{\tilde{m}}_{k,l} = \frac{u_{k,d} - \hat{d}_{k,ml}}{\hat{m}_{k,l}} z_{k,m_k} \eta_{k,2} (1 - \delta_k) + 2\mu_{k,2} \tilde{m}_{k,l}, \quad (65)$$

$$\dot{\tilde{d}}_{k,mr} = -z_{k,m_k} \eta_{k,3} \delta_k + 2\mu_{k,3} \hat{d}_{k,mr}, \quad (66)$$

$$\dot{\tilde{d}}_{k,ml} = z_{k,m_k} \eta_{k,4} (1 - \delta_k) + 2\mu_{k,4} \hat{d}_{k,ml}, \quad (67)$$

where $c_{k,m_k} > 0$, $\mu_{k,1} > 0$, $\mu_{k,2} > 0$, $\mu_{k,3} > 0$, and $\mu_{k,4} > 0$ are design parameters.

Substituting (63)–(67) into (62) results in

$$\begin{aligned} \dot{V}_{m_k} \leq & \sum_{k=1}^M \left\{ -q_{k,1} \|\tilde{x}_k\|^2 \right\} + \sum_{k=1}^M \left[-\sum_{i=1}^{m_k} (c_{k,i} - 1) z_{k,i}^2 + \sum_{i=1}^{m_k-1} \left(\frac{5}{4} - \frac{1}{v_{k,i}} \right) e_{k,i}^2 - \sum_{i=1}^2 \frac{\sigma_{k,i}}{2\gamma_{k,i}} \tilde{\omega}_{k,i}^2 + \sum_{i=1}^2 \frac{2\sigma_{k,i}}{\gamma_{k,i}} \tilde{\omega}_{k,i}^{*2} \right. \\ & \left. + \sum_{i=1}^{m_k-1} \frac{1}{4} \psi_{k,i}^2 + d_{k,1} + \frac{1}{4} \varepsilon_{k,d}^2 + \frac{2\mu_{k,1}}{\eta_{k,1}} \tilde{m}_{k,r} \hat{m}_{k,r} + \frac{2\mu_{k,2}}{\eta_{k,2}} \tilde{m}_{k,i} \hat{m}_{k,i} + \frac{2\mu_{k,3}}{\eta_{k,3}} \tilde{d}_{k,mr} \hat{d}_{k,mr} + \frac{2\mu_{k,4}}{\eta_{k,4}} \tilde{d}_{k,ml} \hat{d}_{k,ml} \right]. \end{aligned} \quad (68)$$

Define

$$\begin{aligned} H = & \sum_{i=1}^2 \frac{\sigma_{k,i}}{\gamma_{k,i}} \tilde{\omega}_{k,i}^{*2} + \frac{\mu_{k,1}}{\eta_{k,1}} m_{k,r}^2 + \frac{\mu_{k,2}}{\eta_{k,2}} m_{k,i}^2 + \frac{\mu_{k,3}}{\eta_{k,3}} d_{k,mr}^2 \\ & + \frac{\mu_{k,4}}{\eta_{k,4}} d_{k,ml}^2 + \sum_{i=1}^{m_k-1} \frac{1}{4} \psi_{k,i}^2 + d_{k,1} + \frac{1}{4} \varepsilon_{k,d}^2, \end{aligned} \quad (69)$$

where $|\varepsilon_{k,d}| < \bar{\varepsilon}_{k,d}$.

Substituting (69) into (68) leads to

$$\begin{aligned} \dot{V}_{m_k} \leq & \sum_{k=1}^M \left\{ -q_{k,1} \|\tilde{x}_k\|^2 \right\} + \sum_{k=1}^M \left[-\sum_{i=1}^{m_k} (c_{k,i} - 1) z_{k,i}^2 \right. \\ & - \sum_{i=1}^{m_k-1} \left(\frac{1}{v_{k,i}} - \frac{5}{4} \right) e_{k,i}^2 - \sum_{i=1}^2 \frac{\sigma_{k,i}}{2\gamma_{k,i}} \tilde{\omega}_{k,i}^2 - \frac{\mu_{k,1}}{\eta_{k,1}} \tilde{m}_{k,r}^2 \\ & \left. - \frac{\mu_{k,2}}{\eta_{k,2}} \tilde{m}_{k,i}^2 - \frac{\mu_{k,3}}{\eta_{k,3}} \tilde{d}_{k,mr}^2 - \frac{\mu_{k,4}}{\eta_{k,4}} \tilde{d}_{k,ml}^2 + H \right]. \end{aligned} \quad (70)$$

6. Stability Analysis

Define Lyapunov function of the closed-loop system as $\hat{V} = V_{m_k}$, and we can have $\dot{\hat{V}} = \dot{V}_{m_k}$. Let the design parameters satisfy

$$\begin{aligned} c_{k,i} - 1 > 0, \quad i = 1, 2, \dots, m_k, \\ \frac{1}{v_{k,i}} - \frac{5}{4} > 0, \quad i = 1, 2, \dots, m_k - 1. \end{aligned} \quad (71)$$

Define

$$\begin{aligned} C_k = \min & \left\{ \frac{2q_{k,1}}{\lambda_{\max}(P_k)}, 2(c_{k,i} - 1), 2\sigma_{k,1}, 2\sigma_{k,2}, \right. \\ & \left. 2\mu_{k,1}, 2\mu_{k,2}, 2\mu_{k,3}, 2\mu_{k,4}, 2\left(\frac{1}{v_{k,i}} - \frac{5}{4} \right) \right\}, \end{aligned} \quad (72)$$

where $i = 1, 2, \dots, m_k$.

Define $C = \min_{k=1, \dots, M} \{C_k\}$, and (70) can be rearranged as

$$\dot{\hat{V}} \leq -C\hat{V} + H. \quad (73)$$

Then, we have

$$e^{Ct} \hat{V} \leq (-C\hat{V} + H)e^{Ct}, \quad (74)$$

$$\frac{d}{dt} (\hat{V} e^{Ct}) \leq H e^{Ct}, \quad (75)$$

$$\hat{V} e^{Ct} - \hat{V}(0) \leq \frac{H}{C} (e^{Ct} - 1), \quad (76)$$

$$0 \leq \hat{V}(t) \leq \hat{V}(0) e^{-Ct} + \frac{H}{C} (1 - e^{-Ct}) \leq \hat{V}(0) + \frac{H}{C}. \quad (77)$$

From (78), we can get that all the signals of the closed system, such as $x_{k,i}(t)$, $\hat{x}_{k,i}(t)$, $z_{k,i}(t)$, $a_{k,i}(t)$, and $u_k(t)$, are semiglobally uniformly ultimately bounded (SGUUB). Moreover, the observer error satisfies $\|\tilde{x}_k\| \leq \sqrt{(2\hat{V}(0)e^{-Ct} + (H/C))/\lambda_{\min}(P_k)}$, and the tracking error satisfies $\|z_{k,1}\| \leq \sqrt{2\hat{V}(0)e^{-Ct} + (H/C)}$.

Remark 3. Note that from (78), we can only conclude that the state observer errors and tracking errors do not converge to zero. However, we can make the state observer errors and tracking errors to be small by choosing design parameters.

Remark 4. It is worth mentioning that the influence of dead zone is compensated by inverse transformation technology, and dead zone parameters can be identified online, which is different from that in [17]. In [17], the problem of the dead zone is solved by constructing a tan-type BLF, which can compensate the influence of the dead zone but cannot get the dead zone parameters.

The configuration of the output feedback control scheme proposed above for the nonstrict feedback system is shown in Figure 1.

7. Comparisons with Some Previous Results

To further illustrate the contributions of this method, some comparisons with previous results in [10–32, 36, 37] will be given in this section.

(1) The control methods in [13, 17, 19–24] can address only nonlinear system (78) without switching signals:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(x_1, x_2, \dots, x_n), & (i = 1, 2, \dots, n-1), \\ \dot{x}_n = u(t) + f_n(x_1, x_2, \dots, x_n), \\ y = x_1. \end{cases} \quad (78)$$

The control methods in [25–28, 32] can only control switched systems (79), not large-scale systems:

FIGURE 1: Block diagram of the control scheme.

$$\begin{cases} \dot{x}_1 = x_2 + f_{1,\sigma(t)}(x_1, x_2, \dots, x_n), \\ \vdots \\ \dot{x}_i = x_{i+1} + f_{i,\sigma(t)}(x_1, x_2, \dots, x_n), \\ \vdots \\ \dot{x}_n = u_{\sigma(t)} + f_{n,\sigma(t)}(x_1, x_2, \dots, x_n), \\ y = x_1. \end{cases} \quad (79)$$

The control methods in [14–16, 18, 29, 36, 37] address the following strict feedback system:

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_n) + g_i(\bar{x}_n)x_{i+1}, & (i = 1, 2, \dots, n-1), \\ \dot{x}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n)u, \\ y = x_1, \end{cases} \quad (80)$$

where $\bar{x}_i = [x_1, x_2, \dots, x_i]^T$ and $f_i(\bar{x}_i)$ and $g_i(\bar{x}_i)$ are the functions of partial state variables.

The control method in this paper is designed for the nonlinear large-scale nonstrict feedback system (81) with arbitrary switching signals:

$$\begin{cases} \dot{x}_{k,1} = x_{k,2} + f_{k,1}^{\sigma(t)}(\underline{x}_k) + h_{k,1}^{\sigma(t)}(\underline{y}), \\ \dot{x}_{k,2} = x_{k,3} + f_{k,2}^{\sigma(t)}(\underline{x}_k) + h_{k,2}^{\sigma(t)}(\underline{y}), \\ \vdots \\ \dot{x}_{k,m_k-1} = x_{k,m_k} + f_{k,m_k-1}^{\sigma(t)}(\underline{x}_k) + h_{k,m_k-1}^{\sigma(t)}(\underline{y}), \\ \dot{x}_{k,m_k} = D_k u_k + f_{k,m_k}^{\sigma(t)}(\underline{x}_k) + h_{k,m_k}^{\sigma(t)}(\underline{y}), \\ y_k = x_{k,1}, \end{cases} \quad (81)$$

where $f_{k,l}^{\sigma(t)}(\cdot)$ is the function of whole state variables and $\sigma(t)$ is the arbitrary switching signal.

Whether it is controller design or stability proof, switched large-scale nonstrict feedback systems are far more difficult than systems (78)–(80). We were unable to use the control methods in [13, 17, 19–24] to control the switched system due to the switching problem. The control methods in [25–28, 32] also cannot be used because of the interconnected influence between subsystems of the large-scale system. Moreover, when the controller of the nonstrict feedback switched system is designed by the control methods in [14–16, 18, 29, 36, 37], the virtual control signal and adaptive law of each subsystem are the functions of whole state variables. Consequently, the algebraic loop problem arises, which makes the controller design of a nonstrict feedback switched system very difficult. Therefore, the controller design method of the nonstrict feedback switched large-scale system (81) considered in this paper is quite different from that of the controller design methods in [13–29, 32, 36, 37].

- (2) Li and Tong and Younsi et al. [10, 11] proposed adaptive control methods for the nonstrict feedback switched large-scale system, but the input constraints were not considered. Though Wang et al. [12] presented the control schemes for a system with input constrained systems, all states of the system should be measurable. The strict limitation makes these control methods difficult to realize in practical applications. Therefore, control methods in [10–12] cannot be used to control a nonlinear system with unmeasurable state and input constraint that is discussed in this paper.
- (3) This proposed adaptive control scheme does not need n -order differentiable and bounded conditions of the input signals and a monotonically increasing condition of unknown functions. However, these strict assumptions are common in the existing references [30, 31]. Moreover, by adopting dynamic surface technology and first-order filters, this control scheme can avoid the problem of the “explosion of

complexity.” Therefore, this control scheme not only conforms to engineering practice but also has a simple algorithm and requires a small number of calculations.

8. Simulations

Consider the following switched nonstrict feedback large-scale system [10]:

$$\begin{cases} \dot{x}_{1,1} = x_{1,2} + f_{1,1}^{\sigma(t)}(x_{1,1}, x_{1,2}) + h_{1,1}^{\sigma(t)}(y_1, y_2), \\ \dot{x}_{1,2} = D_1 u_1 + f_{1,2}^{\sigma(t)}(x_{1,1}, x_{1,2}) + h_{1,2}^{\sigma(t)}(y_1, y_2), \\ y_1 = x_{1,1}, \\ \dot{x}_{2,1} = x_{2,2} + f_{2,1}^{\sigma(t)}(x_{2,1}, x_{2,2}) + h_{2,1}^{\sigma(t)}(y_1, y_2), \\ \dot{x}_{2,2} = D_2 u_2 + f_{2,2}^{\sigma(t)}(x_{2,1}, x_{2,2}) + h_{2,2}^{\sigma(t)}(y_1, y_2), \\ y_2 = x_{2,1}, \end{cases} \quad (82)$$

where $\sigma(t) = 1, 2$, $f_{1,1}^1(x_{1,1}, x_{1,2}) = x_{1,1}^2(1 - \cos(x_{1,1}x_{1,2})) + x_{1,2} - 0.2x_{1,1}\sin(x_{1,1}x_{1,2})$, $h_{1,1}^1(y_1, y_2) = y_1y_2$, $f_{1,2}^1(x_{1,1}, x_{1,2}) = x_{1,1}x_{1,2} + x_{1,1}$, $h_{1,2}^1(y_1, y_2) = y_1^2 + y_2^2$, $f_{2,1}^1(x_{2,1}, x_{2,2}) = x_{2,1}\sin(x_{2,1}x_{2,2}) - 0.5x_{2,1}\sin(x_{2,1}x_{2,2})$, $h_{2,1}^1(y_1, y_2) = y_1^3$, $f_{2,2}^1(x_{2,1}, x_{2,2}) = x_{2,1}x_{2,2}$, $h_{2,2}^1(y_1, y_2) = y_1 - 2y_2$, $f_{1,1}^2(x_{1,1}, x_{1,2}) = x_{1,1}^2(1 - \cos(x_{1,1}x_{1,2})) + x_{1,2} - 0.3x_{1,1}\sin(x_{1,1}x_{1,2})$, $h_{1,1}^2(y_1, y_2) = 4y_1y_2$, $f_{1,2}^2(x_{1,1}, x_{1,2}) = 3x_{1,1}$, $h_{1,2}^2(y_1, y_2) = y_1^2 + 2y_2^2$, $f_{2,1}^2(x_{2,1}, x_{2,2}) = \cos(x_{2,1}x_{2,2})$, $h_{2,1}^2(y_1, y_2) = 3y_1^2y_2^2$, $f_{2,2}^2(x_{2,1}, x_{2,2}) = 0.8x_{2,1}x_{2,2}$, and $h_{2,2}^2(y_1, y_2) = y_1 - 3y_2$. The given tracking signals are $y_{1,d} = y_{2,d} = 0.25\sin(t) + 0.25\sin(0.5t)$.

Remark 5. Note that if system (82) does not contain the switching signal $\sigma(t)$, it turns into a nonlinear large-scale system, which was extensively studied in [17–22].

Remark 6. Note that functions $f_{k,l}^{\sigma(t)}(\underline{x}_k)$ in (82) do not satisfy the assumptions in [28, 29], i.e., we cannot find increasing smooth functions $\phi_{k,l}^{\sigma(t)}(\|\underline{x}_k\|)$ with $\phi_{k,l}^{\sigma(t)}(0) = 0$, such that $f_{k,l}^{\sigma(t)}(\underline{x}_k) \leq \phi_{k,l}^{\sigma(t)}(\|\underline{x}_k\|)$. In addition, (82) is not a switched strict feedback nonlinear system. Therefore, the control methods in [13–16, 27, 34, 35] cannot be used to control the system (82).

Choose the parameters as $\ell_{1,1} = \ell_{1,2} = \ell_{2,1} = \ell_{2,2} = 10$, $c_{1,1} = 40$, $c_{1,2} = 20$, $c_{2,1} = 20$, $c_{2,2} = 20$, $\lambda_1 = \lambda_2 = 1$, $\tau_1 = \tau_2 = 1$, $\gamma_{1,1} = \gamma_{1,2} = 1$, $\gamma_{2,1} = \gamma_{2,2} = 2$, $\sigma_{1,1} = \sigma_{1,2} = 0.05$, $\sigma_{2,1} = \sigma_{2,2} = 0.1$, $v_{1,1} = v_{1,2} = v_{2,1} = v_{2,2} = 0.01$, $m_{1,r} = 2$, $m_{1,l} = 1.3$, $d_{1,r} = 0.4$, $d_{1,l} = 0.6$, $m_{2,r} = 1.1$, $m_{2,l} = 2$, $d_{2,r} = 0.2$, and $d_{2,l} = 0.5$.

Remark 7. It is worth pointing out that the selected design parameters mentioned above are the only sufficient conditions for guaranteeing the stability of the control system. Of course, in the simulation, we can select another group of design parameters based on the selection guidelines of the design parameters.

The simulation results are shown in Figures 2–7, where Figure 2 shows the trajectories of the first subsystem's output y_1 and tracking signal $y_{1,d}$; Figure 3 shows the trajectory of the first subsystem's tracking error $z_{1,1}$; Figure 4 shows the

FIGURE 2: Trajectories of y_1 and $y_{1,d}$. FIGURE 4: Trajectories of y_2 and $y_{2,d}$. FIGURE 3: The tracking error $z_{1,1}$. FIGURE 5: The tracking error $z_{2,1}$.

trajectories of the second subsystem's output y_2 and tracking signal $y_{2,d}$; Figure 5 shows the trajectory of the second subsystem's tracking error $z_{2,1}$; Figure 6 shows the trajectories of control signals $u_{k,d}$ and the output of dead zone $D_k(u_k)$; and Figure 7 shows the trajectory of the switched signal $\sigma(t)$.

From Figures 2, 3, and 7, we can see that a good tracking performance of the first subsystem is achieved under arbitrary switched signal and the absolute value of tracking error is less than 0.014. From Figures 4, 5, and 7, we can see that a

good tracking performance of the second subsystem is achieved under arbitrary switched signal and the absolute value of tracking error is less than 0.04. From Figure 6, we can see that the effect of unknown dead zone can be compensated well by the dead zone inverse technique.

From the simulation results, it can be seen that the proposed control scheme in this paper can guarantee the switched large-scale system to have a good tracking performance and can ensure the tracking errors to be kept in a small neighbourhood of zero. Moreover, all the signals of the closed-loop system are SGUUB.

FIGURE 6: Control inputs $u_{k,d}$ and $D_k(u_k)$. FIGURE 7: The switched signal $\sigma(t)$.

9. Conclusion

In this paper, an adaptive fuzzy dynamic surface decentralized output feedback control scheme is proposed for large-scale interconnected uncertain nonstrict feedback switched systems. A decentralized linear state observer is designed to estimate the unmeasurable states of each subsystem. The dead zone problem is solved by using the dead zone inverse technology. The virtual control law, the real control law, and the adaptive law are designed by using the common Lyapunov function method to deal with the problem of arbitrary switching. Finally, the stability of the large-scale interconnected switched system is proved, and all the signals are guaranteed to be SGUUB. By choosing appropriate observer matrix parameters and controller design parameters, the tracking errors can be kept as small as possible. Future research directions are the extension of the results to stochastic nonlinear switched large-scale uncertain systems.

Data Availability

All the data included in this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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