Research Article

Application of a Fractional Grey Prediction Model Based on a Filtering Algorithm in Image Processing

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Image filtering can change or enhance an image by emphasizing or removing certain features of the image. An image is a system in which some information is known and some information is unknown. Grey system theory is an important method for dealing with this kind of system, and grey correlation analysis and grey prediction modeling are important components of this method. In this paper, a fractional grey prediction model based on a filtering algorithm by combining a grey correlation model and a fractional prediction model is proposed. In this model, first, noise points are identified by comparing the grey correlation and the threshold value of each pixel in the filter window, and then, through the resolution coefficient of the important factor in image processing, a variety of grey correlation methods are compared. Second, the image noise points are used as the original sequence by the filter pane. The grey level of the middle point is predicted by the values of the surrounding pixel points combined with the fractional prediction model, replacing the original noise value to effectively eliminate the noise. Finally, an empirical analysis shows that the PSNR and MSE of the new model are approximately 27 and 140, respectively; these values are better than those of the comparison models and achieve good processing effects.

1. Introduction

Image information is the main source of external information for humans. Image information processing is more important than image acquisition for various reasons, especially in the era of rapid scientific and technological development. In this era, there are stronger requirements for image information processing, and the main objective is to obtain useful information more quickly, accurately, and reliably. Regardless of whether an image is macroscopic or microscopic, the image is inevitably contaminated by various types of noise during acquisition and transmission due to camera sensor failure, transmission errors, the environment, and other factors. Noise not only affects the visual quality of an image but also masks the original useful information in the image, thereby interfering with the analysis and extraction of the target of interest and thus affecting the qualitative or quantitative investigation of the image information by a computer.

When an image is contaminated by salt-and-pepper noise, some of the pixels are replaced by the maximum and minimum values of the greyscale dynamic range of the image, which are denoted by $L_{\max}$ and $L_{\min}$, respectively. This method produces pixels in the image which are too dark or too bright, which severely alters the visual effect of the image. However, such pixels are easily confused with the image edge pixels, which renders the detection of image edges and textures, among other features, difficult. Many denoising algorithms for salt-and-pepper noise are available [1–4]. However, the effective filtering and detailed protection of salt-and-pepper noise in the case of high-density noise remain challenging. These are difficult and hot topics for researchers. Such problems are challenging mainly because, under high-density noise, the amount of useful information in an indefinite image is too small, the noise distribution is uncertain, and the image and noise exhibit grey characteristics.

2. Literature Review

2.1. Image Processing Models. For image processing, Chang et al. [5] introduced a center-weighted median (CWM) filter for adaptive media filtering. Based on the minimum absolute value of four convolutions obtained using one-dimensional Laplacian operators, Zhang et al. [6] proposed switching filters. Khan [7] proposed a new hybrid technology combining gradient calculation with an adaptive trimmed mean filter (ATMF) to reduce Poisson noise in Flickr images. Then, he proposed a new approach for denoising medical images called a weighted gradient filter [8] and used a two-dimensional adaptive trimmed mean autoregressive (ATMAR) model to remove Poisson noise from medical images [9].

2.2. Research Progress of Grey Models in Image Processing. The models in Section 2.1 have achieved good results in image processing, but an image itself is a system in which part of the information is known and part of the information is unknown. The grey model is an important method for dealing with such systems. Grey system theory [10] is a theoretical method for studying a small sample of data with poor information under high uncertainty. The small sample and poor information of uncertain systems in which some information is known and some information is unknown characterize grey system research. If an image has grey characteristics, the grey levels of the image pixels will always be within a certain range, and the image will have an insufficient amount of useful information, which fails to provide a reliable basis for judgment. In recent years, grey systems have received extensive attention from researchers in the field of image engineering. When a grey system is affected by many complicated factors, which renders analysis difficult, the important factors can be quickly analyzed to solve the problem. This section reviews the literature on grey model image processing.

The grey correlation and grey prediction model are important components of a grey system. The grey correlation is mainly used in image compression [11–14], edge detection [15, 16], image denoising [17, 18], and image enhancement [19]. Although the grey correlation degree-based filtering algorithm has yielded satisfactory results, many problems remain to be solved in the noise removal process. At present, there are many kinds of grey correlation methods, and it is unclear which one is best, which makes choosing one difficult for researchers. Therefore, this paper compares and summarizes several typical grey correlation models. The grey prediction model is adaptable and can handle mutation parameter changes, which can effectively solve this problem. Hsieh Cheng Hsiung and Tu Ching Chi [20] regard salt-and-pepper noise in an image as discrete extreme points and use them to establish a grey prediction model for the surrounding pixels which predicts the value of each point. It is a feasible and effective method for removing salt-and-pepper noise. At present, the fractional grey prediction

model is one of the most common prediction models, and it is effective in other applications. Therefore, this paper applies a fractional grey prediction model to remove image noise.

However, since the correlations reflected by the degree of relevance differ, the methods and expressions for the degree of relevance also differ, and the degrees of relevance are diverse. The first definition of the degree of relevance is Deng's grey correlation, which was proposed by Deng Julong in [10]. Subsequently, Liu [21] proposed the absolute grey correlation and the relative grey correlation, which depend on the practical scenario; since then, new correlation methods [22], matrix relevance measures [23], and hundreds of forms of grey correlation have been proposed. Depending on the nature of the response, it is challenging to select the degree of relevance. Image processing mainly involves analyzing the sensitivity of the resolution coefficient to determine the degree of association, and Deng's grey correlation provides such a resolution coefficient. Therefore, as a correlation in image edge detection and denoising, Deng's grey correlation has an advantage.

The grey forecasting model has been widely studied and optimized [24–27] since it was proposed, and it has been widely used in various fields [28–33]. Many forecasting models have been proposed, such as the grey model with one variable and one first-order equation (GM(1,1)), which is suitable for exponential growth; the discrete grey model with one variable and one first-order equation (DGM(1,1)) for homogeneous exponential growth, the nonhomogeneous discrete grey model (NDGM) with one variable and one first-order equation (NDGM(1,1)) for nonhomogeneous exponential growth, the single-variable Verhulst model for saturated S-type or single-peak sequences, the multivariable GM(1, n) model, the multivariable nonlinear grey model (NGM(1, n)) [34–38], and the new grey forecasting model [30, 39–41]. At present, the fractional grey prediction model is widely used to extend integer-order accumulation to fractional-order accumulation. The fractional grey model (FGM) introduced by Wu et al. [42, 43] is the most prominent innovation of nonlinear grey models, and it is very effective in improving the accuracy of the existing grey model. Duan et al. [44] proposed a discrete FGM and an iterative optimization algorithm to predict China's total crude oil production. Zeng et al. [45] established a dynamic grey model by fractional accumulation and demonstrated its applicability in Hong Kong. Ma et al. [46, 47] established a fractional grey prediction model for hysteresis terms by using fractional-order accumulation and made accurate energy and economic predictions. Wu et al. [48] introduced the general formula of the FGM, which is more effective than most existing multivariable linear grey models. Wu et al. [49] proposed a new fractional time delay grey model to predict China's renewable energy consumption. Mao et al. [50, 51] proposed a new fractional GM(1, N , t) model and FGM(q , 1) model and applied them to different fields. Meng et al. [52] applied a fractional discrete model to predict carbon emissions.

2.3. Contribution and Organization. In this paper, the image processing filtering algorithm is combined with the current FGM, and a grey prediction model based on the fractional order of the filtering algorithm is proposed. The main contributions of this paper are as follows.

2.3.1. Image Noise Point and Edge Detection. According to the importance of the resolution coefficient in an image, Deng's grey correlation has an absolute advantage over many grey correlation methods. Comparing Deng's grey correlation with other typical correlation methods in image edge detection experiments verifies the advantage of Deng's grey correlation in image edge processing, and the optimal threshold range is demonstrated through empirical analysis.

2.3.2. Noise Point Processing. By combining the fractional-order model and the particle swarm optimization (PSO) algorithm, an improved fractional-order NDGM is proposed. The original noise value is replaced by the value of the surrounding pixels, and the grey value of the center point is predicted by the new model to effectively eliminate noise.

As shown by the empirical analysis, the Duns and Ros correlation has obvious advantages over other typical grey correlation metrics in image edge detection, and the image processing effect of the fractional-order NDGM is better than that of other models compared in this paper.

In the following sections, we use different abbreviations for different grey prediction models. The abbreviations and their definitions are listed in Table 1.

The remainder of this paper is organized as follows: Section 2 introduces the properties of the proposed grey correlation model and several typical grey correlation models. Section 3 demonstrates how to identify noise in an image by using filtering algorithms and grey correlation models and compares the performances of various filtering algorithms and grey correlation models in image edge detection. Section 4 establishes the fractional filtering NDGM model for processing image noise. The validity of the model is evaluated. In Section 5, the conclusions of this work are presented.

3. Grey Relational Analysis

This section introduces several grey relational models for grey systems.

3.1. Introduction to Grey Relational Analysis. Set the reference sequence of the grey system as

$$x_0 = \{x_0(k) | k = 1, 2, \dots, n\}. \quad (1)$$

Set the comparison sequence of the grey system as

$$x_i = (x_i(1), x_i(2), \dots, x_i(n)), \quad i = 1, 2, \dots, m. \quad (2)$$

Then, the correlation number of reference sequence x_0 with comparison sequence x_i is

$$\xi_{i,0}(k) = \frac{\min_i \min_k |x_0(k) - x_i(k)| + \rho \min_i \min_k |x_0(k) - x_i(k)|}{|x_0(k) - x_i(k)| + \rho \min_i \min_k |x_0(k) - x_i(k)|}. \quad (3)$$

Formula (3) is called Deng's grey correlation, where ρ is a constant (typically 0.5), and $\rho \in (0, 1)$. The correlation coefficient describes the relative difference between x_i and x_0 at time K ; namely, the correlation coefficient is the degree of similarity of various x_i relations with respect to a reference sequence x_0 at time K .

The number of correlation coefficients $\xi_{i,0}(k)$ depends on the number of compared sequences x_i . Therefore, the results of the calculation tend to be scattered and are not advantageous for the overall measurement of the system. Then, the new grey correlation method is defined as follows:

$$r(X_i, X_0) = \frac{1}{n} \sum_{i=1}^n \xi_{i,0}(k). \quad (4)$$

Equation (4) reflects a degree of similarity between the entire comparison sequence and the entire reference sequence.

Using the above definition, the following calculation steps can be performed:

- (1) The normalized sequence, that is, the difference in the numerical value that results from the removal of a dimension from the sequence, is obtained:

$$y_i = x_i D = (y_i(1), y_i(2), \dots, y_i(n)), \quad i = 0, 1, \dots, m. \quad (5)$$

- (2) The difference sequence of the newly obtained sequence is obtained as follows:

$$\Delta_{0,i}(k) = |y_0(k) - y_i(k)|, \quad i = 0, 1, \dots, m, k = 1, 2, \dots, n. \quad (6)$$

- (3) The maximum and minimum differences of the difference sequence are calculated:

$$M = \max_i \max_k \Delta_{0,i}(k), m = \max_i \max_k \Delta_{0,i}(k). \quad (7)$$

- (4) The grey correlation degree of the reference sequence and each of the scattered comparison sequences is calculated:

$$r_{0,i}(k) = \frac{m + \rho M}{\Delta_{0,1}(k) + \rho M}, \quad \rho \in (0, 1). \quad (8)$$

- (5) Finally, the grey correlation degree between the sequences is obtained:

TABLE 1: Abbreviations and corresponding definitions for different grey prediction models.

Index	Abbreviation	Definition
1	MF	Mean filter
2	AMF	Adaptive median filter
3	WMF	Weighted median filter
4	IWT	Joint wavelet transform scheme using the iterative noise density and median filtering
5	SF	Switching filter
6	CA	Image noise filter based on cellular automata
7	GM(1,1)	Grey model with one variable and one first-order equation
8	NDGM(1,1)	Nonhomogeneous discrete grey model with one variable and one first-order equation
9	ATMAR	Adaptive trimmed mean autoregressive
10	ATMF	Adaptive trimmed mean filter
11	NDGM ^{p/q} (1,1)	Nonhomogeneous discrete grey model with one variable and fractional-order accumulation equation

$$r\left(X_i, X_0 = \frac{1}{n} \sum_{i=1}^n r_{0,i}(k), i = 0, 1, 2, \dots, m\right). \quad (9)$$

A few other typical grey correlation methods are defined as follows.

Define complex vectors $X = (x(1), x(2), \dots, x(n))$ and $X_0 = (x_0(1), x_0(2), \dots, x_0(n))$, where the plural number is $x(k) = a(k) + b(k)i$, $k = 1, 2, \dots, n$. Let $|x(k)| = \sqrt{a^2(k) + b^2(k)}$, $k = 1, 2, \dots, n$ be the 2-norm of the specified vector. Then, the following norm is defined:

$$\|X\| = \left[\sum_{k=1}^n |x(k)|^2 \right]^{1/2}. \quad (10)$$

Definition 1. The starting point annihilation operator is denoted by D :

$$XD = (x(1)d, x(2)d, \dots, x(n)d), \quad (11)$$

where $x(k)d = x(k) - x(1)$, $k = 1, 2, \dots, n$. Then, the image of the starting (zero) point $s = XD$ is obtained and

$$\|s\| = \left[\sum_{k=1}^n |x(k) - x(1)|^2 \right]^{1/2}. \quad (12)$$

The initial value is

$$X' = \left(\frac{x(1)}{x(1)}, \frac{x(2)}{x(1)}, \dots, \frac{x(n)}{x(1)} \right). \quad (13)$$

The image of the initial (zero) point after initialization is calculated as $s' = X'D$. Then,

$$\|s'\| = \left[\sum_{k=1}^n |x(k)/x(1) - 1|^2 \right]^{1/2} \quad (14)$$

Definition 2. Let vector $X_i = (x_i(1), x_i(2), \dots, x_i(n))$, $i = 1, 2, \dots, n$, where $s_i = X_i D$. Then,

$$\begin{aligned} \|s_i\| &= \left[\sum_{k=1}^n |x(k) - x_i(1)|^2 \right]^{1/2} \|s_i - s_j\| \\ &= \left[\sum_{k=1}^n |x_i(k) - x_i(1) - x_j(k) + x_j(1)|^2 \right]^{1/2}. \end{aligned} \quad (15)$$

The initial value is

$$X_i^i = \left(\frac{x_i(1)}{x_i(1)}, \frac{x_i(2)}{x_i(1)}, \dots, \frac{x_i(n)}{x_i(1)} \right). \quad (16)$$

Let $s' = X'D$. Then,

$$\begin{aligned} \|s'_i\| &= \left[\sum_{k=1}^n |x_i(k)/x_i(1) - 1|^2 \right]^{1/2} \|s'_j - s'_i\| \\ &= \left[\sum_{k=1}^n |x_i(k)/x_i(1) - x_j(k)/x_j(1)|^2 \right]^{1/2}. \end{aligned} \quad (17)$$

Various grey correlation methods are defined as follows:

(1) Absolute grey correlation:

$$\varepsilon_{ij} = \frac{1 + \|s_i\| + \|s_j\|}{1 + \|s_i\| + \|s_j\| + \|s_i - s_j\|}. \quad (18)$$

(2) Relative grey correlation:

$$r_{ij} = \frac{1 + \|s'_i\| + \|s'_j\|}{1 + \|s'_i\| + \|s'_j\| + \|s'_i - s'_j\|}. \quad (19)$$

(3) Similarity correlation:

$$\varepsilon_{ij} = \frac{1}{1 + \|s_i - s_j\|}. \quad (20)$$

(4) Proximity correlation:

$$r_{ij} = \frac{1}{1 + \|S_i - S_j\|}. \quad (21)$$

- (5) The grey angle correlation is similar to the relevance definition, which satisfies the following formulas:

$$\begin{aligned} \varepsilon_{ij} &= \frac{1}{1 + \theta_{ij}}, \theta_{ij} = \arccos \frac{X_i \cdot X_j}{\|X_i\| \|X_j\|} \in [0, \pi] r(X_i, X_0) \\ &= \frac{1}{n} \sum_{i=1}^n r_{0,i}(k), i = 0, 1, 2, \dots, m. \end{aligned} \quad (22)$$

4. Application of Grey Correlation Analysis in the Image Filtering Process

4.1. Grey Correlation Degree-Based Image Noise Point Detection. The first step is to extract the sequence of features and factors in the system. The system feature sequence is recorded as

$$X_0 = (x_0(1), x_0(2), \dots, x_0(n)) \quad (23)$$

where n represents the number of components in the sequence and $x_0(n)$ represents the value of the first component in the system feature sequence.

The initial sequence of the system feature sequence, x'_0 , and the factor sequence, x'_i , are calculated as follows:

$$\begin{aligned} X'_0 &= \frac{X_0}{x_0(1)}, X'_i = \frac{X_i}{x_i(1)} = \{x'_i(1), x'_i(2), \dots, x'_i(n)\} \\ (i &= 1, 2, \dots, k). \end{aligned} \quad (24)$$

The initial value sequence is obtained using the grey correlation defined above. In an image system, the noise of an image is isolated and has little correlation with the surrounding pixels, which can be expressed as a small degree of grey correlation with the whole image. However, for normal points, the relationship will be very close, and the grey correlation degree will be larger. Therefore, after setting a threshold that satisfies the requirements of the task, the noise points can be removed from the image.

4.2. Image Denoising Algorithm Based on the Grey Correlation. Some of the information in the grey system is known. Corresponding to the image system, an image can be divided into edge points and nonedge points.

The selected filter window is of size 3×3 . The average of the grey values across all the points in the window is regarded as the characteristic sequence of the system, and the grey values of all the pixels are regarded as the sequence of factors.

The window for a normal pixel $x(i, j)$ in the 3×3 image is expressed as follows:

$$\begin{pmatrix} x(i-1, j-1) & x(i-1, j) & x(i-1, j+1) \\ x(i, j-1) & x(i, j) & x(i, j+1) \\ x(i+1, j-1) & x(i+1, j) & x(i+1, j+1) \end{pmatrix}. \quad (25)$$

The sequence of features in the image system can be expressed as

$$X_0 = \frac{\sum_{n=j-1}^{j+1} \sum_{m=i-1}^{i+1} x(m, n)}{9}. \quad (26)$$

The sequence of factors in the image system can be expressed as

$$\begin{aligned} X_1 &= \{x(i-1, j-1)\}, X_2 = \{x(i-1, j)\}, X_3 = \{x(i-1, j+1)\}, \\ X_4 &= \{x(i, j-1)\}, X_5 = \{x(i, j)\}, X_6 = \{x(i, j+1)\}, \\ X_7 &= \{x(i+1, j-1)\}, X_8 = \{x(i+1, j)\}, X_9 = \{x(i+1, j+1)\}. \end{aligned} \quad (27)$$

The average of the grey values across all the pixels in the window is calculated. This average is expanded into a feature sequence of the system, which is one-dimensional. The systematic factor sequence, which is also one-dimensional, corresponds to the grey values of the 9 pixels. Next, using various correlation degree calculation methods, the grey correlation degree of the center point is calculated. If the degree of association is less than the specified threshold value, then the point is classified as salt-and-pepper noise.

Next, the noise is reassigned. First, the association degree sequence is constructed; namely, all the point relevance values in the window are sorted, and the median of the sequence is selected as the second threshold. Then, the correlation values of the points in the window are sequentially compared with the second threshold, and any point for which the correlation value exceeds the threshold is classified as a normal point. Thus, the two threshold constraints eliminate the noise points and normal points not related to the grey value of the window center point. These normal points will be processed later. Next, summation is performed, and the maximum grey correlation value in the window is subtracted from the sum to obtain an average value. The new value is assigned to the center point of the window for reevaluation. A flow diagram of the algorithm is shown in Figure 1.

4.3. Grey Correlation Degree-Based Image Edge Detection Theory. In the improved image, a jump in the maximum grey value in the image indicates that the changed point is an edge point of the image. If the center of the window is an edge point, most of the pixels around it are highly correlated with each other, and their correlation degrees are relatively large. The grey correlation values of individual points are relatively low but similar; if the center of the window is not an edge point, all the points in the window have a high degree of grey correlation.

For the pixel points in the center of the window of interest, if there are five or more points whose grey values are similar and the differences among the grey values of three or more points are large, the center point is judged to be an edge point. Similarly, if the grey values of all the points in the template do not differ substantially, then the center point is classified as a nonedge point. Hence, a threshold can be set for screening. However, the accuracy of the threshold

FIGURE 1: Basic flow of the noise point detection algorithm.

determines the noise detection accuracy. Therefore, the noise threshold value affects the performance of the switching filter (SF) algorithm. However, due to the randomness, uncertainty, and grey level of the noise, the thresholds for a noise-contaminated pixel for a normal image with a consistent noise density or the same image with different noise densities may not be exactly the same. There is uncertainty in the noise threshold within a range, and an ideal threshold does not exist. Since the grey correlation degree is extended by the grey value between pixels, the difference in the grey correlation degree is used as the criterion for threshold judgment, and the dependence on the image can be removed to realize an adaptive algorithm architecture.

Taking the eight-directional filtering window as an example, a flow diagram of the algorithm is shown in Figure 2.

After traversing every pixel of the image, the algorithm is used to classify the edge points and nonedge points in the image; the edge points are assigned a value of 255, whereas the nonedge points are assigned a value of 0.

FIGURE 2: Basic flow of the filtering algorithm.

4.4. Grey Relational Analysis of the Experimental Results of Image Denoising. The image processing experiment was conducted using MATLAB 2016b, and the classic Lena image for image processing was selected for this study.

The original and greyscale Lena images are shown in Figures 3(a) and 3(b), respectively.

The extraction effects of various grey correlation methods for various thresholds are compared in the following.

First, the image denoising threshold of the algorithm is varied to determine a suitable range of values. Here, the threshold for edge extraction is stabilized in the interval by which the edge of the image can be extracted.

For a threshold value of 0.8, the edge extraction effects of each grey correlation calculation method are shown in Figure 4.

For a threshold value of 0.85, the edge extraction effects of each grey correlation calculation method are shown in Figure 5.

For a threshold value of 0.9, the edge extraction effects of each grey correlation calculation method are shown in Figure 6.

For a threshold value of 0.95, the edge extraction effects of each grey correlation calculation method are shown in Figure 7.

According to Figures 4–7, the similarity degree performs consistently well across the four thresholds, and Deng's grey correlation outperforms the other five grey-based image edge detection approaches. The worst-performing approach is the angle of correlation. The experimental results demonstrate that, in image processing, because of the important factor resolution coefficient in Deng's grey correlation, the important factor of image processing is also the resolution coefficient. Moreover, in image processing, the performance of Deng's correlation is objectively superior. In addition, the threshold values of 0.85 and 0.9 outperform the threshold values of 0.8 and 0.95. To further investigate the range of thresholds, the following experimental analysis is conducted on the thresholds from 0.8 to 0.95 for the images processed with Deng's grey correlation and the similarity degree; the results are shown in Figures 8 and 9, respectively.

According to the horizontal comparison within Figures 8 and 9, a threshold that is too low may mix contaminated points and result in blurring during edge extraction, while a

FIGURE 3: (a) Original image. (b) Greyscale image.

FIGURE 4: Various grey correlation-based edge extraction images for a threshold value of 0.8. (a) Absolute relevance. (b) Relative relevance. (c) Deng's grey correlation. (d) Similarity correlation. (e) Proximity correlation. (f) Angle correlation.

threshold that is too high will remove the edge details, and it will be impossible to restore the image edges accurately. When using Deng's grey correlation and the similarity degree, the denoising threshold should be in the range of 0.85~0.9, which is suitable for extracting image edges.

We continue to explore the effect of threshold changes on the final shape of the image when extracting edges. The results under various threshold values are shown in Figure 10.

Comparing the images in Figures 10(a)–10(c), the excessively wide edge extraction threshold results in rough and fuzzy images, and the edges are not clearly observed. Comparing Figures 10(c)–10(f) reveals that an edge

extraction threshold that is too strict will cause the edge points to be largely eliminated such that normal edges cannot be formed. Therefore, it is more suitable to restrict the threshold to a range from 0.88 to 0.99.

5. Image Noise Point Processing with a Fractional Grey Prediction Model Based on a Filtering Algorithm

Definition 3. For a sequence $X^{(0)}$, an r -order accumulation generation sequence can be obtained:

FIGURE 5: Various grey correlation-based edge extraction images for a threshold value of 0.85. (a) Absolute grey correlation. (b) Relative grey correlation. (c) Deng's grey correlation. (d) Similarity correlation. (e) Proximity correlation. (f) Angle correlation.

$$X^{(r)} = (x^{(r)}(1), x^{(r)}(2), \dots, x^{(r)}(n)), r \in R^+. \quad (28)$$

The sequence can be rewritten in the following form:

$$X^{(r)}(K) = \sum_{i=1}^k \frac{\Gamma(r+k-i)}{\Gamma(r-i+1)\Gamma(r)} x^{(0)}(i), \quad k = 1, 2, \dots, n. \quad (29)$$

When $r \in Z^+$, the expansion factor of $x^{(r)}(k)$ is

$$\alpha_k = \frac{\Gamma(r+k-i)}{\Gamma(r-i+1)\Gamma(r)} = \frac{(r+k-i-1)!}{(k-i)!(r-1)!} \quad (30)$$

Equation (29) expresses that the gamma function is a generalization of the factorial function to real numbers. The operator defined by the above formula, which is denoted by r -AGO ($r \in R^+$), is called a fractional-order accumulation generation operator.

Grey subtraction generation and grey accumulation generation are the corresponding processes.

Definition 4. Sequence $X^{(0)}$ is given by definition, and the r -order subtraction generation sequence is

$$X^{(-r)} = (x^{(-r)}(1), x^{(-r)}(2), \dots, x^{(-r)}(n)), \quad r \in R^+. \quad (31)$$

This sequence can be expressed in the following form:

$$X^{(r)}(K) = \sum_{i=0}^{k-1} \frac{\Gamma(r+k)}{\Gamma(r-i+1)\Gamma(i+1)} x^{(0)}(k-i), \quad (32)$$

$$k = 1, 2, \dots, n.$$

The operator that is defined by formula (40) is called a fractional-order subtraction generation operator. It is the inverse operator of r -AGO ($r \in R^+$).

Combined with formula (28), the following transformation is obtained:

$$X^0 D^{(1)} = X^{(1)} = \{x^1(1), x^1(2), \dots, x^1(n)\}. \quad (33)$$

This is the accumulation generation sequence of sequence $X^{(0)}$, where

$$X^{(1)}(k) = \sum_{i=1}^n x^{(1)}(i), \quad k = 1, 2, \dots, n \quad (34)$$

is 1-AGO in the above process. When D is applied r times in the sequence, the following is obtained:

$$X^0 D^{(r)} = X^{(r)} = \{x^r(1), x^r(2), \dots, x^r(n)\}, \quad (35)$$

where

$$X^{(r)}(k) = \sum_{i=1}^n x^{(r-1)}(i), \quad k = 1, 2, \dots, n, r \in Z^+. \quad (36)$$

FIGURE 6: Various grey correlation-based edge extraction images for a threshold value of 0.9. (a) Absolute grey correlation. (b) Relative grey correlation. (c) Deng's grey correlation. (d) Similarity degree. (e) Proximity correlation. (f) Angle correlation.

FIGURE 7: Continued.

FIGURE 7: Various grey correlation-based edge extraction images for a threshold value of 0.95. (a) Absolute grey correlation. (b) Relative grey correlation. (c) Deng's grey correlation. (d) Similarity degree. (e) Proximity correlation. (f) Angle correlation.

FIGURE 8: Deng's grey correlation. (a) Threshold of 0.8. (b) Threshold of 0.85. (c) Threshold of 0.9. (d) Threshold of 0.95.

FIGURE 9: Similarity degree. (a) Threshold of 0.8. (b) Threshold of 0.85. (c) Threshold of 0.9. (d) Threshold of 0.95.

FIGURE 10: Continued.

FIGURE 10: Edge extraction results for various threshold ranges for Deng's grey correlation. (a) Thresholds of 0.82~0.99. (b) Thresholds of 0.85~0.99. (c) Thresholds of 0.88~0.9. (d) Thresholds of 0.91~0.99. (e) Thresholds of 0.94~0.99. (f) Thresholds of 0.97~0.99.

D is called the r -order accumulation sequence of sequence $X^{(0)}$, namely, r -AGO.

Similarly, the inverse accumulation generation operator is the inverse of the accumulation generation operator, and the corresponding transformation is as follows:

$$X^{(0)} D^{(r)} = a^{(r)} X^{(r)} = \{a^{(r)} x^{(0)}(1), a^{(r)} x^{(0)}(2), \dots, a^{(r)} x^{(0)}(n)\}, \quad (37)$$

where

$$a^{(r)} X^{(0)}(k) = \{a^{(r-1)} x^{(0)}(k) - a^{(r-1)} x^{(0)}(k-1)\}, \quad (38)$$

$$k = 2, 3, \dots, n.$$

Definition 5. For sequences $X^{(0)}$ and $X^{(1)}$, which are defined above, the following equations are obtained:

$$\begin{aligned} \hat{x}^{(1)}(k+1) &= \beta_1 \hat{x}^{(1)}(k) + \beta_2(k) + \beta_3, \\ \hat{x}^{(1)}(1) &= x^{(1)}(1) + \beta_3. \end{aligned} \quad (39)$$

This model is called the NDGM model. $\hat{x}^{(1)}(k)$ is the analog value of $x^{(1)}(k)$, $x^{(1)}(1)$ is the iterative value of the NDGM model, and $\beta_1, \beta_2, \beta_3, \beta_4$ are the related parameters of the NDGM model. The following equation is called the reduction equation of the NDGM model:

$$\hat{x}^{(1)}(k+1) = \beta_1^k \hat{x}^{(1)}(1) + \beta_2 \sum_{j=1}^k j \beta_1^{k-j} - \frac{1 - \beta_1^k}{1 - \beta_1} \times \beta_3. \quad (40)$$

Definition 6. The p/q -order accumulation generation sequence of the original nonhomogeneous sequence $X^{(0)}$ ($0 < p/q < 1$) is denoted by $X^{p/q}$. Let $C_{p/q}^0 = 1, C_{p/q}^1 = 0$. Then,

$$x^{p/q}(k) = \sum_{i=1}^k C_{k-i+p/q-1}^{k-i} x^{(0)}(i), \quad k = 1, 2, \dots, n, \quad (41)$$

where

$$C_{k-i+p/q-1}^{k-i} = \frac{(p/q + k - i - 1)(p/q + k - i - 2) \dots (r+1)(p/q)}{(k-i)!}. \quad (42)$$

The p/q -order inverse accumulation generation operator of $X^{(0)}$ ($0 < p/q < 1$) can be expressed as

$$\begin{aligned} a^{(p/q)} X^{(0)} &= a^{(1)} X^{(1-p/q)} \\ &= \{a^{(1)} x^{1-p/q}(1), a^{(1)} x^{1-p/q}(2), \dots, a^{(1)} x^{1-p/q}(n)\}. \end{aligned} \quad (43)$$

Definition 7. Suppose that the following equations hold:

$$\begin{aligned} \hat{x}^{p/q}(k-1) &= \beta_1 \hat{x}^{p/q}(k) + \beta_1 k + \beta_3, \\ \hat{x}^{p/q}(1) &= x^{(1)}(1) + \beta_4. \end{aligned} \quad (44)$$

Sequences $X^{(0)}$ and $X^{p/q}$, as described above, are called the fractional-order accumulation NDGM, which is referred to as the NDGM $^{p/q}$ model, where $\hat{x}^{p/q}(k)$ is the analog value of $x^{(1)}(k)$, $\hat{x}^{p/q}(1)$ is the iteration value of the model, and $\beta_1, \beta_2, \beta_3, \beta_4$ are the correlation coefficients of the model.

The least-squares method is used to obtain an expression for the model parameters. $\beta_1, \beta_2, \beta_3$ satisfy the following matrix equation:

$$\begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = (B^T B)^{-1} B^T Y, \quad (45)$$

where

$$B = \begin{pmatrix} x^{p/q}(1) & 1 & 1 \\ x^{p/q}(2) & 2 & 1 \\ \vdots & \vdots & \vdots \\ x^{p/q}(n-1) & (n-1) & 1 \end{pmatrix}, \quad Y = \begin{pmatrix} x^{p/q}(2) \\ x^{p/q}(3) \\ \vdots \\ x^{p/q}(n) \end{pmatrix}. \quad (46)$$

The reduction equation of NDGM $^{p/q}$ is

$$\hat{x}^{(p/q)}(k+1) = \beta_1^k \hat{x}^{(p/q)}(1) + \beta_2 \sum_{j=1}^k j \beta_1^{k-j} + \frac{1 - \beta_1^k}{\beta_1} \times \beta_3. \quad (47)$$

Similarly, the least-squares method is used to calculate parameter β_4 . Minimizing the errors of $\hat{x}^{p/q}(k)$ and $x^{p/q}(k)$ yields

$$\beta_4 = \frac{[\hat{x}^{(p/q)}(k+1) - \beta_1^k x^{(1)}(1) - \beta_2 \sum_{j=1}^k j \beta_1^{k-j} - \beta_3((1 - \beta_1^k)/(1 - \beta_1))] \beta_1^k}{1 + \sum_{k=1}^{n-1} (\beta_1^k)^2}, \quad (48)$$

where $k = 1, 2, \dots, n-1$.

5.1. Optimization of the Particle Swarm Fractional-Order NDGM^{p/q}. The PSO algorithm is an evolutionary algorithm for global optimization. The PSO algorithm is widely used in function optimization and neural network training. The computational sequence of the optimal-order adaptive PSO algorithm, which is based on [46], is as follows.

In the first step, randomly initialize the particle position and velocity of the particle swarm.

In the second step, select pBest as the current particle position and let gBest become the position of the best particle in the initial generated group.

In the third step, when $r = p$ Best, calculate the average relative error of the predictive model based on the fractional operator. The steps are as follows:

- (1) Solve for the r -order accumulation generation sequence of the original sequence of the target
- (2) Transform sequence $X^{(r)}$ to generate a new sequence, which is denoted by $Z^{(r)}$
- (3) Calculate the first-order accumulation generation sequence of $X^{(r)}$, which is denoted by $X^{(r-1)}$
- (4) Solve for parameters $[\beta_1, \beta_2, \beta_3]^T$
- (5) Determine the time response of $\hat{x}^{(r)}(k)$
- (6) Calculate the analog value of $X^{(r)}$
- (7) The simulated value of $X^{(0)}$, namely, $\hat{X}^{(0)}$, is obtained via a reduction calculation
- (8) Calculate the average relative error, which is denoted by $f(pBest)$
- (9) Determine whether $|f(pBest) - f(gBest)|$ is less than the specified convergence value, which is denoted by δ . If so, jump to the ninth step; otherwise, proceed to the fourth step

In the fourth step, the object is all particles in the particle swarm; perform the following steps:

- (1) Update the speeds and positions of the particles:

$$V = \omega \times V + c_1 \times \text{rand} \times (pBest - \text{Present})$$

$$+ c_2 \times \text{rand} \times (pBest - \text{Present}),$$

$$\text{Present} = \text{Present} + V, \quad (49)$$

$$\omega = \omega_{\max} - \text{run} \frac{\omega_{\max} - \omega_{\min}}{\text{runmax}}.$$

- (2) If the particle fitness is superior to the fitness of pBest, set pBest to the new position.

- (3) If the particle fitness is superior to gBest's fitness, set gBest to the new position.

In the fifth step, calculate the variance of the group fitness, which is denoted by σ^2 , and $f(gBest)$:

$$\sigma^2 = \sum_{i=1}^n \left(\frac{f_i - f_{\text{avg}}}{f} \right)^2, \quad (50)$$

$$f = \begin{cases} \max\{|f_i - f_{\text{avg}}|\}, & \max\{|f_i - f_{\text{avg}}|\} > 1, \\ 0, & \text{others.} \end{cases}$$

In the sixth step, calculate the mutation probability, which is denoted by p_m :

$$p_m = \begin{cases} k, & \sigma^2 < \sigma_d^2 f(gbest) > f_d, \\ 0, & \text{others.} \end{cases} \quad (51)$$

In the seventh step, generate random number $\varepsilon \in [0, 1]$; if $\varepsilon < p_m$, perform the mutation operation; otherwise, proceed to the eighth step:

$$\text{Best}_k = gBest_k \times (1 + 0.5\eta). \quad (52)$$

In the eighth step, judge whether the convergence criterion of the algorithm satisfies the condition. If the condition is satisfied, proceed to the ninth step; otherwise, return to the third step.

In the ninth step, output gBest, the optimum value of r , and the predicted value of fractional operator model when $r = gBest$. The execution of the algorithm is complete.

FIGURE 11: Graphs for the various accumulation times. (a) Original sequence. (b) 1-AGO. (c) 2-AGO. (d) r-AGO.

5.2. Fractional-Order NDGM of the Filtering Algorithm.

The filter window is selected for calculating the average grey level of all the pixels in the window. According to the flow chart, noise points are identified from the original sequence by combining the grey correlation model and filtering algorithm. The grey index rate sequence is obtained according to the cumulative coefficient of the original sequence. The original sequence of noise points obtained from the grey correlation model of the filter window is evaluated below. The exponential law of the sequence is not strong. At the same time, the PSO algorithm is used to determine the optimal fractional order. The minimum mean absolute percentage error (MAPE), 0.00044968, is achieved when $r = 2.50144265244855$. This precision satisfies the specified requirements. Then, the grey exponential law of the cumulative sequence under this fractional order is calculated and studied, as shown in Figure 11.

According to Figure 11, the original sequence does not satisfy the grey exponential law, but the accumulation fractional order does satisfy the grey exponential law. Therefore, the grey prediction model can be used to predict noise points. The basic flow chart of the algorithm is shown in Figure 12.

5.3. Validation of $NDGM^{p/q}$. The algorithm is written in MATLAB 2016b. The original sample image is shown in Figure 13. In Figure 14, salt-and-pepper noise with a density of 0.1 has been added to the original sample image.

The noisy image is processed via the traditional discrete cosine transform-based image denoising algorithm. The resulting image is shown in Figure 15.

As shown in Figure 15, the noise in the results from the traditional discrete cosine transform-based image denoising method is of high frequency, and since the amplitude of the high-frequency portion is small, the corresponding point is utilized to realize image denoising. However, details of the image will be lost.

Each pixel of the image is traversed, and, for pixels that correspond to extreme points, the neighboring pixels are selected to form the original sequence. The original sequence is used by the selected model to predict the noise point. The effects of various image processing techniques (i.e., the GM(1,1), NDGM(1,1) and $NDGM^{p/q}(1,1)$ prediction models) are shown in Figure 16.

According to Figure 16, in terms of the image noise removal effect, $NDGM^{p/q}(1,1)$ outperforms the traditional

FIGURE 12: Flow chart of the algorithm.

FIGURE 13: Original sample image.

FIGURE 14: Original sample image with salt-and-pepper noise with density of 0.1 added.

FIGURE 15: Image that has been processed by the traditional discrete cosine transform-based image denoising method.

cosine transform image denoising algorithm of Figure 15 and the traditional GM(1,1) and NDGM(1,1) models of Figure 16. Most of the noise of the image is removed, and the image is restored. To illustrate the validity of the model, the mean filter (MF), adaptive median filter (AMF), weighted median filter (WMF), and joint wavelet transform scheme using the iterative noise density and median filtering (IWT) are selected for comparison with the above grey models. Since there are no image processing results in paper [53] or [54], the peak signal-to-noise ratio (PSNR) is introduced as an objective evaluation criterion, and the image processing results from the papers are used for comparison. The comparison results are shown in Table 2. From Table 2, the PSNR is the highest for the NDGM^{p/q}(1,1) prediction model, and the mean squared error (MSE) is the lowest for the NDGM^{p/q} prediction model; that is, this model achieves the best denoising effect.

5.4. Application. After checking the accuracy of the model, the new model is applied to Figure 17. The density of the salt-and-pepper noise is shown in Figure 18. The processing effects are shown in Figure 19.

According to Figure 19, the most effective approach for dealing with salt-and-pepper noise is the NDGM^{p/q}(1,1) prediction model, and the least effective approach is the traditional discrete cosine transform-based image denoising method. The three grey prediction models outperform the traditional discrete cosine transform-based image denoising algorithm. The best denoising performance is realized by the NDGM^{p/q}(1,1) prediction model, which has been optimized via PSO. From the above figure, it is possible to directly observe that the new model has the best denoising ability. To further illustrate its advantages, the AMF, MF, SF, and image noise filter based on cellular automata (CA) [55] image processing methods are compared with the grey models. The PSNR is used as an objective comparison standard. Since there is no image processing result in [57], the PSNR results are directly compared. The comparison table is shown in Table 3. Similar to the results in Table 2, the PSNR of the

FIGURE 16: Images processed via three prediction methods. (a) Image predicted by the GM(1,1). (b) Image predicted by the NDGM(1,1). (c) Image predicted by the NDGM p/q (1,1).

TABLE 2: Objective quality measures of the models.

Quality measure	Cosine processing	MF [49]	AMF [49]	WMF [49]	IWT [50]	GM (1,1)	NDGM (1,1)	NDGM p/q (1,1)
PSNR	23.00	21.97	24.73	21.92	23.90	19.67	21.72	26.48
MSE	325.90	413.12	218.82	417.91	264.90	701.599	437.60	146.24

FIGURE 17: Original sample image.

FIGURE 18: Image with salt-and-pepper noise with a density of 0.1.

FIGURE 19: Images processed via four prediction methods. (a) Image that has been processed by the traditional discrete cosine transform-based image denoising method. (b) Image that has been processed by the GM(1,1). (c) Image that has been processed by the HDGM(1,1). (d) Image that has been processed by the NDGM $^{p/q}$ (1,1).

TABLE 3: Objective quality measures of the models.

Quality measure	Cosine processing	AMF [52]	MF [52]	SF [52]	CA [52]	GM (1,1)	NDGM (1,1)	NDGM $^{p/q}$ (1,1)
PSNR	21.25	12.77	12.77	13.44	23.75	21.81	23.70	27.51
MSE	487.62	3436.22	3436.22	2944.97	274.21	428.63	277.38	115.37

NDGM $^{p/q}$ prediction model is much higher than that of the other models, and the MSE of the NDGM $^{p/q}$ prediction model is lower than that of the other models. The result shows that this new model can effectively denoise the image.

6. Conclusions

An image is a system in which some information is known and some information is unknown. This paper combines grey system theory and a fractional-order model to provide a new image denoising algorithm. Additionally, based on the theoretical analysis, the PSO algorithm is introduced to modify the grey system model to further improve the denoising effect. Therefore, this article mainly uses grey correlation analysis and grey prediction modeling to

perform noise point detection and noise point processing on an image. The main contributions are as follows:

- (1) Using the grey correlation model and the filtering algorithm, noise points and the edges of an image can be detected. Many grey correlation methods are available. Which grey correlation method is the most effective in image processing? According to the relationship between the resolution coefficient of image processing and the resolution coefficient of Deng's grey correlation, the experimental results demonstrate that Deng's grey correlation outperforms five other typical grey correlation methods. One of the main difficulties in image processing is the selection of a threshold. In this paper, the threshold range is selected experimentally.

- (2) Noise points are detected using the filtering algorithm and the grey correlation. The noise points are selected as the original sequence, and the NDGM^{p/q} prediction model is established according to the filtering algorithm for the image denoising process. The grey value of the intermediate point is predicted from the values of the surrounding points and is used to replace the original noise value to eliminate the noise points. The empirical analysis results demonstrate that the NDGM^{p/q} prediction model outperforms the traditional discrete cosine transform-based image denoising algorithm and the GM(1,1) and NDGM(1,1) prediction model in terms of the denoising effect.

Data Availability

The data used to support the findings of this study are included within the article. The data are from the figures.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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