

Research Article

A Super-Resolution DOA Estimation Method for Fast-Moving Targets in MIMO Radar

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Received 11 October 2019; Revised 28 December 2019; Accepted 14 January 2020; Published 16 March 2020

Academic Editor: Luis J. Yebra

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Direction of arrival (DOA) estimation is an essential problem in the radar systems. In this paper, the problem of DOA estimation is addressed in the multiple-input and multiple-output (MIMO) radar system for the fast-moving targets. A virtual aperture is provided by orthogonal waveforms in the MIMO radar to improve the DOA estimation performance. Different from the existing methods, we consider the DOA estimation method with only one snapshot for the fast-moving targets and achieve the super-resolution estimation from the snapshot. Based on a least absolute shrinkage and selection operator (LASSO), a denoise method is formulated to obtain a sparse approximation to the received signals, where the sparsity is measured by a new type of atomic norm for the MIMO radar system. However, the denoise problem cannot be solved efficiently. Then, by deriving the dual norm of the new atomic norm, a semidefinite matrix is constructed from the denoise problem to formulate a semidefinite problem with the dual optimization problem. Finally, the DOA is estimated by peak-searching the spatial spectrum. Simulation results show that the proposed method achieves better performance of the DOA estimation in the MIMO radar system with only one snapshot.

1. Introduction

The orthogonal waveforms are transmitted by multiple-input and multiple-output (MIMO) radar systems [1–6], so the performance of target detection, estimation, and tracking can be improved by the waveform diversity. Generally, the MIMO radar system can be classified into two types: (1) the colocated MIMO radars [7–9], where the distance between antennas is comparable with the wavelength, and the waveform diversity is exploited to improve the target estimation performance with the large virtual aperture; (2) the distributed MIMO radar [10–13], where the spacing between antennas is large, and the different view-angles from the antennas to targets are used to improve the detection performance with the diversity of radar cross-section (RCS). Since the direction of arrival (DOA) estimation problem is addressed in this paper, the colocated MIMO radar is adopted to improve the DOA estimation performance with larger virtual aperture than that in traditional phased array [14].

Many papers have investigated the DOA estimation problem [15–18]. Traditionally, the discrete Fourier transform

(DFT) [19] is used to estimate the DOA, where the received signals are sampled by the antennas in the spatial domain, and then the DOA estimation is equal to a corresponding frequency estimation in the transformed domain. Therefore, the frequency (DOA) in the spatial domain is obtained by the DFT methods, but the resolution of DFT method is limited by Rayleigh criterion. The methods that can break through the Rayleigh criterion are called super-resolution methods. Multiple signal classification (MUSIC) method [20–22], Root-MUSIC [23], and the estimation of signal parameters via rotational invariant techniques (ESPRIT) method [24–26] are three most essential super-resolution methods. The noise subspace and signal subspace are obtained in the MUSIC and ESPRIT methods to estimate the DOA, respectively. A TOD-MUSIC algorithm is proposed in [27] to estimate the DOA in the scenario with low signal-to-noise ratio (SNR) with diversity bistatic MIMO radar.

However, the subspaces are obtained from the estimated covariance matrix of the received signals, so the multiple measurements are needed in MUSIC and ESPRIT methods to achieve a reasonable estimation of the covariance matrix.

In the MIMO radar system, the multiple measurements are obtained from the multiple pulses, where the measurements are obtained from the output of the pulse compression. In the scenario with fast-moving targets, the multiple measurements are impossible, so the subspace-based methods cannot be used. For the fast-moving targets, the DOA of targets is changing fast and exceeds the limit of the pulse repetition frequency (PRF) [28], so the statistic information, such as the covariance matrix of received signals, cannot be estimated well from the nonstationary pulse signals, and only the single pulse can be adopted to estimate the DOA. Therefore, the fast-moving targets are defined as the ones with nonstationary DOA between adjacent pulses.

The compressed sensing- (CS-) based methods have been proposed [29, 30] to improve the DOA estimation performance with fewer measurements, where the target sparsity is exploited in the spatial domain [31–35]. Therefore, the DOA estimation for the sparse targets is transformed into the sparse reconstruction problem in the CS-based methods [36, 37, 37], and the sparse-based methods can be used in the scenario with fast-moving targets. In [38], a reweighted ℓ_0 norm minimization method with fast iterations is proposed for the DOA estimation in MIMO radar. A fast sparse DOA estimation algorithm for both the white and colored Gaussian noises is proposed in [39] in the scenario with multiple measurement vectors. Xie et al. [40] developed a covariance-vector sparsity-aware estimator to estimate the DOA from the MIMO radar.

Usually, the CS-based methods can be classified into the greedy methods and the norm-based methods: (1) in the greedy methods, such as orthogonal matching pursuits (OMP) [41], stagewise OMP (StOMP), and CoSaMP [42], iterations are used to reconstruct the sparse signals; (2) in the norm-based method, the ℓ_0 norm minimization problem is transformed into a ℓ_1 norm minimization problem, which can be solved efficiently with the convex optimization tools. Additionally, sparse Bayesian learning- (SBL-) based methods with the prior assumption of sparse signals are also proposed [43], such as SBL method and OGSBI method [44], which can achieve the excellent performance with relatively high computational complexity. Moreover, different from the traditional methods of the sparse reconstruction using the discretized dictionary matrix, the atomic norm-based method as a new type of the norm-based methods realizes the sparse reconstruction in the continuous domain [45–47]. In [48], a semidefinite programming-based method is proposed for the ℓ_1 norm optimization over infinite dictionaries. However, the atomic norm has not studied well in the MIMO radar system for the DOA estimation.

In this paper, the DOA estimation problem in the MIMO radar system is addressed, where a larger virtual aperture is provided by MIMO radar than that in phased array. To estimate the DOA of fast-moving targets, we consider the DOA estimation problem in the scenario with only one measurement from the MIMO radar system. Then, by exploiting both the structure of the MIMO radar system and the target sparsity in the spatial domain, a new type of atomic norm is proposed to obtain the trade-off between the target sparsity and the reconstructed signals. A least absolute

shrinkage and selection operator (LASSO) with the novel atomic norm is formulated to denoise the received signals, but the LASSO cannot be solved efficiently, so a semidefinite matrix is constructed theoretically from the dual norm of the new atomic norm. Therefore, a dual optimization problem is obtained and transformed into a semidefinite problem, which can be solved efficiently. With the denoised signals, the DOA is estimated by peak-searching the spatial spectrum.

The remainder of this paper is organized as follows. The MIMO radar system is shown in Section 2. A continue domain super-resolution method for fast-moving targets is proposed in Section 3. Section 4 shows the simulation results, and Section 5 concludes the paper.

Notations. $\|\cdot\|_1$, and $\|\cdot\|_2$ denote the ℓ_1 norm and the ℓ_2 norm, respectively. $\|\cdot\|^*$ denotes the dual norm. $\Re\{a\}$ denotes the real part of a complex value. $(\cdot)^H$ denotes the Hermitian transpose of a complex matrix/vector.

2. MIMO Radar System with Fast-Moving Targets

In this paper, we consider the DOA estimation problem in the MIMO radar with fast-moving targets, and we assume that the number of transmitting antennas is M and that of receiving antenna is N . As shown in Figure 1, the orthogonal waveforms are transmitted in the MIMO radar system, and the waveform in the m -th transmitting antenna is denoted as $s_m(t)$ ($m = 0, 1, \dots, M - 1$). Since the orthogonal waveforms are transmitted, we have

$$\int_0^T s_{m_1}(t) s_{m_2}^H(t) dt = \begin{cases} 1, & m_1 = m_2, \\ 0, & m_1 \neq m_2, \end{cases} \quad (1)$$

where T is the pulse duration. The MIMO radar system considered in this paper is a type of colocated MIMO radar. Both the transmitting and receiving antennas are equipped in the same system, so the DOA and direction of departure (DOD) are the same. To simplify the analysis, the transmitting and receiving antennas are both uniform linear arrays (ULA). Therefore, in the n -th receiving antenna, the received signal can be expressed as

$$\begin{aligned} y_n(t) = & \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} s_m(t) e^{j2\pi(md/\lambda)\sin(\theta_k)} \alpha_k e^{j2\pi(nd/\lambda)\sin(\theta_k)} \\ & + w_n(t), \end{aligned} \quad (2)$$

where the number of targets is K , α_k denotes the scattering coefficient of the k -th target, d is the distance between adjacent antennas, λ is the wavelength, θ_k is the DOA of the k -th target, and $w_n(t)$ is the additive Gaussian noise.

In the n -th receiving antenna, the matched filters for orthogonal waveforms are adopted to distinguish the waveforms transmitted by different antennas. For the received signals in the MIMO radar system, if we use the signals directly, the SNR can be lower than 10 dB. However, we can use the pulse compression (i.e., matched filters) to

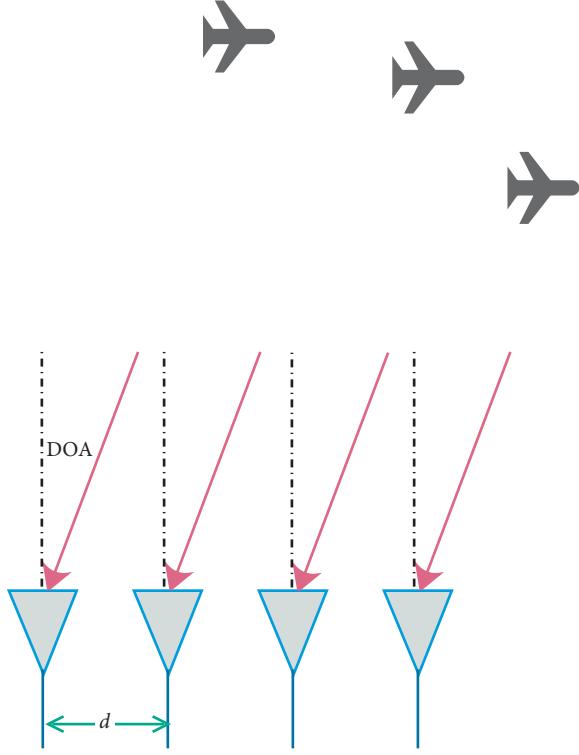


FIGURE 1: The MIMO radar system for DOA estimation.

increase the SNR of the received signals. For the m -th waveform $s_m(t)$, the corresponding matched filter is $h_m(t) = s_m^*(T-t)$, where $s_m^*(t)$ is a conjugate function of $s_m(t)$. Therefore, in the n -th receiving antenna, the signal passing the m -th matched filter and sampled at time T can be given as

$$\begin{aligned} y_{n,m} &= \sum_{k=0}^{K-1} \int_0^T s_m^H(t) s_m(t) e^{j2\pi(md/\lambda)\sin(\theta_k)} \alpha_k e^{j2\pi(nd/\lambda)\sin(\theta_k)} dt \\ &\quad + \underbrace{\int_0^T s_m^H(t) w_n(t) dt}_{v_{n,m}} \\ &= \sum_{k=0}^{K-1} \alpha_k e^{j2\pi((m+n)d/\lambda)\sin(\theta_k)} + v_{n,m}. \end{aligned} \quad (3)$$

We define the steering vectors with the direction θ for the transmitter and receiver, respectively, as follows:

$$\begin{aligned} \mathbf{a}(\theta) &\triangleq [a_0(\theta), a_1(\theta), \dots, a_{M-1}(\theta)]^T, \\ \mathbf{b}(\theta) &\triangleq [b_0(\theta), b_1(\theta), \dots, b_{N-1}(\theta)]^T, \end{aligned} \quad (4)$$

where the m -th entry of $\mathbf{a}(\theta)$ is $a_m(\theta) = e^{j2\pi md/\lambda\sin(\theta)}$ and the n -th entry of $\mathbf{b}(\theta)$ is $b_n(\theta) = e^{j2\pi nd/\lambda\sin(\theta)}$. Therefore, the received signal passing the matched filter can be simplified as

$$y_{n,m} = \sum_{k=0}^{K-1} \alpha_k a_m(\theta_k) b_n(\theta_k) + v_{n,m}, \quad (5)$$

where $v_{n,m} \sim \mathcal{CN}(0, \sigma_w^2)$ denotes the additive white Gaussian noise. Then, we can collect all the signals in

the n -th receiving antenna after the matched filter as follows:

$$\begin{aligned} \mathbf{y}_n &\triangleq [y_{n,0}, y_{n,1}, \dots, y_{n,M-1}]^T \\ &= \sum_{k=0}^{K-1} \alpha_k b_n(\theta_k) \mathbf{a}(\theta_k) + \mathbf{v}_n, \end{aligned} \quad (6)$$

where the noise vector is defined as $\mathbf{v}_n \triangleq [v_{n,0}, v_{n,1}, \dots, v_{n,M-1}]^T$. Collect the signals from all the receiving antennas, and we can get the vector with all the information as

$$\begin{aligned} \mathbf{y} &\triangleq [\mathbf{y}_0^T, \mathbf{y}_1^T, \dots, \mathbf{y}_{N-1}^T]^T \\ &= \sum_{k=0}^{K-1} \alpha_k \mathbf{b}(\theta_k) \otimes \mathbf{a}(\theta_k) + \mathbf{v} \\ &= \mathbf{C}\mathbf{a} + \mathbf{v}, \end{aligned} \quad (7)$$

where we define $\mathbf{v} \triangleq [\mathbf{v}_0^T, \mathbf{v}_1^T, \dots, \mathbf{v}_{N-1}^T]^T$, and \otimes denotes the Kronecker product. Additionally, we define the matrix with steering vectors as

$$\mathbf{C} \triangleq [\mathbf{b}(\theta_0) \otimes \mathbf{a}(\theta_0), \mathbf{b}(\theta_1) \otimes \mathbf{a}(\theta_1), \dots, \mathbf{b}(\theta_{K-1}) \otimes \mathbf{a}(\theta_{K-1})], \quad (8)$$

and we define the vector for target scattering coefficients as

$$\alpha \triangleq [\alpha_0, \alpha_1, \dots, \alpha_{K-1}]^T. \quad (9)$$

Finally, from the system model (7), we try to estimate the DOA θ_k of targets from the received signals \mathbf{y} with only one snapshot. Usually, in the existing methods with super-resolution, the multiple measurements are used, for example, the MUSIC or ESPRIT methods; the multiple measurements are used to estimate the covariance matrix, and the DOAs are estimated from the noise or signal subspaces, which are obtained from the covariance matrix. In this paper, we focus in the scenario with only one measurement, so the proposed DOA estimation method can be used to track the fast-moving targets and can get the target information from just one pulse.

3. Continue Domain Super-Resolution Estimation Method

In this section, we will propose a super-resolution method for the DOA estimation in the continuous domain, named as continue domain super-resolution (CDSR) method. To exploit the target sparsity in the spatial domain, we will formulate a system model from (7) by introducing the sparsity. In the traditional methods to exploit the target sparsity, the compressed sensing- (CS-) based methods have been proposed [49], where the dictionary matrix is formulated by discretizing the spatial domain. Then, the DOA information is obtained from the dictionary matrix. However, the *off-grid* problem will be introduced during discretizing the spatial domain [50, 51], where the targets cannot be precise at the discretized angles. Different from the traditional methods, we will propose a sparse-based method in the continuous domain. First, the preliminary for the CDSR will be given as the background knowledge.

3.1. Preliminary of Dual Norm. For a norm $\|\mathbf{x}\|$ (\mathbf{x} is a vector with the entries being complex), the dual norm of \mathbf{x} is defined as

$$\|\mathbf{x}\|^* = \sup_{\|\mathbf{p}\|_1 \leq 1} \Re\{\mathbf{p}^H \mathbf{x}\}. \quad (10)$$

Therefore, for the traditional LASSO problem [52], we have

$$\min_{\mathbf{x}} \frac{1}{2} \|\mathbf{y} - \mathbf{Ax}\|_2^2 + \lambda \|\mathbf{x}\|_1, \quad (11)$$

where \mathbf{x} is an unknown vector and \mathbf{y} is a known vector. \mathbf{A} is a measurement matrix. The LASSO problem tries to estimate the vector \mathbf{x} from the known vector \mathbf{y} and measurement matrix \mathbf{A} . The first term $1/2\|\mathbf{y} - \mathbf{Ax}\|_2^2$ describes the accuracy and the second term $\|\mathbf{x}\|_1$ is for the reconstruction sparsity. The parameter λ is used to control the tradeoff between the reconstruction accuracy and the sparsity of \mathbf{x} .

Then, the optimization problem in (11) can be rewritten as

$$\min_{\mathbf{x}, \mathbf{z}} \frac{1}{2} \|\mathbf{y} - \mathbf{z}\|_2^2 + \lambda \|\mathbf{x}\|_1 \quad (12)$$

$$\text{s.t. } \mathbf{z} = \mathbf{Ax}.$$

To solve this optimization problem, the corresponding Lagrangian can be expressed as

$$\mathcal{L}(\mathbf{x}, \mathbf{z}, \mathbf{l}) \triangleq \frac{1}{2} \|\mathbf{y} - \mathbf{z}\|_2^2 + \lambda \|\mathbf{x}\|_1 + \Re\{\mathbf{l}^H (\mathbf{z} - \mathbf{Ax})\}, \quad (13)$$

where \mathbf{l} is the vector of Lagrangian parameter, and $\Re\{\cdot\}$ denotes the real part of a complex value. Therefore, the dual optimization problem of (12) can be obtained as

$$\begin{aligned} \max_{\mathbf{l}} \min_{\mathbf{x}, \mathbf{z}} \mathcal{L}(\mathbf{x}, \mathbf{z}, \mathbf{l}) &= \max_{\mathbf{l}} \min_{\mathbf{x}, \mathbf{z}} \frac{1}{2} \|\mathbf{y} - \mathbf{z}\|_2^2 + \lambda \|\mathbf{x}\|_1 \\ &\quad + \Re\{\mathbf{l}^H (\mathbf{z} - \mathbf{Ax})\} \\ &= \max_{\mathbf{l}} \left\{ \min_{\mathbf{x}} \lambda \|\mathbf{x}\|_1 - \Re\{\mathbf{l}^H \mathbf{Ax}\} \right. \\ &\quad \left. + \min_{\mathbf{z}} \frac{1}{2} \|\mathbf{y} - \mathbf{z}\|_2^2 + \Re\{\mathbf{l}^H \mathbf{z}\} \right\}. \end{aligned} \quad (14)$$

Additionally, we have

$$\frac{\partial(1/2)\|\mathbf{y} - \mathbf{z}\|_2^2 + \Re\{\mathbf{l}^H \mathbf{z}\}}{\partial \mathbf{z}} = \frac{1}{2} (\mathbf{l} - \mathbf{y} + \mathbf{z})^H. \quad (15)$$

Let $1/2(\mathbf{l} - \mathbf{y} + \mathbf{z})^H = 0$, and we can obtain

$$\mathbf{z} = \mathbf{y} - \mathbf{l}. \quad (16)$$

Moreover, from the definition of dual norm, we can obtain [53]

$$\begin{aligned} \min_{\mathbf{x}} \lambda \|\mathbf{x}\|_1 - \Re\{\mathbf{l}^H \mathbf{Ax}\} &= -\lambda \max_{\mathbf{x}} \left\{ \Re\left\{\frac{1}{\lambda} (\mathbf{A}^H \mathbf{l})^H \mathbf{x}\right\} - \|\mathbf{x}\|_1 \right\} \\ &= I\left(\|\mathbf{A}^H \mathbf{l}\|_1^* \leq \lambda\right), \end{aligned} \quad (17)$$

where $\|\cdot\|_1^*$ denotes the dual norm of ℓ_1 norm. $I(\cdot)$ is an indicator function, which is defined as

$$I(f(x)) = \begin{cases} 0, & f(x) \text{ is true} \\ +\infty, & \text{otherwise.} \end{cases} \quad (18)$$

Therefore, substitute (16) and (17) in (14), and we can simplify the dual optimization problem as follows:

$$\max_{\mathbf{l}} \left\{ I\left(\|\mathbf{A}^H \mathbf{l}\|_1^* \leq \lambda\right) + \frac{1}{2} \|\mathbf{l}\|_2^2 + \Re\{\mathbf{l}^H (\mathbf{y} - \mathbf{l})\} \right\}, \quad (19)$$

which can be rewritten as

$$\begin{aligned} \min_{\mathbf{l}} & \|\mathbf{y} - \mathbf{l}\|_2^2 \\ \text{s.t. } & \|\mathbf{A}^H \mathbf{l}\|_1^* \leq \lambda. \end{aligned} \quad (20)$$

The original LASSO problem is transformed into the dual optimization problem using the dual norm, which can be solved more efficiently.

3.2. CDSR Method. We will define a new type of atomic norm, which can exploit the target sparsity in the spatial domain and also use the MIMO advantage. The new type of atomic norm [54–57] for MIMO radar is named as MIMO Atomic Norm (AN-MIMO), which is defined as

$$\|\mathbf{x}\|_{\text{AN-MIMO}, 0} \triangleq \inf \left\{ P: \mathbf{x} = \sum_{p=0}^{P-1} d_p \mathbf{b}(\theta_p) \otimes \mathbf{a}(\theta_p), \theta_p \in [0, 2\pi) \right\}, \quad (21)$$

where we use the subscript “AN-MIMO, 0” to show the decomposition for the signal \mathbf{x} with minimum number of terms. The AN-MIMO shows that we can use the minimum number of terms formulated by $\mathbf{b}(\theta_p) \otimes \mathbf{a}(\theta_p)$ to describe the signal \mathbf{x} . However, as shown in (21), we must determine the number of terms, i.e., the parameter P , to obtain the decomposition, which is nondeterministic polynomial (NP) hard and cannot be solved easily.

Inspired by the ℓ_1 norm used in CS theory, we can relax the original AN-MIMO by the following equation:

$$\begin{aligned} \|\mathbf{x}\|_{\text{AN-MIMO}} & \\ &\triangleq \inf \left\{ \sum_{p=0}^{P-1} |d_p|: \mathbf{x} = \sum_{p=0}^{P-1} d_p \mathbf{b}(\theta_p) \otimes \mathbf{a}(\theta_p), \theta_p \in [0, 2\pi) \right\}, \end{aligned} \quad (22)$$

where we use the footnote “AN-MIMO” to show that the relaxation equation is different from the original one in (21). The AN-MIMO is described by the sum of absolute values and can be solved by the following semidefinite programming-(SDP-) based method. The number of terms P can be obtained by searching the number of peaks in $|\mathbf{x}^H [\mathbf{b}(\theta) \otimes \mathbf{a}(\theta)]|^2$.

With the AN-MIMO, the received signal \mathbf{y} can be approximated by a signal \mathbf{x} with the sparse consideration. We can formulate the following optimization (denoising) problem as

$$\min \frac{1}{2} \|\mathbf{y} - \mathbf{x}\|_2^2 + \lambda \|\mathbf{x}\|_{\text{AN-MIMO}}, \quad (23)$$

which is a type of LASSO problem, but we use the AN-MIMO to replace the traditional ℓ_1 norm. As shown in (22), the AN-MIMO is defined as the sum of the absolute values $|d_p|$, which

are the coefficients in the atomic decomposition. In the ℓ_1 norm minimization problem with the vector \mathbf{x} , the ℓ_1 norm is defined as the absolute sum of the entries in \mathbf{x} . Therefore, the AN-MIMO norm is similar to the ℓ_1 norm, and both use the sum of the absolute values. Then, the method used in the ℓ_1 norm minimization problem can be introduced in the AN-MIMO problem. Similar to the LASSO problem in Section 3.1, the denoising problem in (24) can be simplified as

$$\begin{aligned} \min_{\mathbf{x}} \quad & \|\mathbf{y} - \mathbf{x}\|_2^2 \\ \text{s.t.} \quad & \|\mathbf{x}\|_{\text{AN-MIMO}}^* \leq \lambda, \end{aligned} \quad (24)$$

where $\|\mathbf{x}\|_{\text{AN-MIMO}}^*$ denotes the dual norm of AN-MIMO $\|\mathbf{x}\|_{\text{AN-MIMO}}$.

Proposition 1. *The optimization problem in (23) with the dual norm of AN-MIMO can be solved by the SDP method.*

$$\min_{\mathbf{x}, \mathbf{F}, g} \|\mathbf{y} - \mathbf{x}\|_2^2$$

$$\text{s.t. } \begin{bmatrix} \mathbf{F} & \mathbf{x} \\ \mathbf{x}^H & g \end{bmatrix} \geq 0$$

$$\mathbf{F} \succeq 0$$

$$0 < g \leq \lambda^2$$

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{0,0} & \mathbf{F}_{0,1} & \dots & \mathbf{F}_{0,N-1} \\ \mathbf{F}_{1,0} & \mathbf{F}_{1,1} & \dots & \mathbf{F}_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{F}_{N-1,0} & \mathbf{F}_{N-1,1} & \dots & \mathbf{F}_{N-1,N-1} \end{bmatrix} \quad (25)$$

$$\sum_{m_1} F_{n_1, n_2}^{m_1, m_1 + \Delta} = \begin{cases} \frac{1}{N}, & n_1 = n_2, \Delta = 0 \\ 0, & \text{otherwise,} \end{cases}$$

where \mathbf{x} denotes the approximation of received signal \mathbf{y} with the sparse constraint. The matrix \mathbf{F} is a semidefinite matrix and is composed of the block matrix \mathbf{F}_{n_1, n_2} . For the diagonal block matrices \mathbf{F}_{n_1, n_2} ($n_1 = n_2$), the sum of diagonal entries is $1/N$, and the sum of subdiagonal entries is 0. However, for the off-diagonal block matrices \mathbf{F}_{n_1, n_2} ($n_1 \neq n_2$), both the sum of the diagonal entries and that of the subdiagonal entries are 0.

In the optimization problem (25), the parameters \mathbf{x} , \mathbf{F} , and g are optimized and solved by minimizing $\|\mathbf{y} - \mathbf{x}\|_2^2$. The

optimal \mathbf{x} is denoted as the denoised signal from \mathbf{y} . The optimization problem in (25) is a type of SDP problem and can be solved by the interior-point method. According to Section 6.4 in [58], the computation complexity is $\mathcal{O}\{(MN)^3\}$ using the interior-point method to solve the SDP problem (25).

After obtaining the denoised signal \mathbf{x} , the DOA can be estimated by the spatial spectrum, which is calculated by $|\mathbf{x}^H [\mathbf{b}(\theta) \otimes \mathbf{a}(\theta)]|^2$.

3.3. Proof of Proposition 1. With the definition of dual norm in (10), the dual norm of AN-MIMO can be expressed as

$$\begin{aligned} \|\mathbf{x}\|_{\text{AN-MIMO}}^* &= \sup_{\|\bar{\mathbf{x}}\|_{\text{AN-MIMO}} \leq 1} \mathcal{R}\{\bar{\rho}^H \mathbf{x}\} \\ &\stackrel{(a)}{=} \sup_{\sum_{p=0}^{P-1} |d_p| \leq 1, \theta_p \in [0, 2\pi)} \mathcal{R}\left\{\left(\sum_{p=0}^{P-1} d_p \mathbf{b}(\theta_p) \otimes \mathbf{a}(\theta_p)\right)^H \mathbf{x}\right\} \\ &\stackrel{(b)}{=} \sup_{|d'| \leq 1, \theta' \in [0, 2\pi)} \mathcal{R}\{(d' \mathbf{b}(\theta') \otimes \mathbf{a}(\theta'))^H \mathbf{x}\} \\ &\stackrel{(c)}{=} \sup_{\theta \in [0, 2\pi)} |(\mathbf{b}(\theta) \otimes \mathbf{a}(\theta))^H \mathbf{x}|. \end{aligned} \quad (26)$$

The equation (a) is obtained from the definition of AN-MIMO, and the equation (b) is obtained by choosing the DOA θ' to maximize the correlation between $\mathbf{b}(\theta') \otimes \mathbf{a}(\theta')$ and \mathbf{x} . The equation (c) is obtained by letting

$$d' = \frac{\mathbf{x}^H (\mathbf{b}(\theta') \otimes \mathbf{a}(\theta'))}{|(\mathbf{b}(\theta') \otimes \mathbf{a}(\theta'))^H \mathbf{x}|}. \quad (27)$$

Therefore, based on (26), the constraint $\|\mathbf{x}\|_{\text{AN-MIMO}}^* \leq \lambda$ in the dual problem (24) can be simplified as

$$\sup_{\theta \in [0, 2\pi)} |(\mathbf{b}(\theta) \otimes \mathbf{a}(\theta))^H \mathbf{x}| \leq \lambda. \quad (28)$$

This inequality can be further simplified as a SDP problem. Before getting the SDP problem, we first introduce the Schur complement as follows.

Lemma 2. *Given a block matrix $\begin{bmatrix} \mathbf{A} & \mathbf{C} \\ \mathbf{B} & \mathbf{D} \end{bmatrix}$, the matrix is a semidefinite matrix if and only if the following conditions are satisfied: the block \mathbf{A} is a semidefinite matrix and $(\mathbf{A} - \mathbf{C}\mathbf{D}^{-1}\mathbf{B})$ is also a semidefinite matrix.*

With the Schur complement, we can formulate a semidefinite matrix $\begin{bmatrix} \mathbf{F} & \mathbf{x} \\ \mathbf{x}^H & g \end{bmatrix}$ ($\mathbf{F} \succeq 0$ and $g > 0$), then we have

$$\mathbf{F} - \mathbf{x}\mathbf{g}^{-1}\mathbf{x}^H \succeq 0. \quad (29)$$

Hence, from the definition of semidefinite matrix, we can find that for any vector \mathbf{u} , we have $\mathbf{u}^H (\mathbf{F} - \mathbf{x}\mathbf{g}^{-1}\mathbf{x}^H) \mathbf{u} \geq 0$, which can be rewritten as

$$\mathbf{u}^H \mathbf{x} \mathbf{x}^H \mathbf{u} \leq g \mathbf{u}^H \mathbf{F} \mathbf{u}. \quad (30)$$

By choosing $\mathbf{u} = \mathbf{b}(\theta) \otimes \mathbf{a}(\theta)$, (30) can be simplified as

$$\left| [\mathbf{b}(\theta) \otimes \mathbf{a}(\theta)]^H \mathbf{x} \right|^2 \leq g [\mathbf{b}(\theta) \otimes \mathbf{a}(\theta)]^H \mathbf{F} [\mathbf{b}(\theta) \otimes \mathbf{a}(\theta)], \quad (31)$$

where the left side is equal to the square of left side in (28).

Therefore, if we can let $g [\mathbf{b}(\theta) \otimes \mathbf{a}(\theta)]^H \mathbf{F} [\mathbf{b}(\theta) \otimes \mathbf{a}(\theta)] \leq \lambda^2$, the constraint (28) can be satisfied. Then, the dual optimization problem (24) can be simplified as

$$\begin{aligned} & \min_{\mathbf{x}, \mathbf{F}, g} \|\mathbf{y} - \mathbf{x}\|_2^2 \\ \text{s.t. } & \begin{bmatrix} \mathbf{F} & \mathbf{x} \\ \mathbf{x}^H & g \end{bmatrix} \geq 0 \\ & g > 0 \\ & \mathbf{F} \geq 0 \\ & g [\mathbf{b}(\theta) \otimes \mathbf{a}(\theta)]^H \mathbf{F} [\mathbf{b}(\theta) \otimes \mathbf{a}(\theta)] \leq \lambda^2. \end{aligned} \quad (32)$$

This optimization problem is a type of SDP problem, but cannot be solved directly, so we need to further simplify the expressions. Let $g \leq \lambda^2$, and we need to formulate the block matrix \mathbf{F} to satisfy

$$[\mathbf{b}(\theta) \otimes \mathbf{a}(\theta)]^H \mathbf{F} [\mathbf{b}(\theta) \otimes \mathbf{a}(\theta)] \leq 1. \quad (33)$$

We can express the left side as

$$\begin{aligned} [\mathbf{b}(\theta) \otimes \mathbf{a}(\theta)]^H \mathbf{F} [\mathbf{b}(\theta) \otimes \mathbf{a}(\theta)] &= \sum_{m_1=0}^{M-1} \sum_{n_1=0}^{N-1} \sum_{m_2=0}^{M-1} \sum_{n_2=0}^{N-1} [\mathbf{b}_{n_1}(\theta) \mathbf{a}_{m_1}(\theta)]^H F_{n_1 M + m_1, n_2 M + m_2} [\mathbf{b}_{n_2}(\theta) \mathbf{a}_{m_2}(\theta)] \\ &= \sum_{\Delta_1=1}^{M-1} \sum_{m_1=0}^{M-1-\Delta_1} \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} [\mathbf{b}_{n_1}(\theta) \mathbf{a}_{m_1}(\theta)]^H F_{n_1 M + m_1, n_2 M + m_1 + \Delta_1} [\mathbf{b}_{n_2}(\theta) \mathbf{a}_{m_1 + \Delta_1}(\theta)] \\ &\quad + \sum_{\Delta_1=-(M-1)}^{-1} \sum_{m_1=-\Delta_1}^{M-1} \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} [\mathbf{b}_{n_1}(\theta) \mathbf{a}_{m_1}(\theta)]^H F_{n_1 M + m_1, n_2 M + m_1 + \Delta_1} [\mathbf{b}_{n_2}(\theta) \mathbf{a}_{m_1 + \Delta_1}(\theta)] \\ &\quad + \sum_{m_1=0}^{M-1} \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} F_{n_1 M + m_1, n_2 M + m_1} b_{n_1}^H(\theta) b_{n_2}(\theta) \\ &= \sum_{\Delta_1=1}^{M-1} a_{\Delta_1}(\theta) \sum_{m_1=0}^{M-1-\Delta_1} \left\{ \sum_{\Delta_2=1}^{N-1} b_{\Delta_2}(\theta) \sum_{n_1=0}^{N-1-\Delta_2} F_{n_1 M + m_1, (n_1 + \Delta_2) M + m_1 + \Delta_1} \right. \\ &\quad \left. + \sum_{\Delta_2=-(N-1)}^{-1} b_{\Delta_2}(\theta) \sum_{n_1=-\Delta_2}^{N-1} F_{n_1 M + m_1, (n_1 + \Delta_2) M + m_1 + \Delta_1} + \sum_{n_1=0}^{N-1} F_{n_1 M + m_1, n_1 M + m_1 + \Delta_1} \right\} \\ &\quad + \sum_{\Delta_1=-(M-1)}^{-1} a_{\Delta_1}(\theta) \sum_{m_1=-\Delta_1}^{M-1} \left\{ \sum_{\Delta_2=1}^{N-1} b_{\Delta_2}(\theta) \sum_{n_1=0}^{N-1-\Delta_2} F_{n_1 M + m_1, (n_1 + \Delta_2) M + m_1 + \Delta_1} \right. \\ &\quad \left. + \sum_{\Delta_2=-(N-1)}^{-1} b_{\Delta_2}(\theta) \sum_{n_1=-\Delta_2}^{N-1} F_{n_1 M + m_1, (n_1 + \Delta_2) M + m_1 + \Delta_1} + \sum_{n_1=0}^{N-1} F_{n_1 M + m_1, n_1 M + m_1 + \Delta_1} \right\} \\ &\quad + \sum_{m_1=0}^{M-1} \left\{ \sum_{\Delta_2=1}^{N-1} b_{\Delta_2}(\theta) \sum_{n_1=0}^{N-1-\Delta_2} F_{n_1 M + m_1, (n_1 + \Delta_2) M + m_1} + \sum_{\Delta_2=-(N-1)}^{-1} b_{\Delta_2}(\theta) \sum_{n_1=-\Delta_2}^{N-1} F_{n_1 M + m_1, (n_1 + \Delta_2) M + m_1} \right. \\ &\quad \left. + \sum_{n_1=0}^{N-1} F_{n_1 M + m_1, n_1 M + m_1} \right\}. \end{aligned} \quad (34)$$

Therefore, we can formulate the matrix \mathbf{F} as a following block matrix

$$\mathbf{F} \triangleq \begin{bmatrix} \mathbf{F}_{0,0} & \mathbf{F}_{0,1} & \dots & \mathbf{F}_{0,N-1} \\ \mathbf{F}_{1,0} & \mathbf{F}_{1,1} & \dots & \mathbf{F}_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{F}_{N-1,0} & \mathbf{F}_{N-1,1} & \dots & \mathbf{F}_{N-1,N-1} \end{bmatrix}, \quad (35)$$

where $\mathbf{F}_{n_1,n_2} \in \mathbb{C}^{M \times M}$ ($n_1 = 0, 1, \dots, N-1$ and $n_2 = 0, 1, \dots, N-1$). For all blocks, \mathbf{F}_{n_1,n_2} has the following characteristics:

$$\sum_{m_1} F_{n_1,n_2}^{m_1,m_1+\Delta} = \begin{cases} \frac{1}{N}, & n_1 = n_2, \Delta = 0 \\ 0, & \text{otherwise,} \end{cases} \quad (36)$$

where $F_{n_1,n_2}^{m_1,m_1+\Delta}$ denotes the entry of \mathbf{F}_{n_1,n_2} at the m_1 -th row and $(m_1 + \Delta)$ -th column. When the characteristics in (36) is satisfied, we can find that the result in (34) is 1. Therefore, the condition in (33) is also satisfied.

Finally, the optimization problem in (32) is simplified as

$$\min_{\mathbf{x}, \mathbf{F}, g} \|\mathbf{y} - \mathbf{x}\|_2^2$$

$$\text{s.t. } \begin{bmatrix} \mathbf{F} & \mathbf{x} \\ \mathbf{x}^H & g \end{bmatrix} \geq 0$$

$$\mathbf{F} \geq 0$$

$$0 < g \leq \lambda^2$$

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{0,0} & \mathbf{F}_{0,1} & \dots & \mathbf{F}_{0,N-1} \\ \mathbf{F}_{1,0} & \mathbf{F}_{1,1} & \dots & \mathbf{F}_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{F}_{N-1,0} & \mathbf{F}_{N-1,1} & \dots & \mathbf{F}_{N-1,N-1} \end{bmatrix} \quad (37)$$

$$\sum_{m_1} F_{n_1,n_2}^{m_1,m_1+\Delta} = \begin{cases} \frac{1}{N}, & n_1 = n_2, \Delta = 0 \\ 0, & \text{otherwise.} \end{cases}$$

4. Simulation Results

We will show the simulation results in this section, where the simulation parameters are shown in Table 1. We consider the

TABLE 1: Simulation parameters.

Parameter	Value
The SNR of received signals	20 dB
The number of receiving antennas N	10
The number of transmitting antennas M	5
The number of targets K	3
The space between antennas d	0.5 wavelength
The spatial angles	$[-80^\circ, 80^\circ]$

DOA estimation problem in the scenario with the MIMO radar system, where the transmitter and the receiver are colocated. Without additional explanation, the default parameters are the same with Table 1. All the simulation results are obtained from a Work Station with 64G Byte RAM and Intel i7 CPU (3.40 GHz). The MATLAB code for the proposed method is available online: https://drive.google.com/drive/folders/1_iFF1XEQhKOI7-6Y9uKcy8kK6h6tRbtD?usp=sharing.

In this section, three state-of-art methods are compared with the proposed method:

- (i) *Direct Method.* In this method, the correlation between the received signal \mathbf{y} and the steering vector $\mathbf{b}(\theta) \otimes \mathbf{a}(\theta)$ is calculated as $\mathbf{y}^H [\mathbf{b}(\theta) \otimes \mathbf{a}(\theta)]$ to get the spatial spectrum for the DOA estimation.
- (ii) *OMP Method.* Orthogonal matching pursuits (OMP) method [59] is a typical method for sparse reconstruction. Since our method is a sparse-based DOA estimation method, this comparison is needed.
- (iii) *SBL Method.* Sparse Bayesian learning (SBL) method [44] is a Bayesian-based method for the sparse reconstruction. Usually, this method can achieve better reconstruction performance, but with relatively high computational complexity.

First, we show the estimated spatial spectrum for the DOA estimation in Figure 2, where the number of receiving antennas is $N = 10$. The estimated DOAs using different methods are given in Table 2. The parameter λ is chosen as $\lambda = 2\sqrt{MN \log MN} \sigma_n$. As shown in this figure, the DOAs of targets are, respectively, -29.474° , 0.68532° and 40.836° . When we use the proposed method, we can get the estimated DOAs as -29.57° , 0.58° and 41.01° . To describe the DOA estimation performance, we use the root mean square error (RMSE) to measure the errors, and the RMSE is defined as

$$\text{RMSE} \triangleq \sqrt{\frac{1}{N_{mc} K} \sum_{n_{mc}=0}^{N_{mc}-1} \left\| \hat{\theta}_{n_{mc}} - \theta \right\|_2^2}, \quad (38)$$

where N_{mc} denotes the number of Monte Carlo simulations and $N_{mc} = 10^3$ in this paper. $\hat{\theta}_{n_{mc}}$ is the estimated DOA during the n_{mc} -th Monte Carlo simulation, and θ is the ground-truth DOA. We can calculate the RMSE (in degree) using the proposed 0.1300. When the direct method is used to estimate the DOA, the RMSE is 0.4508. The RMSEs of OMP and SBL methods are, respectively, 0.4082 and 0.3985.

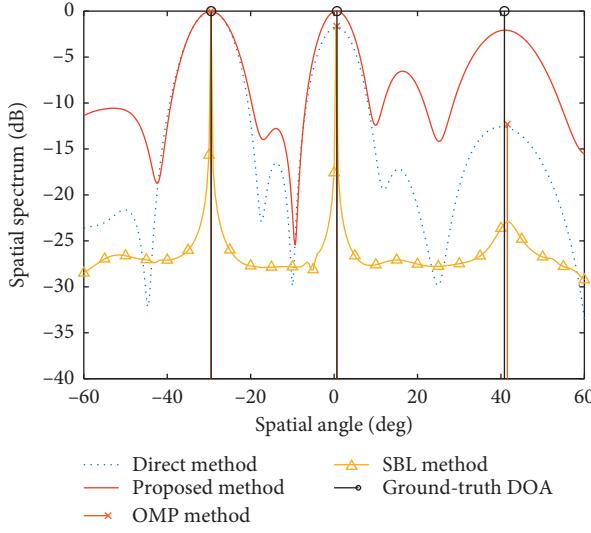


FIGURE 2: The spatial spectrum for DOA estimation using 10 receiving antennas.

TABLE 2: DOA estimation (10 receiving antennas).

Methods	Target 1 (°)	Target 2 (°)	Target 3 (°)	RMSE (deg)
Ground-truth DOA	-29.474	0.68532	40.836°	—
Proposed method	-29.57	0.58	41.01°	0.1300
Direct method	-29.5	0.6	40.06°	0.4508
OMP method	-29.5	0.63	41.54°	0.4082
SBL method	-29.5	0.5	41.5°	0.3985

We can find that the proposed method achieves better DOA estimation performance.

Second, in the scenario with less receiving antennas, we give the simulation results for the DOA estimation in Figure 3, where the number of receiving antennas is $N = 4$. As shown in this figure, we can find that the spatial spectrum is worse than the scenario with $N = 10$ receiving antennas (Figure 2). The corresponding DOA estimation results are given in Table 3. The RMSE of the proposed method is 0.7009 in degree. The RMSEs of existing methods including the direct method, OMP method, and SBL method are 1.0756, 1.0591, and 0.7623, respectively. Therefore, in the scenario with less receiving antennas, the proposed method also achieves better performance of the DOA estimation.

Then, Figures 4 and 5 are, respectively, the DOA estimation performance with different SNRs of the received signals, where the number of receiving antennas is $M = 10$ in Figure 4 and that in Figure 5 is $M = 4$. From these figures, we can find that the proposed method can achieve better performance of DOA estimation in the scenarios with relative high SNR of received signals. Additionally, with $M = 10$, the proposed method has better performance at $\text{SNR} > 25 \text{ dB}$, but the better performance can be achieved at $\text{SNR} > 15 \text{ dB}$ with $N = 4$. Therefore, with more antennas, the performance improvement is significant at higher SNR. The corresponding Cramér-Rao lower bound (CRLB) for the

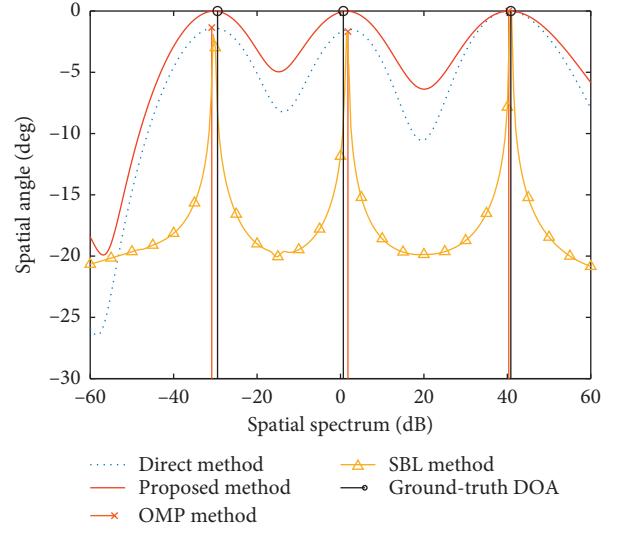


FIGURE 3: The spatial spectrum for DOA estimation using 4 receiving antennas.

TABLE 3: DOA estimation (4 receiving antennas).

Methods	Target 1 (°)	Target 2 (°)	Target 3 (°)	RMSE (deg)
Ground-truth DOA	-29.474	0.68532	40.836	—
Proposed method	-30.28	1.54	40.53	0.7009
Direct method	-30.28	2.29	40.34	1.0756
OMP method	-30.86	1.78	40.34	1.0591
SBL method	-30.5	1.5	41	0.7623

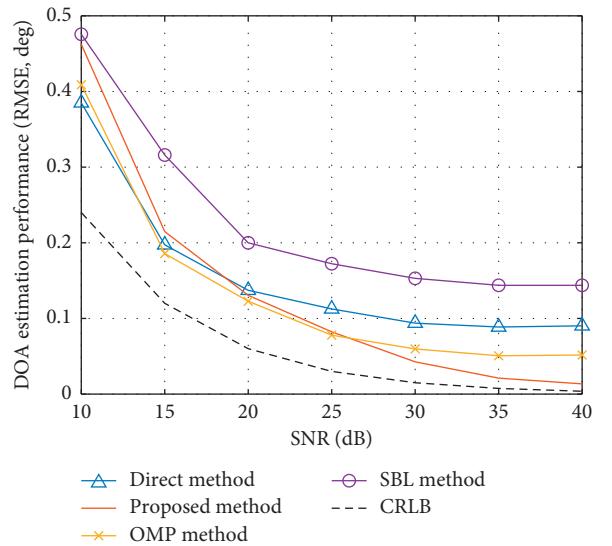


FIGURE 4: The DOA estimation performance with different SNRs (10 receiving antennas).

DOA estimation can be obtained from [60, 61]. In both Figures 4 and 5, the CRLBs for the DOA estimation are shown. As shown in the figures, the proposed method can

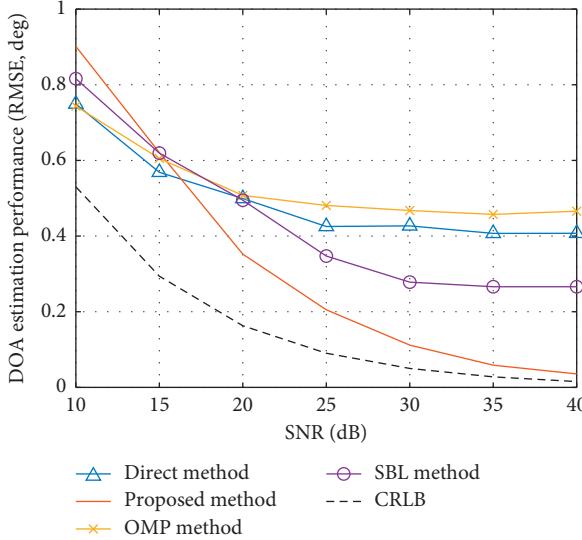


FIGURE 5: The DOA estimation performance with different SNRs (4 receiving antennas).

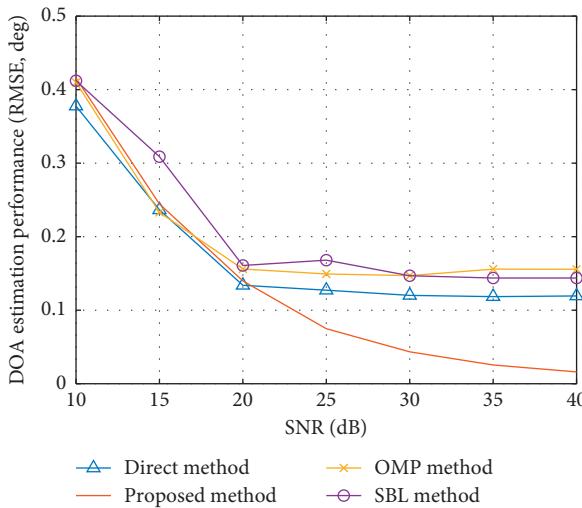


FIGURE 6: The DOA estimation performance with different SNRs (10 transmitting antennas).

approach the CRLB in the scenario with higher SNR ($\text{SNR} \geq 30 \text{ dB}$).

Finally, the DOA estimation performance with more transmitting antennas is shown in Figure 6, where 10 transmitting antennas are adopted. As shown in this figure, we can find that the proposed method achieves better DOA estimation performance at $\text{SNR} > 20 \text{ dB}$. Additionally, the performance improvement is more significant in the scenario with higher SNR of the received signals. Compared with Figure 5 using only 5 transmitting antennas, the DOA estimation performance is improved with more antennas. Therefore, the proposed method can achieve better performance of DOA estimation in the scenario with higher SNR, especially using fewer antennas. In Table 4, the computational time is given for the different methods. The computational time of the proposed method is 0.6037 s.

TABLE 4: Computational time.

Methods	Computational time (s)
Proposed method	0.6037
Direct method	0.02321
OMP method	0.1051
SBL method	1.0702

The direct and OMP methods have lower computational time than the proposed method. The SBL method has higher computational complexity with the computational time being 1.0702 s. Therefore, the proposed method can achieve better performance for the DOA estimation with the acceptable computational complexity.

5. Conclusion

The DOA estimation problem in the MIMO radar system with fast-moving targets has been addressed, and the new type of atomic norm has been formulated to measure the target sparsity in the spatial domain. The received signals have been denoised by the LASSO-based model. Then, the semidefinite matrix has been constructed to transform the atomic norm minimization problem into the SDP problem, which can be solved efficiently. Simulation results show that the proposed method achieves better DOA estimation performance in the MIMO radar with fast-moving targets. The future work will focus on the radar system optimization to further improve the DOA estimation performance.

Data Availability

The simulation data used to support the findings of this study are included within the article. Additionally, the Matlab codes for the simulations are available online: https://drive.google.com/drive/folders/1_iFF1XEQhKOI7-6Y9uKcy8kK6h6tRbtD?usp=sharing.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China (Grant no. 61976113).

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